PDE Apps for Acoustic Ducts
(or Elastic Shafts, Historic Structures, Thermal Fins,. . .)

Anthony T Patera
Massachusetts Institute of Technology

SIAM Annual Meeting 2016

Boston, USA
July 11-14, 2016

The Akselos, S.A.,[1] Team: computational methods, software, and pedagogy.

Ho Chi Minh City: *Phuong Huynh*, *Loi Nguyen*, Tuyen Hoang, Thuc Nguyen

Boston: David Knezevic, Brian Sabbey

Lausanne: Thomas Leurent, Sylvain Vallaghé

[1]Akselos, S.A., licenses technology developed in the MIT research group of ATP. ATP has *no financial interest* in Akselos, S.A.

Academic Collaborators[2]

P Dahl	U Washington
SC Joyce	SUTD
JD Penn	MIT
K Smetana	(U Münster, MIT)
T Taddei	MIT
D Vignon	MIT
M Yano	U Toronto

[2] None of the academic collaborators has any financial interest in Akselos, S.A.

Sponsors

US AFOSR/Office of Secretary of Defense (— 9/2015)

US Office of Naval Research (— 5/2016)

Singapore University of Technology and Design

Ford Professorship, MIT

Parametrized Partial Differential Equations (PDEs)

- General Setting
- PDE Apps

Parametrized Partial Differential Equations (PDEs)

- General Setting
- PDE Apps

Examples of Parametrized PDEs

Heat Transfer (Conduction):

$$-\nabla(\kappa\nabla u\ ) = q \text{ in } \Omega, \quad s\ \equiv \bar{u}_{\text{root}}\ .$$

Linear Elasticity:

$$-\frac{\partial}{\partial x_j} E_{ij\ell m} \frac{\partial u_\ell}{\partial x_m} = f_i \text{ in } \Omega, \quad s\ \equiv \text{SCF}\ \ .$$

Helmholtz Acoustics:

$$-(1 + \mathrm{i}\epsilon k)\nabla^2 u\ - k^2 u\ = f \text{ in } \Omega_\lambda, \quad s\ \equiv Z^{\text{inlet}}\ .$$

INPUT PARAMETER $\mu \equiv (k, \lambda) \in \mathbb{R}^P$

$\rightarrow$ FIELD $u_\mu(x)$ and OUTPUT (QoI) s_μ

Given $\mu \in \mathcal{P}$ (compact) $\subset \mathbb{R}^P$, find

field $u_\mu \in X(\Omega_\mu)$ (say) scalar, real

$$\begin{aligned} A_\mu u_\mu &= F_\mu \text{ in } \Omega_\mu \text{ , or} \\ \langle A_\mu u_\mu, v\rangle &= \langle F_\mu, v\rangle, \forall v \in X \text{ , or} \\ a_\mu(u_\mu, v) &= f_\mu(v), \forall v \in X \text{ ,} \end{aligned}$$

output(s) $s_\mu \in \mathbb{R}$

$$s_\mu = \langle L_\mu, u_\mu\rangle, \text{ or } s_\mu = \ell_\mu(u_\mu) \text{ ,}$$

where $\Omega_\mu \subset \mathbb{R}^3$, $X = H^1_{(0)}(\Omega_\mu)$, and $F, L \in X'$.

Note boundary conditions are included in a_μ and f_μ.

A *Model* is a particular problem definition:

parametrization: $\mu \in \mathcal{P} \subset \mathbb{R}^P$;

spatial domain: $x \in \Omega_\mu \subset \mathbb{R}^3$;

physical discipline: a_μ, f_μ;

engineering outputs (QoI): ℓ_μ.

A Model maps parameter $\mu \in \mathcal{P}$ to
field $u_\mu(x)$ and output(s) s_μ.

A *Family* is a set of Models which share
a physical discipline and engineering context.

Acoustic Ducts, Elastic Shafts, Historic Structures,. . .

Parametrized Partial Differential Equations (PDEs)

- General Setting
- PDE Apps

PDE App: Definition

A PDE App is

software associated to a Model

which maps any $\mu \in \mathcal{P}$ to an

$$\text{approximate} \begin{cases} \text{field } \tilde{u}_\mu(x) & \approx u_\mu(x) \\ \text{output } \tilde{s}_\mu = \ell_\mu(\tilde{u}_\mu) & \approx s_\mu \end{cases}$$

subject to performance requirements:

response time and accuracy.

PDE App: Performance Requirements

A *deployed* PDE App should satisfy:

- $\lessapprox$ 5-second problem set-up time; "app-ification"
- $\lessapprox$ 5-second problem solution time, field and outputs;
- $\lessapprox$ 5% solution error, specified metrics;
- $\lessapprox$ 5-second field visualization time.

The choice of 5 seconds is informed by
the human attention span: *interaction*.

Offline I: Very Slow — Days

Given Family, form associated Online Dataset $\mathbb{D}$.

Offline II: Slow — Hours

Given Model $\in$ Family, script PDE App.

Online: Fast — Seconds

Given PDE App, evaluate $\mu \in \mathcal{P} \xrightarrow{\mathbb{D}} \tilde{u}_\mu(x), \tilde{s}_\mu$.

The PDE App Offline-Online approach
is computationally competitive in
the many-query context — Offline amortized, and
the interactive context — Offline "irrelevant."

Computational Methodology

- Perspective
- Components and System Synthesis
- Finite Element (FE) Approximation
- Static Condensation Reformulation of FE
- Model Order Reduction
- Remarks
- Computational Procedure: PDE App Workflow

Computational Methodology

- Perspective
- Components and System Synthesis
- Finite Element (FE) Approximation
- Static Condensation Reformulation of FE
- Model Order Reduction
- Remarks
- Computational Procedure: PDE App Workflow

Component Mode Synthesis, 1960s PR

Hurty, Craig-Bampton, Bourquin, Hetmaniuk,. . .

Static Condensation 1970s SC

Reduced Basis Methods, 1980s RB

Almroth, Noor, Porsching, Gunzburger,. . .

Post-Modern Reduced Basis Methods, 2000s

MoRePaS I-III: *a priori/posteriori* error estimation,
Weak Greedy sampling,
(approximate) affine expansions,
strict Offline-Online decomposition,. . .

