CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

CSP 1

lunedì 18 maggio 2015
18:13

ON THE CONTROL OF MOREAU'S SWEEPING PROCESS

(joint work with R. Henrion, B. Mordukhovich, Nguyen D. Hoang)

Plan of the talk:

- The dynamics of the sweeping process
- Some control problems for the sweeping process

MOREAU'S SWEEPING PROCESS

(joint work with R. Henrion, B. Mordukhovich, Nguyen D. Hoang)

Plan of the talk :

- The dynamics of the sweeping process

- Some control problems for the sweeping process

- A shape optimization problem involving the sweeping process

MOREAU'S SWEEPING PROCESS

(joint work with R. Henrion, B. Mordukhovich, Nguyen D. Hoang)

Plan of the talk:

- The dynamics of the sweeping process

- Some control problems for the sweeping process

- A shape optimization problem involving the sweeping process

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

CSP 1

lunedì 18 maggio 2015
18:13

ON THE CONTROL OF MOREAU'S SWEEPING PROCESS

(joint work with R. Henrion, B. Mordukhovich, Nguyen D. Hoang)

Plan of the talk:

- The dynamics of the sweeping process

- Some control problems for the sweeping process

Microsoft OneNote
File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

CSP 1

lunedì 18 maggio 2015
18:13

ON THE CONTROL OF MOREAU'S SWEEPING PROCESS

(joint work with R. Henrion, B. Mordukhovich, Nguyen D. Hoang)

Plan of the talk:

- The dynamics of the sweeping process

- Some control problems for the sweeping process

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

- Some control problems for the sweeping process

- A shape optimization problem involving the sweeping process

$$(SP)\quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) & x \in H \text{, Hilbert space} \\ x(0) = x_0 \in C(0) \end{cases}$$

$C(t)$ is closed and convex (mildly non convex)

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

- Some control problems for the sweeping process

- A shape optimization problem involving the sweeping process

$$(SP)\quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) & x \in H \text{, Hilbert space} \\ x(0) = x_0 \in C(0) \end{cases}$$

$C(t)$ is closed and convex (mildly non convex)

Lipschitz with respect to t

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

- A shape optimization problem involving the sweeping process

$$(SP) \quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) & x \in H \text{, Hilbert space} \\ x(0) = x_0 \in C(0) \end{cases}$$

$C(t)$ is closed and convex (mildly non convex)

Lipschitz with respect to t

$$(PSP) \quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t)) \quad [+ F(x(t))] \end{cases}$$

IT 17: 20/05/

CSP 2 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Nuova scheda Nuova scheda

Generale seminari

Cerca in tutti i blocchi appunti (CTR

Personale Analisi_1

CSP 2

lunedì 18 maggio 2015

18:21

On the control of Moreau's sweeping process

Dynamics of the sweeping process

$$\begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) \\ x(0) = x_0 \in C(0) \end{cases}$$

Highly discontinuous w.r.t. the state

Nuova pagina
OneNote conse
Informazioni di
Utilizzi principal
Novità
Lezione 1 ottob
Colombe 6 mag
Colombo 8 mag
CSP 1
CSP 2

IT 17: 20/05/

- A shape optimization problem involving the sweeping

$$(SP)\quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) & x \in H, \text{ Hil} \\ x(0) = x_0 \in C(0) \end{cases}$$

$C(t)$ is closed and convex (mildly non convex)

Lipschitz with respect to t

$$\left\{ \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t)) \right. \qquad \int + F$$

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

- A shape optimization problem involving the sweeping process

$$(SP)\quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) \\ x(0) = x_0 \in C(0) \end{cases}$$

$x \in H$, Hilbert space

$C(t)$ is closed and convex (mildly non convex)

Lipschitz with respect to t

$$(PSP)\quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t)) \quad [+ F(x(t))] \end{cases}$$

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

$$(SP) \quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) \\ x(0) = x_0 \in C(0) \end{cases}$$

$x \in H$, Hilbert space

$C(t)$ is closed and convex (mildly non convex)

Lipschitz with respect to t

$$(PSP) \quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t)) \\ x(0) = x_0 \in C(0) \end{cases}$$

$[+ F(x(t))]$

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

$C(t)$

(SP) $\quad x(0) = x_0 \in C(0)$

$C(t)$ is closed and convex (mildly non convex)

Lipschitz with respect to t

$$(PSP) \quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t)) & [+F(x(t))] \\ x(0) = x_0 \in C(0) \end{cases}$$

$\dot{x}(t) \in -N \ldots (x(t)) + f(x(t), u(t))$

IT

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

$$\left\{ x(0) = x_0 \in C(0) \right.$$

$C(t)$ is closed and convex (mildly non convex)

Lipschitz with respect to t

$$(PSP) \quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t)) & [+ F(x(t))] \\ x(0) = x_0 \in C(0) \end{cases}$$

$$\left\{ \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t), u(t)) \right.$$

(CSP)

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

$x(0) = x_0 \in C(0)$

$C(t)$ is closed and convex (mildly non convex)

Lipschitz with respect to t

$$(PSP) \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t)) & [+F(x(t))] \\ x(0) = x_0 \in C(0) \end{cases}$$

the state constraint $x(t) \in C(t)$ is built in the dynamics

IT 17: 20/05

Lipschitz with respect to t

$$(PSP)\quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t)) \quad \left[+ F(x(t))\right] \\ x(0) = x_0 \in C(0) \end{cases}$$

the state constraint $x(t) \in C(t)$ is built in the dynamics

$$(CSP)\quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t), u(t)) \end{cases}$$

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

Lipschitz with respect to t

$$(PSP)\quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t)) & [+F(x(t))] \\ x(0) = x_0 \in C(0) \end{cases}$$

the state constraint $x(t) \in C(t)$ is built in the dynamics

$$(CSP)\quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t), u(t)) \\ x(0) = x_0 \in C(0) \end{cases}$$

