Predictability of Extreme Events

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Outline

1) Motivation

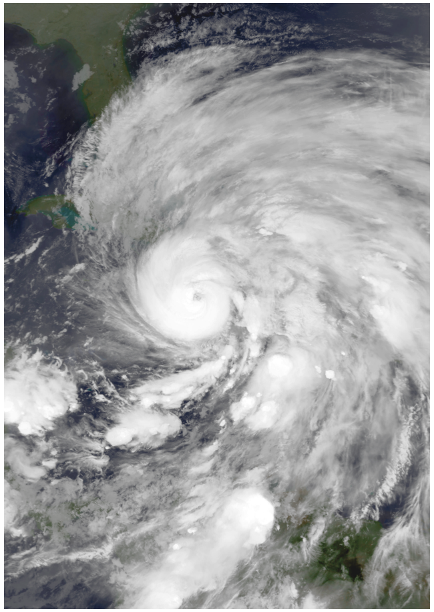
2) Dynamic-Stochastic Models for Extremes

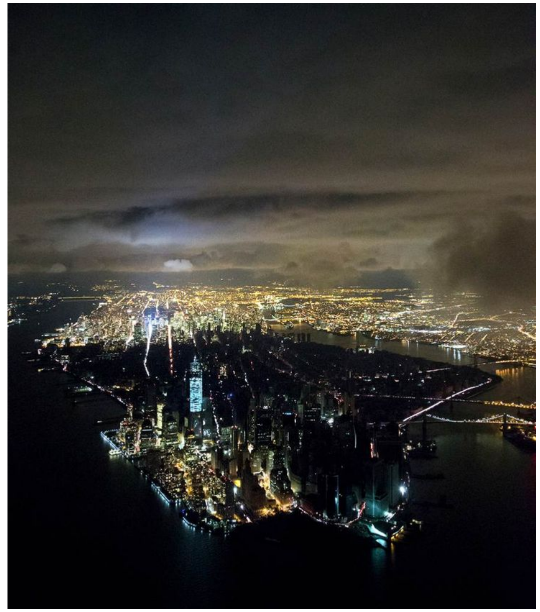
3) CAM Noise

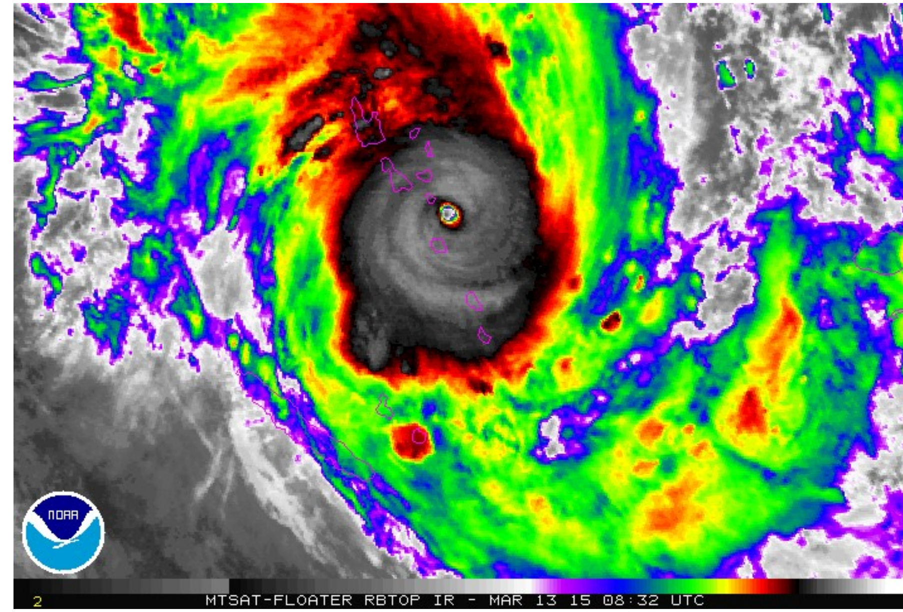
4) Are extremes better predictable the larger they are?