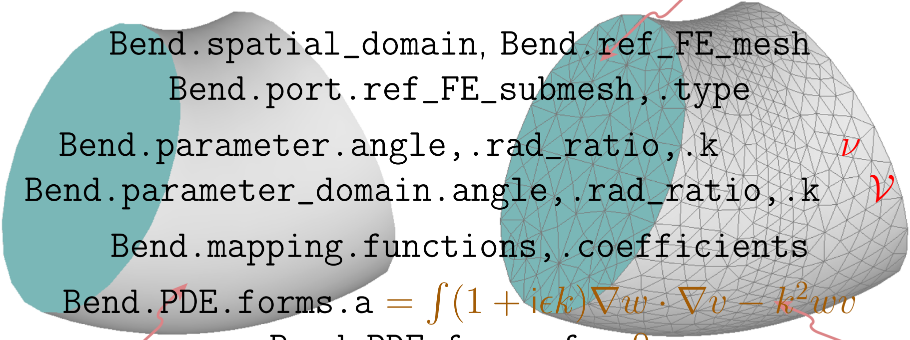
Reduced Basis Element Method, 2000s E

Maday-Rønquist

Computational Methodology

- Perspective
- Components and System Synthesis
- Finite Element (FE) Approximation
- Static Condensation Reformulation of FE
- Model Order Reduction
- Remarks
- Computational Procedure: PDE App Workflow

Parametrized Archetype Component



Bend

Bend.spatial_domain, Bend.ref_FE_mesh

Bend.port.ref_FE_submesh, .type

Bend.parameter.angle, .rad_ratio, .k ν

Bend.parameter_domain.angle, .rad_ratio, .k $\mathcal{V}$

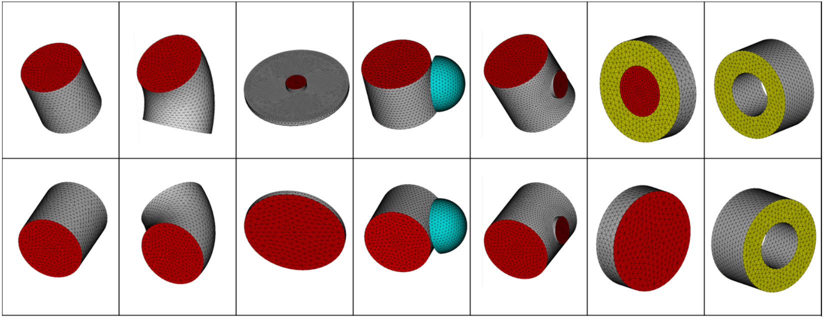
Bend.mapping.functions, .coefficients

Bend.PDE.forms.a $= \int (1 + \mathrm{i}\epsilon k) \nabla w \cdot \nabla v - k^2 w v$

Bend.PDE.forms.f $= 0$

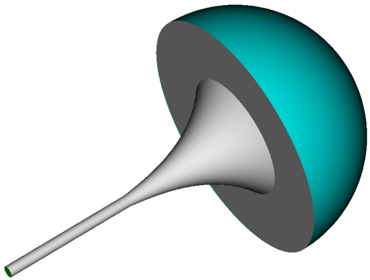
Library of Parametrized Archetype Components ↔ Family

Acoustic Ducts
(selected archetype components)



Admissible connections:
ports of common color ↔ common port type.

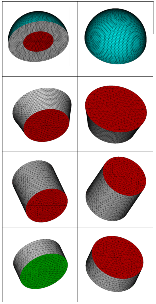
Synthesis: A Model

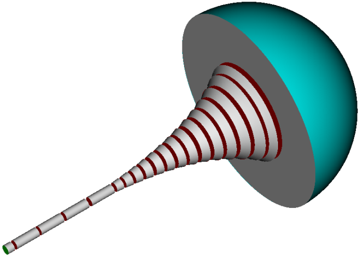


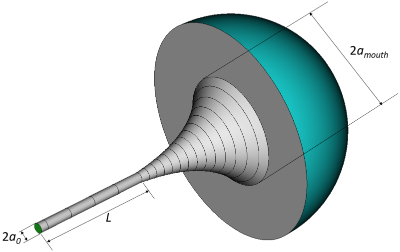
`Model_Exponential_Horn` (Flanged)

$$\mu \equiv (L/a_0, m^{\text{horn}}, a_{\text{mouth}}/a_0, ka_0)$$
$$\in \mathcal{P} \equiv [2, 20] \times [0.0334, 0.1666] \times [4, 12] \times [0, 1]$$

Synthesis: Instantiation and Connection





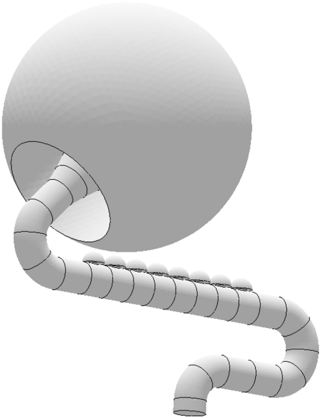


Instantiation

$\mu_{\text{model}} \in \mathcal{P} \rightarrow$
$\{\nu_{\text{local}} \in \mathcal{V}\}_{\text{instantiated components}}$

Connection

local port pairs $\rightarrow$
global ports $\Gamma \in G$



`Model_Nguyenophone`

$$\mu \equiv (\texttt{Hole\_Location}, \texttt{Hole\_Open}, k) \in \mathcal{P} \equiv \texttt{Wedge} \subset \mathbb{R}^8 \times \{0,1\}^8 \times [0,2]$$

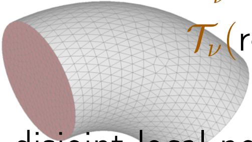
Computational Methodology

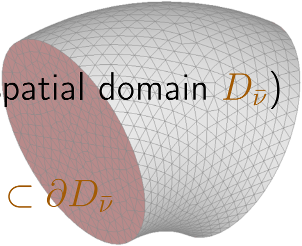
- Perspective
- Components and System Synthesis
- Finite Element (FE) Approximation
- Static Condensation Reformulation of FE
- Model Order Reduction
- Remarks
- Computational Procedure: PDE App Workflow

An archetype component is characterized by

spatial domain $D_\nu =$

$\mathcal{T}_\nu$(reference spatial domain $D_{\bar{\nu}}$)

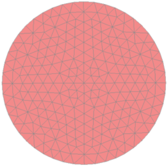




and

two disjoint local ports $\gamma_1, \gamma_2 \subset \partial D_{\bar{\nu}}$

such that



$$\gamma_i = \tau_i \gamma_0,\ i = 1, 2\ ,$$

for γ_0 a fiducial port (type).

We may easily consider more than two local ports.