$(\cdot)$ u are the controls

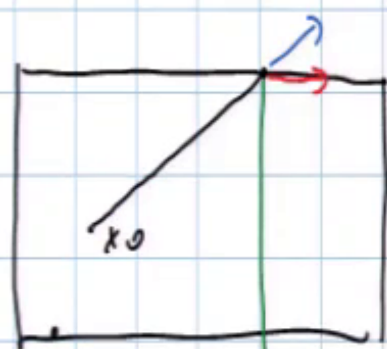
IT 17: 20/05/

Lipschitz with respect to t

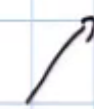
$$(PSP)\quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t)) \quad [+F(x(t))] \\ x(0) = x_0 \in C(0) \end{cases}$$

the state constraint $x(t) \in C(t)$ is built in the dynamics

$C(t) \equiv$



$f(x) \equiv (1,1)$



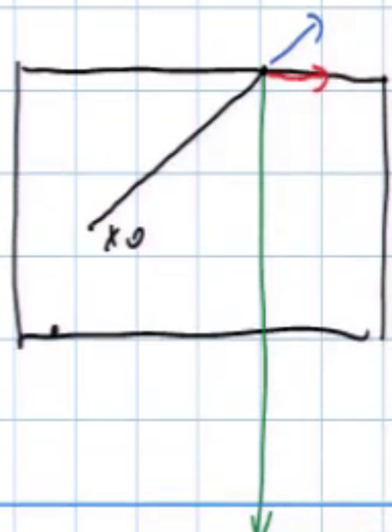
$(\dot{x}(t) \in -N \; x(t) + f(x(t), u(t))$

Lipschitz with respect to t

$$(PSP) \quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t)) \quad [+F(x(t))] \\ x(0) = x_0 \in C(0) \end{cases}$$

the state constraint $x(t) \in C(t)$ is built in the dynamics

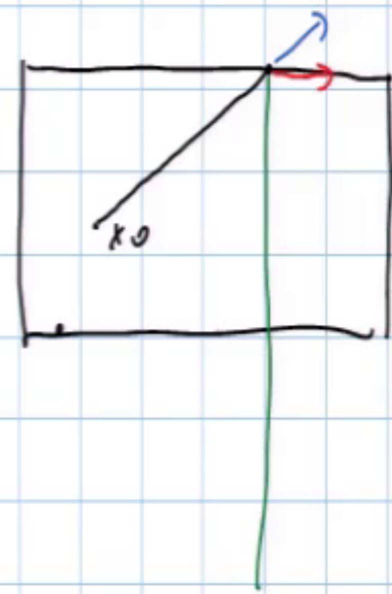
$C(t) \equiv$



$f(x) \equiv (1,1)$ ↗

$$\dot{x}(t) \in -N_{\ldots}(x(t)) + f_0(x(t), u(t))$$

the state constraint $x(t) \in C(t)$ is built in the dynamics

$C(t) \equiv$ 

$f(x) \equiv (1,1)$

Let $C(t) \equiv C$ mildly nonconvex

The solution set of

$$\begin{cases} \dot{x} \in -N_C(x) + f(x) \\ x(0) = x_0 \in C \end{cases}$$

and

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

$$(CSP)\quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t), u(t)) \\ x(0) = x_0 \in C(0) \end{cases}$$

C, u are the controls

(joint work with Henrion, Mordukhovich, Nguyen Hoang)

$$C(t) = \{ x \in \mathbb{R}^n : \langle x(t), u_i(t) \rangle \leq b_i(t),\ t \in [0,T],\ i = 1, \dots, m \}$$

$\|u_i\| = 1$

(a moving polyhedron to be designed)

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

The solution set of

$$\begin{cases} \dot{x} \in -N_C(x) + f(x) \\ x(0) = x_0 \in C \end{cases}$$

and

(CSP)
$$\begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t), u(t)) \\ x(0) = x_0 \in C(0) \end{cases}$$

C, u are the controls

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

The solution set of

$$\begin{cases} \dot{x} \in -N_C(x) + f(x) \\ x(0) = x_0 \in C \end{cases}$$

and

(CSP)
$$\begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t), u(t)) \\ x(0) = x_0 \in C(0) \end{cases}$$

C, u are the controls

IT

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

The solution set of

$$\begin{cases} \dot{x} \in -N_C(x) + f(x) \\ x(0) = x_0 \in C \end{cases}$$

and

$$\begin{cases} \dot{x} \in \pi_{T_C(x)}(f(x)) \\ x(0) = x_0 \end{cases}$$

coincide

(CSP)

$$\begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t), u(t)) \\ x(0) = x_0 \in C(0) \end{cases}$$

C, u are the controls

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

$$\begin{cases} \\ x(0) = x_0 \in C \end{cases}$$

$$\begin{cases} \dot{x} \in \pi_{T_C(x)}(f(x)) \\ x(0) = x_0 \end{cases}$$

coincide

$$(CSP) \quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t), u(t)) \\ x(0) = x_0 \in C(0) \end{cases}$$

C, u are the controls

— — — — — — — — — — — — —

The result

$$\begin{cases} \dots \\ x(0) = x_0 \in C \end{cases}$$

$$\begin{cases} \dot{x} \in \pi_{T_C(x)}(f(t)) \\ x(0) = x_0 \end{cases}$$

Coincide

$$(CSP) \quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t), u(t)) \\ x(0) = x_0 \in C(0) \end{cases}$$

C, u are the <u>controls</u>

The result

$$\begin{cases} \dot{x} \in \pi_{T_C(x)}(f(x)) \\ x(0) = x_0 \end{cases}$$

coincide

$$(CSP) \quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t), u(t)) \\ x(0) = x_0 \in C(0) \end{cases}$$

C, u are the controls

The result

(joint work with Henrion, Mordukhovich, Nguyen Hoang)

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

$$\begin{cases} \dot{x} \in \pi_{T_C(x)}(f(x)) \\ x(0) = x_0 \end{cases}$$

coincide

$$(CSP) \quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t), u(t)) \\ x(0) = x_0 \in C(0) \end{cases}$$

C, u are the controls

The result

(joint work with Henrion, Mordukhovich, Nguyen Hoang)

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

$$x(0) = x_0$$

$$(CSP)\quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t), u(t)) \\ x(0) = x_0 \in C(0) \end{cases}$$

C, u are the controls

The result

(joint work with Henrion, Mordukhovich, Nguyen Hoang)

$$C(t) = \{ x \in \mathbb{R}^n : \langle x(t), u_i(t) \rangle \le b_i(t),\ t \in [0,T],\ i = 1, \dots, m \}$$

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

$$(CSP)\quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t), u(t)) \\ x(0) = x_0 \in C(0) \end{cases}$$

C, u are the controls

- - - - - - - - - - - - - -

The result

(joint work with Henrion, Mordukhovich, Nguyen Hoang)

$$C(t) = \{ x \in \mathbb{R}^n : \langle x(t), u_i(t) \rangle \leq b_i(t), \ t \in [0,T], i = 1, \dots, m \}$$

(a moving polyhedron to be designed)

$\| u_i \| = 1$

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

$$(CSP)\quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t), u(t)) \\ x(0) = x_0 \in C(0) \end{cases}$$

C, u are the controls

The result

(joint work with Henrion, Mordukhovich, Nguyen Hoang)

$$C(t) = \{ x \in \mathbb{R}^n : \langle x(t), u_i(t) \rangle \le b_i(t),\ t \in [0,T],\ i = 1, \dots, m \}$$

$$\|u_i\| \equiv 1$$

(a moving polyhedron to be designed)

