Data-Driven Applications Inspiring Linear Algebra

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- To clear up misconceptions that the material in these courses is only useful to the students going on to graduate school.
- To introduce students to techniques we as researchers use.
- Students preparing to graduate need skills for graduate school and/or a career in industry.

Overview of project

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- Modules are being tested at several types of schools and classroom set ups, for transportability.
- After each implementation, we are editing and adding to the materials.

Module Concept

Common Application-Based Learning	

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Mathematical Tools Development	

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Mathematical Tools Development $\Downarrow$ Explore Pedagogical Examples	

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Radiography/Tomography Example

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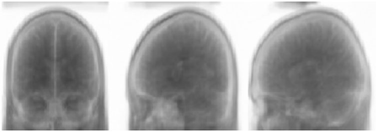

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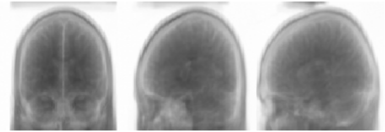

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- Vector Spaces
- Linear Transformations
- Invertibility
- Nullspace, Range Space
- Singular Value Decomposition

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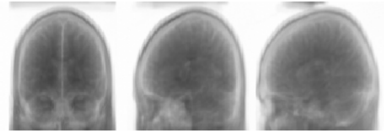

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Project: Reconstruct

Given radiographs with Noise
Reconstruct the object that produced them

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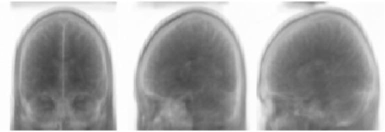

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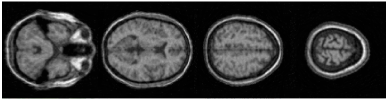

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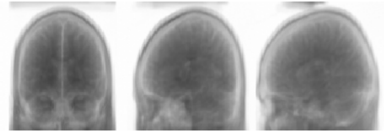

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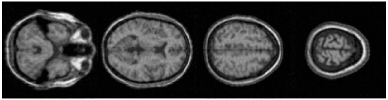

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Tools Learned:

- **Exploration**
- **Linear Algebra Concepts**
- **Writing**
- **Matlab/Octave Commands**

Common Application-Based Learning: The Nullspace

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Example N: Let us now consider the real-life example of the linear transformation called the radiographic transformation...

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- Lab2: Students create example radiographic transformations and find out later that these transformations are linear transformations.

Application-Inspired Learning: The Nullspace

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This lab is followed up with a discussion of “invisible" vectors. Because of their importance, we define the space of “invisible" vectors as the nullspace.

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In our second year, we have plans to collect a larger sample of data to get a better picture.

What Students Say

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"I have loved learning about both the Heat Equation application and the Radiography/Tomography application in class this semester. Studying to be a middle school math teacher, it can be very frustrating taking difficult upper level classes. However, it is incredible to see how the math we learn in class can be used in the real world and makes the topics we learn much more interesting. I think adding this aspect to the class is extremely helpful and a great way to keep students engaged in the class."

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"I enjoyed learning about radiography–tomography. It was cool to be able to apply math and understand where this is used in real life."

Contact Us

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We’re looking for beta testers for this spring.