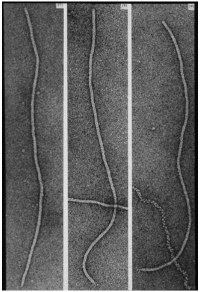
Chiral Edge Fluctuations of Colloidal Membranes

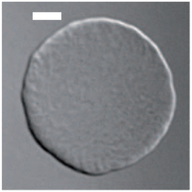
Leroy Jia (w/ T. Powers & R. Pelcovits)

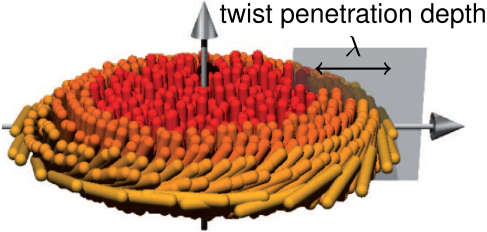
SIAM Conference on the Life Sciences

August 6, 2018

Colloidal membranes are composed of rod-like viruses







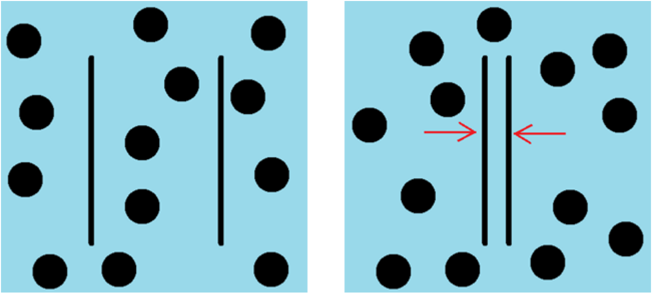
- Left: fd virus[1], 870 nm$\times$ 6 nm
- Center: DIC image [2], scale bar = 5 μm
- Right: Color indicates angle relative to normal[2]

[1] Model and Russel, *The Bacteriophages*. R.New York:Plenum (1988)

[2] Gibaud et. al, *Nature* **481**, 348 (2012)

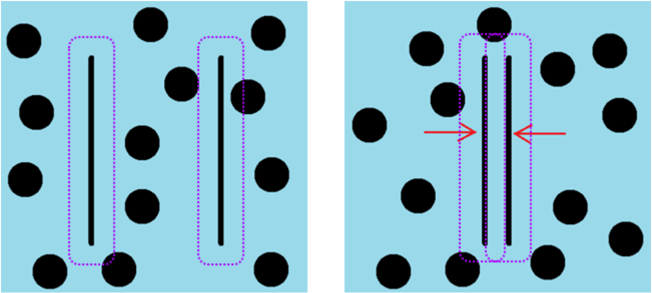
Polymer depletant induces attractive forces

First described by Asakura and Oosawa (1954)



Polymer depletant induces attractive forces

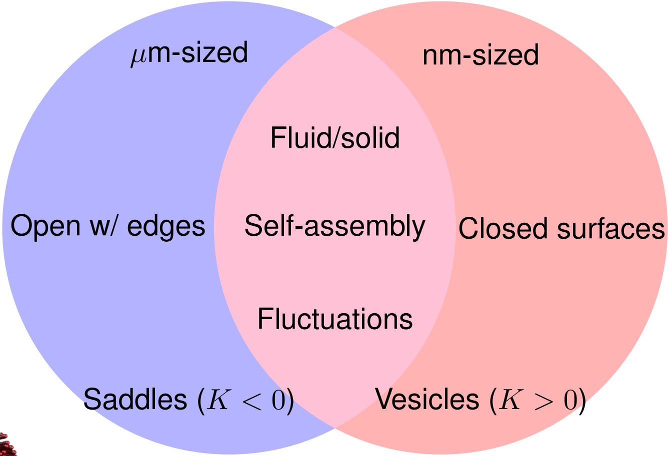
First described by Asakura and Oosawa (1954)



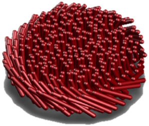
$$F = -Nk_BT\log(V - V_{ex}) \simeq nk_BTV_{ex}$$

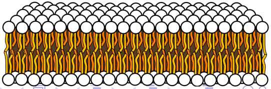
Colloidal membranes are familiar and different

