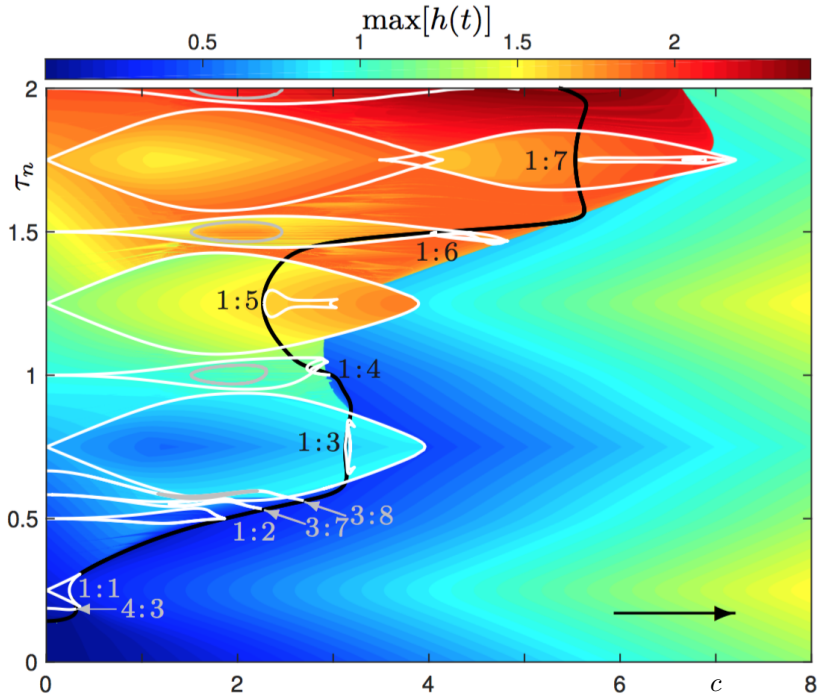
Conceptual climate models with global feedback mechanisms

Bernd Krauskopf, University of Auckland



- conceptual modeling
- energy balance and global transport
- an ENSO DDE model

SIAM DS 2017, Snowbird, May 2017

Climate of Earth: the big picture

<--- **radiation in**

---> **radiation out**

↓ ↑

internal mechanisms

Climate of Earth: the big picture

<--- **radiation in**

- **solar forcing**

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Climate of Earth: the big picture

<--- **radiation in**
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- **absorption/reflection**

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Climate of Earth: the big picture

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---> **radiation out**
- black body radiation

↓ ↑

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Climate of Earth: the big picture

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- colored body radiation

↓ ↑

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Climate of Earth: the big picture

<--- radiation in
- solar forcing
- absorption/reflection

---> radiation out
- colored body radiation
- cloud cover

↓ ↑

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- CO_2 cycles: volcanos, oceans and plants

Climate of Earth: the big picture

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internal mechanisms
- CO_2 cycles: volcanos, oceans and plants
- oceans, atmosphere and their coupling

Climate of Earth: the big picture

Every climate model
is a conceptual model!

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- solar forcing
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- colored body radiation
- cloud cover

↓ ↑

internal mechanisms
- CO_2 cycles: volcanos, oceans and plants
- oceans, atmosphere and their coupling

[Engler & Kaper, Dijkstra, ...] plus MS123/129

Climate of Earth: the big picture

which conceptual model?
the time scales matter!

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- CO_2 cycles: volcanos, oceans and plants
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Energy balance models: without time scale

<--- **radiation in**

- **constant solar forcing**

---> **radiation out**

- **considered constant but given by CO_2 level**

↓ ↑

internal mechanisms

- **none considered**

Energy balance models: without time scale

<--- **radiation in**
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↓ ↑

internal mechanisms
- **none considered**

=> **resulting constant temperature of Earth**

Energy balance models: large time scale

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- constant solar forcing
- global glaciation (different albedo)

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internal mechanisms
- none considered

Energy balance models: large time scale

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internal mechanisms

- **none considered**

=> **bistability between a hot and a cold Earth**

Energy balance models: large time scale

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- orbital solar forcing
- global glaciation (different albedo)

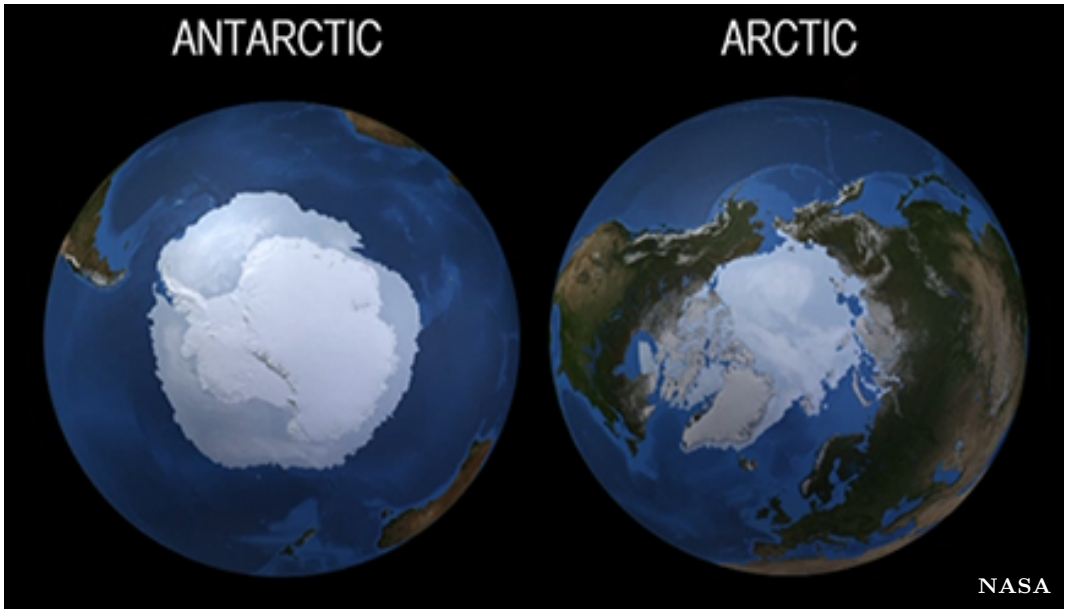
---> radiation out
- considered constant

↓ ↑

internal mechanisms
- none considered

=> periodicity of glaciation cycles

How to model feedback by glaciation?



feedback mechanism:

less ice => lower albedo
lower albedo => ice melts

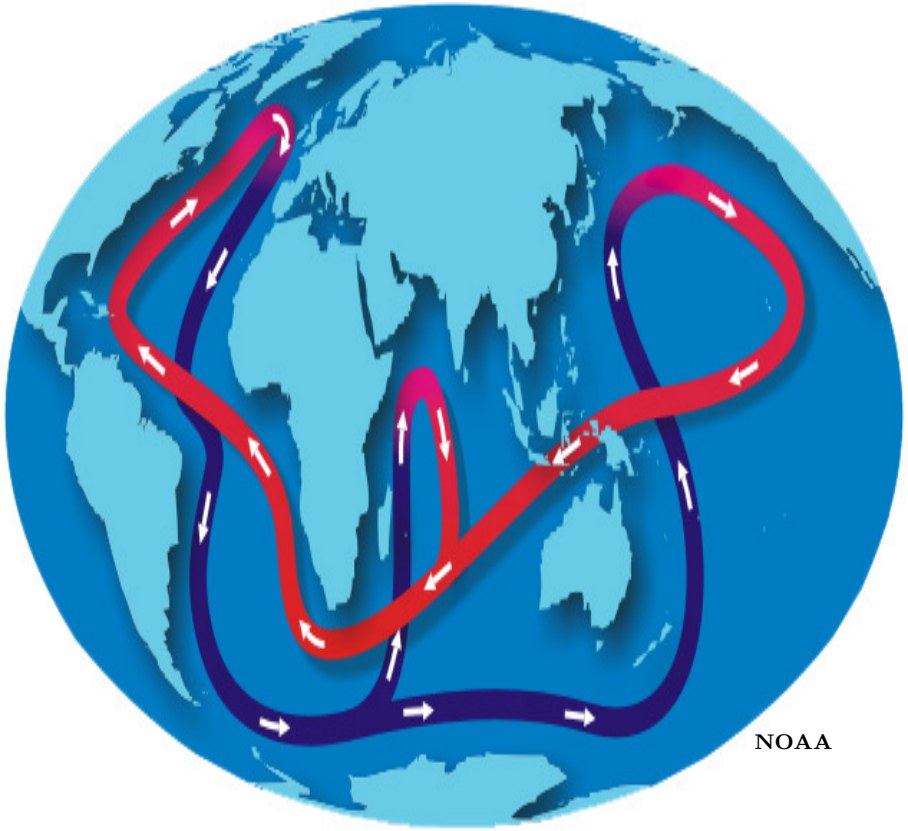
complicated process
to model fully

- 0D Earth? 1D Earth? 2D Earth?
- Type of model?