5) Summary

Motivation



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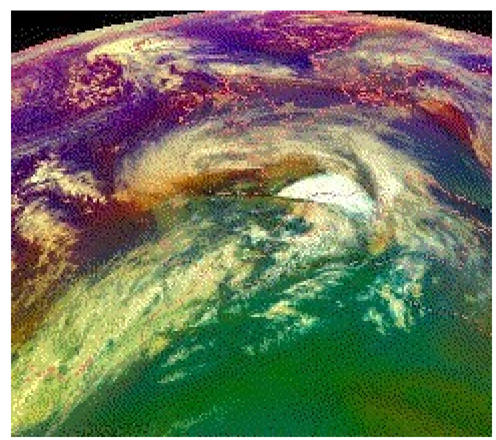
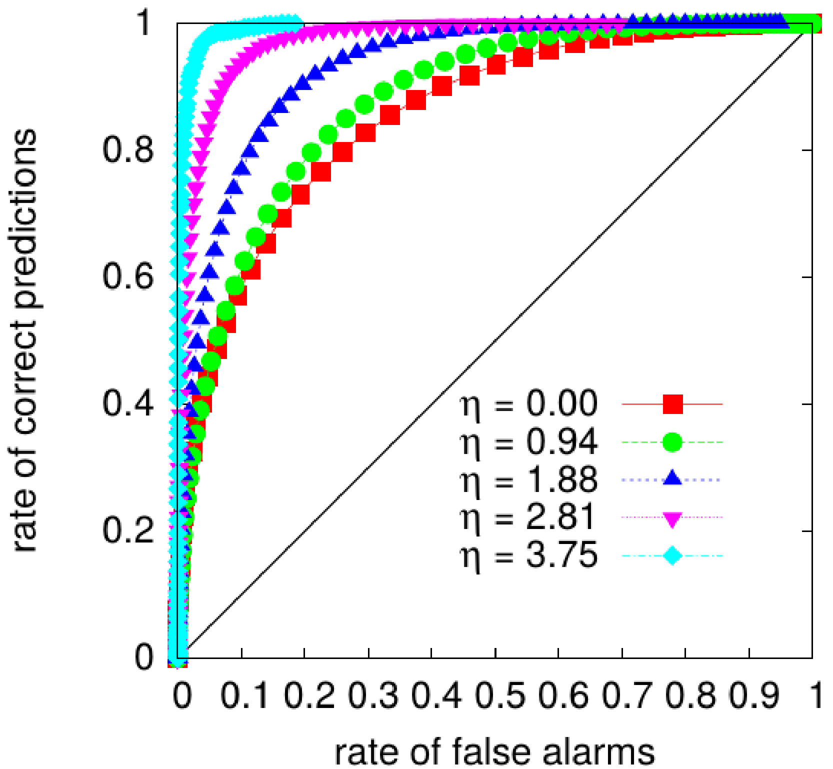
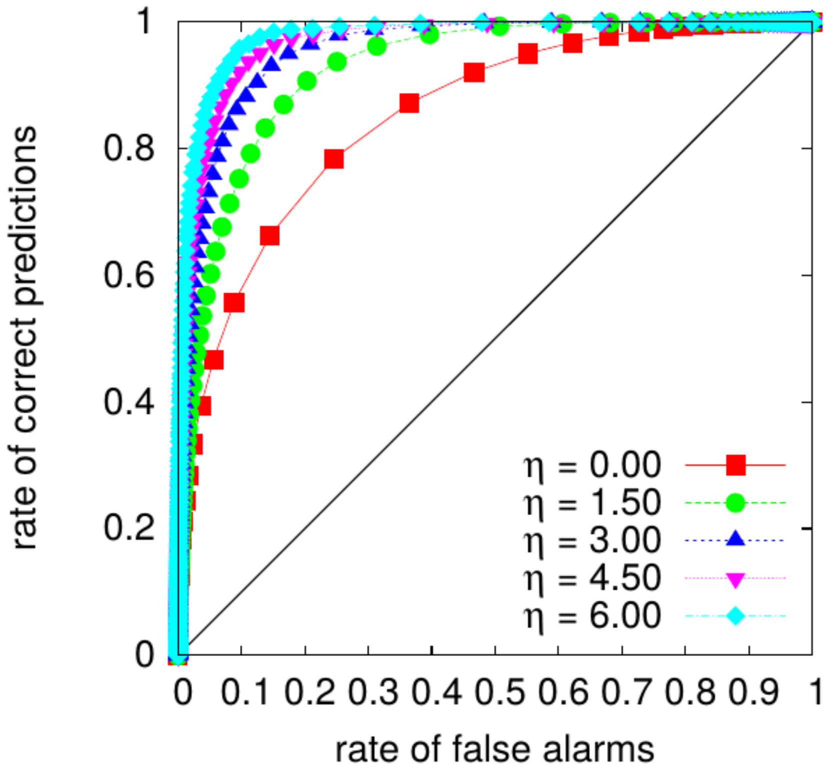
Climate-related disasters affect over
200 million people
every year



Motivation

Are extremes better predictable the larger they are?





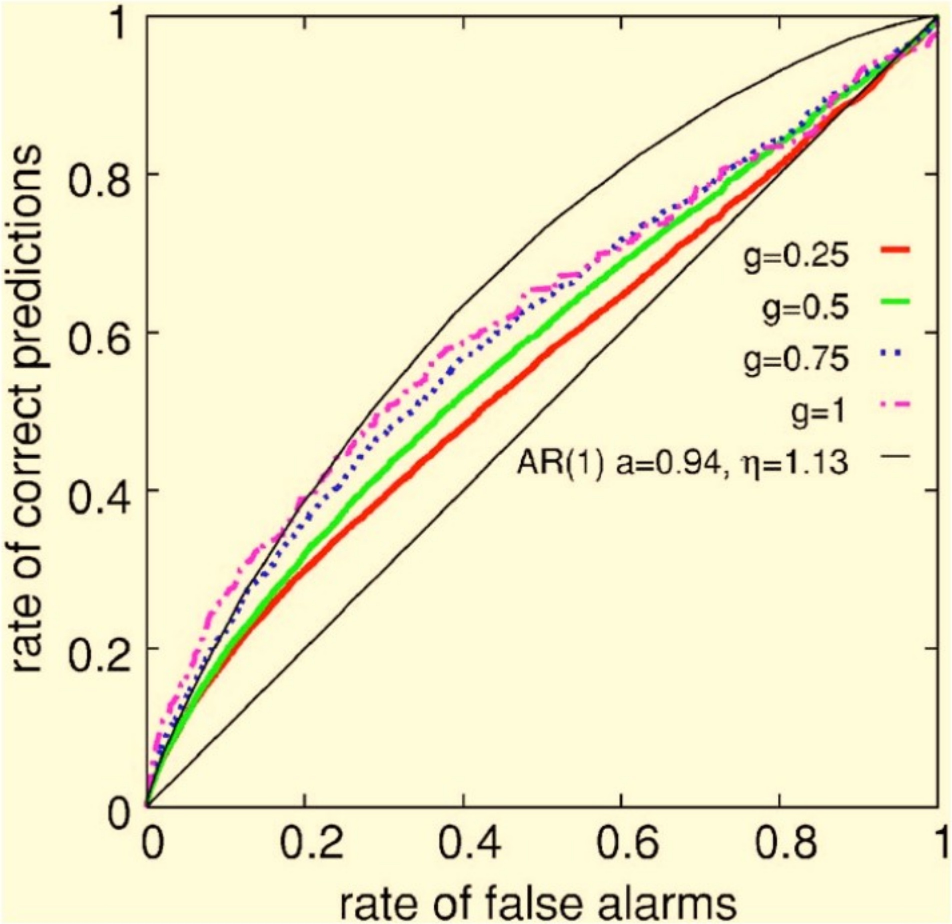
AR(1) with Gaussian and power-law distributed noise

Hallerberg and Kantz 2008

Motivation

Are extremes better predictable the larger they are?

Wind gust data



Hallerberg et al. 2007

How to model extremes?