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Nuova scheda Nuova scheda

Generale seminari

$$(CSP)\quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t), u(t)) \\ x(0) = x_0 \in C(0) \end{cases}$$

C, u are the con

The result

(joint work with Henrion, Mordukhovich, Nguyen Hoa

$C(t) = \{ x \in \mathbb{R}^n : \langle x(t), u_i(t) \rangle \le b_i(t),\ t \in [0,T],\ i =$

$\| u_i \| = 1$ (a moving polyhedron to be de

Nuova pagina
OneNote conse
Informazioni di
Utilizzi principal
Novità
Lezione 1 ottob
Colombe 6 mag
Colombo 8 mag
CSP 1
CSP 2

On the control of Moreau's sweeping process

lunedì 18 maggio 2015
18:21

Dynamics of the sweeping process

$$\begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) \\ x(0) = x_0 \in C(0) \end{cases}$$

Highly discontinuous w.r.t. the state

CSP 2 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

Some control problems

Problems	Results
Controllability / stabilization	ε
Existence / relaxation optimal control problems	something
Necessary conditions	today's talk
Hamilton - Jacobi theory	ε^2 ↗

Necessary conditions

IT 17: 20/05/

CSP 2 - Microsoft OneNote

File | Home | Inserisci | Condividi | Disegno | Revisione | Visualizza | Pagine | Nuova scheda | Nuova scheda

Problems

Controllability / stabilization

Existence / relaxation optimal control problems

Necessary conditions

Hamilton - Jacobi theory

Results

ε

something

today's talk

ε^2 ↗

Necessary conditions

IT

CSP 2 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

Controllability / stabilization

Existence / relaxation optimal control problems

Necessary conditions

Hamilton - Jacobi theory

ε something

today's talk

$\varepsilon^2 \nearrow$

Necessary conditions

CSP 2 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Nuova scheda Nuova scheda

Generale seminari

Cerca in tutti i blocchi appunti (CTR

Controllability / stabilization

Existence / relaxation optimal control problems

Necessary conditions

Hamilton - Jacobi theory

ε something today's $\varepsilon^2 \nearrow$

Necessary conditions

Nuova pagina
OneNote conse
Informazioni di
Utilizzi principal
Novità
Lezione 1 ottob
Colombe 6 mag
Colombo 8 mag
CSP 1
CSP 2

Personale
Analisi_1

IT 17: 20/05/

CSP 1 - Microsoft OneNote

$$(CSP)\quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t), u(t)) \\ x(0) = x_0 \in C(0) \end{cases}$$

C, u are the con

- - - - - - - - - - - - -

The result

(joint work with Henrion, Mordukhovich, Nguyen Hoa

$$C(t) = \{ x \in \mathbb{R}^n : \langle x(t), u_i(t) \rangle \le b_i(t),\ t \in [0,T],\ i =$$

$\| u_i \| = 1$ (a moving polyhedron to be de

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

$$(CSP)\quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t), u(t)) \\ x(0) = x_0 \in C(0) \end{cases}$$

C, u are the controls

The result

(joint work with Henrion, Mordukhovich, Nguyen Hoang)

$$C(t) = \{ x \in \mathbb{R}^n : \langle x(t), u_i(t) \rangle \le b_i(t),\ t \in [0,T],\ i = 1, \dots, m \}$$

$$\|u_i\| \equiv 1$$

(a moving polyhedron to be designed)

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

(CSP) $$\begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t), u(t)) \\ x(0) = x_0 \in C(0) \end{cases}$$

C, u are the controls

The result

(joint work with Henrion, Mordukhovich, Nguyen Hoang)

$$C(t) = \{ x \in \mathbb{R}^n : \langle x(t), u_i(t) \rangle \leq b_i(t), \; t \in [0,T], i = 1, \dots, m \}$$

$\| u_i \| \equiv 1$

(a moving polyhedron to be designed)

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

$x(0) = x_0 \in C(0)$ C, u are the controls

- - - - - - - - - - - - - -

The result

(joint work with Henrion, Mordukhovich, Nguyen Hoang)

$$C(t) = \{ x \in \mathbb{R}^n : \langle x(t), u_i(t) \rangle \le b_i(t),\ t \in [0,T],\ i = 1, \dots, m \}$$

$\|u_i\| \equiv 1$ (a moving polyhedron to be designed)

Set $z = (x, u, b)$

Minimize $J(z) = \varphi(x(T)) + \int^T \ell(z(t), \dot z(t))\, dt$

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

$x(0) = x_0 \in C(0)$ C, u are the controls

The result

(joint work with Henrion, Mordukhovich, Nguyen Hoang)

$$C(t) = \{ x \in \mathbb{R}^n : \langle x(t), u_i(t) \rangle \le b_i(t),\ t \in [0,T],\ i = 1, \dots, m \}$$

$\| u_i \| \equiv 1$ (a moving polyhedron to be designed)

Set $z = (x, u, b)$

Minimize $$J(z) = \varphi(x(T)) + \int^T \ell(z(t), \dot z(t))\, dt$$

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

The result

(joint work with Henrion, Mordukhovich, Nguyen Hoang)

$$C(t) = \{ x \in \mathbb{R}^n : \langle x(t), u_i(t) \rangle \le b_i(t),\ t \in [0,T],\ i = 1, \dots, m \}$$

$\| u_i \| \equiv 1$ (a moving polyhedron to be designed)

Set $z = (x, u, b)$

Minimize $J(z) = \varphi(x(T)) + \int_0^T \ell(z(t), \dot{z}(t))\, dt$

φ smooth, ℓ ... well.

IT 17: 20/05/

The result

(joint work with Henrion, Mordukhovich, Nguyen Hoang)

$$C(t) = \{ x \in \mathbb{R}^n : \langle x(t), u_i(t) \rangle \le b_i(t), \ t \in [0,T], i = 1, \dots, m \}$$

$\|u_i\| \equiv 1$ (a moving polyhedron to be designed)

Set $z = (x, u, b)$

Minimize $J(z) = \varphi(x(T)) + \int_0^T \ell(z(t), \dot{z}(t))\, dt$

φ smooth, ℓ reasonable

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

The result

(joint work with Henrion, Mordukhovich, Nguyen Hoang)

$$C(t) = \{ x \in \mathbb{R}^n : \langle x(t), u_i(t) \rangle \le b_i(t), \; t \in [0,T], i = 1, \dots, m \}$$

$\| u_i \| \equiv 1$

(a moving polyhedron to be designed)