FE Approximation Spaces

Associate to

each archetype component

a reference FE mesh,

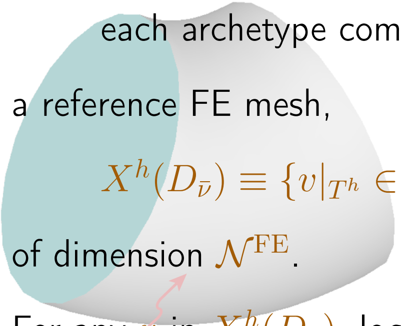
$$X^h(D_{\bar{\nu}}) \equiv \{v|_{T^h} \in \mathbb{P}_p(T^h), \forall T^h \in \mathbb{T}^h\}$$

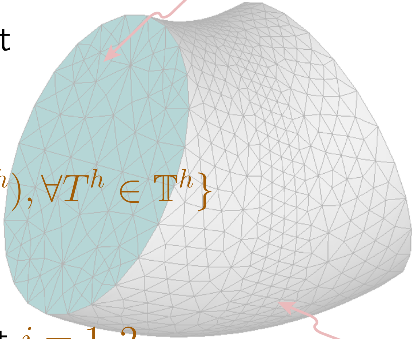
of dimension $\mathcal{N}^{\mathrm{FE}}$.

For any v in $X^h(D_{\bar{\nu}})$, local port $i = 1, 2$,

$$v|_{\gamma_i} \in \{ \underbrace{\chi_j^h}_{\text{fiducial port modes}} \circ \tau_i^{-1}, 1 \le j \le J^{\mathsf{FE}} \};$$

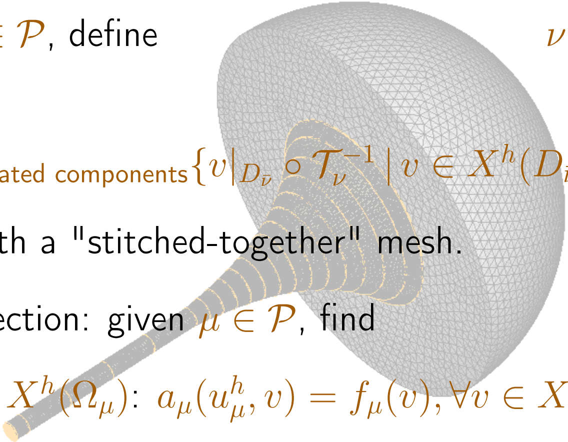
implicit conforming condition on ports of common type.





Finite Element (FE) Approximation of Model

For given $\mu \in \mathcal{P}$, define $\nu = \nu(\mu)$



$X^h(\Omega_\mu) \equiv$

$$\bigoplus_{\text{instantiated components}} \{ v|_{D_{\bar{\nu}}} \circ \mathcal{T}_\nu^{-1} \,|\, v \in X^h(D_{\bar{\nu}}) \} \cap X$$

associated with a "stitched-together" mesh.

Galerkin projection: given $\mu \in \mathcal{P}$, find

field $u_\mu^h \in X^h(\Omega_\mu)$: $a_\mu(u_\mu^h, v) = f_\mu(v), \forall v \in X^h(\Omega_\mu)$,

and subsequently

output $s_\mu^h = \ell_\mu(u_\mu^h)$.

Computational Methodology

- Perspective
- Components and System Synthesis
- Finite Element (FE) Approximation
- Static Condensation Reformulation of FE
- Model Order Reduction
- Remarks
- Computational Procedure: PDE App Workflow

Static Condensation (SC): Component Level . . .

In *given instantiated component*, $\nu = \nu(\mu)$

for local port $i = 1, 2$

for port mode $j = 1, \ldots, J^{\mathrm{FE}}$:

$\psi^h_{i,j} = \mathcal{L}_i(\chi^h_j \circ \tau_i^{-1})$ is lifting to reference domain of port mode j on port i, and

$\varphi^h_{i,j;\nu} = \psi^h_{i,j} + \eta^h_{i,j;\nu} \in X^h_{[\gamma_1,\gamma_2:0]}(D_{\bar{\nu}})$ satisfies

$$a^{D_{\bar{\nu}}}_{\nu}(\varphi^h_{i,j;\nu}, v) = 0, \forall v \in X^h_{[\gamma_1,\gamma_2:0]}(D_{\bar{\nu}}), \text{ subject to}$$

$$\varphi^h_{i,j;\nu}|_{\gamma_{i'}} = \chi^h_j \, \delta_{ii'}, \qquad \mathcal{N}^{\mathrm{FE}} \times \mathcal{N}^{\mathrm{FE}}$$

where for simplicity all sources reside on ports.

In *given instantiated component*, LINEARITY

$$u_\mu^h|_{D_{\nu(\mu)}} = \sum_{i=1}^{2} \sum_{j=1}^{J} u_{i,j;\nu}^h \, (\varphi_{i,j;\nu}^h \circ \mathcal{T}_\nu^{-1})$$

for appropriate coefficients $u_{i,j;\nu}^h, 1 \leq j \leq J, i = 1, 2$.

Form $2J^{\mathrm{FE}} \times 2J^{\mathrm{FE}}$ stiffness matrix $A^{h\ \text{Galerkin}}_{[i,j],[k,\ell];\nu}$:

normal velocity moment on local port i flux

with respect to test port mode j

expressed in terms of

pressure coefficient on local port k

associated with trial port mode ℓ.

SC: System Level

Require on global ports $\Gamma \in G$

continuity of pressure, and

weak continuity of normal velocity

implemented as direct stiffness assembly:

$$\{A^h_{\nu(\mu)}\}_{\text{instantiated components}} \to \mathcal{A}^h_\mu \quad ; \qquad \mathcal{F}^h_\mu$$

here $\mathcal{A}^h_\mu$ is $|G|J^{\mathsf{FE}} \times |G|J^{\mathsf{FE}}$ block-sparse Schur complement.

Issues: J^{FE} will be large, and

$\mathcal{N}^{\mathrm{FE}}$ will be large,

such that $\mathcal{A}^h_\mu$ costly to form and to "invert."

Computational Methodology

- Perspective
- Components and System Synthesis
- Finite Element (FE) Approximation
- Static Condensation Reformulation of FE
- Model Order Reduction
- Remarks
- Computational Procedure: PDE App Workflow

In *given instantiated component*, $\nu = \nu(\mu)$

for local port $i = 1, 2$

for port mode $j = 1, \ldots, M$:

$\psi^h_{i,j} = \mathcal{L}_i(\chi^h_j \circ \tau_i^{-1})$ is lifting to reference domain of port mode j on port i, and

$\varphi^{h,N}_{i,j;\nu} = \psi^h_{i,j} + \eta^{h,N}_{i,j;\nu} \in Z^{h,N}_{i,j\,[\gamma_1,\gamma_2:0]}(D_{\bar{\nu}})$ satisfies

$$a^{D_{\bar{\nu}}}_{\nu}(\varphi^h_{i,j;\nu}, v) = 0, \forall v \in Z^{h,N}_{i,j\,[\gamma_1,\gamma_2:0]}(D_{\bar{\nu}}), \text{ subject to}$$

$$\varphi^h_{i,j;\nu}|_{\gamma_{i'}} = \chi^h_j\, \delta_{ii'}; \qquad N \times N$$

where for simplicity all sources reside on ports.