  finite dim: PWS [Widiasih@MS80; Baek@MS123]
  slow-fast ODEs [Engler;Kaper@MS129]

  infinite dim: DDE models [Quinn@MS129]

Ocean circulation models: mid time scale



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- (constant) solar forcing
- global/local glaciation (e.g., Greenland)

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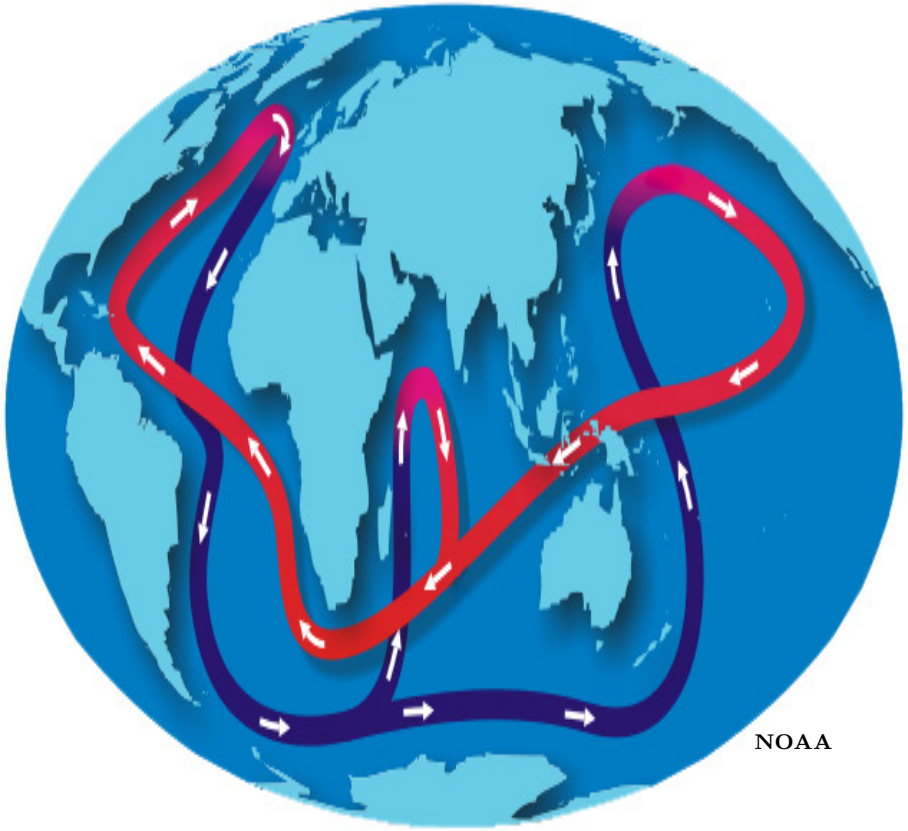
↓ ↑

internal mechanisms

- ocean currents/conveyors
- ocean/atmosphere coupling

Ocean circulation models: mid time scale



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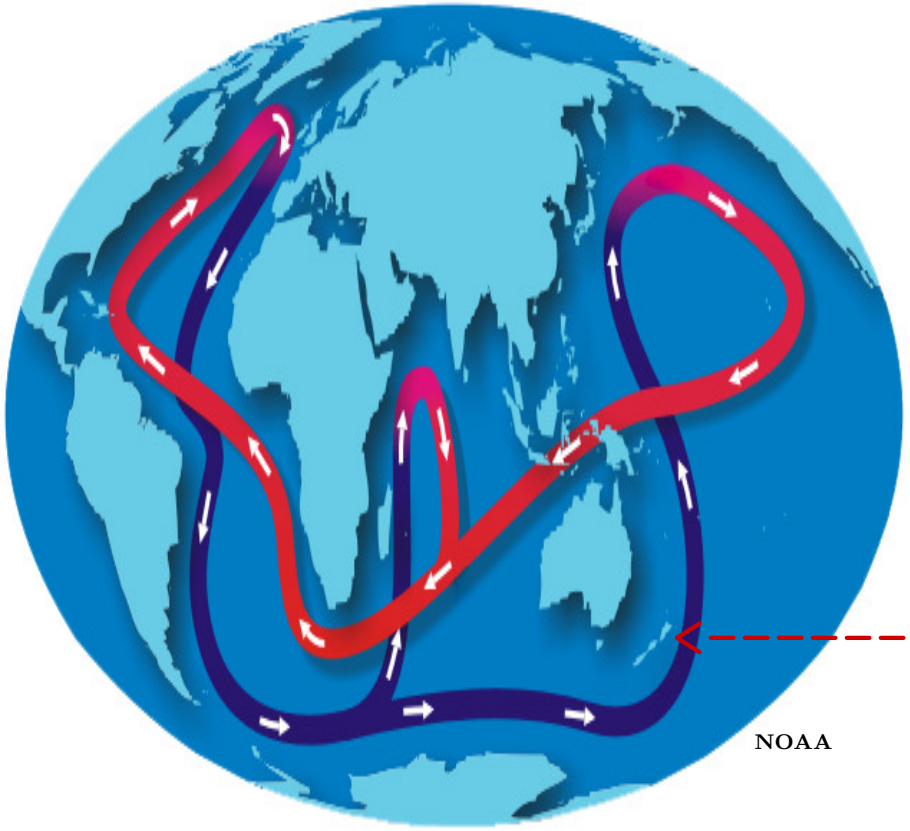
=> tipping points
e.g., of Gulf stream
[MS80/93]

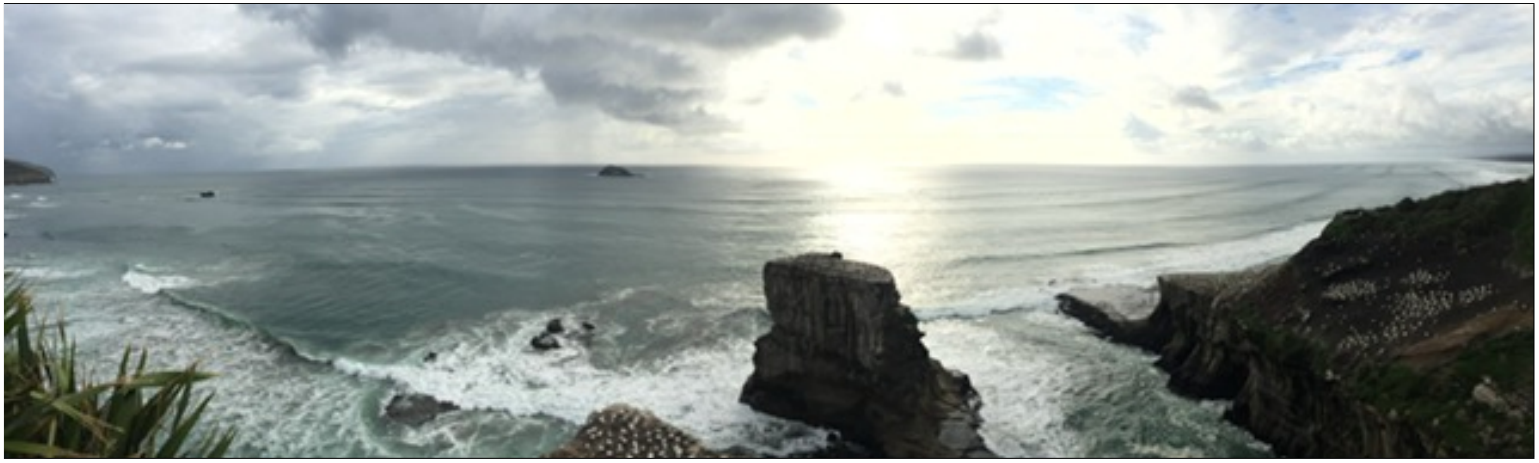
Advertising break: Lectureship in Applied Maths