Set $z = (x, u, b)$

Minimize $$J(z) = \varphi(x(T)) + \int_0^T \ell(z(t), \dot z(t))\, dt$$

φ smooth, ℓ reasonable

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

The result

(joint work with Henrion, Mordukhovich, Nguyen Hoang)

$$C(t) = \{ x \in \mathbb{R}^n : \langle x(t), u_i(t) \rangle \le b_i(t),\ t \in [0,T],\ i = 1,\dots,m \}$$

$\|u_i\| \equiv 1$ (a moving polyhedron to be designed)

Set $z = (x, u, b)$

Minimize $$J(z) = \varphi(x(T)) + \int_0^T \ell(z(t), \dot z(t))\, dt$$

φ smooth, ℓ reasonable

subject to $\quad (\dot x(t) \in -N \quad (x(t)) \quad t \in [0,T]$

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

The result

(joint work with Henrion, Mordukhovich, Nguyen Hoang)

$$C(t) = \{ x \in \mathbb{R}^n : \langle x(t), u_i(t) \rangle \le b_i(t),\ t \in [0,T],\ i = 1, \dots, m \}$$

$\| u_i \| \equiv 1$ (a moving polyhedron to be designed)

Set $z = (x, u, b)$

Minimize $$J(z) = \varphi(x(T)) + \int_0^T \ell(z(t), \dot z(t))\, dt$$

φ smooth, ℓ reasonable

subject to $\quad (\ \dot x(t) \in - N \quad (x(t)) \qquad t \in [0,T]$

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

$$\begin{cases} \dot{x} \in \pi_{T_C(x)}(f(x)) \\ x(0) = x_0 \end{cases}$$

convicole

(CSP) $$\begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) ~~+ f(x(t), u(t))~~ \\ x(0) = x_0 \in C(0) \end{cases}$$

C, u are the controls

- - - - - - - - - - - - - -

The result

(joint work with Henrion Mordukhovich Nguyen Hoang)

IT 17: 20/05

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

$$\begin{cases} x \in \Pi_{T_C(x)}(f(x)) \\ x(0) = x_0 \end{cases}$$

Camino (?)

$$(CSP) \quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) ~~+ f(x(t), u(t))~~ \\ x(0) = x_0 \in C(0) \end{cases}$$

C, u are the controls

- - - - - - - - - - - - - - -

The result

(joint work with Henrion, Mordukhovich, Nguyen Hoang)

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

$$(CSP)\quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + \cancel{f(x(t), u(t))} \\ x(0) = x_0 \in C(0) \end{cases}$$

C, u are the controls

- - - - - - - - - - - - - -

The result

(joint work with Henrion, Mordukhovich, Nguyen Hoang)

$$C(t) = \left\{ x \in \mathbb{R}^n : \langle x(t), u_i(t) \rangle \leq b_i(t), \; t \in [0,T], i = 1, \dots, m \right\}$$

IT 17: 20/05/

$$(CSP)\quad \begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(\cancel{x(t)}, \cancel{u(t)}) \\ x(0) = x_0 \in C(0) \end{cases}$$

C, u are the controls

The result

(joint work with Henrion, Mordukhovich, Nguyen Hoang)

$$C(t) = \{ x \in \mathbb{R}^n : \langle x(t), u_i(t) \rangle \le b_i(t),\ t \in [0,T],\ i = 1, \dots, m \}$$

$$\|u_i\| = 1$$

(a moving polyhedron to be designed)

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

The result

(joint work with Henrion, Mordukhovich, Nguyen Hoang)

$$C(t) = \{ x \in \mathbb{R}^n : \langle x(t), u_i(t) \rangle \leq b_i(t), \; t \in [0,T], \; i = 1, \dots, m \}$$

$$\| u_i \| \equiv 1$$

(a moving polyhedron to be designed)

Set $z = (x, u, b)$

Minimize $J(z) = \varphi(x(T)) + \int_0^T \ell(z(t), \dot z(t))\, dt$

φ smooth, ℓ reasonable

subject to $\dot x(t) \in - N_{C(t)}(x(t))$, $t \in [0,T]$

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

$$C(t) = \{ x \in \mathbb{R}^n : \langle x(t), u_i(t) \rangle \leq b_i(t), \ t \in [0,T], i = 1, \dots, m \}$$

$$\| u_i \| \equiv 1$$

(a moving polyhedron to be designed)

Set $z = (x, u, b)$

Minimize $$J(z) = \varphi(x(T)) + \int_0^T \ell(z(t), \dot{z}(t)) \, dt$$

φ smooth, ℓ reasonable

subject to
$$\begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)), & t \in [0,T] \\ x(0) = x_0 \in C(0) & (x_0 \text{ given}) \end{cases}$$

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

$$C(t) = \{ x \in \mathbb{R}^n : \langle x(t), u_i(t) \rangle \le b_i(t), \; t \in [0,T], i = 1, \dots, m \}$$

$$\| u_i \| \equiv 1$$

(a moving polyhedron to be designed)

Set $z = (x, u, b)$

Minimize $\displaystyle J(z) = \varphi(x(T)) + \int_0^T \ell(z(t), \dot z(t))\, dt$

φ smooth, ℓ reasonable

subject to

$$\begin{cases} \dot x(t) \in -N_{C(t)}(x(t)), & t \in [0,T] \\ x(0) = x_0 \in C(0) & (x_0 \text{ given}) \end{cases}$$

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

$\|u_i\| \equiv 1$ (a moving polyhedron to be designed)

Set $z = (x, u, b)$

Minimize $J(z) = \varphi(x(T)) + \int_0^T \ell(z(t), \dot z(t))\, dt$

φ smooth, ℓ reasonable

subject to

$$\begin{cases} \dot x(t) \in -N_{C(t)}(x(t)), & t \in [0,T] \\ x(0) = x_0 \in C(0) & (x_0 \text{ given}) \end{cases}$$

Theorem [for the case $m = 1$, $\ell(z, \dot z) = \frac{1}{2}\|\dot z\|^2$]

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

$\| u_i \| \equiv 1$ (a moving polyhedron to be designed)

Set $z = (x, u, b)$

Minimize $J(z) = \varphi(x(T)) + \int_0^T \ell(z(t), \dot z(t))\, dt$

φ smooth, ℓ reasonable

subject to
$$\begin{cases} \dot x(t) \in -N_{C(t)}(x(t)), & t \in [0,T] \\ x(0) = x_0 \in C(0) & (x_0 \text{ given}) \end{cases}$$

Theorem [for the case $m = 1$, $\ell(z, \dot z) = \frac{1}{2} \| \dot z \|^2$]

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

Minimize $J(z) = \varphi(x(T)) + \int_0^T \ell(z(t), \dot{z}(t))\, dt$

φ smooth, ℓ reasonable

subject to $\begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)), & t \in [0,T] \\ x(0) = x_0 \in C(0) & (x_0 \text{ given}) \end{cases}$

