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BILEVEL MIXED-BINARY PROGRAMMING FOR IDENTIFYING CRITICAL CONTINGENCY EVENTS IN AC POWER SYSTEMS

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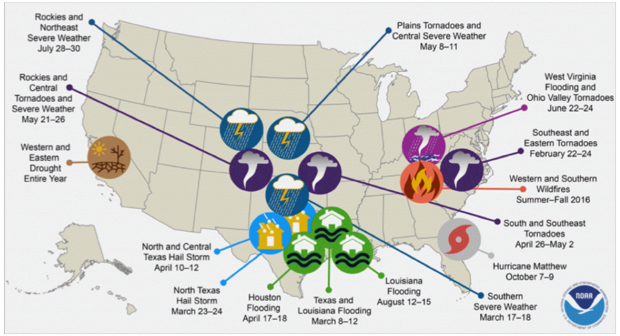
TOWARD THE RESILIENT POWER SYSTEM

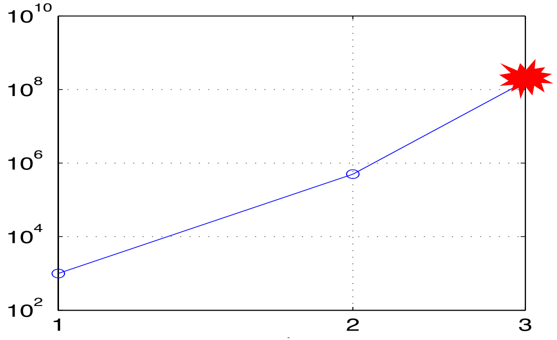
Why this work is needed?

- **Contingencies in power systems:**
  - Combinations of weather and manmade threats
  - Rare, unpredictable – no probability distribution
  - Astronomical scenarios: Nearly one billion scenarios for N-3 contingency events of 1,000 electric grid assets

- **Importance of resilient power system planning and operations:**
  - Valuable information to government agencies to make targeted investment decisions for resilient infrastructures
  - Developing suitable mitigation and restoration strategies
  - But, no techniques and tools for identifying and prioritizing critical contingency events



Billion USD damage events in 2016



Possible failure scenarios with an *N-3* contingency for 1,000 electric power assets

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BILEVEL OPTIMIZATION APPROACH

Representing an Attacker-Defender Framework

- We developed bilevel optimization models to identify and prioritize a set of critical contingency events.
- Binary variable x_i represents the contingency at component *i*.
- Continuous variables y represent power system operation.

Upper Level

$$\begin{aligned} \max \quad & f(y) \\ \text{s.t.} \quad & x \in X^{(1)}, \\ & y \in S(x) \end{aligned}$$

Adverse consequence (e.g., load shedding, operating cost)

Attacker's constraints (e.g., budget)

Lower Level

$$\begin{aligned} S(x) := \arg\min \quad & f(y) \\ \text{s.t.} \quad & g(x, y) \leq 0 \end{aligned}$$

Optimal power flow for a given disruption x

Assumptions:

- An attacker's attack consists of the cutting/impairment of transmission lines/transformers within the restriction of an attack budget.
- The defender takes corrective action in response to an attack, adjusting voltage and power injection states to recover system stability.

BILEVEL OPTIMIZATION APPROACH

Existing Approaches Similar to Our Model

- Existing work is limited and need strong assumptions.
- Pinar et al. “Optimization Strategies for the Vulnerability Analysis of the Electric Power Grid” *SIAM Journal on Optimization*, 2010
  - Our model *generalizes* the model used in Pinar et al, where the power system is formulated as a lossless power system, as compared with ACOPF.
- Bienstock and Verma. “The *n-k* Problem in Power Grids: New Models, Formulations, and Numerical Experiments” *SIAM Journal on Optimization*, 2010
  - Different ways of modeling attacker problem
  - Their model is based on DC power system.

EXISTING SOLUTION APPROACH

KKT Reformulations to a Single-Level Problem

- The lower level problem can be replaced with a system of equations, including:
  - Primal constraints
  - Dual constraints
  - Complementarity slackness equations

KKT Conditions

- The KKT reformulation leads to a single-level problem.