COLLOIDAL MEMBRANES



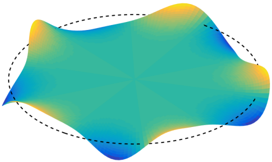
LIPID BILAYER MEMBRANES



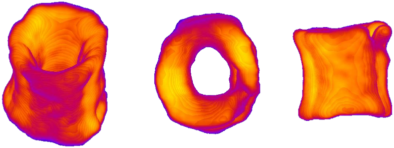


Colloidal membranes exhibit diverse phenomena

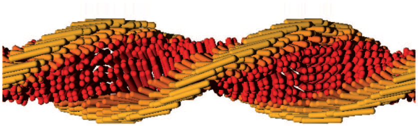
Chiral edge fluctuations



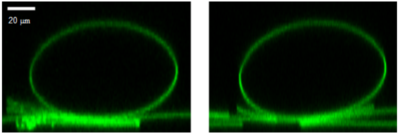
Unduloid formation



Ribbon force vs. extension



Colloidal vesicles



Canham-Helfrich energy models membrane bending

$$E_{CH} = \frac{\kappa}{2}\int (2H)^2 dA \;+\; \bar{\kappa}\int K dA \;+\; \mu \int dA$$

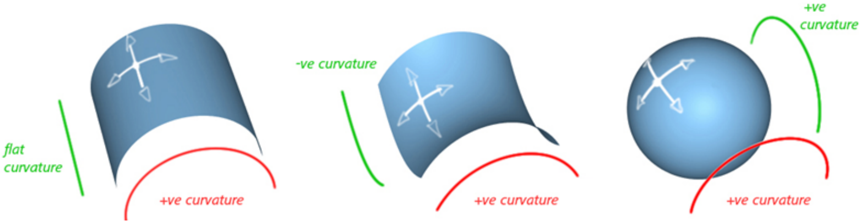


Image: Website, Autodesk Alias Workbench

Canham-Helfrich energy models membrane bending

$$E_{CH} = \frac{\kappa}{2}\int (2H)^2 dA + \bar{\kappa}\int K dA + \mu \int dA$$

- Mean curvature ($\kappa \sim 10^5 k_B T$)

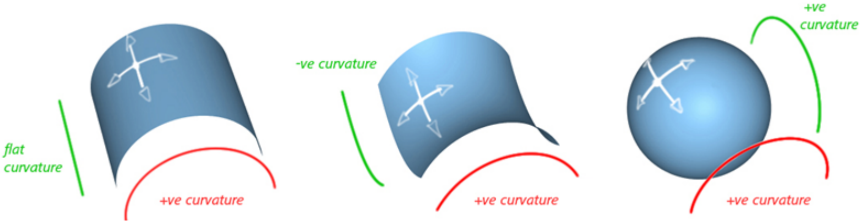


Image: Website, Autodesk Alias Workbench

Canham-Helfrich energy models membrane bending

$$E_{CH} = \frac{\kappa}{2}\int (2H)^2 dA + \bar{\kappa}\int K dA + \mu \int dA$$

- Mean curvature ($\kappa \sim 10^5 k_B T$)
- Gaussian curvature ($\bar{\kappa} \sim 10^2 k_B T$)

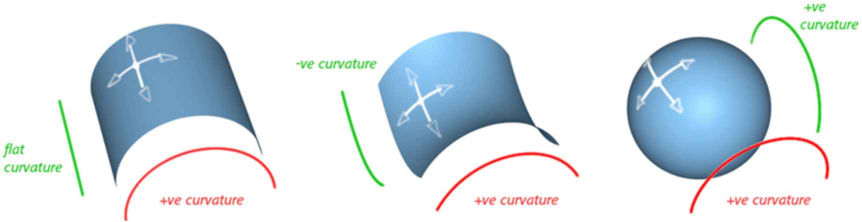


Image: Website, Autodesk Alias Workbench

Canham-Helfrich energy models membrane bending

$$E_{CH} = \frac{\kappa}{2}\int (2H)^2 dA + \bar{\kappa}\int K dA + \mu \int dA$$

- Mean curvature ($\kappa \sim 10^5 k_B T$)
- Gaussian curvature ($\bar{\kappa} \sim 10^2 k_B T$)
- Lagrange multiplier