Theorem [for the case $m=1$, $\ell(z,\dot{z}) = \frac{1}{2}\|\dot{z}\|^2$]

Let $\bar{z} = (\bar{x}, \bar{u}, \bar{b})$ be an optimal triple, and assume

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

Minimize $J(z) = \varphi(x(T)) + \int_0^T \ell(z(t), \dot z(t))\, dt$

φ smooth, ℓ reasonable

subject to
$$\begin{cases} \dot x(t) \in -N_{C(t)}(x(t)), & t \in [0,T] \\ x(0) = x_0 \in C(0) & (x_0 \text{ given}) \end{cases}$$

Theorem [for the case $m = 1$, $\ell(z, \dot z) = \frac{1}{2}\|\dot z\|^2$]

Let $\bar z = (\bar x, \bar u, \bar b)$ be an optimal triple, and assume

IT 17: 20/05/

$\|u_i\| \equiv 1$ (a moving polyhedron to be designed)

Set $z = (x, u, b)$

Minimize $$J(z) = \varphi(x(T)) + \int_0^T \ell(z(t), \dot z(t))\, dt$$

φ smooth, ℓ reasonable

subject to
$$\begin{cases} \dot x(t) \in -N_{C(t)}(x(t)), & t \in [0,T] \\ x(0) = x_0 \in C(0) & (x_0 \text{ given}) \end{cases}$$

Theorem [for the case $m = 1$, $\ell(z, \dot z) = \frac{1}{2}\|\dot z\|^2$]

Minimize $\quad J(z) = \varphi(x(T)) + \int_0^T \ell(z(t), \dot{z}(t))\,dt$

φ smooth, ℓ reasonable

subject to
$$\begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)), & t \in [0,T] \\ x(0) = x_0 \in C(0) & (x_0 \text{ given}) \end{cases}$$

Theorem [for the case $n = 1$, $\ell(z,\dot{z}) = \frac{1}{2}\|\dot{z}\|^2$]

Let $\bar{z} = (\bar{x}, \bar{u}, \bar{b})$ be an optimal triple, and assume

$W^{2,\infty}$

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

Minimize $J(z) = \varphi(x(T)) + \int_0^T \ell(z(t), \dot{z}(t))\, dt$

φ smooth, ℓ reasonable

subject to

$$\begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)), & t \in [0,T] \\ x(0) = x_0 \in C(0) & (x_0 \text{ given}) \end{cases}$$

Theorem [for the case $m = 1$, $\ell(z, \dot{z}) = \frac{1}{2} \|\dot{z}\|^2$]

Let $\bar{z} = (\bar{x}, \bar{u}, \bar{b})$ be an optimal triple, and assume

$W^{2,\infty}$

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

Minimize $J(\cdot) = \varphi(x(T)) + \int_0^T \ell(t, x(t), \dot{x}(t))\,dt$

φ smooth, ℓ reasonable

subject to
$$\begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)), & t \in [0,T] \\ x(0) = x_0 \in C(0) & (x_0 \text{ given}) \end{cases}$$

Theorem [for the case $m=1$, $\ell(z,\dot{z}) = \frac{1}{2}\|\dot{z}\|^2$]

Let $\bar{z} = (\bar{x}, \bar{u}, \bar{b})$ be an optimal triple, and assume that $\bar{z} \in W^{2,\infty}(0,T)$ (piecewise).

IT 17: 20/05/

subject to
$$\begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)), & t \in [0,T] \\ x(0) = x_0 \in C(0) & (x_0 \text{ given}) \end{cases}$$

Theorem [for the case $m=1$, $\ell(z,\dot{z}) = \frac{1}{2}\|\dot{z}\|^2$]

Let $\bar{z} = (\bar{x}, \bar{u}, \bar{b})$ be an optimal triple, and assume that $\bar{z} \in W^{2,\infty}(0,T)$ (piecewise).

Then there exist

$\lambda \geq 0$ (Lagrange multiplier)

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

subject to
$$\begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)), & t \in [0,T] \\ x(0) = x_0 \in C(0) & (x_0 \text{ given}) \end{cases}$$

Theorem [for the case $m = 1$, $\ell(z,\dot{z}) = \frac{1}{2}\|\dot{z}\|^2$]

Let $\bar{z} = (\bar{x}, \bar{u}, \bar{b})$ be an optimal triple, and assume that $\bar{z} \in W^{2,\infty}(0,T)$ (piecewise).

Then there exist

$\lambda \geq 0$ (Lagrange multiplier)

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

Minimize $J(z) = \varphi(x(T)) + \int_0^T \ell(z(t), \dot{z}(t))\, dt$

φ smooth, ℓ reasonable

subject to
$$\begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)), & t \in [0,T] \\ x(0) = x_0 \in C(0) & (x_0 \text{ given}) \end{cases}$$

Theorem [for the case $m = 1$, $\ell(z, \dot{z}) = \frac{1}{2}\|\dot{z}\|^2$]

Let $\bar{z} = (\bar{x}, \bar{u}, \bar{b})$ be an optimal triple, and assume

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

Set $z = (x, u, b)$

Minimize $J(z) = \varphi(x(T)) + \int_0^T \ell(z(t), \dot{z}(t))\, dt$

φ smooth, ℓ reasonable

subject to
$$\begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)), & t \in [0,T] \\ x(0) = x_0 \in C(0) & (x_0 \text{ given}) \end{cases}$$

Theorem [for the case $n=1$, $\ell(z,\dot{z}) = \frac{1}{2}\|\dot{z}\|^2$]

IT 17: 20/05/

(joint work with Henrion, Mordukhovich, Nguyen Hoang)

$$C(t) = \{ x \in \mathbb{R}^n : \langle x(t), u_i(t) \rangle \le b_i(t),\ t \in [0,T],\ i=1,\dots,m \}$$

$\|u_i\| \equiv 1$ (a moving polyhedron to be designed)

Set $z = (x, u, b)$

Minimize $J(z) = \varphi(x(T)) + \int_0^T \ell(z(t), \dot z(t))\, dt$

φ smooth, ℓ reasonable

subject to $\dot x(t) \in -N_{C(t)}(x(t)),\ t \in [0,T]$

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

$$C(t) = \{ x \in \mathbb{R} : \langle x(t), u_i(t) \rangle \leq b_i(t), \ t \in [0,T], i=1,\dots,m \}$$

$$\|u_i\| = 1$$

(a moving polyhedron to be designed)

