SIAM Conference on Applications of Dynamical Systems
MS59 Synchronization of Chaos and its Applications
May 17-21, 2015, Snowbird

# Chimera States in Networks with Symmetry-Breaking Coupling

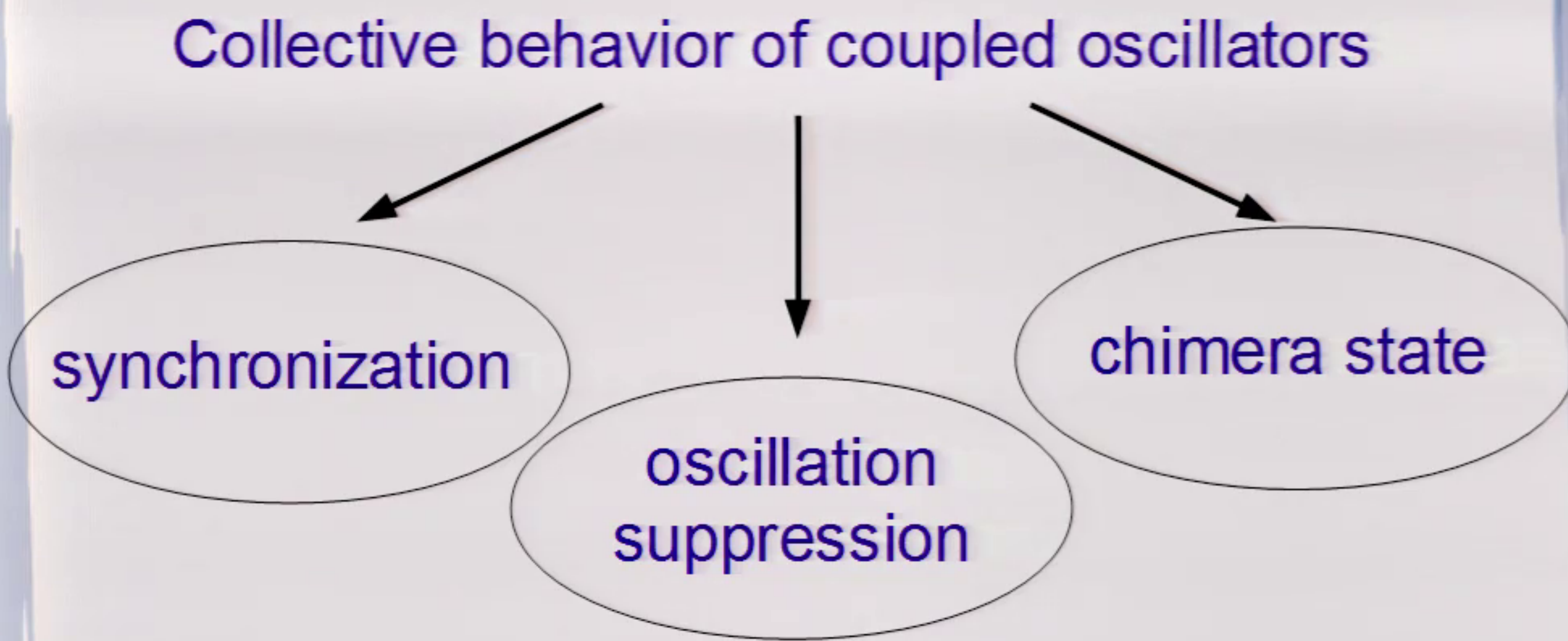
**Anna Zakharova**

Institute of Theoretical Physics
SFB 910 Control of self-organizing nonlinear systems
Technische Universität Berlin
Germany