NOAA

University of Auckland

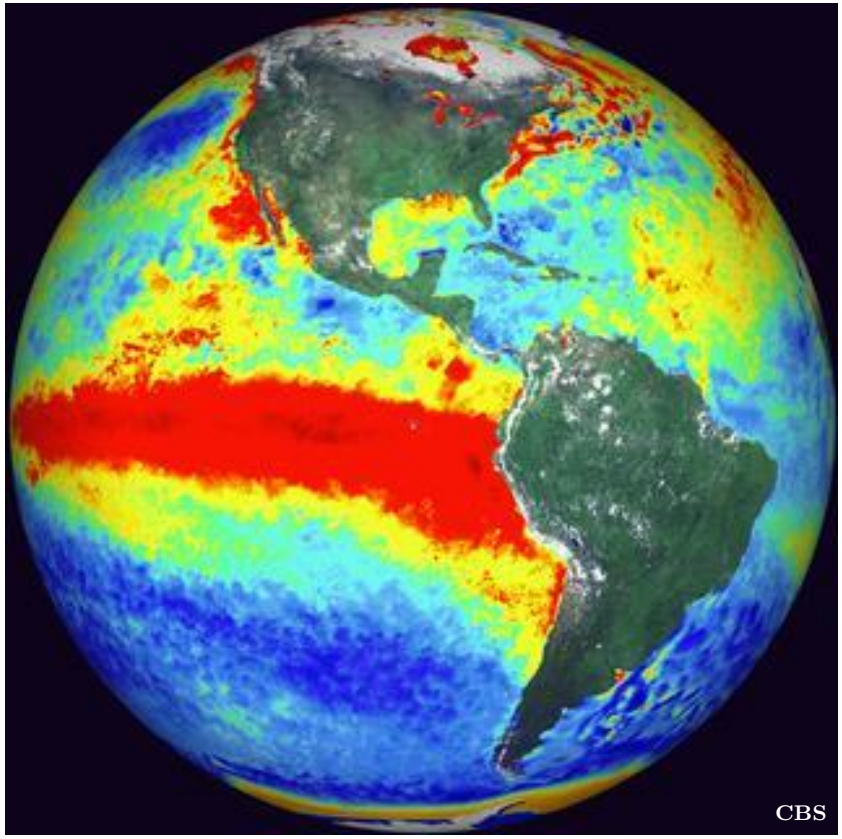
[View from the office]



apply by
11 June 2017

[details are on DsWeb]

El Niño models: human time scale



<--- radiation in

- periodic annual solar forcing cycle

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- transport by ocean waves
- ocean/atmosphere coupling

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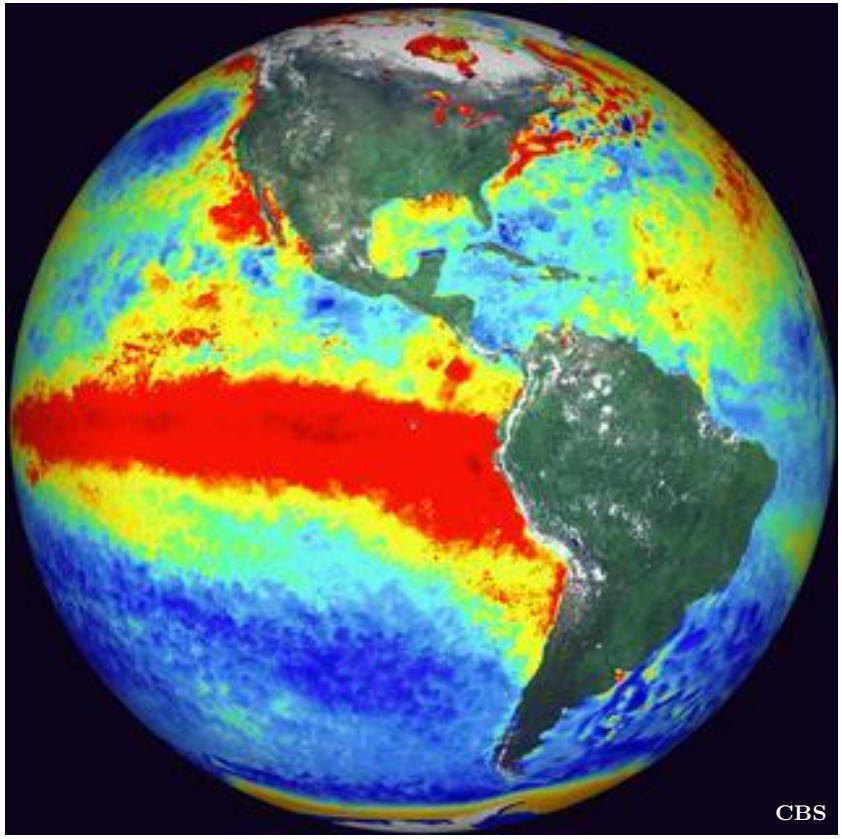

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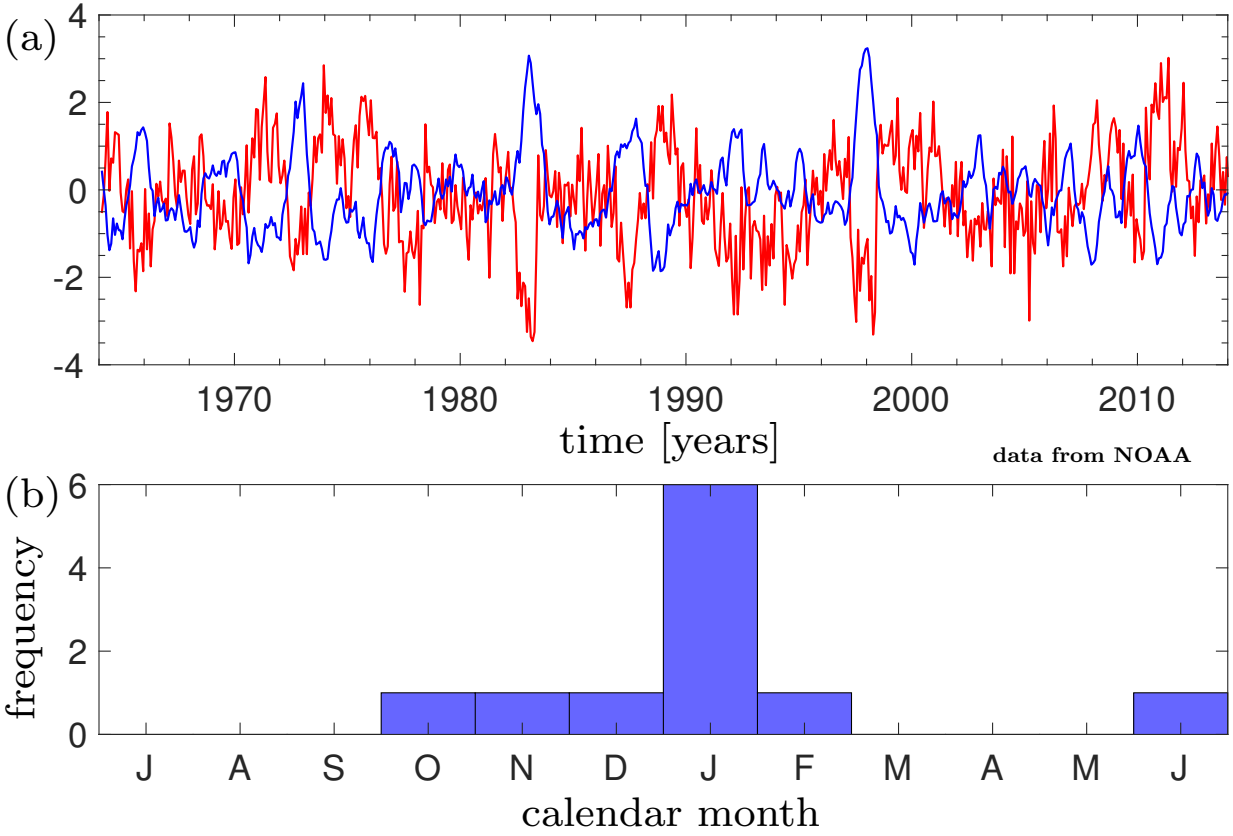
internal mechanisms
- transport by ocean waves
- ocean/atmosphere coupling

=> complicated dynamics with much higher SST about every 4–7 year

El Niño: the phenomenon



SST off Peru
SO index

seasonal locking of large events

Overall: **coupled system called ENSO, with El Niño events about every 4–7 year**