Set $z = (x, u, b)$

Minimize $$J(z) = \varphi(x(T)) + \int_0^T \ell(z(t), \dot{z}(t))\, dt$$

φ smooth, ℓ reasonable

subject to
$$\begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)), & t \in [0,T] \\ x(0) = x_0 \in C(0) & (x_0 \text{ given}) \end{cases}$$

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

$\|u_i\| \equiv 1$ (a moving polyhedron to be designed)

Set $z = (x, u, b)$

Minimize $J(z) = \varphi(x(T)) + \int_0^T \ell(z(t), \dot{z}(t))\, dt$

φ smooth, ℓ reasonable

subject to

$$\begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)), & t \in [0,T] \\ x(0) = x_0 \in C(0) & (x_0 \text{ given}) \end{cases}$$

Theorem

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

Minimize $J(z) = \varphi(x(T)) + \int_0^T \ell(z(t), \dot z(t))\, dt$

φ smooth, ℓ reasonable

subject to
$$\begin{cases} \dot x(t) \in -N_{C(t)}(x(t)), & t \in [0,T] \\ x(0) = x_0 \in C(0) & (x_0 \text{ given}) \end{cases}$$

Theorem [for the case $m=1$, $\ell(z,\dot z) = \frac{1}{2}\|\dot z\|^2$]

Let $\bar z = (\bar x, \bar u, \bar b)$ be an optimal triple, and assume

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

Minimize $J(z) = \varphi(x(T)) + \int_0 \ell(z(t), \dot{z}(t))\, dt$

φ smooth, ℓ reasonable

subject to
$$\begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)), & t \in [0,T] \\ x(0) = x_0 \in C(0) & (x_0 \text{ given}) \end{cases}$$

Theorem [for the case $m=1$, $\ell(z,\dot{z}) = \frac{1}{2}\|\dot{z}\|^2$]

Let $\bar{z} = (\bar{x}, \bar{u}, \bar{b})$ be an optimal triple, and assume

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

subject to
$$\begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)), & t \in [0,T] \\ x(0) = x_0 \in C(0) & (x_0 \text{ given}) \end{cases}$$

Theorem [for the case $m=1$, $\ell(z,\dot{z}) = \frac{1}{2}\|\dot{z}\|^2$]

Let $\bar{z} = (\bar{x}, \bar{u}, \bar{b})$ be an optimal triple, and assume that $\bar{z} \in W^{2,\infty}(0,T)$ (piecewise).

Then there exist

- $\lambda \geq 0$ (Lagrange multiplier)

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

subject to
$$\begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)), & t \in [0,T] \\ x(0) = x_0 \in C(0) & (x_0 \text{ given}) \end{cases}$$

Theorem [for the case $m=1$, $\ell(z,\dot{z}) = \frac{1}{2}\|\dot{z}\|^2$]

Let $\bar{z} = (\bar{x}, \bar{u}, \bar{b})$ be an optimal triple, and assume that $\bar{z} \in W^{2,\infty}(0,T)$ (piecewise).

Then there exist

- $\lambda \geq 0$ (Lagrange multiplier)

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

Theorem [for the case $m = 1$, $\ell(z, \dot z) = \frac{1}{2} \|\dot z\|^2$]

Let $\bar z = (\bar x, \bar u, \bar b)$ be an optimal triple, and assume that $\bar z \in W^{2,\infty}(0,T)$ (piecewise).

Then there exist

- $\lambda \geq 0$ (Lagrange multiplier)
- $p = (p^x, p^u, p^b) : [0,T] \longrightarrow \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$ absolutely continuous (adjoint vector)

IT 17: 20/05

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

Theorem [for the case $m = 1$, $\ell(z, \dot z) = \frac{1}{2}\|\dot z\|^2$]

Let $\bar z = (\bar x, \bar u, \bar b)$ be an optimal triple, and assume that $\bar z \in W^{2,\infty}(0,T)$ (piecewise).

Then there exist

- $\lambda \geq 0$ (Lagrange multiplier)
- $p = (p^x, p^u, p^b) : [0,T] \to \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$ absolutely continuous (adjoint vector)

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

Theorem

[for the case $m=1$, $\ell(z,\dot z)=\frac{1}{2}\|\dot z\|^2$]

Let $\bar z = (\bar x, \bar u, \bar b)$ be an optimal triple, and assume that $\bar z \in W^{2,\infty}(0,T)$ (piecewise).

Then there exist

- $\lambda \geq 0$ (Lagrange multiplier)
- $p = (p^x, p^u, p^b) : [0,T] \to \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$ absolutely continuo

(adjoint vector)

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

Theorem [for the case $m=1$, $\ell(z,\dot z)=\frac{1}{2}\|\dot z\|^2$]

Let $\bar z=(\bar x,\bar u,\bar b)$ be an optimal triple, and assume that $\bar z\in W^{2,\infty}(0,T)$ (piecewise).

Then there exist

- $\lambda\geq 0$ (Lagrange multiplier)
- $p=(p^x,p^u,p^b):[0,T]\to\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}$ absolutely continuous (adjoint vector)

signed measures γ $\zeta\in(\mathcal{C}^0([0,T]))^*$

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

Theorem [for the case $m = 1$, $\ell(z, \dot{z}) = \frac{1}{2}\|z\|$]

Let $\bar{z} = (\bar{x}, \bar{u}, \bar{b})$ be an optimal triple, and assume that $\bar{z} \in W^{2,\infty}(0,T)$ (piecewise).

Then there exist

- $\lambda \geq 0$ (Lagrange multiplier)
- $p = (p^x, p^u, p^b) : [0,T] \longrightarrow \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$ absolutely continuous (adjoint vector)
- signed measures $\gamma, \xi \in (\mathcal{C}^0([0,T]))^*$

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

that $\bar{z} \in W^{2,\infty}(0,T)$ (piecewise).

Then there exist

- $\lambda \geq 0$ (Lagrange multiplier)
- $p = (p^x, p^u, p^b) : [0,T] \to \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$ absolutely continuous (adjoint vector)
- signed measures $\gamma, \xi \in (\mathcal{C}^0([0,T]))^*$ (usual in state constrained problems)

such that

IT

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

Then there exist

- $\lambda \geq 0$ (Lagrange multiplier)
- $p = (p^x, p^u, p^b) : [0,T] \longrightarrow \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$ absolutely continuous

  (adjoint vector)
- signed measures $\gamma, \xi \in \left(\mathcal{C}^0([0,T])\right)^*$

  (usual in state constrained problems)

such that

$$\left(-p^x(T) \in \lambda \nabla \varphi(\bar{x}(T)) + p^b(T)\,\bar{u}(T) \right.$$

(transversality conditions)

IT

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

that $\bar{z} \in W^{1,\infty}(0,T)$ (piecewise).