In *given instantiated component*,

$$u_\mu^{h,M,N}|_{D_{\nu(\mu)}} = \sum_{i=1}^{2} \sum_{j=1}^{M} u_{i,j;\nu}^{h,M,N} \; (\varphi_{i,j;\nu}^{h,N} \circ \mathcal{T}_\nu^{-1})$$

for appropriate coefficients $u_{i,j;\nu}^{h,M,N}, 1 \le j \le M, i = 1, 2$.

Form $2M \times 2M$ stiffness matrix $A_{[i,j],[k,\ell];\nu}^{h,M,N\ \text{Petrov-Galerkin}}$:

normal velocity moment on local port i flux

with respect to test port mode j

expressed in terms of

pressure coefficient on local port k

associated with trial port mode ℓ.

Require on global ports $\Gamma \in G$

continuity of pressure, and

weak continuity of normal velocity

implemented as direct stiffness assembly:

$$\{A^{h,M,N}_{\nu(\mu)}\}_{\text{instantiated components}} \rightarrow \mathcal{A}^{h,M,N}_{\mu} \qquad \mathcal{F}^{h,M,N}_{\mu}$$

where $\mathcal{A}^{h,M,N}_{\mu}$ is $|G|M \times |G|M$ block-sparse.

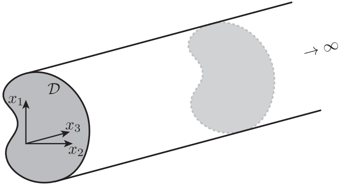
Issues "resolved": $M \ll J^{\mathrm{FE}}$, and

$N \ll \mathcal{N}^{\mathsf{FE}}$,

such that $\mathcal{A}^{h,M,N}_{\mu}$ is inexpensive to form and to "invert."

Port Reduction, $M \ll J^{\mathrm{FE}}$: Rationale...

Consider a waveguide $\mathcal{D} \times (0, \infty)$,



and find $p(x_1, x_2, x_3)$ such that

$$-\nabla^2 p - k^2 p = 0 \text{ in } \mathcal{D} \times (0, \infty) \quad ,$$

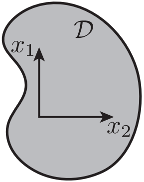
and

$$p = g \text{ on } (x_1, x_2) \in \mathcal{D}, x_3 = 0,$$

$$\frac{\partial p}{\partial n} = 0 \text{ on } (x_1, x_2) \in \partial\mathcal{D}, 0 < x_3 < \infty,$$

$$p \text{ (say) outgoing bounded wave as } x_3 \to \infty.$$

Restrict attention to the transverse domain $\mathcal{D}$,



and find $(\chi_i(x_1, x_2), \lambda_i)_{i=1,\ldots}$ solution of eigenproblem

$$-\nabla^2_{x_1,x_2}\chi = \lambda\chi, \text{ in } \mathcal{D} \quad ,$$

$$\frac{\partial\chi}{\partial n} = 0 \text{ on } \partial\mathcal{D};$$

order (real) eigenvalues $\lambda_1 = 0 < \lambda_2 \leq \lambda_3 \leq \ldots$

Consider $k \in [\sqrt{\lambda_n}, \sqrt{\lambda_{n+1}})$: then $\Re\{\,\cdot\, e^{\mathrm{i}\omega t}\}$

$$p = \sum_{j=1}^{n} \alpha_j^g\, \chi_j(x_1, x_2)\, e^{-\mathrm{i}\sqrt{k^2 - \lambda_j}\, x_3}$$

$$+ \sum_{j=n+1}^{\infty\ (\text{or } J^{\mathrm{FE}})} \alpha_j^g\, \chi_j(x_1, x_2)\, e^{-\sqrt{\lambda_j - k^2}\, x_3}$$

for coefficients α_j^g chosen to realize $p(\cdot, \cdot, x_3 = 0) = g$.

For any global port $\Gamma \in G$, *higher modes introduced*

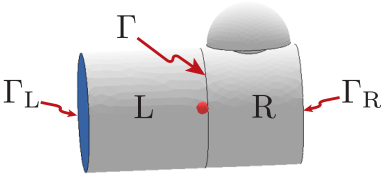
in neighboring components, and

at neighboring global ports,

will be filtered prior to "arrival" at Γ.

Port Reduction, $M \ll J^{\mathrm{FE}}$: A *Library* Training Procedure

For all compatible archetype component pairs in Library,

form *test subsystem* 

and find $z \in X^h(\mathtt{L}, \mathtt{R})$ such that

$$a^{(\mathtt{L},\mathtt{R})}_{\nu_{\mathtt{L}},\nu_{\mathtt{R}}}(z, v) = 0, \ \forall v \in X^h(\mathtt{L}, \mathtt{R}) \ ,$$

for a rich set of Dirichlet conditions on $\Gamma_{\mathtt{L}}$, $\Gamma_{\mathtt{R}}$, and admissible parameters $\nu_{\mathtt{L}}$ and $\nu_{\mathtt{R}}$.

Collect $z|_\Gamma \circ \mathcal{T}_\nu \circ \tau.$ *from all test subsystems* in a set S.

Apply POD to S: fiducial port modes $\{\chi_j\}_{j=1,\ldots,M}$.

Bubble Reduction, $N \ll \mathcal{N}^{\mathrm{FE}}$: Rationale

For any archetype component in Library,

for local port $i = 1, 2$,

for port mode $j = 1, \ldots, M$,

$$\eta^h_{i,j;\nu} \in \underbrace{\{\eta^h_{i,j;\nu} \,|\, \nu \in \mathcal{V}\}}_{\text{low-dimensional smooth manifold}} \subset \underbrace{X^h_{[\gamma_1,\gamma_2;0]}(D_{\bar{\nu}})}_{\text{high-dimensional space}} ;$$

note that

$$\nu_{\text{local}} \in \mathcal{V} \subset \mathbb{R}^V, \; \mu_{\text{model}} \in \mathcal{P} \subset \mathbb{R}^P$$

for (typically) $V \ll P$ — *components divide and conquer.*

Bubble Reduction, $N \ll \mathcal{N}^{\mathrm{FE}}$: A *Library* Training Procedure

For each archetype component in Library,

for local port $i = 1, 2$,

for port mode $j = 1, \ldots, M$,

form $Z^{h,N}_{i,j[\gamma_1,\gamma_2:0]}$ as RB Lagrangian snapshot space $\perp$-ized

$$Z^{h,N}_{i,j[\gamma_1,\gamma_2:0]} \equiv \text{ span}\{\eta^h_{i,j;\nu^n_{i,j}}, 1 \le n \le N_{i,j}\}$$

for quasi-optimal parameter values

$$\{\nu^1_{i,j} \in \mathcal{V}, \ldots, \nu^N_{i,j} \in \mathcal{V}\}$$

selected by the RB Weak-Greedy procedure.