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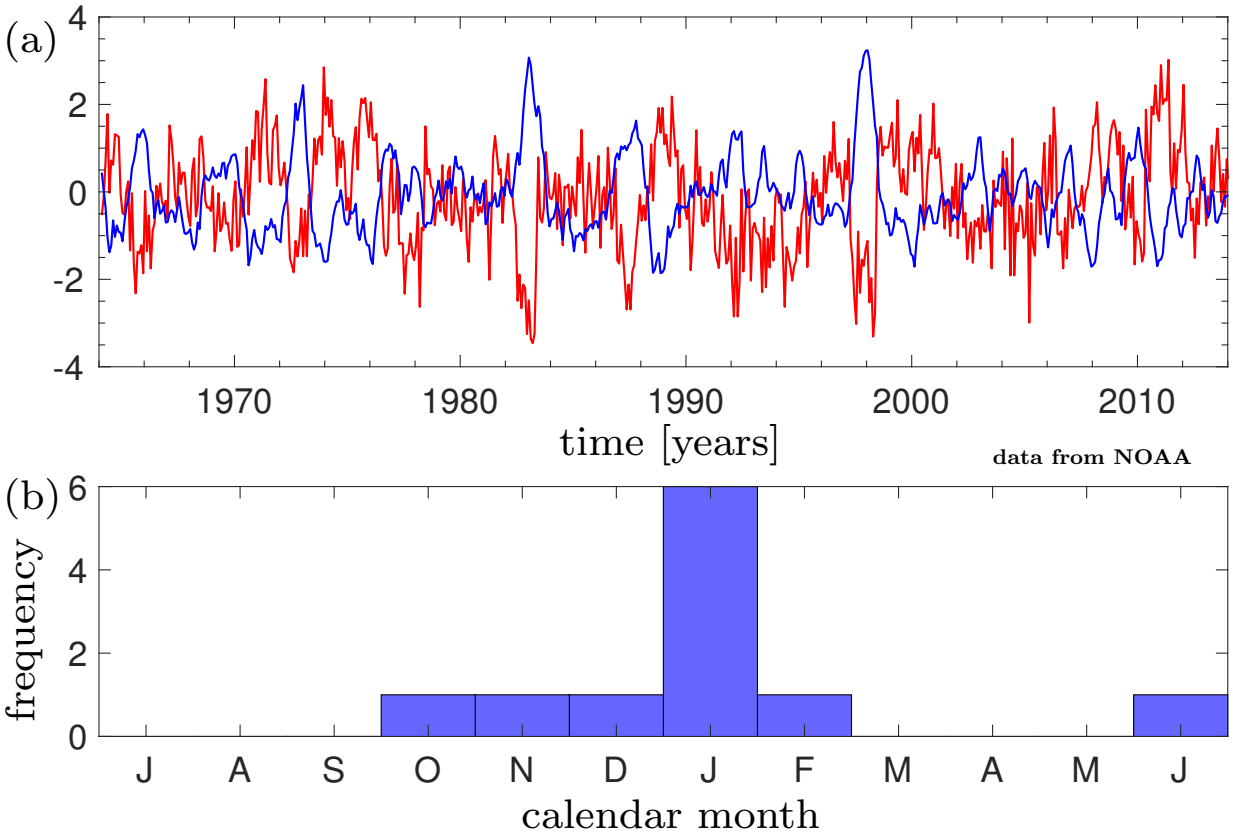

SST off Peru
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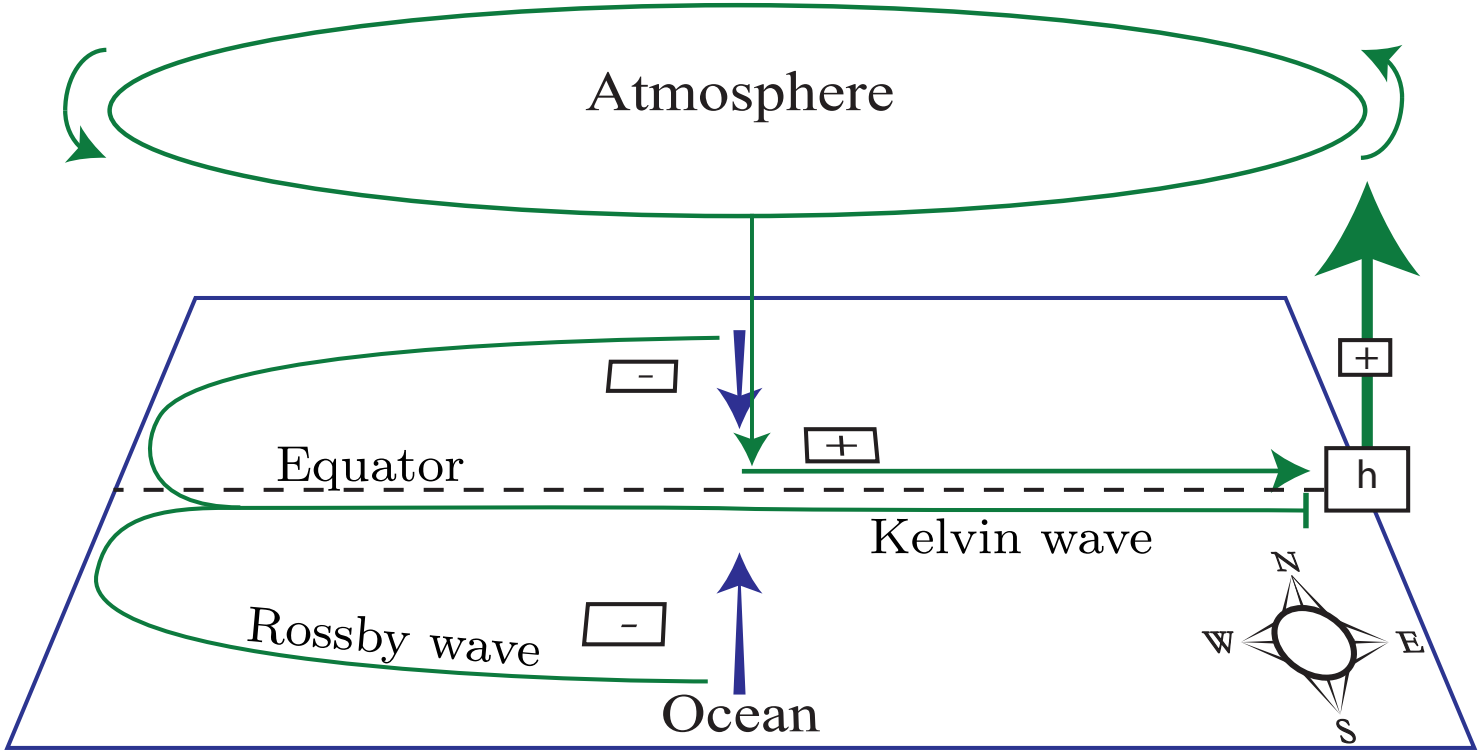
seasonal locking of large events

Overall: **coupled system called ENSO, with large El Niño events about every 4–7 year**