- Challenges:
  - Nonconvex problem, even if the lower level were convex.
  - Complementarity constraints
  - Constraint qualification is violated at every feasible solution.

BILEVEL MODEL FORMULATION

$$\max_{x} \quad \phi(x)$$ *Attacker maximizes the minimum infeasibility of the system.*

$$\text{s.t.} \quad \sum_{l \in L} x_l \leq K,$$ *Attacker's budget*

$$x_l \in \{0,1\}, \; l \in L$$

Difficult (if possible) to obtain subgradients

Nonconvex Lower-Level Problem: *System operator minimizes the system infeasibility.*

$$\phi(x) := \min_{e_i^R, e_i^I, w, p_i^G, q_i^G, u_i} \quad f(p^G) + \rho \sum_{i \in N} (u_i^p + u_i^q)$$

Power flow and balance equations

$$\text{s.t.} \quad w_l^R = e_i^R e_j^R + e_i^I e_j^I, \quad w_i^I = e_j^R e_j^I - e_i^R e_j^I \quad \forall (i,j) = l \in L,$$

$$-u_i^p \leq p_i^N(w, p^G, x) \leq u_i^p \quad \forall i \in N,$$

$$-u_i^q \leq q_i^N(w, q^G, x) \leq u_i^q \quad \forall i \in N$$

Line status after attack

$$(V_i^{min})^2 \leq w_{ii} \leq (V_i^{max})^2 \quad \forall i \in N$$

$$P_i^{min} \leq p_i^G \leq P_i^{max}, \quad Q_i^{min} \leq q_i^G \leq Q_i^{max} \quad \forall i \in N$$

LAGRANGIAN DUAL APPROACH

Lagrangian Relaxation of the Nonconvex Lower-Level Problem

$$\begin{aligned}\phi_D(x,\alpha,\beta,\gamma) := \min_{e_i^R, e_i^I, w, p_i^G, q_i^G, u_i} \quad & f(p^G) \\ & + \sum_{i \in N} \left[\left(\alpha_i^+ - \alpha_i^-\right) p_i^N(w, p^G, x) + \left(\rho - \alpha_i^+ - \alpha_i^-\right) u_i^p \right] \\ & + \sum_{i \in N} \left[\left(\beta_i^+ - \beta_i^-\right) q_i^N(w, q^G, x) + \left(\rho - \beta_i^+ - \beta_i^-\right) u_i^q \right] \\ & + \sum_{i \in N} \left[\gamma_i^+ \left(w_{ii} - (V_i^{max})^2\right) - \gamma_i^+ \left(w_{ii} - (V_i^{min})^2\right) \right] \\ \text{s.t.} \quad & \text{Nonconvex constraints}\end{aligned}$$

The u terms vanish for the boundedness with

$$\alpha_i \in [0,1], \quad \beta_i \in [0,1] \quad \forall i \in N$$

$$\begin{aligned}\phi_D(x,\alpha,\beta,\gamma) := \min_{e_i^R, e_i^I, w, p_i^G, q_i^G} \quad & f(p^G) \\ & + \sum_{i \in N} \left[\alpha_i p_i^N(w, p^G, x) + \beta_i q_i^N(w, q^G, x) \right] \\ & + \sum_{i \in N} \left[\gamma_i^+ \left(w_{ii} - (V_i^{max})^2\right) - \gamma_i^+ \left(w_{ii} - (V_i^{min})^2\right) \right] \\ \text{s.t.} \quad & \text{Nonconvex constraints}\end{aligned}$$

LAGRANGIAN DUAL APPROACH

Lagrangian Relaxation of the Nonconvex Lower-Level Problem

$$\begin{aligned}
\phi_D(x,\alpha,\beta,\gamma) := \min_{e_i^R, e_i^I, w, p_i^G, q_i^G} \quad & f(p^G) \\
& + \sum_{i \in N} \left[\alpha_i p_i^N(w,p^G,x) + \beta_i q_i^N(w,q^G,x)\right] \\
& + \sum_{i \in N} \left[\gamma_i^+ \left(w_{ii} - (V_i^{max})^2\right) - \gamma_i^+ \left(w_{ii} - (V_i^{min})^2\right)\right] \\
\text{s.t.} \quad & \text{Nonconvex constraints}
\end{aligned}$$