Computational Methodology

- Perspective
- Components and System Synthesis
- Finite Element (FE) Approximation
- Static Condensation Reformulation of FE
- Model Order Reduction
- Remarks
- Computational Procedure: PDE App Workflow

Optimality

Under certain hypotheses, the best fits associated with

the port reduced spaces, span$\{\chi_j, 1 \leq j \leq M\}$,

and

the bubble reduced spaces, $Z_{i,j}^{h,N}$,

converge at rates similar to the corresponding

Kolmogorov M (respectively, N) width.

The Petrov-Galerkin projections are optimal

to within a (Model,μ)-dependent stability constant.

Verification (and Validation)

A posteriori error indicators play a role in

optimal choice of snapshots $\rightarrow Z_{i,j}^{h,N}$

and

optimal choice of M and N.

Each Model is verified over $\Xi_{\text{verification}} \in \mathcal{P}$:

refinement in $h \downarrow, M \uparrow$, and $N \uparrow$;

reference to appropriate closed-form approximations;

comparison to 3rd-party computations and experiments.

Verification of each Model improves

archetype components: convergence of *Library*.

Computational Methodology

- Perspective
- Components and System Synthesis
- Finite Element (FE) Approximation
- Static Condensation Reformulation of FE
- Model Order Reduction
- Remarks
- Computational Procedure: PDE App Workflow

Offline I: Library → Online Dataset $\mathbb{D}$ Expensive

Prepare Online Dataset $\mathbb{D}$ for Library:

- archetype *affine* component mappings $\mathcal{T}_\nu$; EIM
- archetype component reference FE meshes;
- port modes $\chi_j, 1 \leq j \leq M$ (for each port type);
- RB spaces $Z_{i,j}^{h,N}$ for each archetype component,
  local port i, and port mode j;
- Petrov-Galerkin parameter-independent inner products.

Role of components:

- *no* Models formed or evaluated in Offline I stage;
- *all* Models in Online stage amortize Offline I effort.

Online: $\mathbb{D}$; Model; $\mu \in \mathcal{P} \rightarrow u_\mu^{h,M,N}, s_\mu^{h,M,N}$ Fast

Web-User-Interface (WUI) Cloud Implementation

Query the PDE App:

input $\mu \in \mathcal{P}$,	User
synthesize Model from (say) script,	Model Server
invoke Online Dataset $\mathbb{D}$	Compute Server
form and solve Schur complement,	Compute Server
calculate field and output,	Compute Server
download and display solution.	User, Servers

(Offline II — prepare Model Server for each Model: parametrization, instantiation, connections, and outputs.)

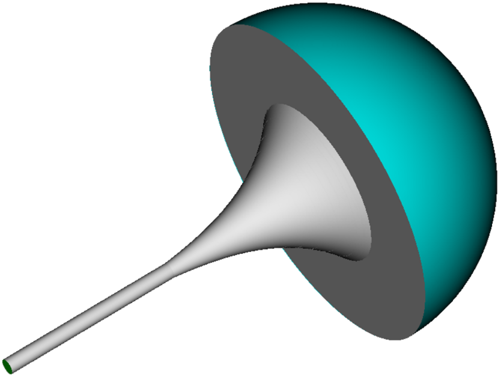
Acoustics Ducts: PDE App Examples

- A Flanged Exponential Horn
- An Expansion Chamber
- An Extended-Tube Expansion Chamber (ETEC)
- A Circular Duct with Toroidal Bend

Acoustics Ducts: PDE App Examples

- A Flanged Exponential Horn
- An Expansion Chamber
- An Extended-Tube Expansion Chamber (ETEC)
- A Circular Duct with Toroidal Bend

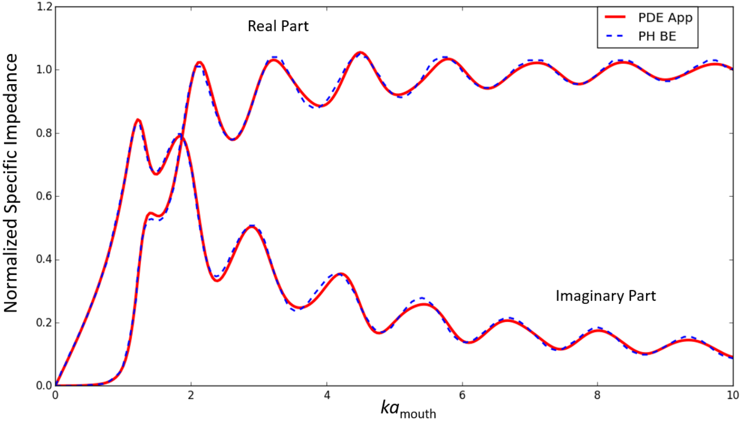
Model: Parametrization and Spatial Domain



$$\mu \equiv (L/a_0, m^{\text{horn}}, a_{\text{mouth}}/a_0, ka_0) \in \mathcal{P} \equiv [2, 20] \times [0.0334, 0.1666] \times [4, 12] \times [0, 1]$$

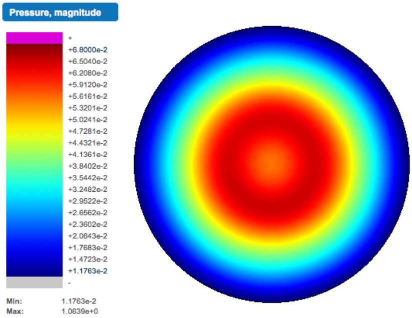
Throat Impedance

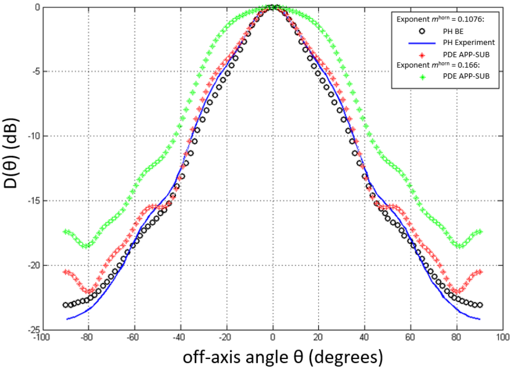
Parameters: $m^{\text{horn}} = 0.1076, a_{\text{mouth}}/a_0 = 10.67$.