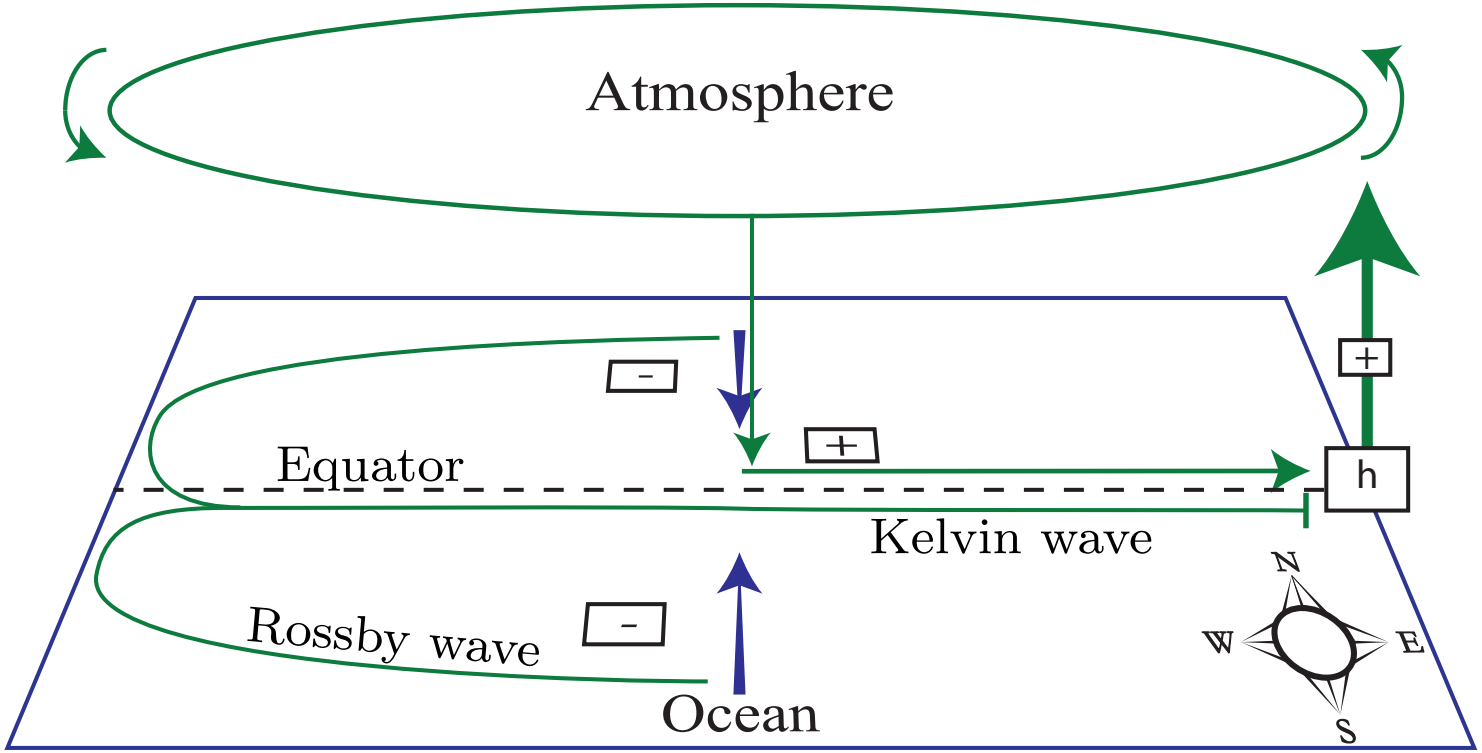
For its global importance, watch the news!

El Niño: mechanisms behind models



SST $h(t)$ at equatorial western edge of Pacific Ocean

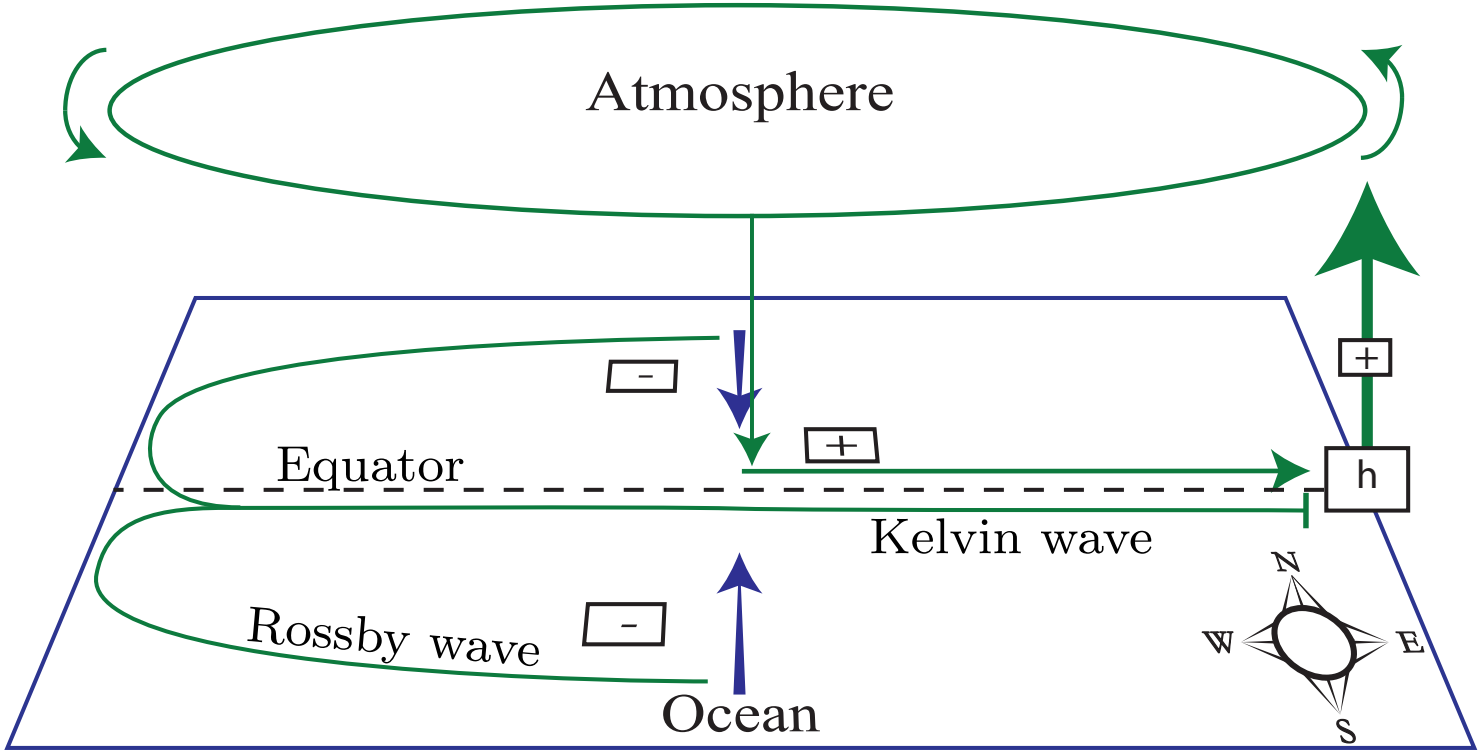
El Niño: mechanisms behind models



SST $h(t)$ at equatorial western edge of Pacific Ocean

- **interaction zone in central Pacific Ocean**

El Niño: mechanisms behind models



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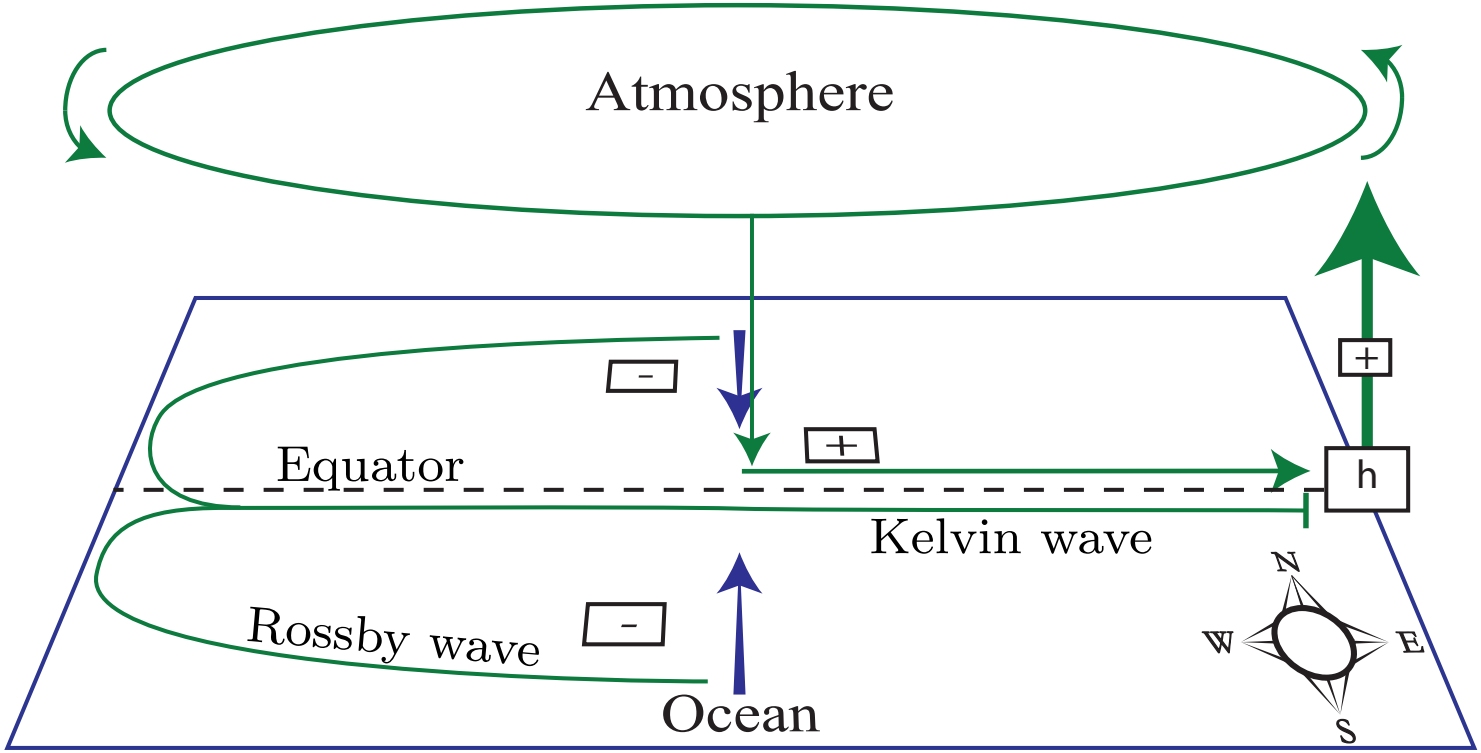