In collaboration with
M. Kapeller, S. Loos, A. Gjurchinovski[1] and E. Schöll

[1]Sts. Cyril and Methodius University, Skopje, Macedonia

# Introduction



# Symmetry-breaking coupling

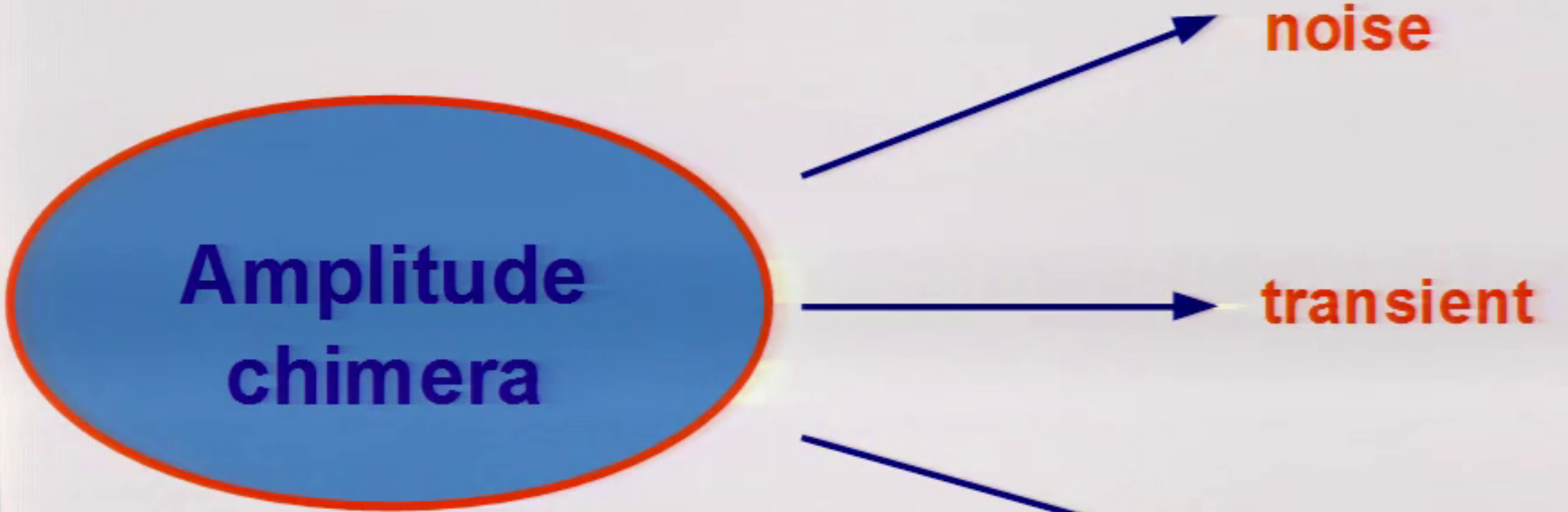
**Amplitude chimera**

# Symmetry-breaking coupling

**Amplitude chimera**

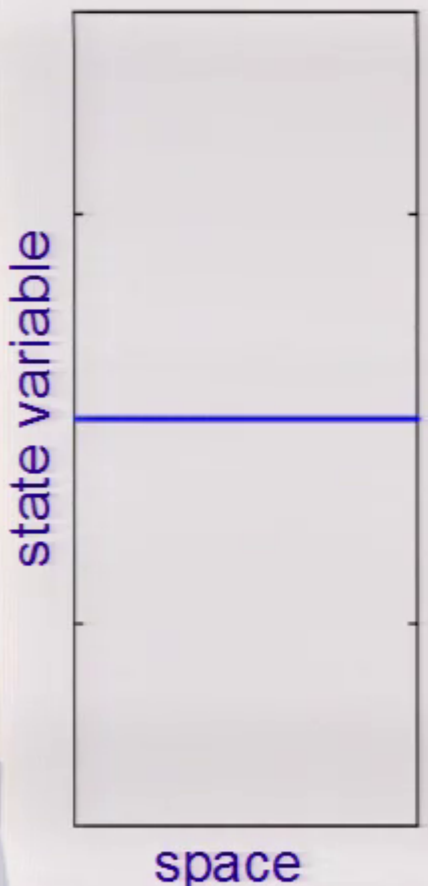
**Chimera death**

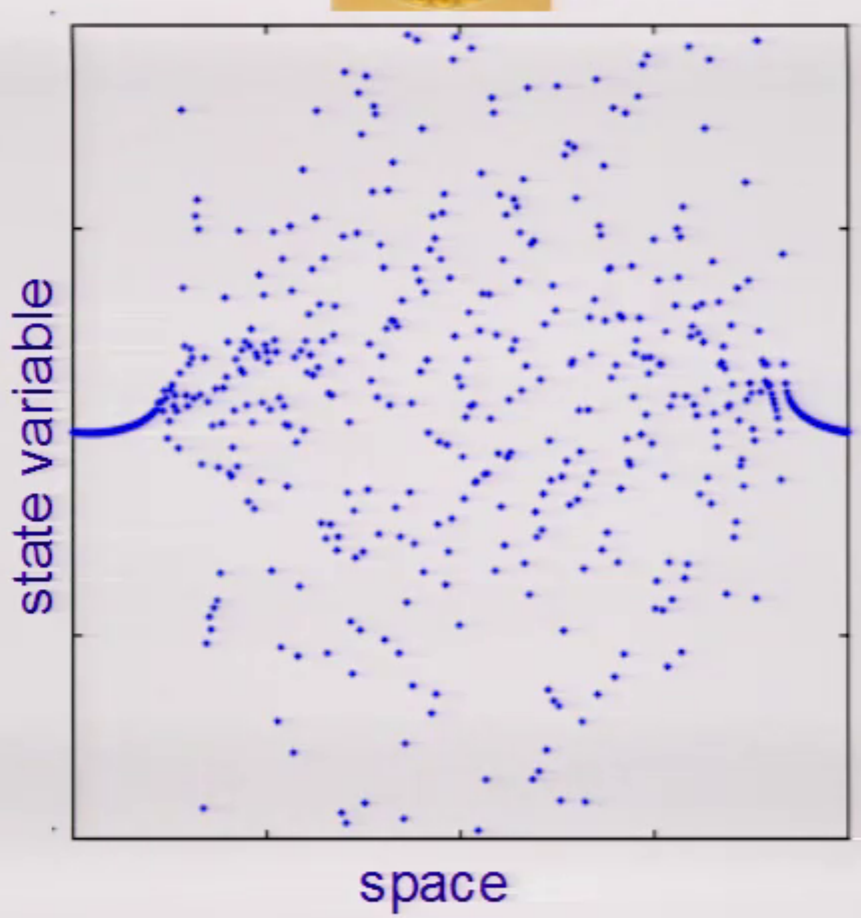
# Symmetry-breaking coupling



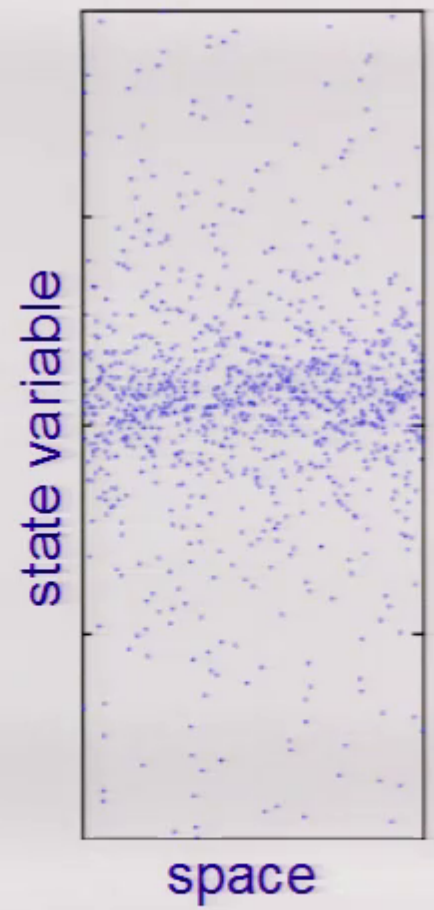
# What is a chimera state?

coherence




incoherence




Recent review:

M. J. Panaggio and D. M. Abrams, Nonlinearity 28, R67 (2015)

# Different chimera types

# Different chimera types

# Amplitude chimera

| | Amplitude | Phase |
|---|---|---|
| ◦ **Classical chimeras** Kuramoto and Battogtokh (2002), Abrams and Strogatz (2004) | no amplitude dynamics |  |
| ◦ **Amplitude-mediated chimeras** Sethia and Sen (2014) |  |  |
| ◦ **Amplitude chimeras** Zakharova, Kapeller, Schöll (2014) |  |  |