Then there exist

- $\lambda \geq 0$ (Lagrange multiplier)
- $p = (p^x, p^u, p^b) : [0,T] \longrightarrow \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$ absolutely continuous (adjoint vector)
- signed measures $\gamma, \xi \in \left(\mathcal{C}^0([0,T])\right)^*$ (usual in state constrained problems)

such that

Theorem [for the case $m=1$, $\ell(t,z) = \frac{1}{2}\|z\|$]

Let $\bar{z} = (\bar{x}, \bar{u}, \bar{b})$ be an optimal triple, and assume that $\bar{z} \in W^{2,\infty}(0,T)$ (piecewise).

Then there exist

- $\lambda \geq 0$ (Lagrange multiplier)
- $p = (p^x, p^u, p^b) : [0,T] \to \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$ absolutely continuous (adjoint vector)
- signed measures $\gamma, \xi \in \left(\mathcal{C}^0([0,T])\right)^*$

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

Let $\bar{z} = (\bar{x}, \bar{u}, \bar{b})$ be an optimal triple, and assume that $\bar{z} \in W^{2,\infty}(0,T)$ (piecewise).

Then there exist

- $\lambda \geq 0$ (Lagrange multiplier)
- $p = (p^x, p^u, p^b) : [0,T] \to \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$ absolutely continuous (adjoint vector)
- signed measures $\gamma, \xi \in \left(\mathcal{C}^0([0,T])\right)^*$ (usual in state constrained problems)

such that

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

Let $\bar{z} = (\bar{x}, \bar{u}, \bar{b})$ be an optimal triple, and assume that $\bar{z} \in W^{2,\infty}(0,T)$ (piecewise).

Then there exist

- $\lambda \geq 0$ (Lagrange multiplier)
- $p = (p^x, p^u, p^b) : [0,T] \longrightarrow \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$ absolutely continuous (adjoint vector)
- signed measures $\gamma, \xi \in \left(\mathcal{C}^0([0,T])\right)^*$ (usual in state constrained problems)

such that

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

Then there exist

- $\lambda \geq 0$ (Lagrange multiplier)
- $p = (p^x, p^u, p^b) : [0,T] \longrightarrow \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$ absolutely continuous
  (adjoint vector)
- signed measures $\gamma, \xi \in (\mathcal{C}^0([0,T]))^*$
  (usual in state constrained problems)

such that

$$\begin{cases} -p^x(T) \in \lambda \nabla \varphi(\bar{x}(T)) + p^b(T)\,\bar{u}(T) \end{cases}$$

IT 17: 20/05/

- $\lambda \geq 0$ (Lagrange multiplier)

- $p = (p^x, p^u, p^b) : [0,T] \longrightarrow \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$ absolutely continuou[s]

  (adjoint vector)

- signed measures $\gamma, \xi \in \left(\mathcal{C}^0([0,T])\right)^*$

  (usual in state constrained problems)

<u>such that</u>

$$\begin{cases} -p^x(T) \in \lambda \nabla\varphi(\bar{x}(T)) + p^b(T)\,\bar{u}(T) \\ p^u(T) + p^b\,\bar{x}(T) \parallel \bar{u}(T) \end{cases}$$

(transversality conditions)

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

- $\lambda \geq 0$ (Lagrange multiplier)

- $p = (p^x, p^u, p^b) : [0,T] \longrightarrow \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$ absolutely continuous

(adjoint vector)

- signed measures $\gamma, \xi \in \left(\mathcal{C}^0([0,T])\right)^*$

(usual in state constrained problems)

<u>such that</u>

$$
\begin{cases}
-p^x(T) \in \lambda \nabla \varphi(\bar{x}(T)) + p^b(T)\, \bar{u}(T) \\
p^u(T) + p^b \bar{x}(T) \parallel \bar{u}(T)
\end{cases}
$$

(transversality conditions)

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

Then there exist

- $\lambda \geq 0$ (Lagrange multiplier)
- $p = (p^x, p^u, p^b) : [0,T] \longrightarrow \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$ absolutely continuous
  (adjoint vector)
- signed measures $\gamma, \xi \in (\mathcal{C}^0([0,T]))^*$
  (usual in state constrained problems)

<u>such that</u>

$$\left(-p^x(T) \in \lambda \nabla \varphi(\bar{x}(T)) + p^b(T)\,\bar{u}(T) \right.$$

(transversality conditions)

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

Then there exist

- $\lambda \geq 0$ (Lagrange multiplier)
- $p = (p^x, p^u, p^b) : [0,T] \longrightarrow \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$ absolutely continuous

  (adjoint vector)
- signed measures $\gamma, \xi \in (\mathcal{C}^0([0,T]))^*$

  (usual in state constrained problems)

<u>such that</u>

$$\begin{cases} -p^x(T) \in \lambda \nabla \varphi(\bar{x}(T)) + p^b(T)\,\bar{u}(T) \\ p^u(T) + p^b\,\bar{x}(T) \parallel \bar{u}(T) \end{cases}$$

(transversality conditions)

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File | Home | Inserisci | Condividi | Disegno | Revisione | Visualizza | Pagine | Nuova scheda | Nuova scheda

- $p = (p^x, p^u, p^b) : [0,T] \to \mathbb{R} \times \mathbb{R} \times \mathbb{R}$ absolutely continuous

(adjoint vector)

- signed measures $\gamma, \xi \in \left(\mathcal{C}^0([0,T])\right)^*$

(usual in state constrained problems)

<u>such that</u>

$$\begin{cases} -p^x(T) \in \lambda \nabla \varphi(\bar{x}(T)) + p^b(T)\,\bar{u}(T) \\ p^u(T) + p^b \bar{x}(T) \parallel \bar{u}(T) \\ p^b(T) \geq 0 \text{ and } \langle \bar{u}(T), \bar{x}(T) \rangle < \bar{b}(T) \Rightarrow p^b(T) = 0 \end{cases}$$

(transversality conditions)

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

- $p = (p^x, p^u, p^b) : [0,T] \longrightarrow \mathbb{R} \times \mathbb{R} \times \mathbb{R}$ absolutely continuous

(adjoint vector)

- signed measures $\gamma, \xi \in (\mathcal{C}^0([0,T]))^*$

(usual in state constrained problems)

<u>such that</u>

$$\begin{cases} -p^x(T) \in \lambda \nabla \varphi(\bar{x}(T)) + p^b(T)\, \bar{u}(T) \\ p^u(T) + p^b \bar{x}(T) \parallel \bar{u}(T) \\ p^b(T) \geq 0 \quad \text{and} \quad \langle \bar{u}(T), \bar{x}(T) \rangle < \bar{b}(T) \Rightarrow p^b(T) = 0 \end{cases}$$

(transversality conditions)