- Classical Extreme Value Theory (Coles 2001)
- Dynamical Systems Theory of Extremes (Lucarini et al. 2016)
- Stochastic Models of Extremes

Dynamic-Stochastic Modeling

Fundamental form of weather and climate models:

$$d\mathbf{z} = \mathbf{F}dt + L\mathbf{z}dt + B(\mathbf{z},\mathbf{z})dt, \quad \mathbf{z} = \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix},$$

X: Slow mode
y: Fast mode

Dynamic-Stochastic Modeling

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Fast modes:

$$\begin{aligned} d\mathbf{y} = {} & \mathbf{F}_y dt + L_y \mathbf{y} dt + L_x \mathbf{x} dt + B_{yxx}(\mathbf{x},\mathbf{x})dt \\ & + B_{yxy}(\mathbf{x},\mathbf{y})dt + B_{yyy}(\mathbf{y},\mathbf{y})dt \end{aligned}$$

Dynamic-Stochastic Modeling

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Stochastic Modeling Assumption:

$$B_{yyy}(\mathbf{y},\mathbf{y})dt \sim -\frac{\gamma}{\varepsilon}\mathbf{y} + \frac{\sigma}{\sqrt{\varepsilon}}d\mathbf{W}.$$

Dynamic-Stochastic Modeling

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Stochastic Climate Models:

$$d\mathbf{x} = \tilde{F}dt + \tilde{L}\mathbf{x}dt + \tilde{B}(\mathbf{x},\mathbf{x})dt + \tilde{M}(\mathbf{x},\mathbf{x},\mathbf{x})dt + \tilde{\sigma}_1 dW_1 + \tilde{\sigma}_2(\mathbf{x})dW_2.$$

$$\tilde{\sigma}_2(\mathbf{x})dW_2 = (a + bx_2)dW_2,$$

Dynamic-Stochastic Modeling

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$$\tilde{\sigma}_2(\mathbf{x})dW_2 = (a + bx_2)dW_2,$$

Correlated Additive Multiplicative Noise (CAM Noise)

CAM Noise

Normal form of stochastic climate models:

$$\frac{dx}{dt} = F + ax + bx^2 - cx^3 + \sum_p \frac{\sigma_p}{\gamma_p}(L_{1p} - I_{1p}^M x)\dot{W}_p + \sigma_A \dot{W}_A$$

Corresponding Fokker-Planck Equation:

$$\frac{\partial p}{\partial t} = -\frac{\partial}{\partial x}[(F + ax + bx^2 - cx^3)p] + \frac{1}{2}\frac{\partial^2}{\partial x^2}\left[\left(\sum_p \left((L_{1p} - I_{1p}^M x)\frac{\sigma_p}{\gamma_P}\right)^2 + \sigma_A^2\right)p\right]$$

Majda et al. 2009

CAM Noise

Stationary distribution:

$$p_{\text{Stat}} = \frac{N_0}{((\tilde{A} - \tilde{B}x)^2 + g^2)^{\tilde{a}}}$$

$$\times \exp\left(\tilde{d} \arctan\left(\frac{\tilde{B}x - \tilde{A}}{g}\right)\right)$$

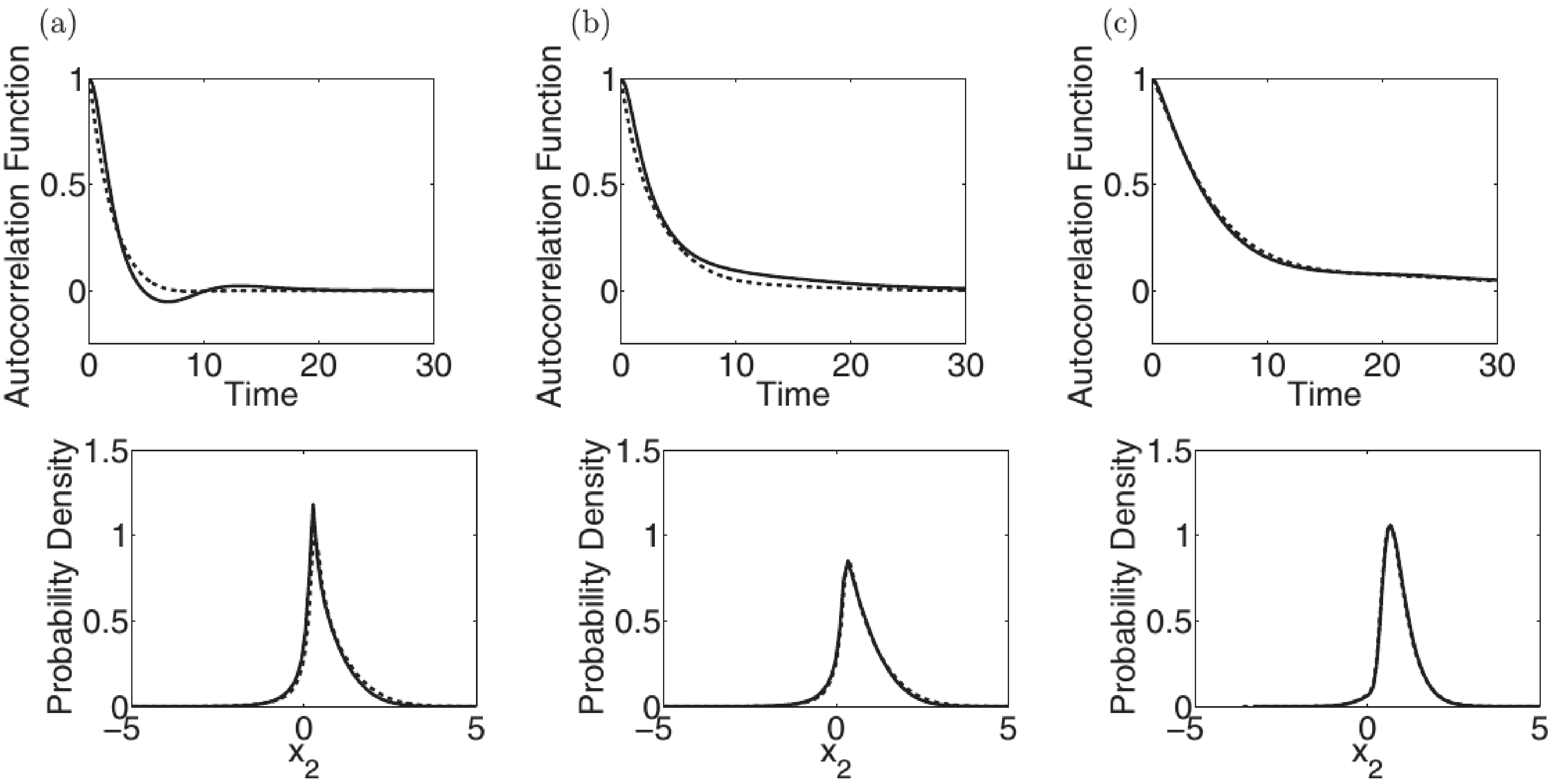
$$\times \exp\left(\frac{1}{\tilde{B}^4}\left(-\tilde{c}x^2 + \tilde{b}x\right)\right)$$

Power law component

Exponential component

Majda et al. 2009

Conceptual Climate Model



. (a) $\varepsilon = 1.0$, (b) $\varepsilon = 0.5$, and (c) $\varepsilon = 0.1$.