# Model

Stuart-Landau oscillator

$$\dot{z} = f(z) \equiv (\lambda + i\omega - |z|^2)z$$

single node dynamics

$$z(t) = x(t) + iy(t),$$

$$z \in C; \lambda, \omega \in R.$$

local

# Model

Stuart-Landau oscillator

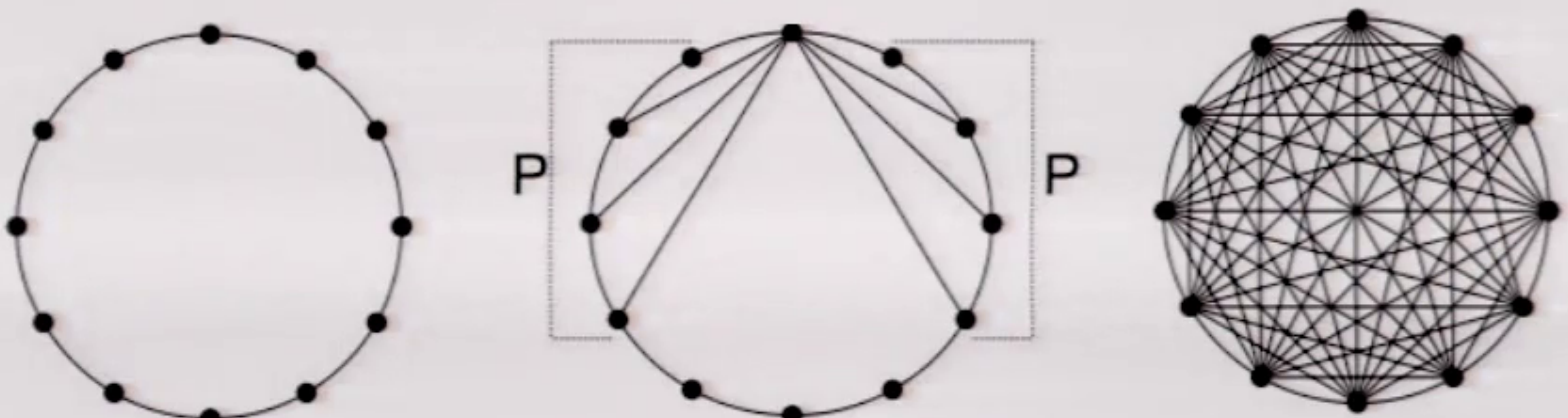
$$\dot{z} = f(z) \equiv (\lambda + i\omega - |z|^2)z$$

single node dynamics

$z(t) = x(t) + iy(t),$

$z \in C;\ \lambda, \omega \in R.$

***P* – number of nearest neighbors in each direction on a ring**




local　　**nonlocal**　　global

# Model

Network of nonlocally coupled Stuart-Landau oscillators

$$\dot{z}_j = f(z_j) + \frac{\sigma}{2P} \sum_{k=j-P}^{j+P} (\mathrm{Re}\, z_k - \mathrm{Re}\, z_j)$$

$j = 1, 2, \ldots, N$

$f(z_j)$ – dynamics of individual element

$P$ – number of coupled neighbors in each direction

$\sigma$ – coupling strength

$N$ – total number of elements

$P/N$ – coupling range



A. Zakharova, M. Kapeller, E. Schöll, *Phys. Rev. Lett.* 112, 154101 (2014)

# Model

Network of nonlocally coupled Stuart-Landau oscillators

$$\dot{z}_j = f(z_j) + \frac{\sigma}{2P} \sum_{k=j-P}^{j+P} (\operatorname{Re} z_k - \operatorname{Re} z_j)$$