IT 17: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

– signed measures $\gamma, \xi \in \left(\mathcal{C}^0([0,T])\right)^*$

(usual in state constrained problems)

such that

$$\begin{cases} -p^x(T) \in \lambda \nabla \varphi(\bar{x}(T)) + p^b(T)\,\bar{u}(T) \\ p^u(T) + p^b \bar{x}(T) \parallel \bar{u}(T) \\ p^b(T) \geq 0 \text{ and } \langle \bar{u}(T), \bar{x}(T) \rangle < \bar{b}(T) \Rightarrow p^b(T) = 0 \end{cases}$$

(transversality conditions)

$$\lambda + \|q(0)\| + \|p(T)\| \neq 0$$

(nontriviality condition)

IT

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

$$
\begin{cases}
-p^x(T) \in \lambda \nabla \varphi(\bar{x}(T)) + p^b(T)\,\bar{u}(T) \\
p^u(T) + p^b \bar{x}(T) \parallel \bar{u}(T) \\
p^b(T) \geq 0 \quad \text{and} \quad \langle \bar{u}(T), \bar{x}(T) \rangle < \bar{b}(T) \Rightarrow p^b(T) = 0
\end{cases}
$$

(transversality conditions)

$$\lambda + \|q(0)\| + \|p(T)\| \neq 0$$

(nontriviality condition)

($q = (q^x, q^u, q^b) : [0,T] \to \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$ a BV function)

and

$$\dot{p}^x \equiv 0, \quad \dot{p}^b \equiv 0$$

IT 18: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

$$\begin{cases} -p^x(T) \in \lambda \nabla \varphi(\bar{x}(T)) + p^b(T)\,\bar{u}(T) \\ p^u(T) + p^b \bar{x}(T) \parallel \bar{u}(T) \\ p^b(T) \geq 0 \quad \text{and} \quad \langle \bar{u}(T), \bar{x}(T) \rangle < \bar{b}(T) \Rightarrow p^b(T) = 0 \end{cases}$$

(transversality conditions)

$$\lambda + \|q(0)\| + \|p(T)\| \neq 0$$

(nontriviality condition)

($q = (q^x, q^u, q^b) : [0,T] \to \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$ a BV function)

and

$$\dot{p}^x \equiv 0, \qquad \dot{p}^b \equiv 0$$

- $p = (p^x, p^u, p^b) : [0,T] \longrightarrow \mathbb{R}^k \times \mathbb{R}^k \times \mathbb{R}$ absolutely continuous

(adjoint vector)

- signed measures $\gamma, \xi \in \left(\mathcal{C}^0([0,T])\right)^*$

(usual in state constrained problems)

<u>such that</u>

$$\begin{cases} -p^x(T) \in \lambda \nabla \varphi(\bar{x}(T)) + p^b(T)\,\bar{u}(T) \\ p^u(T) + p^b\,\bar{x}(T) \parallel \bar{u}(T) \\ p^b(T) \geq 0 \text{ and } \langle \bar{u}(T), \bar{x}(T) \rangle < \bar{b}(T) \Rightarrow p^b(T) = 0 \end{cases}$$

(transversality conditions)

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

- $p = (p^x, p^u, p^b) : [0,T] \longrightarrow \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$ absolutely continuous

(adjoint vector)

- signed measures $\gamma, \xi \in (\mathcal{C}^0([0,T]))^*$

(usual in state constrained problems)

such that

$$\begin{cases} -p^x(T) \in \lambda \nabla\varphi(\bar{x}(T)) + p^b(T)\,\bar{u}(T) \\ p^u(T) + p^b\,\bar{x}(T) \parallel \bar{u}(T) \\ p^b(T) \geq 0 \quad \text{and} \quad \langle \bar{u}(T), \bar{x}(T) \rangle < \bar{b}(T) \Rightarrow p^b(T) = 0 \end{cases}$$

(transversality conditions)

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

Theorem [for the case $m = 1$, $\ell(z, \dot z) = \frac{1}{2}\|\dot z\|^2$]

Let $\bar z = (\bar x, \bar u, \bar b)$ be an optimal triple, and assume that $\bar z \in W^{2,\infty}(0,T)$ (piecewise).

Then there exist

- $\lambda \geq 0$ (Lagrange multiplier)
- $p = (p^x, p^u, p^b) : [0,T] \longrightarrow \mathbb{R}^{\tilde n} \times \mathbb{R}^n \times \mathbb{R}$ absolutely continuous

(adjoint vector)

IT 18: 20/05/

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

φ smooth, ℓ reasonable

subject to
$$\begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)), & t \in [0,T] \\ x(0) = x_0 \in C(0) & (x_0 \text{ given}) \end{cases}$$

Theorem [for the case $m=1$, $\ell(z,\dot{z}) = \frac{1}{2}\|\dot{z}\|^2$]

Let $\bar{z} = (\bar{x}, \bar{u}, \bar{b})$ be an optimal triple, and assume that $\bar{z} \in W^{2,\infty}(0,T)$ (piecewise).

Then there exist

g smooth, ℓ reasonable

subject to
$$\begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)), & t \in [0,T] \\ x(0) = x_0 \in C(0) & (x_0 \text{ given}) \end{cases}$$

Theorem [for the case $m=1$, $\ell(z,\dot{z}) = \frac{1}{2}\|\dot{z}\|^2$]

Let $\bar{z} = (\bar{x}, \bar{u}, \bar{b})$ be an optimal triple, and assume that $\bar{z} \in W^{2,\infty}(0,T)$ (piecewise).

Then there exist

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

$$x(0) = x_0 \in C(0) \qquad (x_0 \text{ given})$$

Theorem [for the case $m = 1$, $\ell(z, \dot z) = \frac{1}{2}\|\dot z\|^2$]

Let $\bar z = (\bar x, \bar u, \bar b)$ be an optimal triple, and assume that $\bar z \in W^{2,\infty}(0,T)$ (piecewise).

Then there exist

- $\lambda \geq 0$ (Lagrange multiplier)
- $p = (p^x, p^u, p^b) : [0,T] \longrightarrow \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$ absolutely continuou[s]

CSP 1 - Microsoft OneNote

File Home Inserisci Condividi Disegno Revisione Visualizza Pagine Nuova scheda Nuova scheda

Theorem [for the case $m=1$, $\ell(z,\dot z)=\frac{1}{2}\|\dot z\|^2$]

Let $\bar z=(\bar x,\bar u,\bar b)$ be an optimal triple, and assume that $\bar z\in W^{2,\infty}(0,T)$ (piecewise).

Then there exist

- $\lambda\geq 0$ (Lagrange multiplier)
- $p=(p^x,p^u,p^b):[0,T]\longrightarrow\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}$ absolutely continuous (adjoint vector)

IT 18: 20/05/