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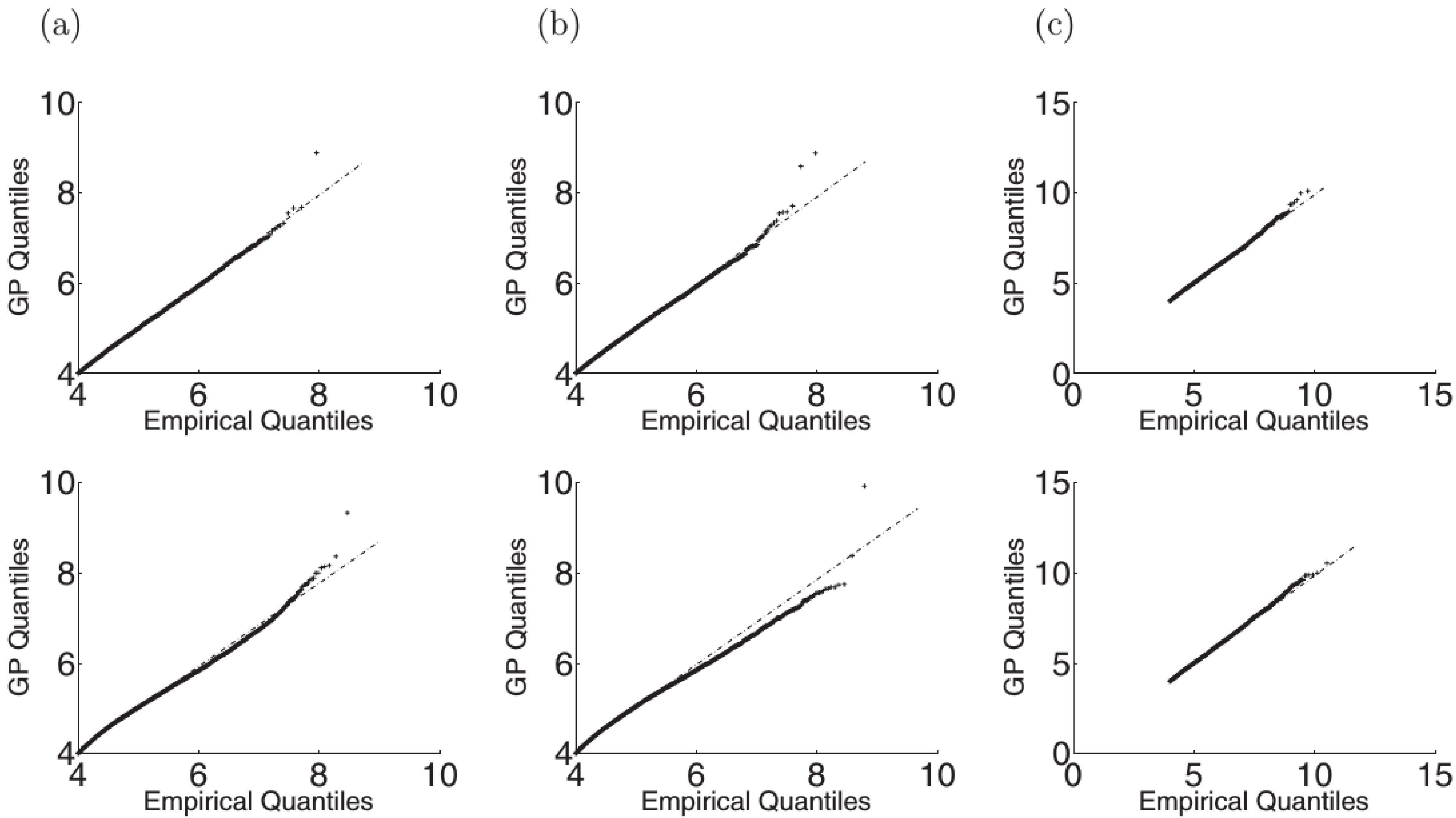
Conceptual Climate Model



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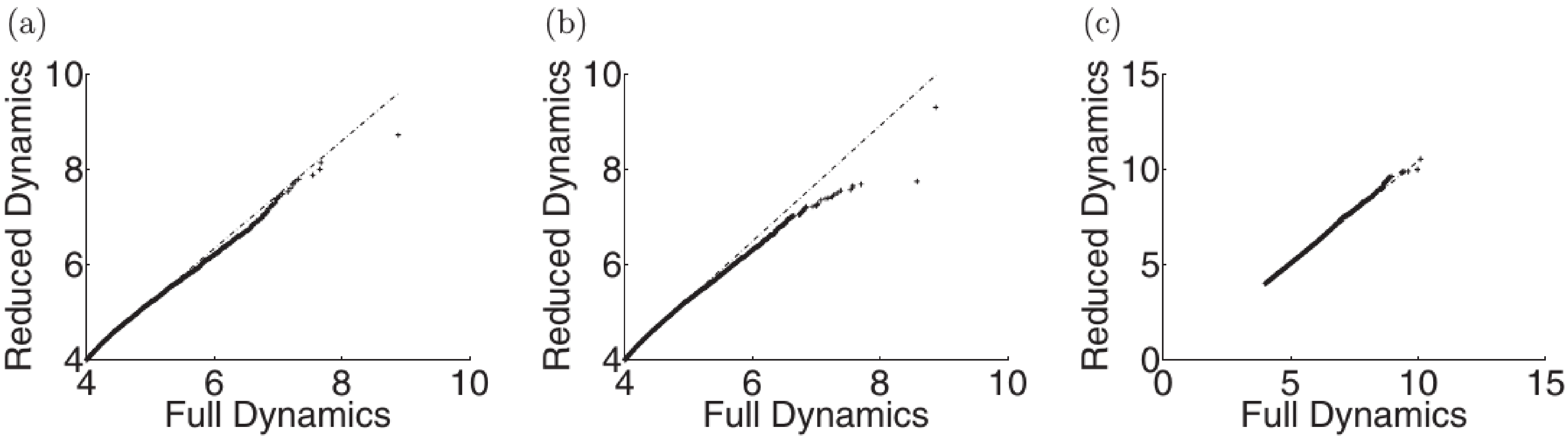