$$\alpha_i p_i^N(w,p^G,x) := \alpha_i \left[P_i^D - p_i^G + p_i^{sh}(w_{ii})\right] + \sum_{l \in L_i^f} \alpha_i (1-x_l) p_l^f(w) + \sum_{l \in L_i^t} \alpha_i (1-x_l) p_l^t(w)$$

$$\beta_i q_i^N(w,p^G,x) := \beta_i \left[Q_i^D - q_i^G + q_i^{sh}(w_{ii})\right] + \sum_{l \in L_i^f} \beta_i (1-x_l) q_l^f(w) + \sum_{l \in L_i^t} \beta_i (1-x_l) q_l^t(w)$$

The bilinear coefficients are the upper-level problem variables and can be linearized by McCormick relaxation.

SINGLE-LEVEL FORMULATION

Single-Level Mixed-Binary Convex Programming Relaxation

$$\max_{x,\alpha,\beta,\lambda,\mu,\gamma} \quad \hat{\phi}_D(\alpha,\beta,\lambda,\mu,\gamma)$$
$$\text{s.t.} \quad \sum_{l \in L} x_l \leq K$$
$$x_l \in \{0,1\} \quad \forall l \in L$$
$$(x,\alpha,\beta,\lambda,\mu,\gamma) \in \Pi \text{ (Polyhedron)}$$

Well, this is a relaxation of the original bilevel problem, but works as a good approximation of the value function.

Phan's ellipsoidal branching can guarantee the global optimum.

$$\hat{\phi}_D(\alpha,\beta,\lambda,\mu,\gamma) := \min_{e_i^R, e_i^I, w, p_i^G, q_i^G} \quad f(p^G)$$
$$+ \sum_{i \in N} \alpha_i \left[P_i^D - p_i^G + p_i^{sh}(w_{ii})\right] + \sum_{l \in L_i^f} \lambda_l^f p_l^f(w) + \sum_{l \in L_i^t} \lambda_l^t p_l^t(w)$$
$$+ \sum_{i \in N} \beta_i \left[Q_i^D - q_i^G + q_i^{sh}(w_{ii})\right] + \sum_{l \in L_i^f} \mu_l^f q_l^f(w) + \sum_{l \in L_i^t} \mu_l^t p_l^t(w)$$
$$+ \sum_{i \in N} \left[\gamma_i^+ \left(w_{ii} - (V_i^{max})^2\right) - \gamma_i^+ \left(w_{ii} - (V_i^{min})^2\right)\right]$$
$$\text{s.t.} \quad \text{Nonconvex constraints}$$

Piecewise linear concave function, but…

Challenge:

We need a global optimal solution for this nonconvex problem!

SOLUTION APPROACHES

Decomposition of the Lagrangian Dual Function

$$\hat{\phi}_D(\alpha,\beta,\lambda,\mu,\gamma) = \eta_0(\alpha,\beta,\lambda,\mu,\gamma) + \sum_{i\in N} \eta_i^G(\alpha,\beta) + \sum_{i\in N} \left[-\gamma_o^+ (V_i^{max})^2 + \gamma_i^- (V_i^{min})^2\right]$$

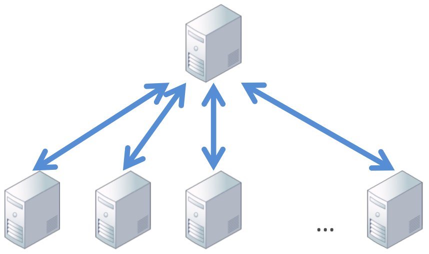
A nonconvex quadratic program

$$\eta_0(\alpha,\beta,\lambda,\mu,\gamma) = \min_{e^R,e^I,w} \quad \rho \sum_{i\in N} \left[\alpha_i p_i^{sh}(w) + \beta_i q_i^{sh}(w) + (\gamma_i^+ - \gamma_i^-) w_{ii}\right]$$

$$+ \rho \sum_{l\in L} \left[\lambda_l^f p_l^f(w) + \lambda_l^t p_l^t(w) + \mu_l^f q_l^f(w) + \mu_l^t q_l^t(w)\right]$$

$$\text{s.t.} \quad w_l^R = e_i^R e_j^R + e_i^I e_j^I, \quad w_l^I = e_j^R e_i^I - e_i^R e_j^I \quad \forall (i,j) = l \in L$$

The decomposition can lead to parallel computing.