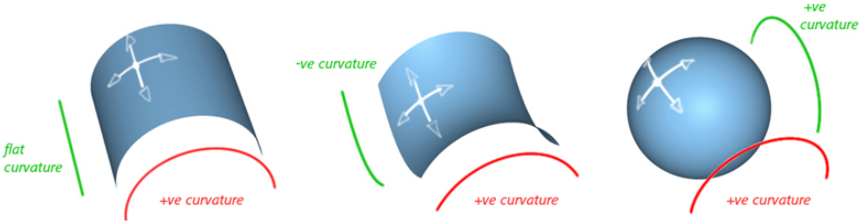
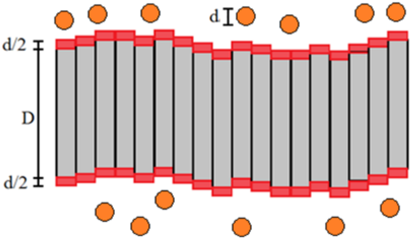
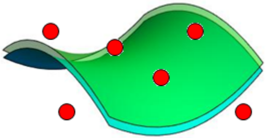


Image: Website, Autodesk Alias Workbench

Entropy favors positive Gaussian curvature modulus

$$V_{ex} \approx D \int dA + \frac{D^2 d}{4} \int K dA$$

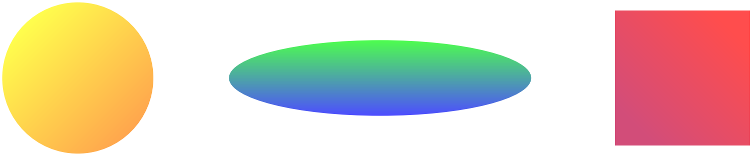




$$\bar{\kappa} = \frac{1}{4} n k_B T D^2 d + \bar{\kappa}_{in} \approx 200 k_B T$$

Line tension limits membrane perimeter

$$E_{LT} = \gamma \int ds$$



γ can depend on

- depletant concentration
- temperature
- chirality

Frank energy models liquid crystalline (LC) order

$$E_F = \frac{1}{2}\int \Big[K_1(\nabla\cdot\hat{n})^2 + K_2(\hat{n}\cdot(\nabla\times\hat{n}) - q)^2 + K_3(\hat{n}\times(\nabla\times\hat{n}))^2 + C\sin^2\theta \Big]\, dA$$

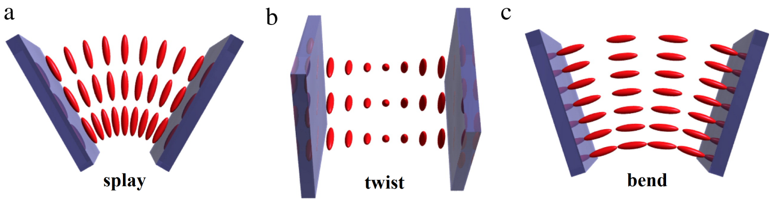


Image: S. Copar, *Physics Reports* **538**, 1 (2014)

Frank energy models liquid crystalline (LC) order

$$E_F = \frac{1}{2}\int \Big[\, K_1(\nabla\cdot\hat{n})^2 \;+\; K_2(\hat{n}\cdot(\nabla\times\hat{n})-q)^2 \;+\; K_3(\hat{n}\times(\nabla\times\hat{n}))^2 + C\sin^2\theta \Big]\, dA$$

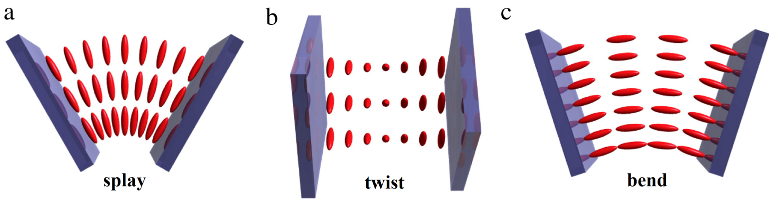


Image: S. Copar, *Physics Reports* **538**, 1 (2014)

Frank energy models liquid crystalline (LC) order

$$E_F = \frac{1}{2}\int \Big[K_1(\nabla \cdot \hat{n})^2 + K_2(\hat{n}\cdot(\nabla \times \hat{n}) - q)^2 + K_3(\hat{n}\times(\nabla\times\hat{n}))^2 + C\sin^2\theta \Big] dA$$