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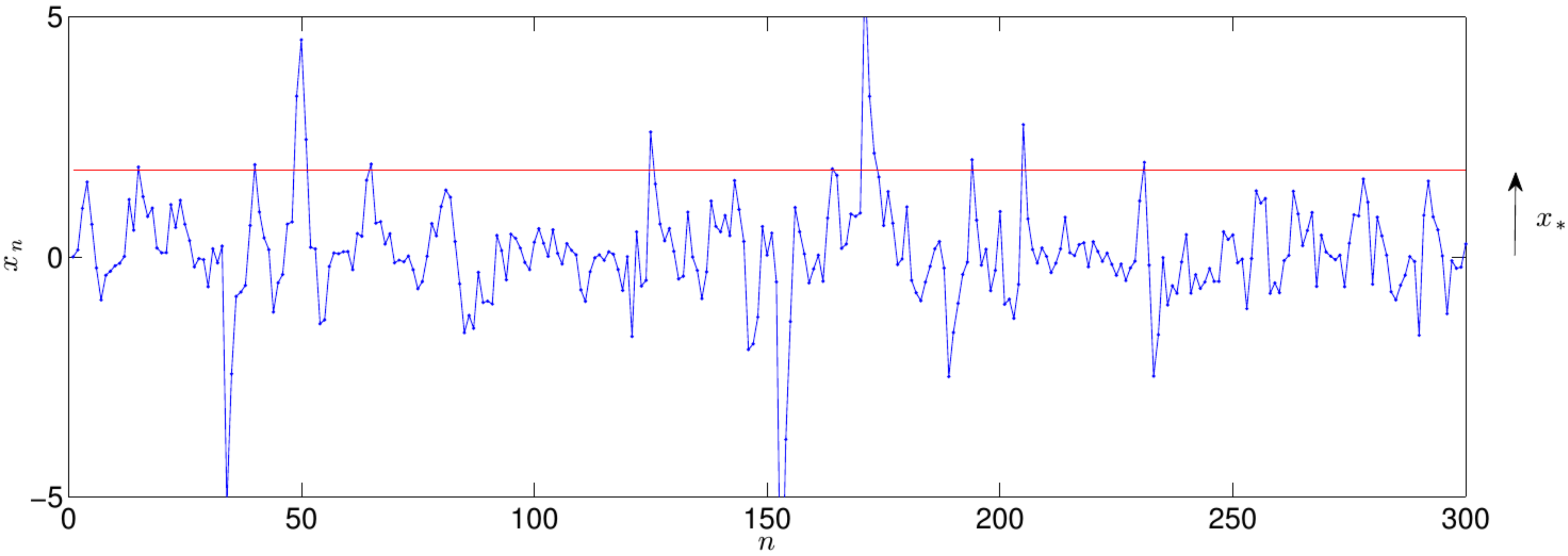
Conceptual Climate Model

Are extremes better predictable the larger they are?

(Hallerberg et al. 2007, 2008; Franzke 2012; Miotto and Altmann 2014)

Predictability



Prediction Scheme

Binary event variable:

$$\chi_n = \begin{cases} 1, & x(t_n) > x_* \\ 0, & x(t_n) < x_* \end{cases}$$

Binary prediction at t(n-d) is defined as:

$$\hat{\chi}_n = \begin{cases} 1, & \mathcal{L}(\mathbf{x}_n) > \mathcal{L}_* \\ 0, & \mathcal{L}(\mathbf{x}_n) < \mathcal{L}_* \end{cases}$$

Hallerberg et al. 2008, Bodai 2015

Prediction Scheme

Hit rate

$$H(\mathcal{L}_*) = \frac{\int_{\mathbb{R}^M} dV_{\mathbf{x}} \mathcal{P}(\mathbf{x}) \mathcal{H}(\mathcal{L}(\mathbf{x}) - \mathcal{L}_*)}{\int_{\mathbb{R}^M} dV_{\mathbf{x}} \mathcal{P}(\mathbf{x})}.$$

False alarm rate

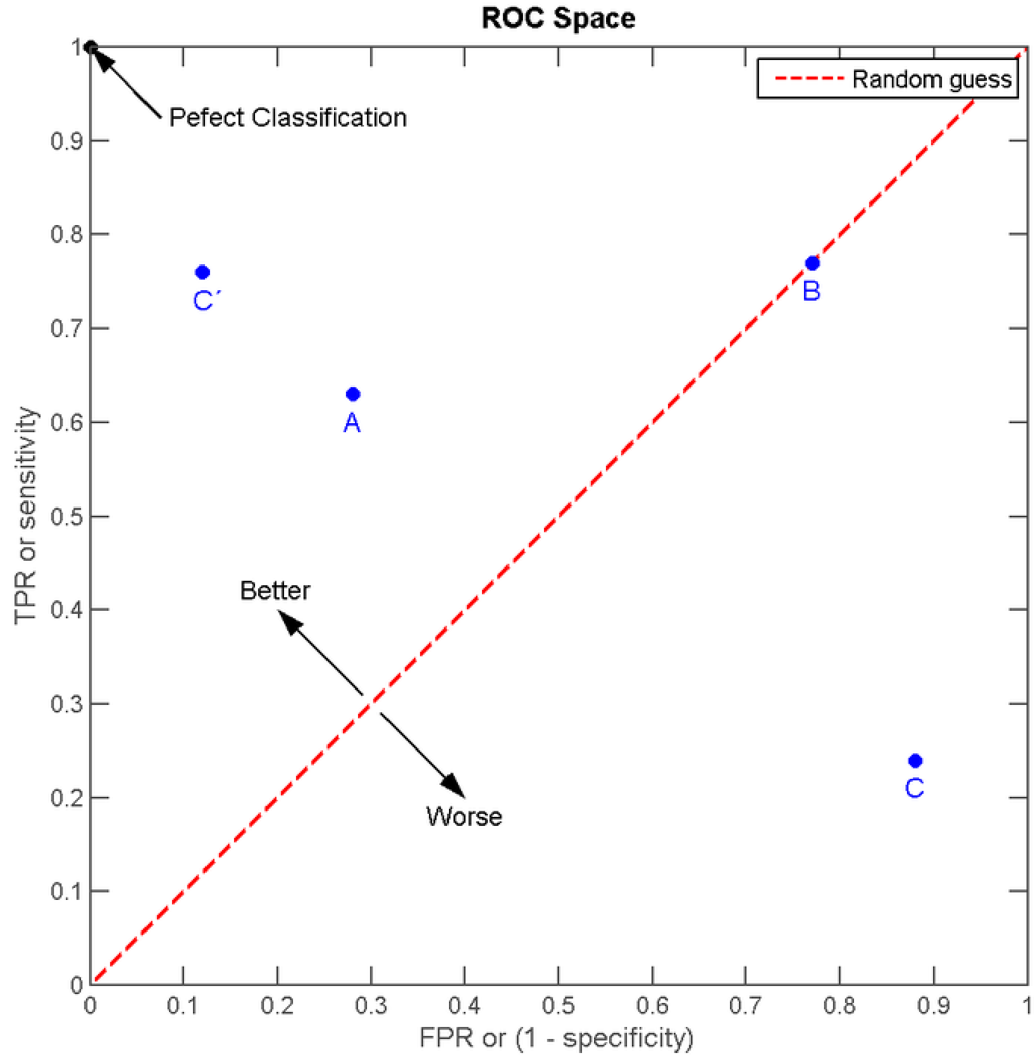
$$F(\mathcal{L}_*) = \frac{\int_{\mathbb{R}^M} dV_{\mathbf{x}} [p(\mathbf{x}) - \mathcal{P}(\mathbf{x})] \mathcal{H}(\mathcal{L}(\mathbf{x}) - \mathcal{L}_*)}{\int_{\mathbb{R}^M} dV_{\mathbf{x}} [p(\mathbf{x}) - \mathcal{P}(\mathbf{x})]}.$$