**Symmetry-breaking coupling through the real variable**

*j* =1,2,…,*N*

$f(z_j)$ – dynamics of individual element

*P* – number of coupled neighbors in each direction

σ – coupling strength

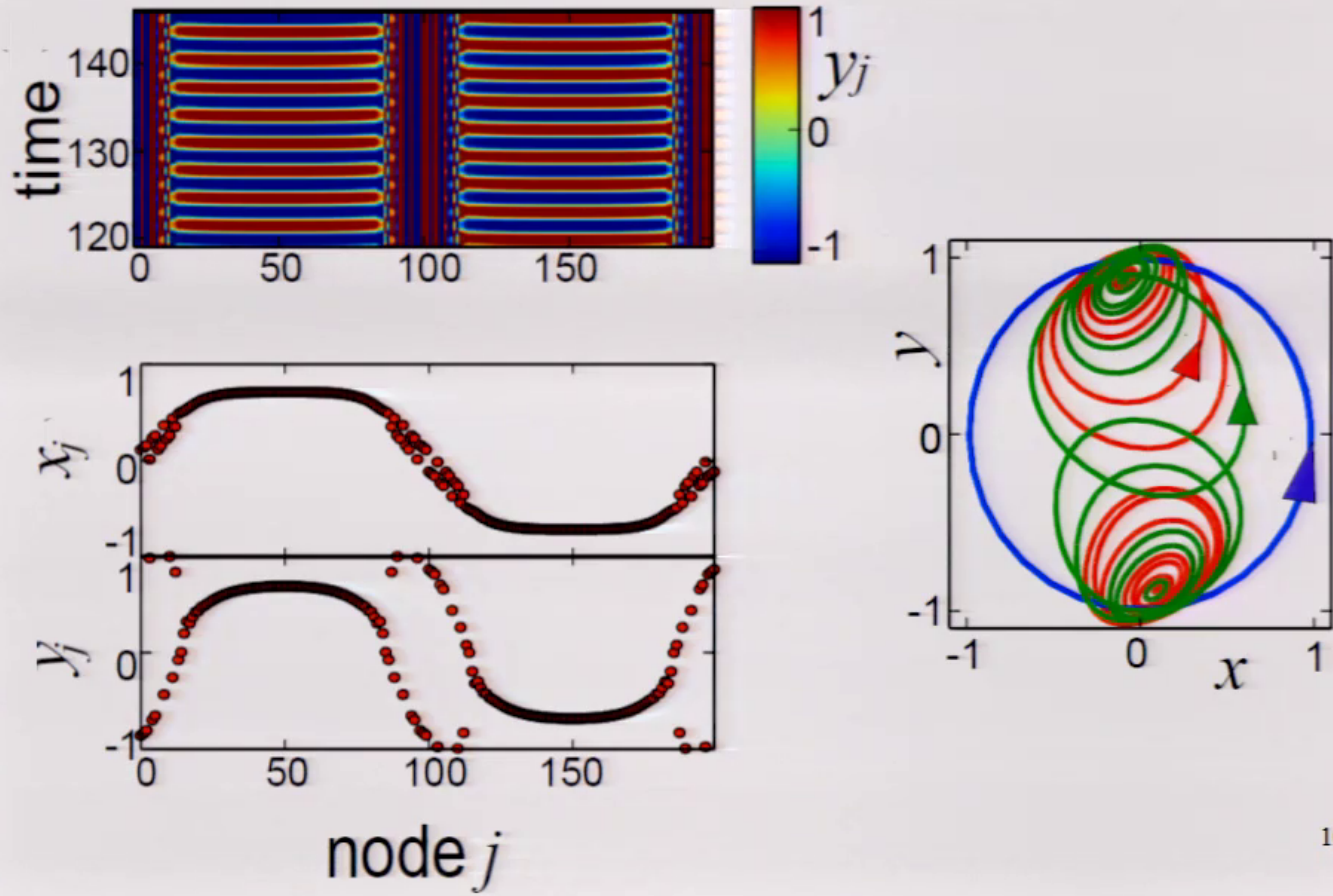
*N* – total number of elements

*P/N* – coupling range