Many linear programs

$$\eta_i^G(\alpha,\beta) = \min_{p^G,q^G} \quad f_i(p_i^G) + \rho\left[\alpha_i(P_i^D - p_i^G) + \beta_i(Q_i^D - q_i^G)\right]$$

$$\text{s.t.} \quad P_i^{min} \le p_i^G \le P_i^{max}, \quad Q_i^{min} \le q_i^G \le Q_i^{max} \quad \forall i \in N$$

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SOCP RELAXATION OF THE LOWER LEVEL

A SOCP Relaxation of the Nonconvex Lower-Level Problem:

$$\phi^{SOCP}(x) := \min_{e_i^R, e_i^I, w, p_i^G, q_i^G, u_i} \quad f(p^G) + \rho \sum_{i \in N} (u_i^p + u_i^q)$$

$$\text{s.t.} \quad \sout{(w_l^R)^2 + (w_l^I)^2 = w_{ii} w_{jj} \quad \forall (i,j) = l \in L,}$$

$$(w_l^R)^2 + (w_l^I)^2 \leq w_{ii} w_{jj} \quad \forall (i,j) = l \in L,$$

$$-u_i^p \leq p_i^N(w, p^G, x) \leq u_i^p \quad \forall i \in N,$$

$$-u_i^q \leq q_i^N(w, q^G, x) \leq u_i^q \quad \forall i \in N$$

$$(V_i^{min})^2 \leq w_{ii} \leq (V_i^{max})^2 \quad \forall i \in N$$

$$P_i^{min} \leq p_i^G \leq P_i^{max}, \quad Q_i^{min} \leq q_i^G \leq Q_i^{max} \quad \forall i \in N$$

The quadratic equality constraints are relaxed to the inequalities.

- The Lagrangian relaxation of the SOCP variant does not need to relax the voltage bound constraints, unlike that of the AC variant.
- So, the resulting Lagrangian dual is *not necessarily a relaxation* of the AC counterpart.

STRUCTURED DUAL DIMENSION REDUCTION

- The exact method may suffer from the large dimension in the dual solution space.
- Challenging to scale in the size of power system network
- We develop the two approaches to reducing the dual dimensions based on the structure of power flow functions.
  - ***H1:*** Relaxing any restrictions on the imaginary component of the conjugate multiplication of two complex voltage
  - ***H2:*** Relaxing the directional restrictions on Kirchoff's current law

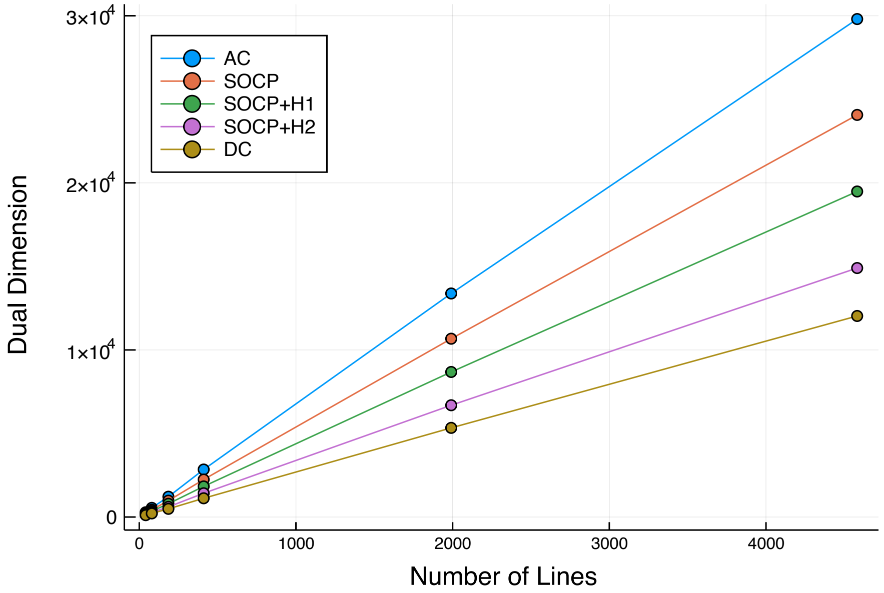
Power flow functions:

$$p_l^f(w) := \left[Y_l^{ffR} w_{ii} + Y_l^{ftR} w_l^R + Y_l^{ftI} w_l^I\right]$$

$$p_l^t(w) := \left[Y_l^{ttR} w_{jj} + Y_l^{tfR} w_l^R - Y_l^{tfI} w_l^I\right]$$

$$q_l^f(w) := -\left[Y_l^{ffI} w_{ii} + Y_l^{ftR} w_l^R - Y_l^{ftR} w_l^I\right]$$

$$q_l^t(w) := -\left[Y_l^{ttR} w_{jj} + Y_l^{tfR} w_l^R + Y_l^{tfR} w_l^I\right]$$

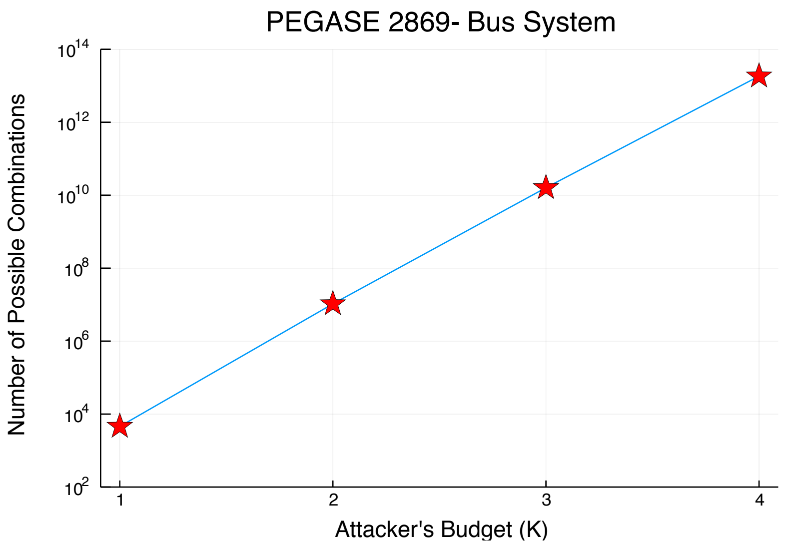


NUMERICAL EXPERIMENTS

Experimental Setup

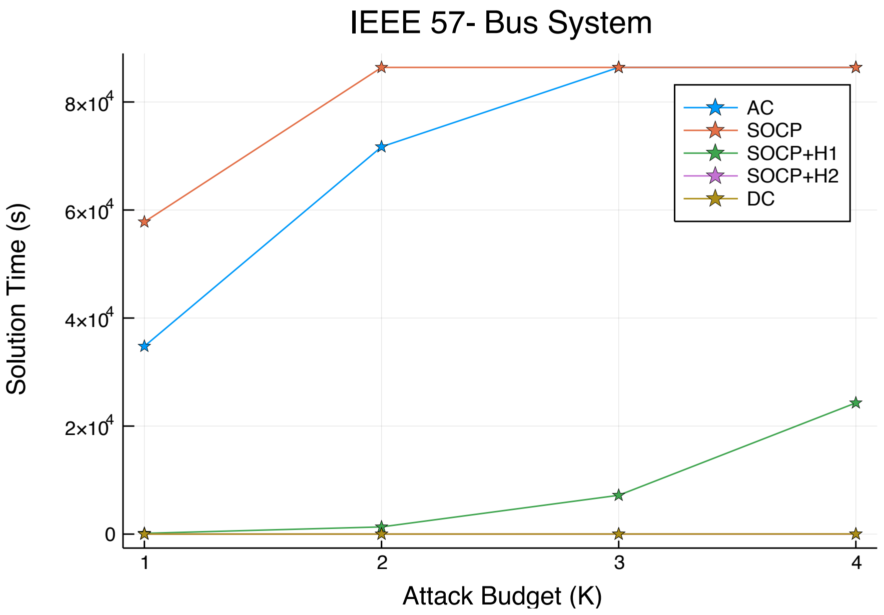
- We implemented the Benders-type method in Julia with CPLEX and Ipopt.
  – Branch-and-cut method with outer approximation
- Computations were run on a single core.
- Test cases with budgets K = 1, 2, 3, 4:

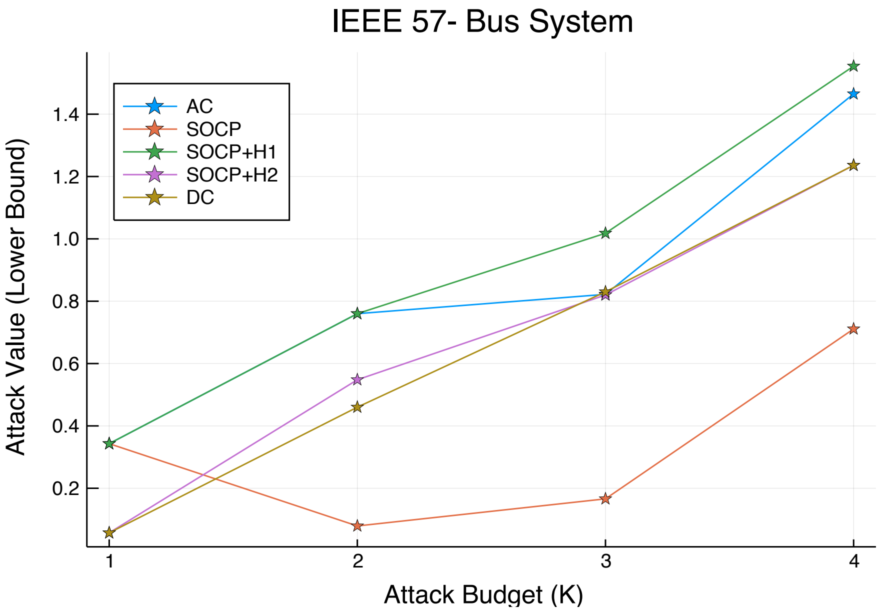
System	Lines	$k = 4$
IEEE 30	41	1×10^5
IEEE 118	186	5×10^7
IEEE 300	411	1×10^9
PEGASE 1354	1991	6×10^{11}
PEGASE 2869	4582	2×10^{13}