$\mathcal{H}(\cdot)$ is the Heaviside step function

Hallerberg et al. 2008, Bodai 2015

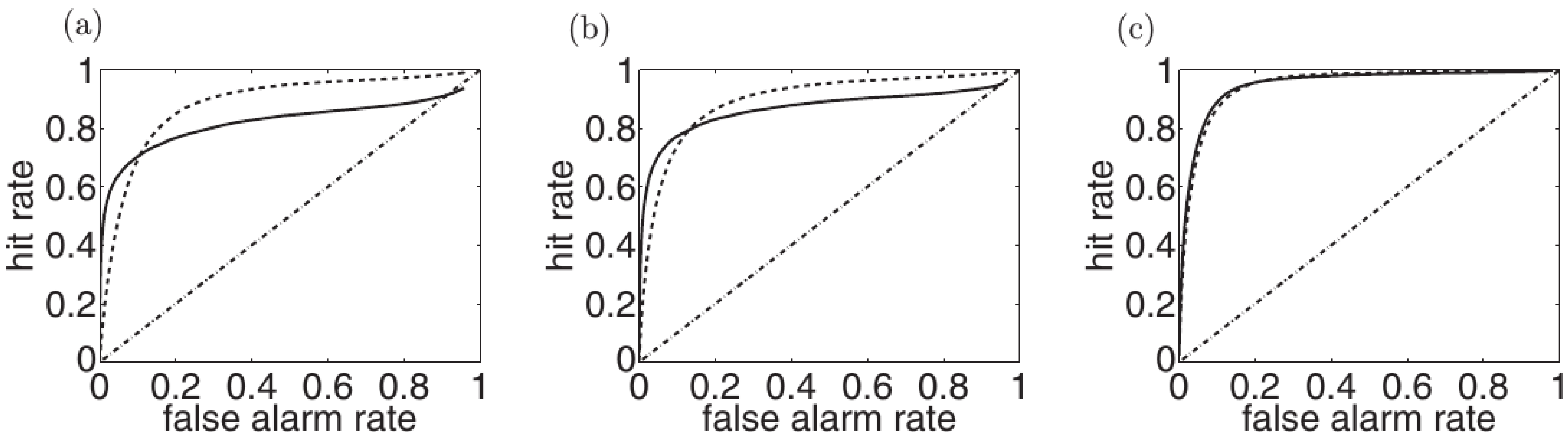
ROC Curves

Hit rate



False alarm rate

Predictability of Extremes



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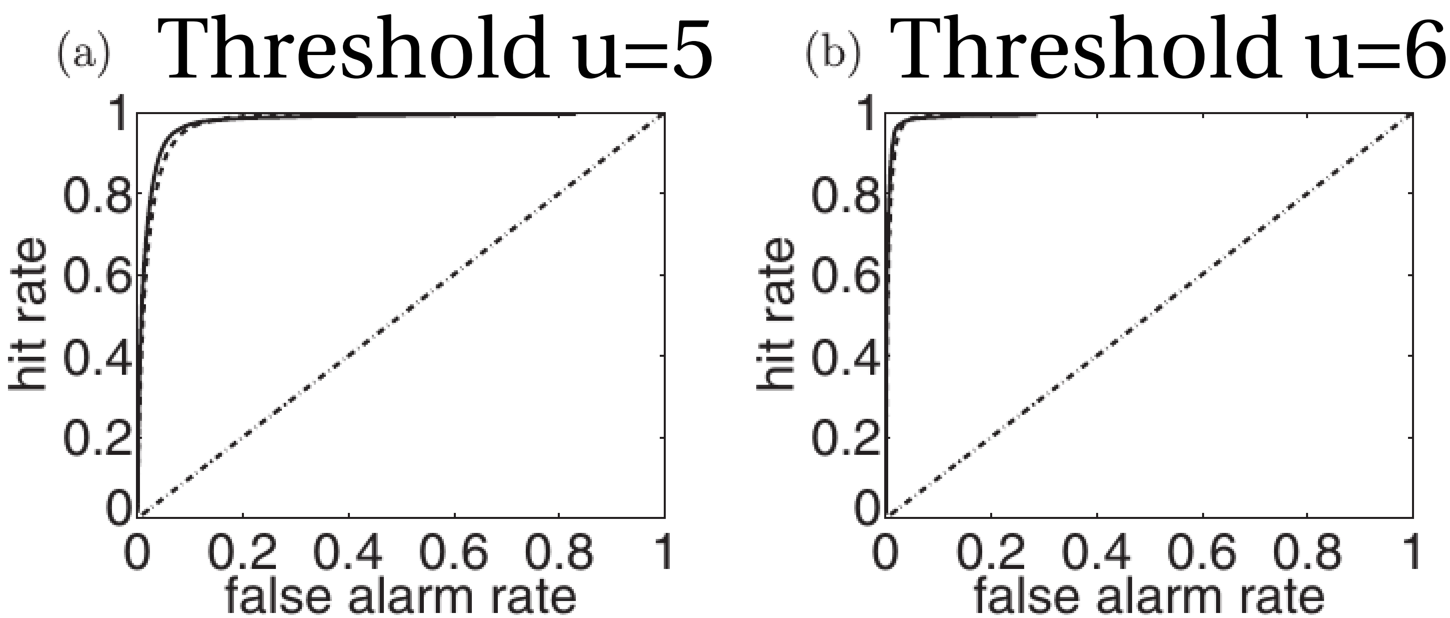
Predictability of Extremes