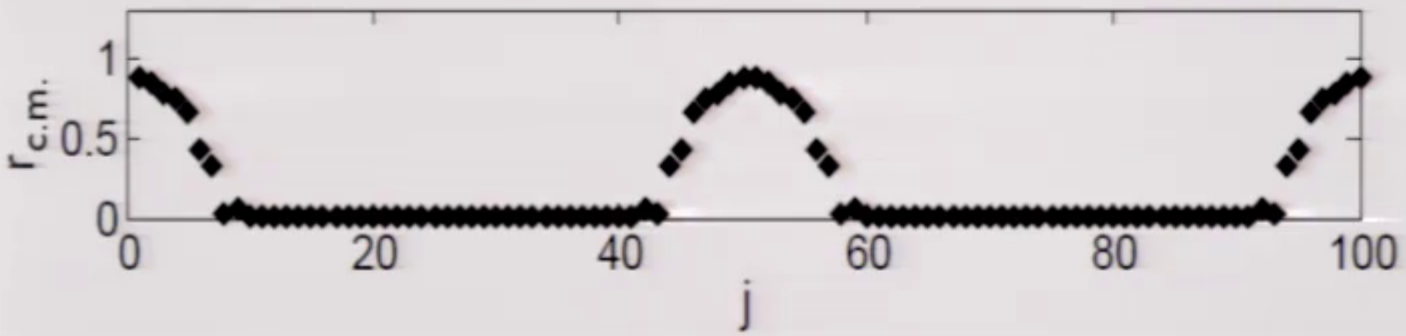

**A. Zakharova, M. Kapeller, E. Schöll, *Phys. Rev. Lett.* 112, 154101 (2014)**

# Amplitude chimera



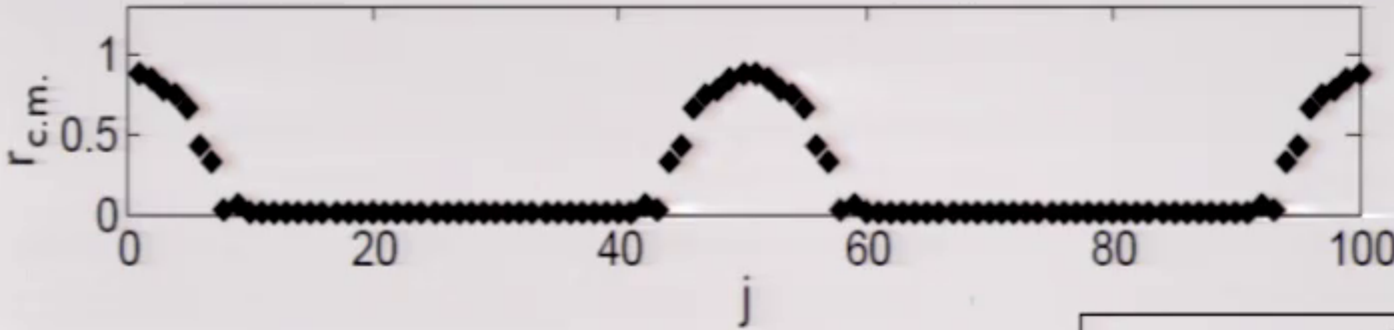
# Amplitude chimera

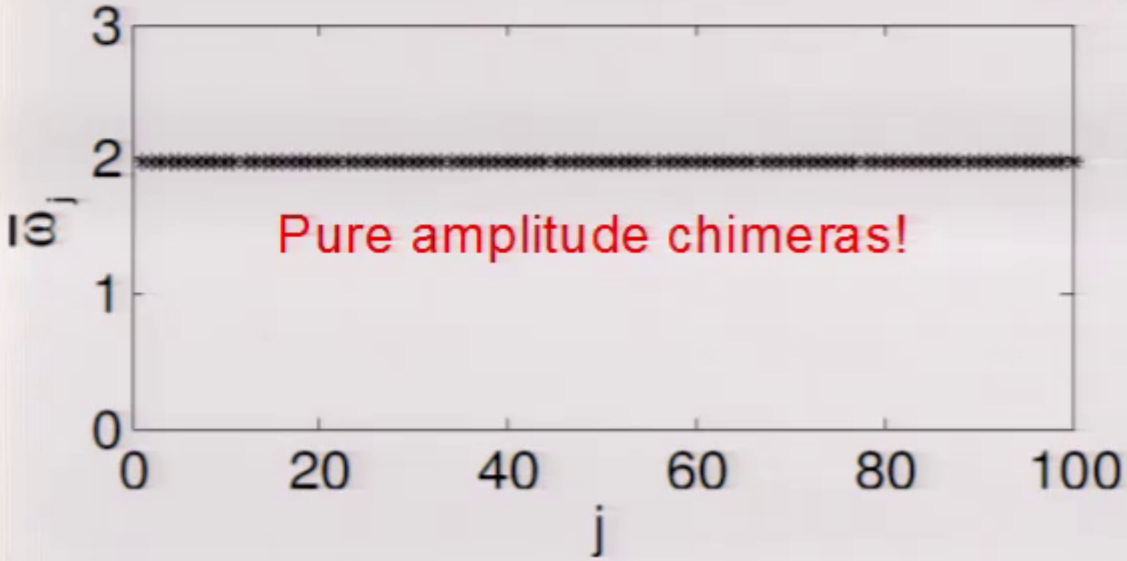
Distance between the center of mass and the origin