NUMERICAL EXPERIMENTS

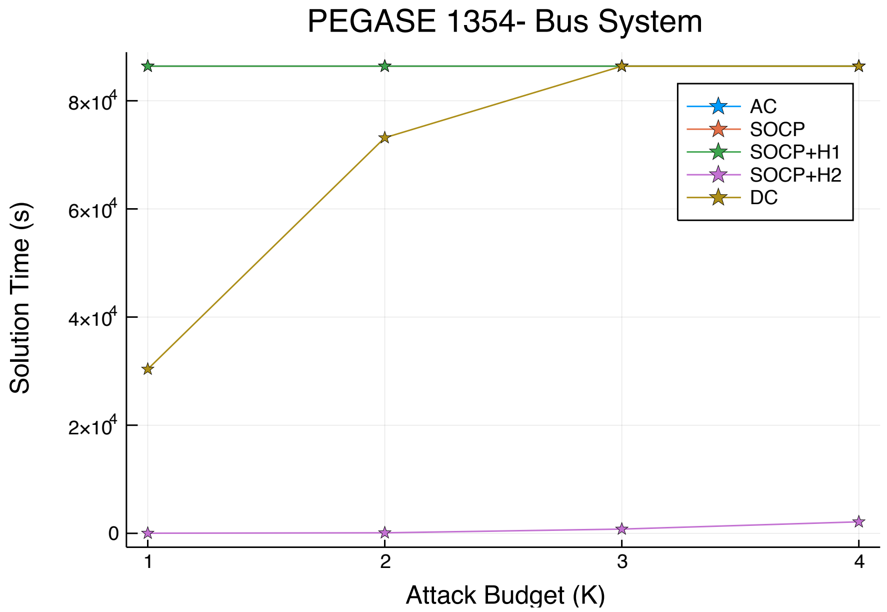
Small System: IEEE 57-Bus System

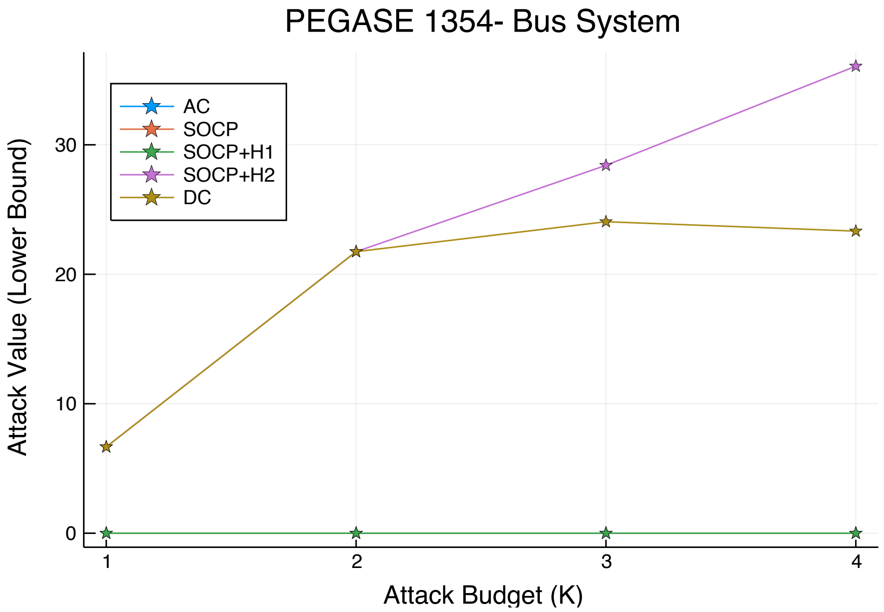




NUMERICAL EXPERIMENTS

Large System: PEGASE 1354-Bus System





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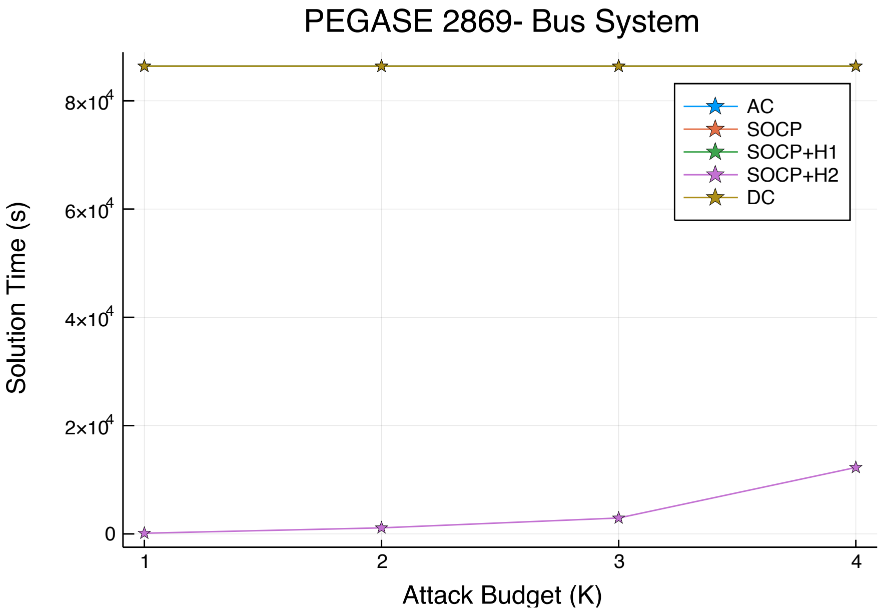
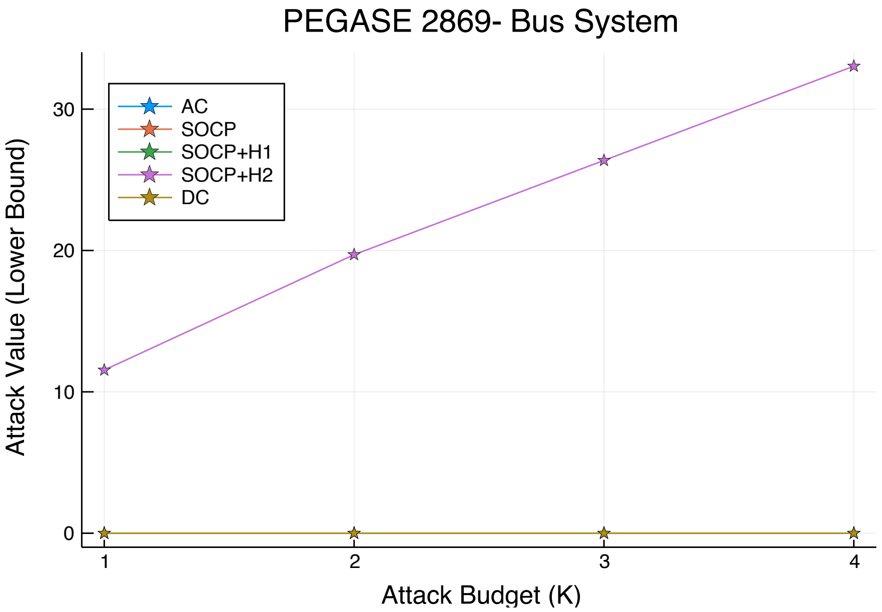
NUMERICAL EXPERIMENTS