- $\sqrt{K_2/C}$ defines the twist penetration depth λ

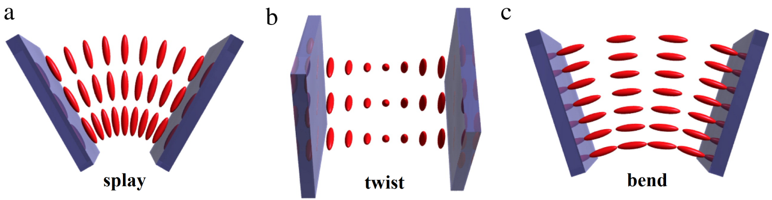
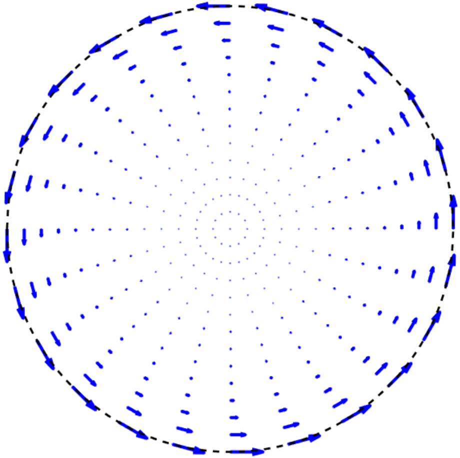
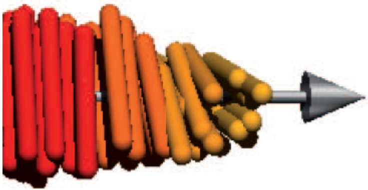


Image: S. Copar, *Physics Reports* **538**, 1 (2014)

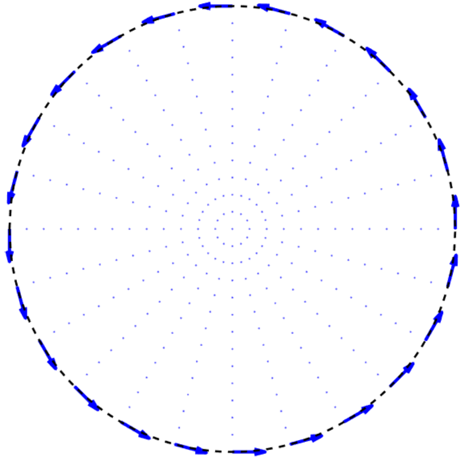
An effective term accounts for the LC bend



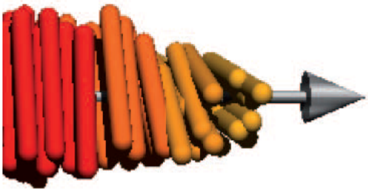
$$\theta \sim \frac{\pi}{2} \exp\left(-\frac{a-r}{\lambda}\right)$$



An effective term accounts for the LC bend

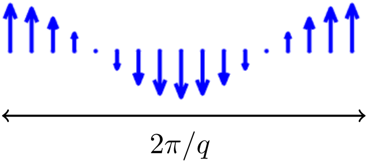


$$\frac{K_3}{2}\int(\hat{n}\times(\nabla\times\hat{n}))^2 dA \simeq \frac{B}{2}\int k^2 ds$$



An effective term accounts for the LC twist

The chirality q is represented as an inherent geodesic torsion τ_g^*.

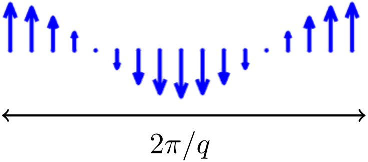


$$\tau_g = \hat{t} \cdot \left(\hat{N} \times \frac{d\hat{N}}{ds} \right) = q$$

$$\frac{K_2}{2} \int (\hat{n} \cdot (\nabla \times \hat{n}) - q)^2 dV \simeq \frac{B'}{2} \int (\tau_g - \tau_g^*)^2 ds$$

An effective term accounts for the LC twist

The chirality q is represented as an inherent geodesic torsion τ_g^*.



$$\tau_g = \hat{t} \cdot \left(\hat{N} \times \frac{d\hat{N}}{ds} \right) = q$$

$$\frac{K_2}{2} \int (\hat{n} \cdot (\nabla \times \hat{n}) - q)^2 dV \simeq \frac{B'}{2} \int (\tau_g - \tau_g^*)^2 ds$$

We define the effective chirality modulus $c^* = -B' \tau_g^*$.