PH: Post & Hixson, PhD Thesis, 1974.

Visualization: Radiation Directivity $ka_{\text{mouth}} = 10$





Nearfield

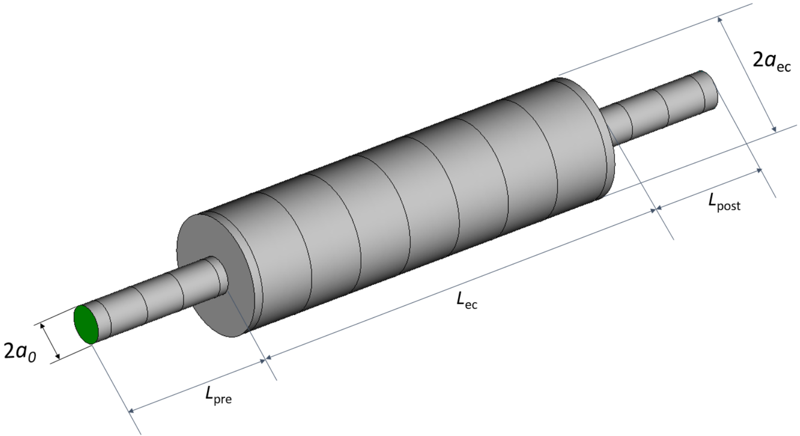
Farfield

Modulus of Pressure

Acoustics Ducts: PDE App Examples

- A Flanged Exponential Horn
- An Expansion Chamber
- An Extended-Tube Expansion Chamber (ETEC)
- A Circular Duct with Toroidal Bend

Model: Parametrization and Spatial Domain

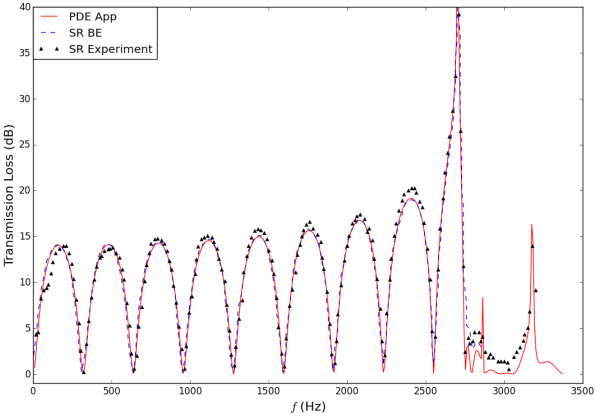


$$\mu \equiv (L_{\text{pre}}/a_0, L_{\text{post}}/a_0, L_{\text{ec}}/a_0, a_{\text{ec}}/a_0, ka_0)$$
$$\in \mathcal{P} \equiv [4, 12]^2 \times [1.5, 25] \times [1.5, 6.5] \times [0, 1.5]$$

Transmission Loss

Parameters: $L_{\mathrm{ec}}/a_0 = 22.26, a_{\mathrm{ec}}/a_0 = 3.152;$

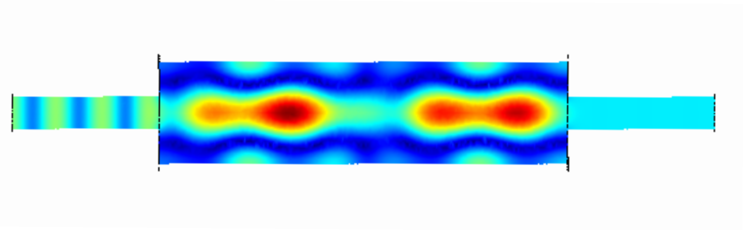
$a_0 = 0.0243$ cm.



SR: Selamet and Radavich, J Sound Vibration, 1997.

Visualization: Excitation of (Axisymmetric) Higher Modes

Parameters: $L_{\text{ec}}/a_0 = 22.26, a_{\text{ec}}/a_0 = 3.1525;$
$f = 2.8$ kHz, $a_0 = 0.0243$ cm.

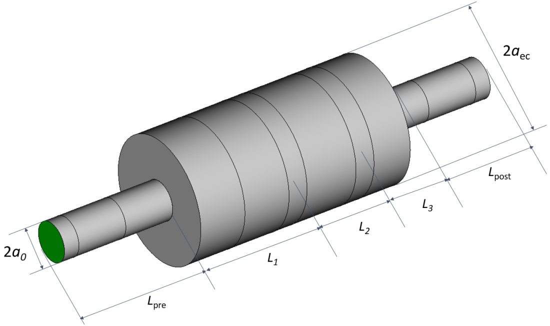


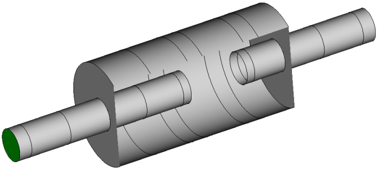
Modulus of Pressure

Acoustics Ducts: PDE App Examples

- A Flanged Exponential Horn
- An Expansion Chamber
- An Extended-Tube Expansion Chamber (ETEC)
- A Circular Duct with Toroidal Bend

Model: Parametrization and Spatial Domain

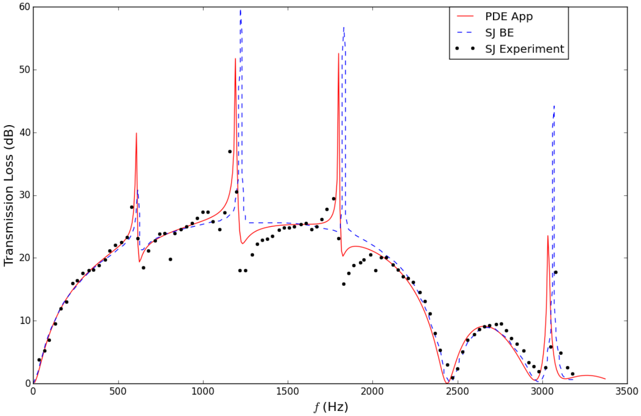




$$\mu \equiv (L_{\mathrm{pre}}/a_0, L_{\mathrm{post}}/a_0, L_1/a_0, L_2/a_0, L_3/a_0, a_{\mathrm{ec}}/a_0, ka_0)$$
$$\in \mathcal{P} \equiv [2,6]^2 \times [2,16]^3 \times [1.5, 4.0] \times [0, 1.5]$$

Transmission Loss

Parameters: $L_1/a_0 = 5.391, L_2/a_0 = 3.716,$
$L_3/a_0 = 2.510, a_{ec}/a_0 = 3.152; a_0 = 0.0243$ cm.