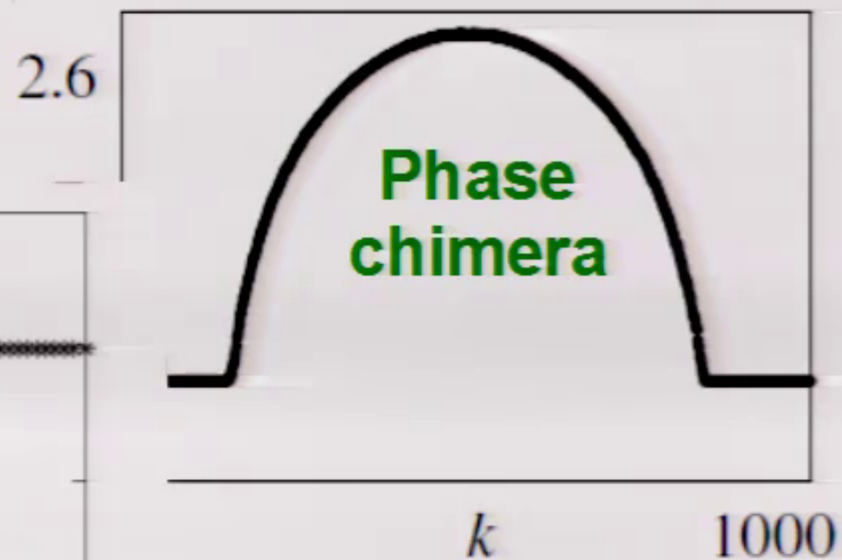


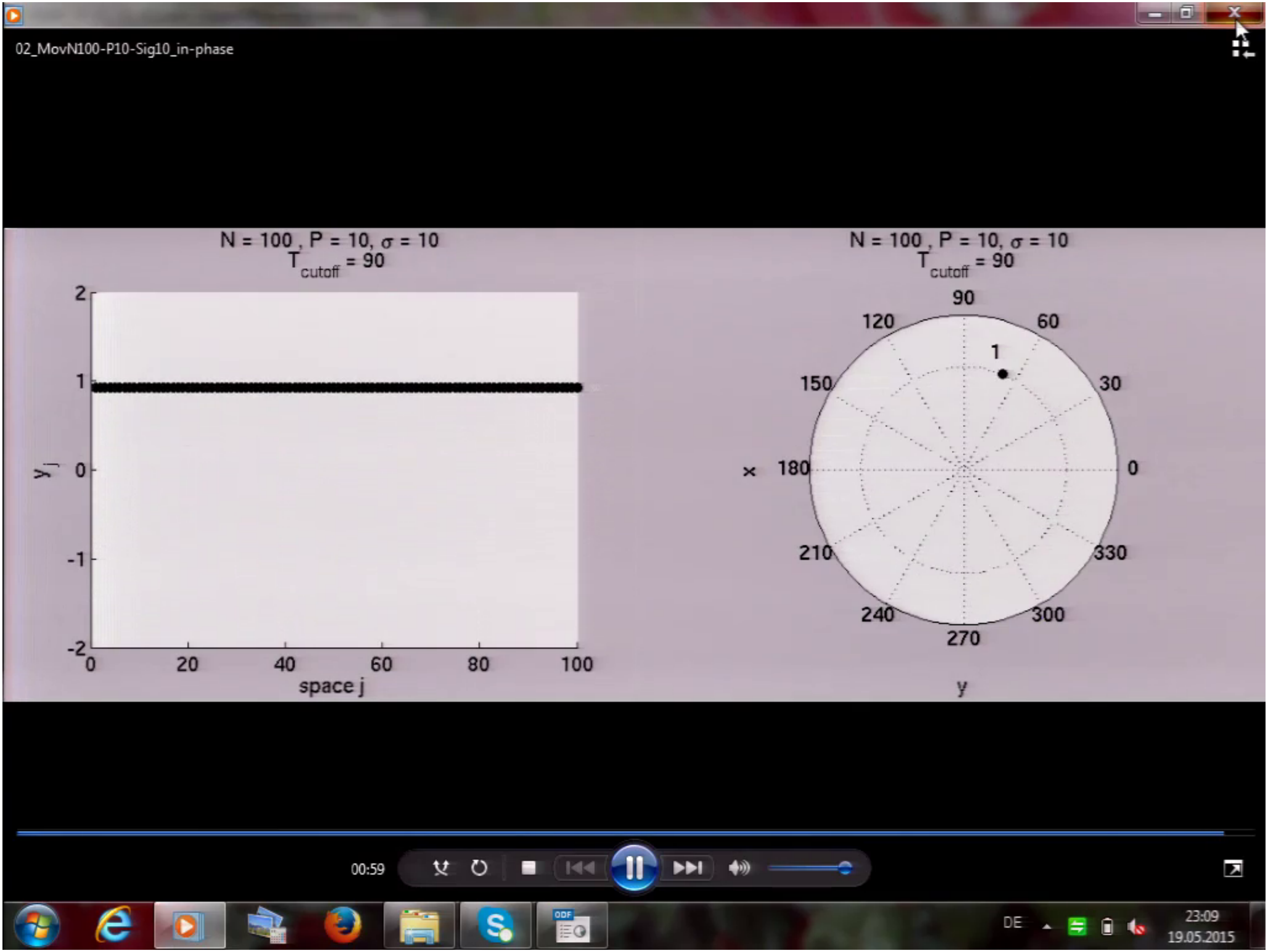
# Amplitude chimera

Distance between the center of mass and the origin




02_MovN100-P10-Sig10_in-phase
N = 100 , P = 10, σ = 10
T cutoff = 90
y j
space j
N = 100 , P = 10, σ = 10
T cutoff = 90
x
y
00:59
DE
23:09
19.05.2015