Membrane shapes are found by minimizing energy

$$E = \int_{\mathcal{S}} \left[\frac{\kappa}{2}(2H)^2 + \bar{\kappa}K + \mu \right] dA + \int_{\partial\mathcal{S}} \left[\gamma + \frac{B}{2}k^2 + \frac{B'}{2}\tau_g^2 + c^*\tau_g \right] ds$$

[3] Website, ''The geometry of soap films and soap bubbles"

[4] Website, ''An ant rolling a sphere of water across the surface of a garden pond"

[5] Website, "Willmore Day at Durham" (Blog of Kevin Houston)

Membrane shapes are found by minimizing energy

$$\boxed{E = \int_{\mathcal{S}} \left[\frac{\kappa}{2}(2H)^2 + \bar{\kappa}K + \mu\right] dA + \int_{\partial\mathcal{S}} \left[\gamma + \frac{B}{2}k^2 + \frac{B'}{2}\tau_g^2 + c^*\tau_g\right] ds}$$

Euler-Lagrange equation:

$$\kappa(\nabla^2 H + 2H^3 - 2HK) - \mu H = 0$$

[3] Website, ''The geometry of soap films and soap bubbles"

[4] Website, ''An ant rolling a sphere of water across the surface of a garden pond"

[5] Website, "Willmore Day at Durham" (Blog of Kevin Houston)

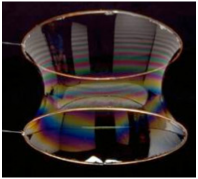
Membrane shapes are found by minimizing energy

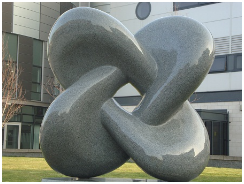
$$E = \int_{\mathcal{S}} \left[\frac{\kappa}{2}(2H)^2 + \bar{\kappa}K + \mu\right] dA + \int_{\partial\mathcal{S}} \left[\gamma + \frac{B}{2}k^2 + \frac{B'}{2}\tau_g^2 + c^*\tau_g\right] ds$$

Euler-Lagrange equation:

$$\kappa(\nabla^2 H + 2H^3 - 2HK) - \mu H = 0$$

Solutions: minimal surfaces[3], CMC surfaces[4], Willmore surfaces[5]





[3] Website, ''The geometry of soap films and soap bubbles"

[4] Website, ''An ant rolling a sphere of water across the surface of a garden pond"

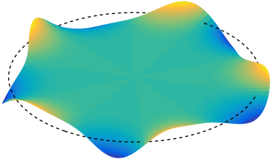
[5] Website, "Willmore Day at Durham" (Blog of Kevin Houston)

Outline

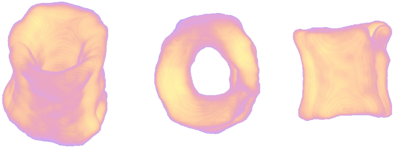
0. Motivation and mathematical model

I. Chiral edge fluctuations

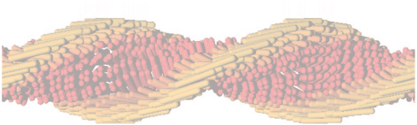
w/ M. J. Zakhary, Z. Dogic. R. A. Pelcovits, and T. R. Powers