Predictability of Extremes

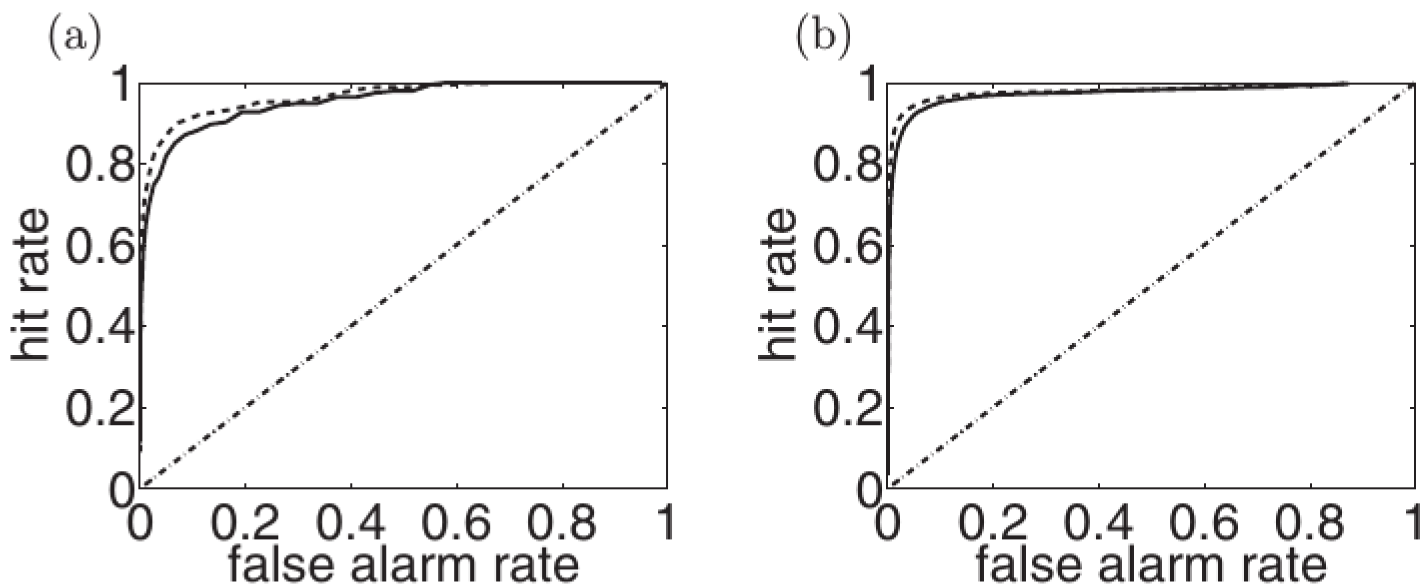


FIG. 9. Receiver-operator characteristic curves: (a) model simulation with no multiplicative triads active and (b) model simulation with no bare truncation active (case with $\varepsilon = 0.1$).

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Process Models

AR(1) model with Pareto noise

$$x_{n+1} = ax_n + \xi_n,$$

$$p_\xi(\xi) = p_{sP}(\xi; \alpha, \xi_m) = \alpha\xi_m^\alpha / |\xi + \xi_m|^{\alpha+1}$$

$$\alpha \geq 2 \longrightarrow p(x) \approx N(0, \sigma) = \frac{1}{2\pi\sigma} e^{-\frac{x^2}{2\sigma^2}}, \sigma = \sigma_\xi / \sqrt{1 - a^2}.$$

$\alpha < 2 \longrightarrow \sigma_\xi$ does not exist,

Process Models

AR(1) model with Pareto noise

$$x_{n+1} = a x_n + \xi_n,$$

$$p_\xi(\xi) = p_{sP}(\xi; \alpha, \xi_m) = \alpha \xi_m^\alpha / |\xi + \xi_m|^{\alpha+1}$$