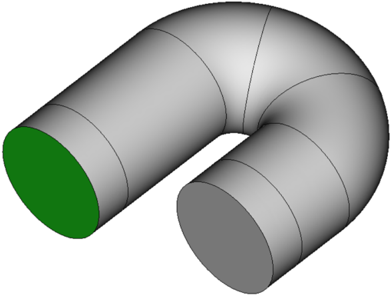


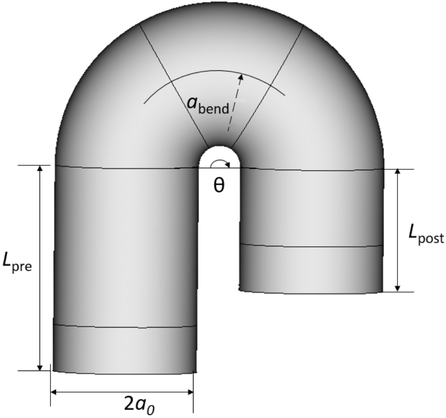
SJ: Selamet and Ji, J Sound Vibration, 1999.

Acoustics Ducts: PDE App Examples

- A Flanged Exponential Horn
- An Expansion Chamber
- An Extended-Tube Expansion Chamber (ETEC)
- A Circular Duct with Toroidal Bend

Model: Parametrization and Spatial Domain





$$\mu \equiv (L_{\text{pre}}/a_0, L_{\text{post}}/a_0, a_{\text{bend}}/a_0, \theta_{\text{bend}}, ka_0)$$
$$\in \mathcal{P} \equiv [1.5, 15]^2 \times [1.2, 3] \times [30°, 180°] \times [0, 1.8412]$$

WUI: Model Selection

PDE Apps for Education
The PDE App Project Team

Quick Start

- List of Models
  - Acoustic Ducts
    - Circular Duct with Bend
    - Expansion Chamber
    - ETEC Muffler
  - Acoustic Ducts SUB

The PDE App Project Team

Principal Investigators

DBP Huynh (Akselos S.A.) and AT Patera* (MIT)

Academic Contributors

P Dahl (U Washington)

SC Joyce (SUTD)

D Vignon (MIT)

M Yano (U Toronto)

None of the academic contributors has any financial interest in Akselos, S.A.

Akselos Contributors

D Knezevic, B Sabbey (Akselos, Boston)

T Hoang, L Nguyen, T Nguyen (Akselos, Ho Chi Minh City)

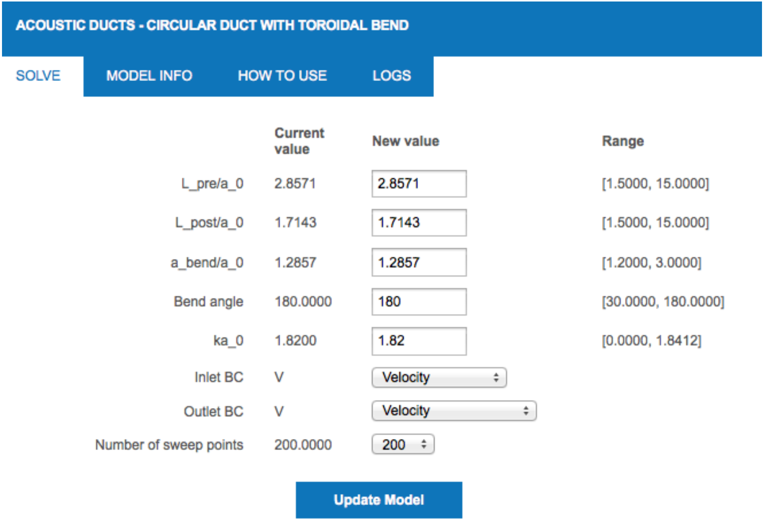
T Leurent, S Vallaghé (Akselos, Lausanne)

Sponsors

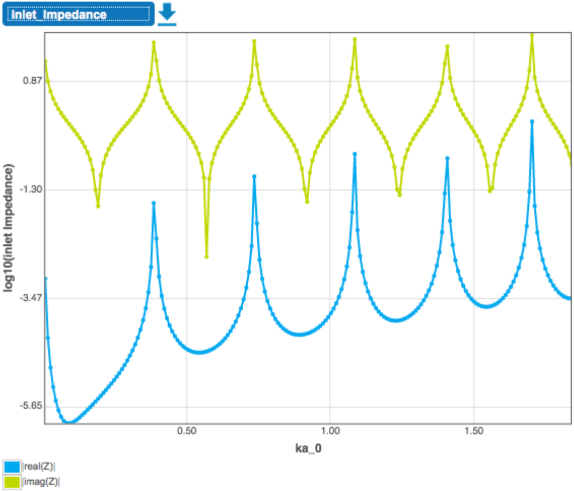
The PDE App methodology, software, server, and website is, or has been, funded by the AFOSR/OSD, the ONR, the Deshpande Center at MIT, SUTD and the SUTD-MIT International Design Center, a Ford Professorship of Engineering at MIT, the MIT Department of Mechanical Engineering, and Akselos, S.A.

**Akselos S.A. licenses technology developed in the MIT research group of AT Patera. AT Patera has no financial interest in Akselos S.A.*

WUI: Parameter Specification



WUI: Output

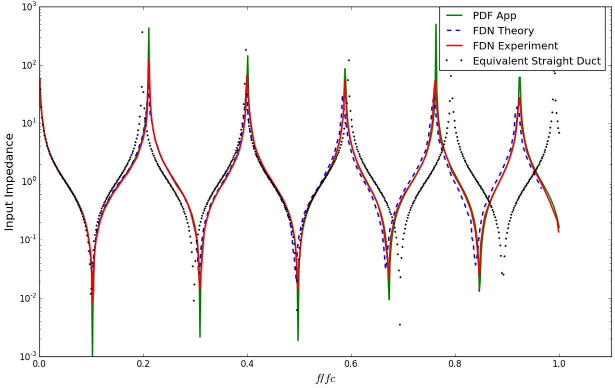


Response Time (all-inclusive) 8.4 seconds:
4-core GCE instance and commodity Internet.