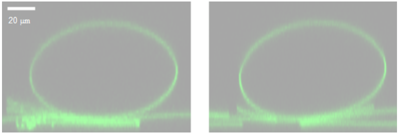
II. Unduloids



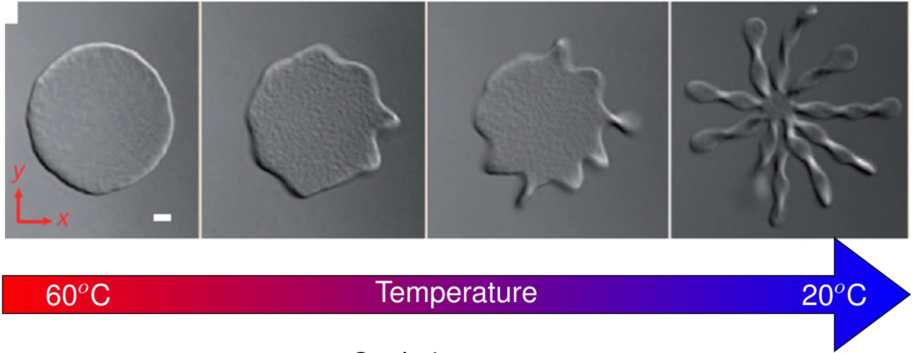
III. Force vs. extension



IV. Colloidal vesicles



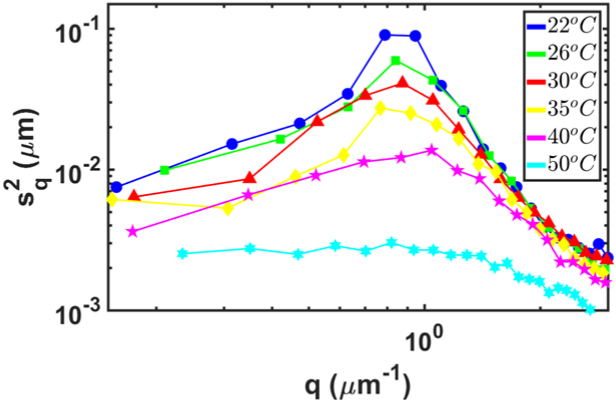
Temperature quench induces disk-to-ribbon transition



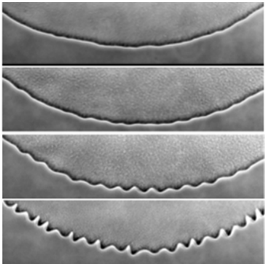
Scale bar = 2μm

Image: Gibaud et. al, *Nature* **481**, 348 (2012)

Edge fluctuation spectrum exhibits an anomalous peak



$$\langle s_q^2 \rangle = \frac{k_B T}{Bq^2 + \gamma}$$

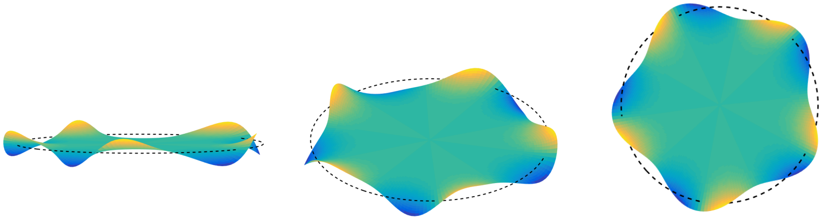


Data and photo: Mark Zakhary

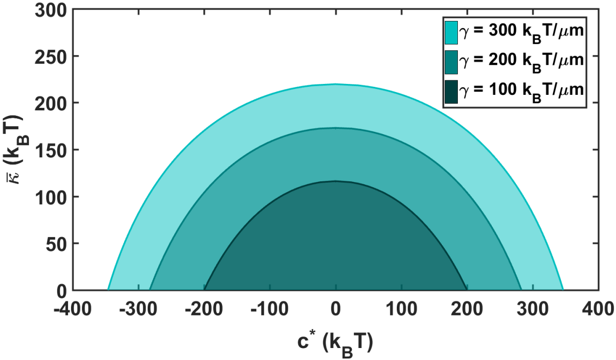
Perturb a disk with a helical ripple

$$E_{flat} = 2\pi R\gamma + \frac{B\pi}{R}$$

$$E = \int_{\mathcal{S}} \left[\frac{\kappa}{2}(2H)^2 + \bar{\kappa}K + \mu\right] dA + \int_{\partial\mathcal{S}} \left[\gamma + \frac{B}{2}k^2 + \frac{B'}{2}\tau_g^2 + c^*\tau_g\right] ds$$