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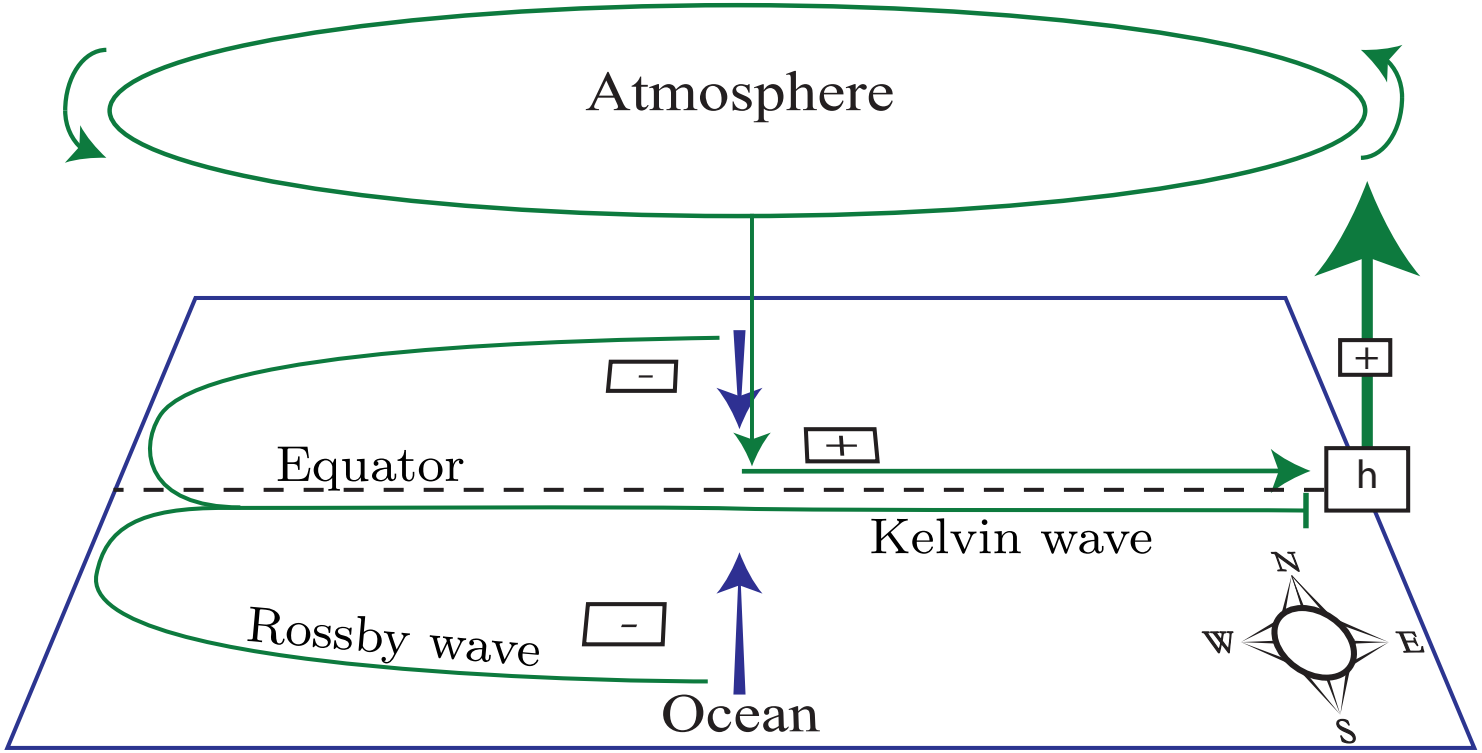
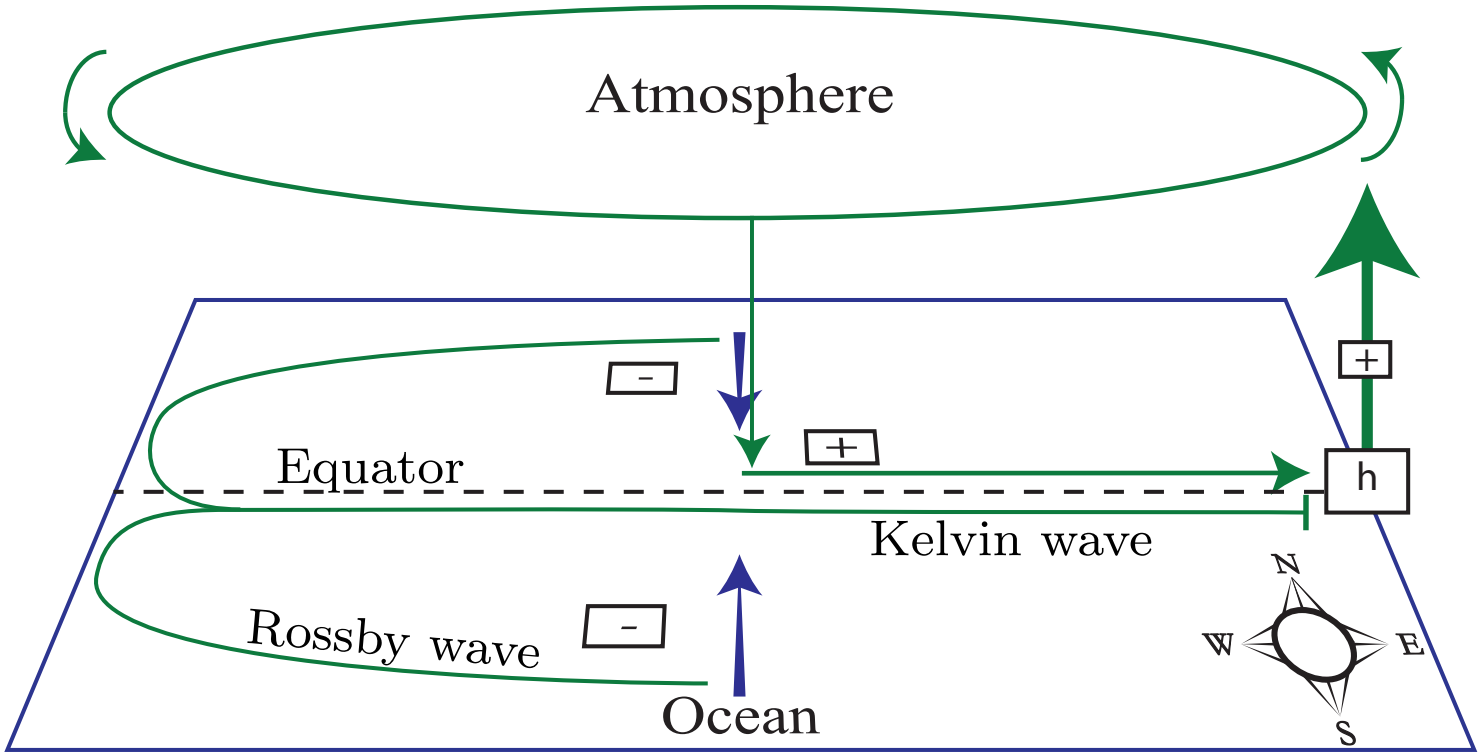


SST $h(t)$ at equatorial western edge of Pacific Ocean

- **interaction zone in central Pacific Ocean**
- **positive feedback via Kelvin waves**
- **negative feedback via Rossby/Kelvin waves**
- **annual periodic forcing by the sun**