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Process Models

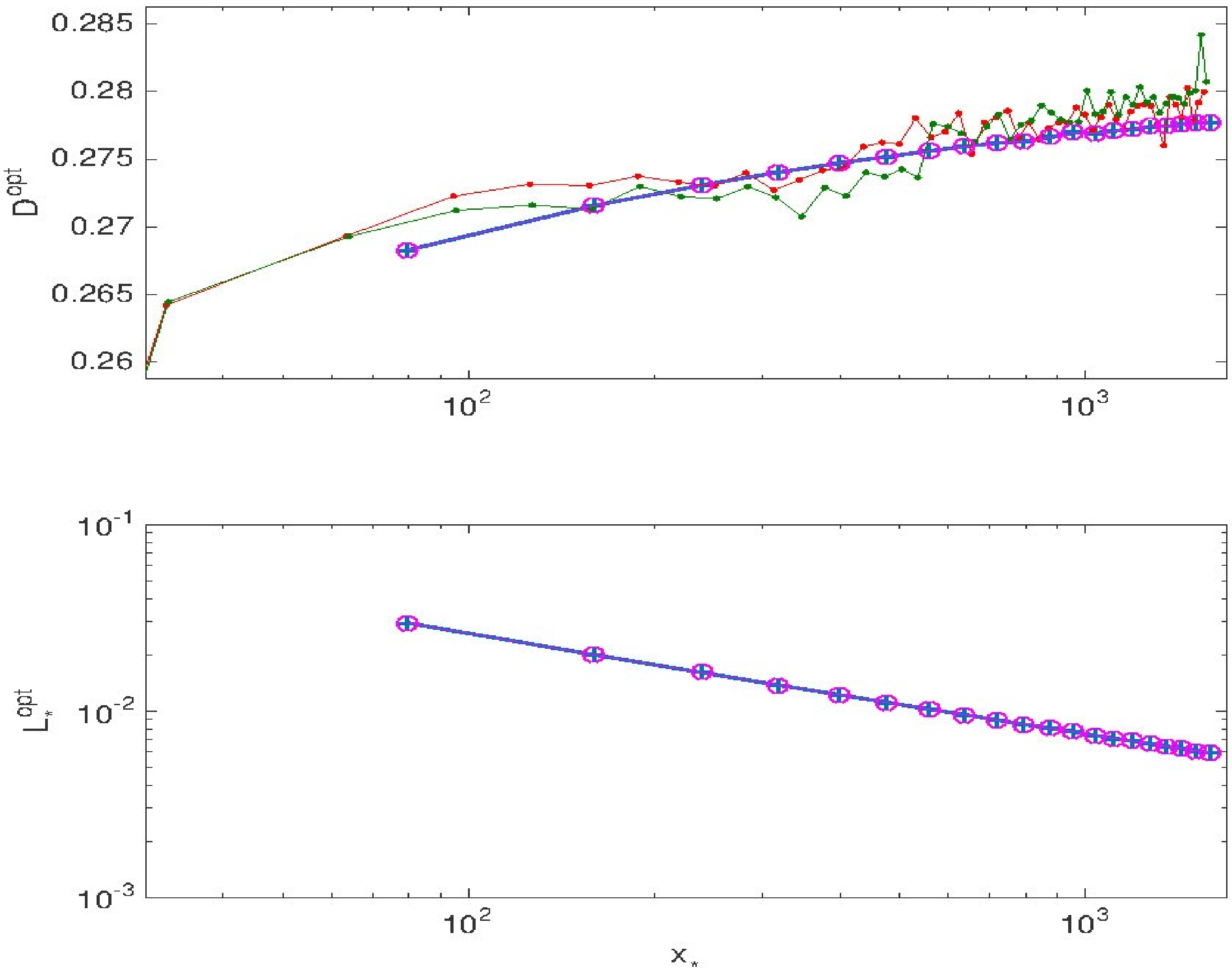
CAM model with α-stable noise

$$dx = (b + ax)dt + (d + cx)dW$$

Solution of the stationary Fokker-Planck equation:

$$p(x) = N_0 \frac{2e^{2\frac{ad-bc}{c^2(d+cx)}}}{(d+cx)^{2(1-a/c^2)}}$$

Process Models



AR(1) model
with
Pareto noise

Distance (for categorical prediction):
$D = \min_{\mathcal{L}*}(\sqrt{F^2 + (H-1)^2})$

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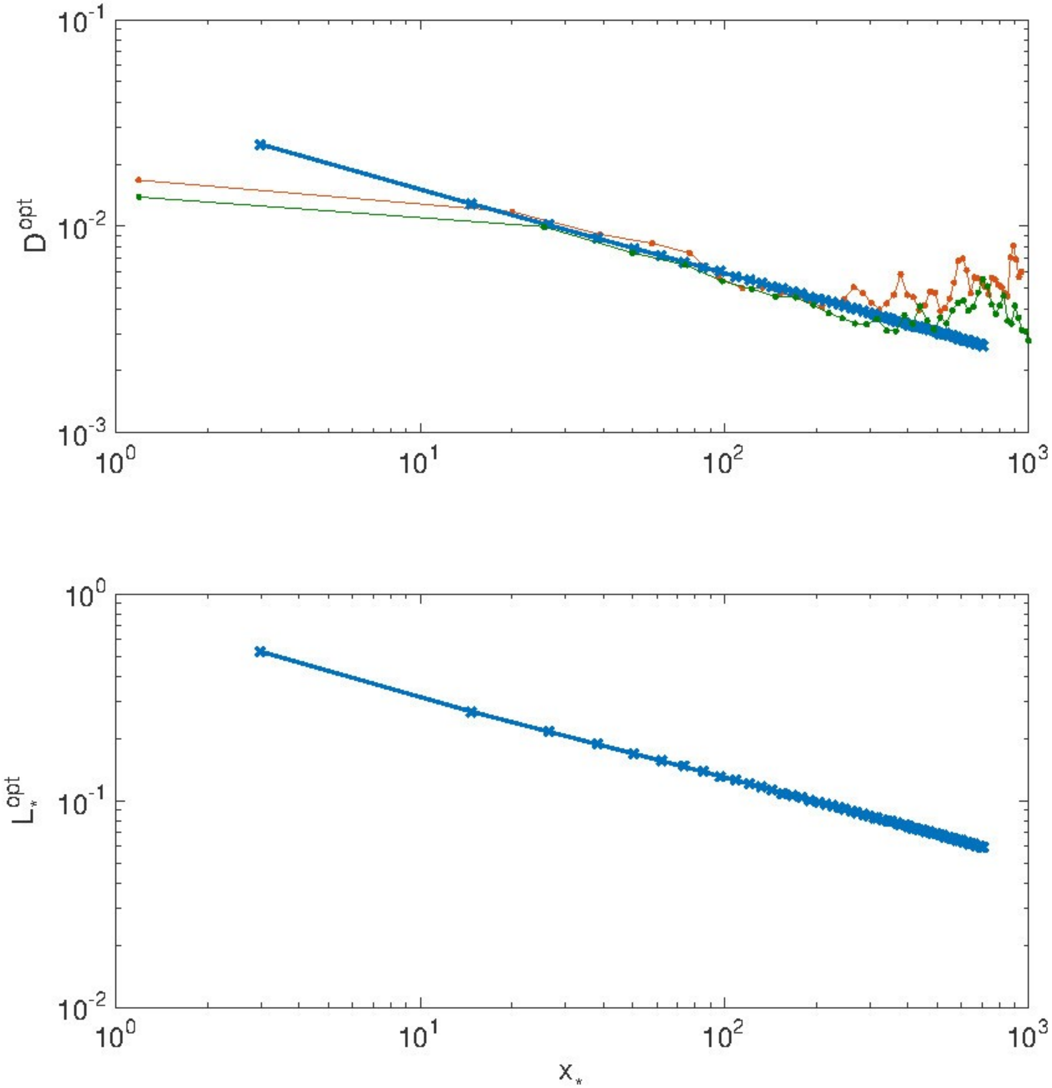
Process Models



CAM model
with α-stable noise

Summary

- Dynamic-Stochastic models have predictive skill when forecasting extreme events
- Extreme events are not necessarily better predictable the more extreme they are
- Predictability seems to be determined by the process

References:

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- Franzke, C., 2012: Predictability of Extreme Events in a Nonlinear Stochastic-Dynamical Model. Phys. Rev. E, 85, DOI: 10.1103/PhysRevE.85.031134.
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- Bodai, T. and C. Franzke, 2016: Predictabiliy of extremes, in preparation.