Inlet Impedance (Reactive)

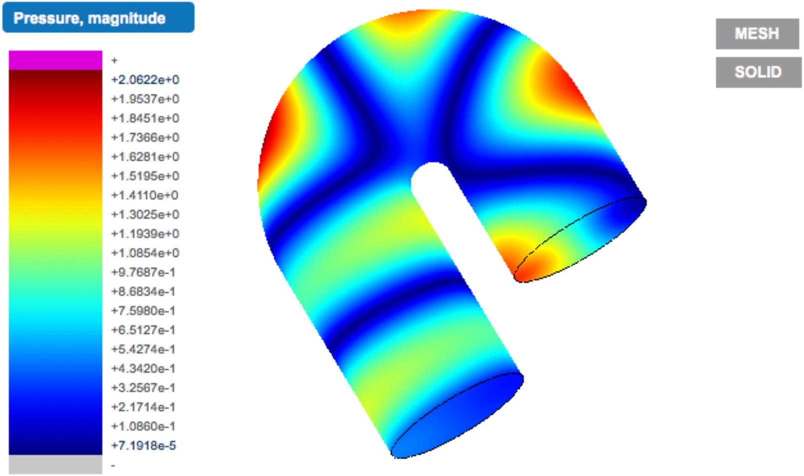
Parameters: $L_{\text{pre}}/a_0 = 2.8571, L_{\text{post}}/a_0 = 1.7143,$
$a_{\text{bend}}/a_0 = 1.2857, \theta_{\text{bend}} = 180°.$

Boundary Conditions: velocity-velocity.



FDN: Félix, Dalmont, and Nederveen, JASA, 2012.

WUI: Visualization — Azimuthal Excitation $ka_0 = 1.82$



Response Time (all-inclusive) 8.4 seconds:
4-core GCE instance and commodity Internet.

References

A Few References. . .

BO Almroth, P Stern, and FA Brogan. Automatic choice of global shape functions in structural analysis. AIAA J, 1978.

I Babuska and R Lipton. Optimal local approximation spaces for generalized finite element methods with application to multiscale problems. Multiscale Modeling and Simulation, 9:373–406, 2011.

M Barrault, Y Maday, N. Nguyen, and A Patera, An empirical interpolation method: Application to efficient reduced-basis discretization of partial differential equations. CR Acad Sci Paris, Série I 339:667–672, 2004.

P Binev, A Cohen, W Dahmen, R DeVore, G Petrova, and P Wojtaszczyk. Convergence rates for greedy algorithms in reduced basis methods. SIAM J Math Anal, 43(3):1457–1472, 2011.

F Bourquin. Component mode synthesis and eigenvalues of second order operators: discretization and algorithm. M2AN, 26(3):385–423,1992.

R Craig and M Bampton. Coupling of substructures for dynamic analyses. AIAA J, 1968.

JL Eftang and AT Patera. A port-reduced static condensation reduced basis element method for large component-synthesized structures: Approximation and a posteriori error estimation. Advanced Modeling and Simulation in Engineering Sciences, 1:3, 2013.

...A Few References...

JL Eftang and AT Patera. Port reduction in component-based static condensation for parametrized problems: Approximation and a posteriori error estimation. IJNME, 96(5):269–302, 2013.

S Félix, J-P Dalmont, and CJ Nederveen. Effects of bending portions of the air column on the acoustical resonances of a wind instrument. JASA, 131 (5):4164–4172, 2012.

B Haasdonk and M Ohlberger. Reduced basis method for finite volume approximations of parametrized linear evolution equations. M2AN, 42(2):277–302, 2008.

JS Hesthaven, G Rozza, and B Stamm. Certified Reduced Basis Methods for Parametrized Partial Differential Equations. Series: Springer Briefs in Mathematics, Edition Number 1, Springer International Publishing, 2016.

U Hetmaniuk and R Lehoucq. A special finite element method based on component mode synthesis. M2AN, 44(3):401–420, 2010.

DBP Huynh, DJ Knezevic, and AT Patera. A Laplace transform certified reduced basis method; Application to the heat equation and wave equation. CR Acad Sci Paris Series I, 349(7-8):401–405, 2011.

DBP Huynh, DJ Knezevic, JW Peterson, and AT Patera. High-fidelity real-time simulation on deployed platforms. Computers and Fluids, 43(1):74–81, 2011.

. . . A Few References. . .

DBP Huynh, DJ Knezevic, and AT Patera. A static condensation reduced basis element method: Approximation and a posteriori error estimation. M2AN, 47(1): 213–251, 2013.

DBP Huynh, DJ Knezevic, and AT Patera. A static condensation reduced basis element method: Complex problems. CMAME, 259:197–216, 2013.

WC Hurty. On the dynamics of structural systems using component modes. AIAA paper no 64–487, 1964.

H Jakobsson, F Bengzon, and M Larson. Adaptive component mode synthesis in linear elasticity. IJNME, 86(7):829–844, 2011.

Y Maday and EM Rønquist. The reduced basis element method: Application to a thermal fin problem. SIAM JSC, 26(1):240–258, 2004.

ML Munjal, Acoustics of Ducts and Mufflers, Second Edition, Wiley, 2014.

AK Noor and JM Peters. Reduced basis technique for nonlinear analysis of structures. AIAA J, 1980.

JT Post and EL Hixson. A modeling and meausrement study of acoustic horns. PhD Thesis, University of Texas at Austin, 1994.

...A Few References...

TA Porsching. Estimation of the error in the reduced basis method solution of nonlinear equations. Math Comp, 45:487–496, 1985.

A Quarteroni, A Manzoni, and F Negri. Reduced Basis Methods for Partial Differential Equations. An Introduction. Series: La Matematica per il 3+2, Volume 92, Springer International Publishing, 2016.

WC Rheinboldt. On the theory and error estimation of the reduced basis method for multi-parameter problems. Nonlinear Anal, 21:849–858, 1993.

G Rozza, DBP Huynh, and AT Patera. Reduced basis approximation and a posteriori error estimation for affinely parametrized elliptic coercive partial differential equations. Arch Comput Methods Eng, 15(3):229–275, 2008.

A Selamet and ZL Ji. Acoustic attenuation performance of circular expansion chambers with extended inlet/outlet. Journal of Sound and Vibration, 223, 197-212, 1999.

A Selamet and PM Radavich. The effect of length on the acoustic attenuation performance of concentric expansion chambers: An analytical, computational and experimental investigation. Journal of Sound and Vibration, 201(4):407–426, 1997.

K Smetana and AT Patera. Optimal local approximation spaces for component-based static condensation procedures. SIAM JSC (submitted February 2015).

... A Few References

S Vallaghé, DBP Huynh, D Knezevic, TL Nguyen, and AT Patera. Component-based reduced basis for parametrized symmetric eigenproblems. Advanced modeling and simulation in engineering sciences, 2:7, 2015.

K Veroy, C Prud'homme, DV Rovas, and AT Patera. *A posteriori* error bounds for reduced-basis approximation of parametrized noncoercive and nonlinear elliptic partial differential equations. AIAA Paper No. 2003-3847: 1-18, 2003.

EL Wilson. The static condensation algorithm. Int J Num Methods Engr, 8(1):198-203, 1974.

K Wong. The "Appification" of Simulation. Desktop Engineering, April 1, 2015.