El Niño: mechanisms behind models



SST $h(t)$ at equatorial western edge of Pacific Ocean

- **Type of model?**

  finite dim: slow-fast ODEs **[Guckenheimer@MS99]**

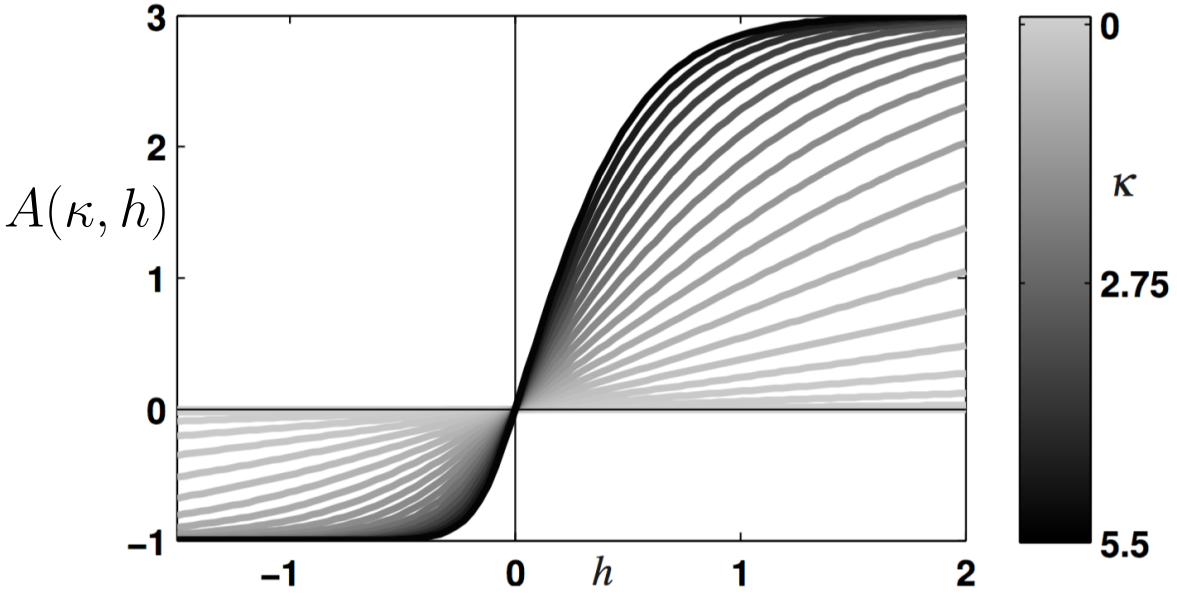
  infinite dim: DDE models **[Keane@MS72]**

El Niño: a delay differential equation model

$$\dot{h}(t) = aA(\kappa, h(t-\tau_p)) - bA(\kappa, h(t-\tau_n)) + c\cos(2\pi t)$$

$$\textbf{with } A(\kappa, h) = \begin{cases} d_u \tanh(\frac{\kappa}{d_u} h) & \textbf{if } h \geq 0 \\ d_l \tanh(\frac{\kappa}{d_l} h) & \textbf{if } h < 0 \end{cases}$$

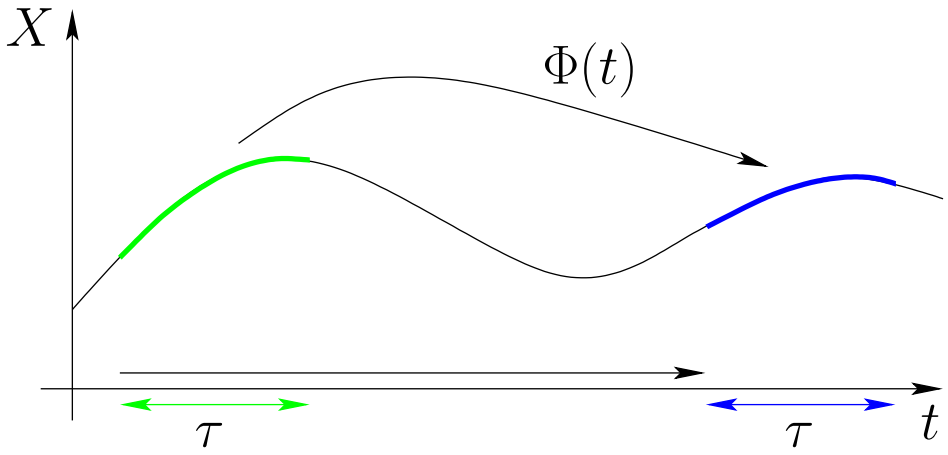
[Tziperman, Stone, Cane & Jarosh, Science-AAAS 264, 1994]



a **strength of pos f/b**
τ_p **delay of pos f/b**
b **strength of neg f/b**
τ_n **delay of neg f/b**
κ **ocean/atmos coupling**
d_l **upper asymptote**
d_l **lower asymptote**
c **strength of forcing**

DDEs with constant delays

$\frac{dx(t)}{dt} = F(x(t), x(t-\tau), \lambda)$ **where τ is a constant delay**



The evolution operator $\Phi(t)$ describes how an initial condition (history segment of length τ) evolves in time.

- **The phase space is the infinite-dimensional space of continuous functions with values in X-space.**
- **Discrete spectra of equilibria and periodic orbits**
- **One finds the standard bifurcations as for ODEs**
- **Continuation software exists**

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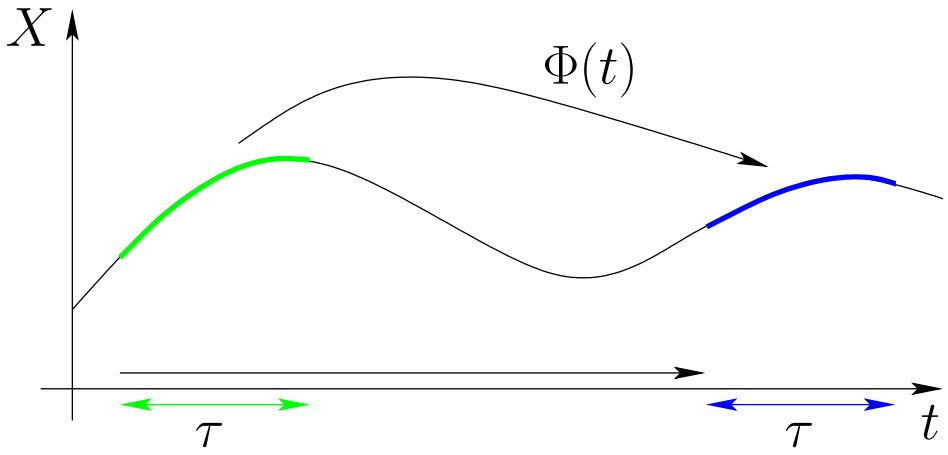