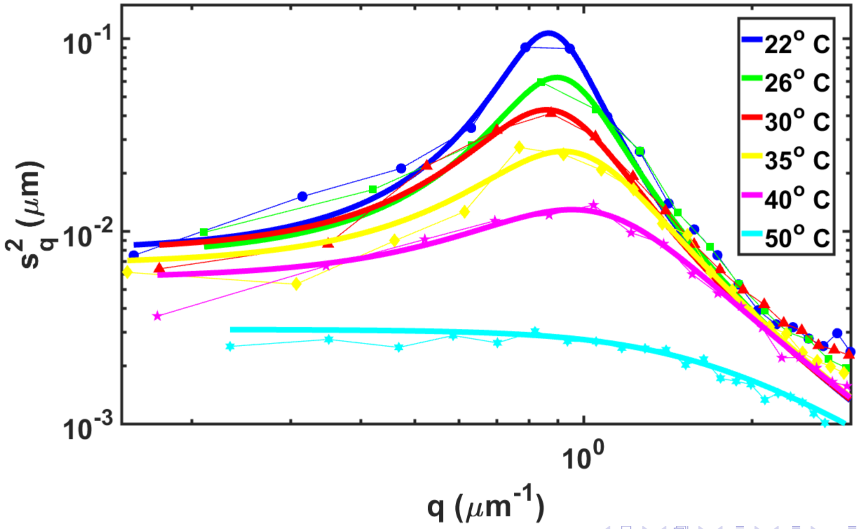
Flat membane is unstable when c^* or $\bar{\kappa}$ is large



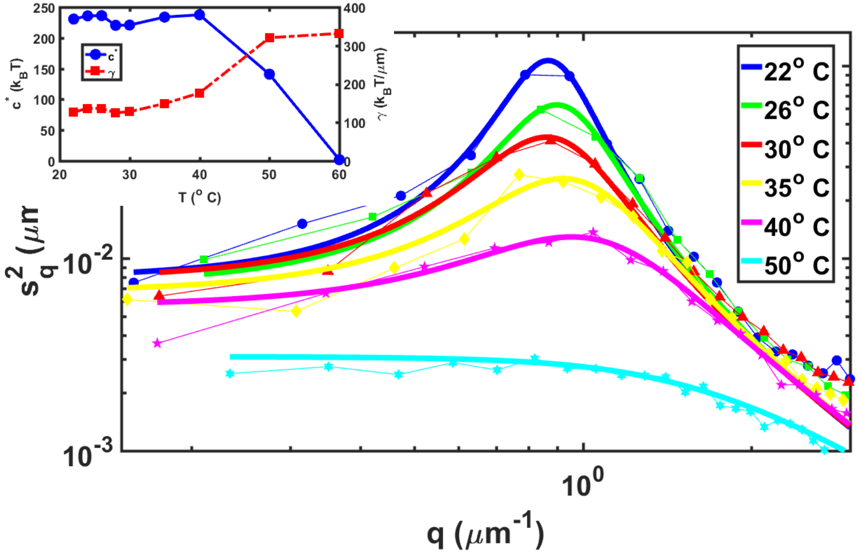
$$\langle u_q^2 \rangle = \frac{k_B T}{Bq^4 + \gamma q^2 - {c^*}^2 q^4 / (2Bq^2 + \gamma - 2\bar{\kappa}|q|)}$$

$$\langle v_q^2 \rangle = \frac{k_B T}{2Bq^4 + \gamma q^2 - 2\bar{\kappa}|q|^3 - {c^*}^2 q^4 / (Bq^2 + \gamma))}$$

Peak forms near instability with chirality



Peak forms near instability with chirality



Conclusion

- Out-of-plane fluctuation spectrum displays an anomalous peak in chiral samples
- Peak corresponds to onset of instability and grows as chirality and/or Gaussian curvature modulus increase
- Our **effective model that accounts for liquid crystalline order through geometric properties of the edge** quantifies the coupling of in- and out-of-plane fluctuations to produce the peak

Acknowledgements

BROWN

- Tom Powers (ENGN & PHYS)
- Bob Pelcovits (PHYS)

This work was supported by National Science Foundation grants

- **MRSEC DMR-1420382**
- **CMMI-1634552**

Brandeis BioInspired Soft Materials MRSEC

- Andrew Balchunas (Brandeis)
- Joanna Robaszewski (Brandeis)
- Prerna Sharma (Indian Institute of Science)
- Mark Zakhary (Mayo Clinic)
- Zvonimir Dogic (UCSB/Brandeis)

We are grateful to the **Brandeis U. MRSEC** for funding and resources.

Thank you for listening!