LARGER System: PEGASE 2869-Bus System





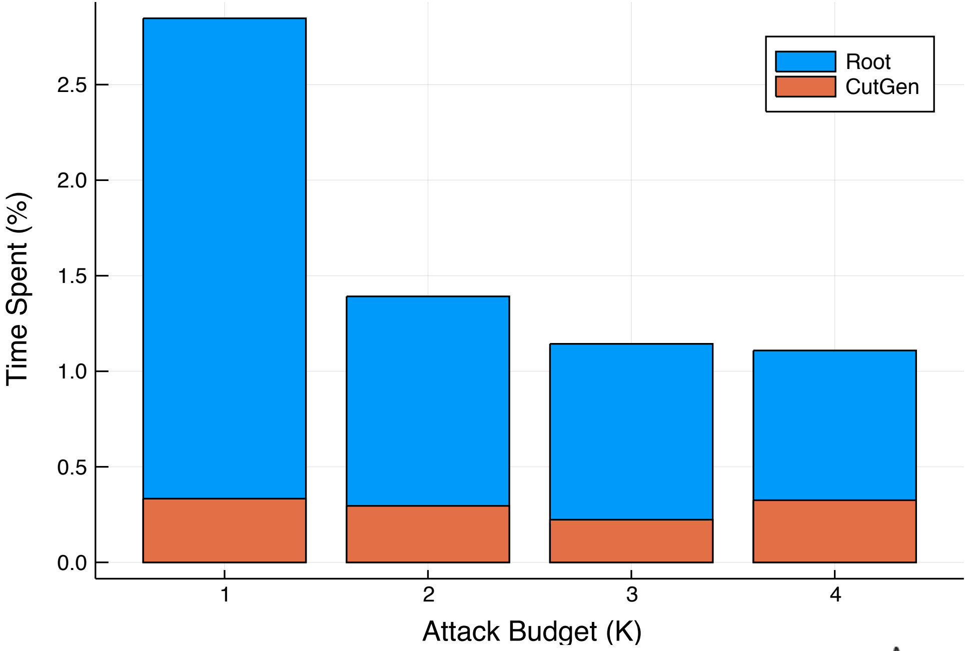
NUMERICAL EXPERIMENTS

Extremely Dense Linear Programs

- Cutting plane methods in large dimension result in extremely dense linear programs.

- More than 97% of the solution time were spent on solving LPs.

- ***Key Bottlenecks:***
  – Generating dense cuts
  – Solving dense linear programs
  – Serial tree search



CONCLUDING REMARKS

- Developed the bilevel model to identify critical line contingency
- Single-level mixed-binary convex programming relaxation
  - Base on the AC nonconvex system
  - Base on the SOCP relaxation (Note: the resulting single-level is not a relaxation of the AC counterpart.)
  - Based on the structure-based dimension reductions
- The AC-based single-level MBCP can find a global optimum (not really practical).
- The SOCP variant did not alleviate the computational cost.
- The structure-based dimension reductions outperforms the DC counterpart.

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