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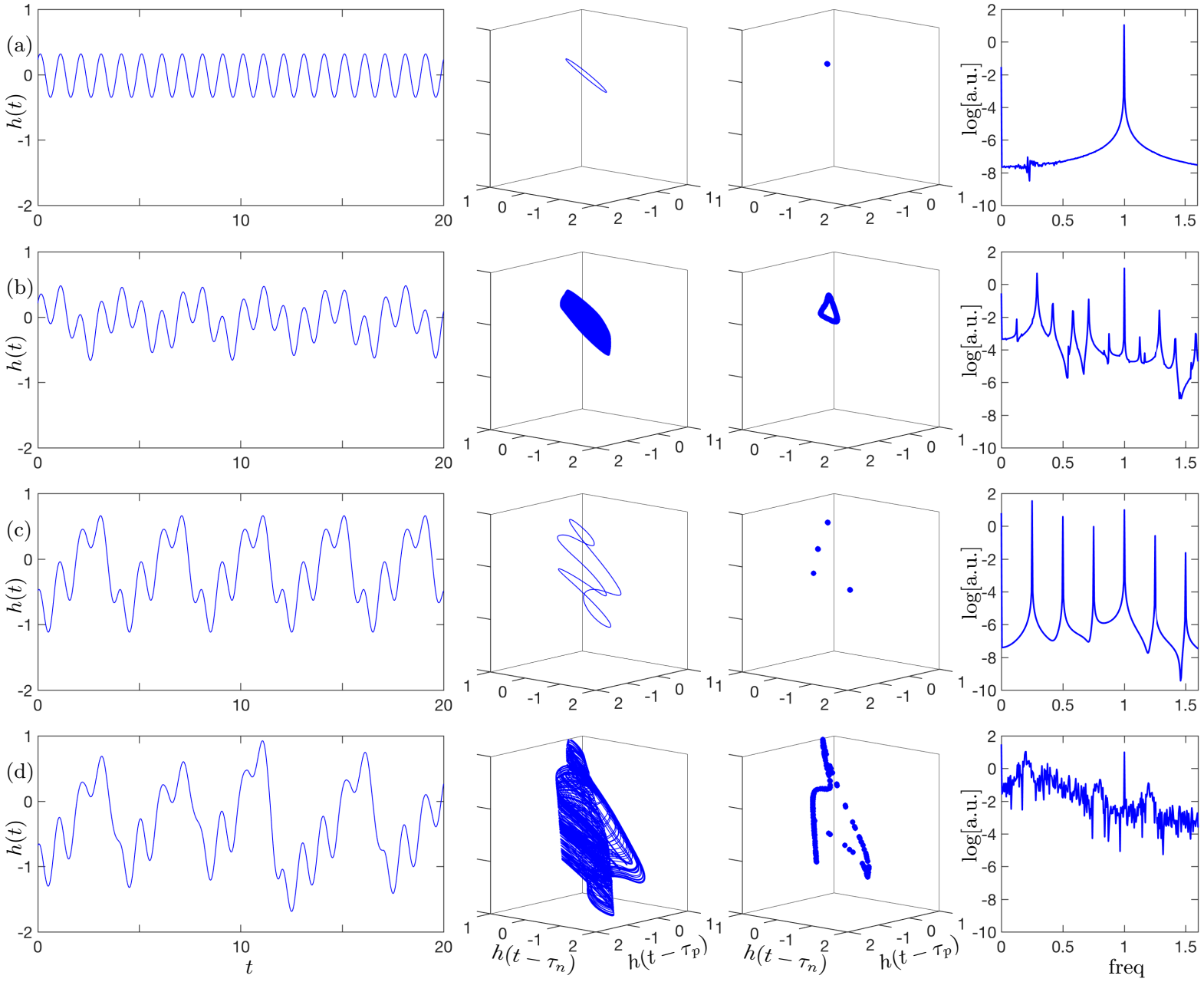
DDE (for fixed τ_i) = ODE++

El Niño: solutions of DDE model



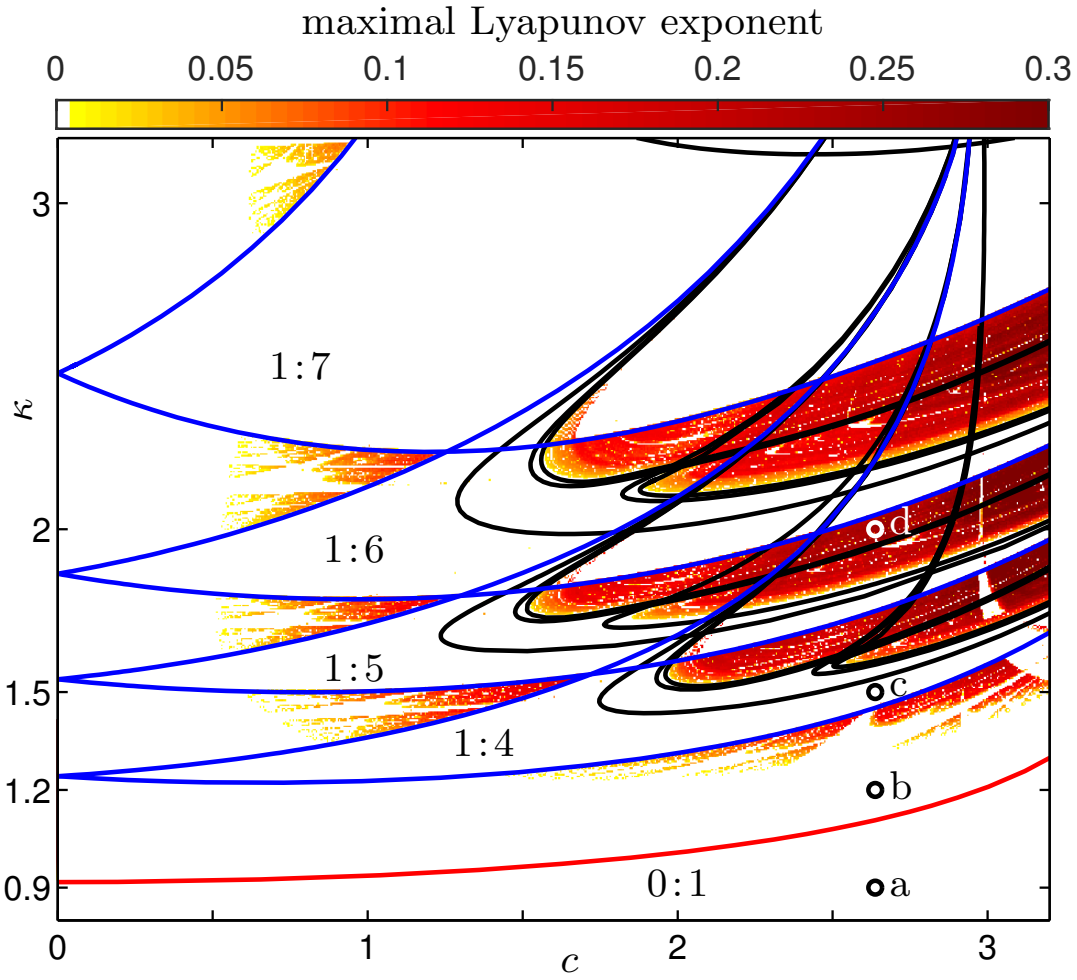
forcing dominated

q/p torus

locked torus

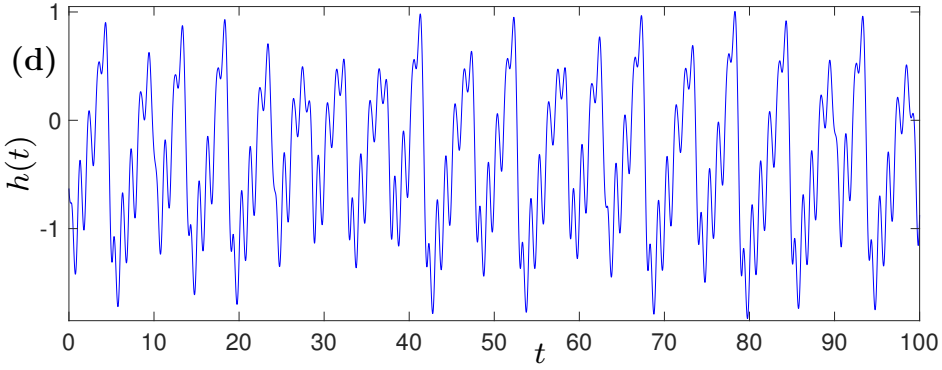
chaos

El Niño: bifurcation analysis of DDE model



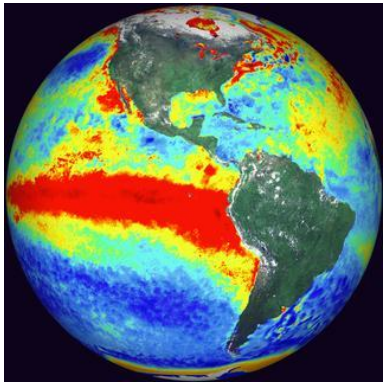
[Keane, K & Postlethwaite, SIADS, 2017] and MS72

[Keane, K & Postlethwaite, Review, 2017, submitted]



- resonance structure
- transition to chaos
- large 'regions' of chaos
- chaotic time series with ENSO characteristics
- one can determine roles of individual feedback mechanisms

Conclusions and outlook



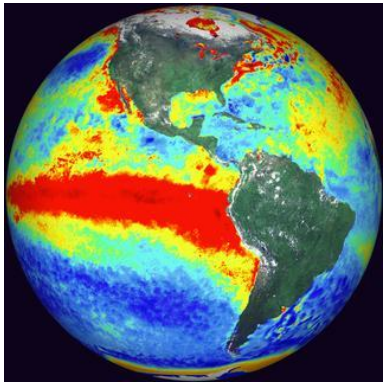
Conceptual models have a role to play

- choose your time scale of interest
- choose your mechanisms of interest
- choose your favourite model class:
  piecewise smooth or slow-fast;
  delay equations or PDEs

Challenges:

- compare different modeling approaches
- non-constant nature of f/b mechanisms
- comparison with measurements
- inclusion of actual data

Conclusions and outlook



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Enjoy MS123/129!