# Classical chimeras:
# the dynamics is chaotic in time

M. Wolfrum and O. Omel'chenko, Chimera states are chaotic transients, *Phys Rev E 84*, 015201 (2011)

**Classical chimeras:**

**the dynamics is chaotic in time**

M. Wolfrum and O. Omel'chenko, Chimera states are chaotic transients, *Phys Rev E 84*, 015201 (2011)

**Amplitude chimeras:**

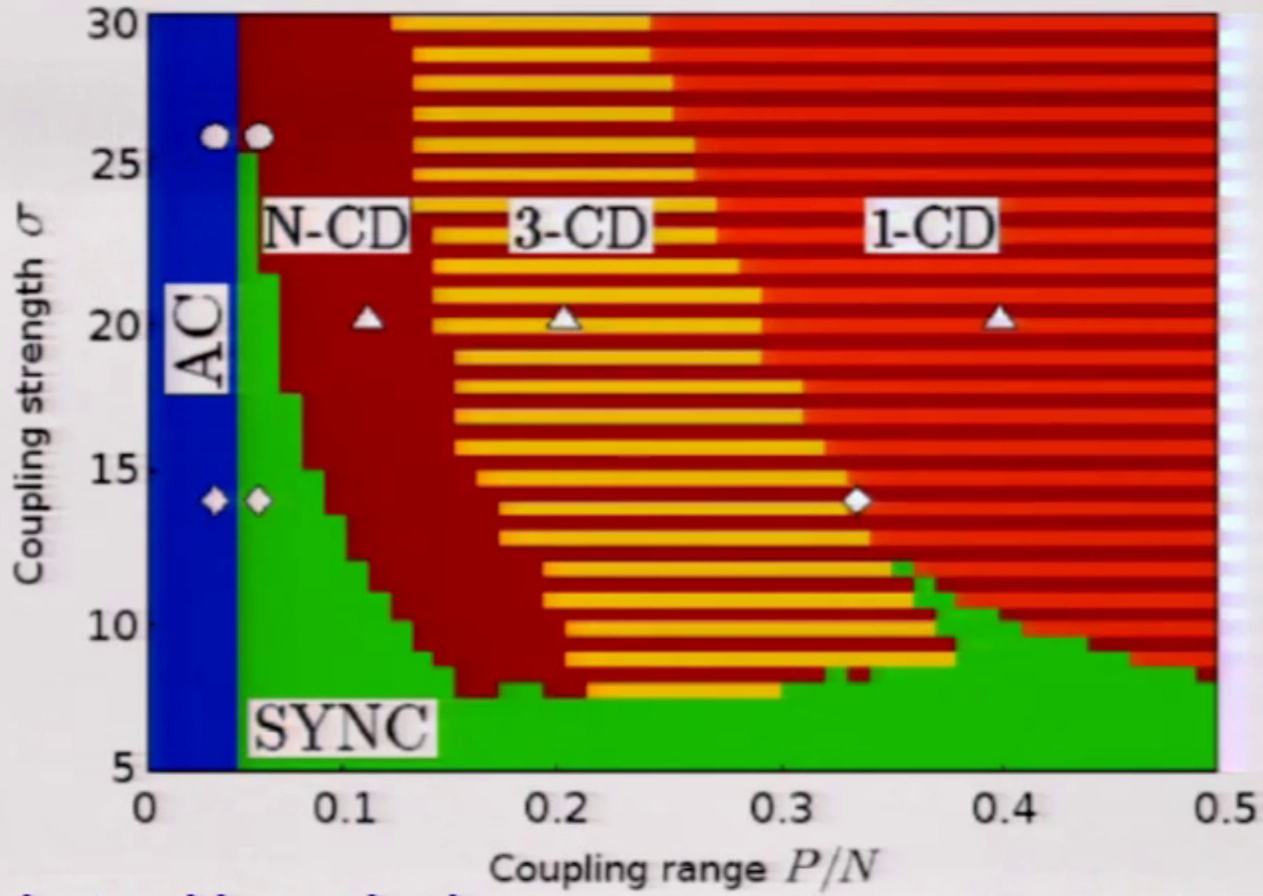
**the dynamics is periodic in time!**

**Amplitude chimeras are long-living transients!**

A. Zakharova, M. Kapeller, E. Schöll, Amplitude chimeras and chimera death in dynamical networks, arXiv:1503.03371 (2015)

# Dynamic regimes

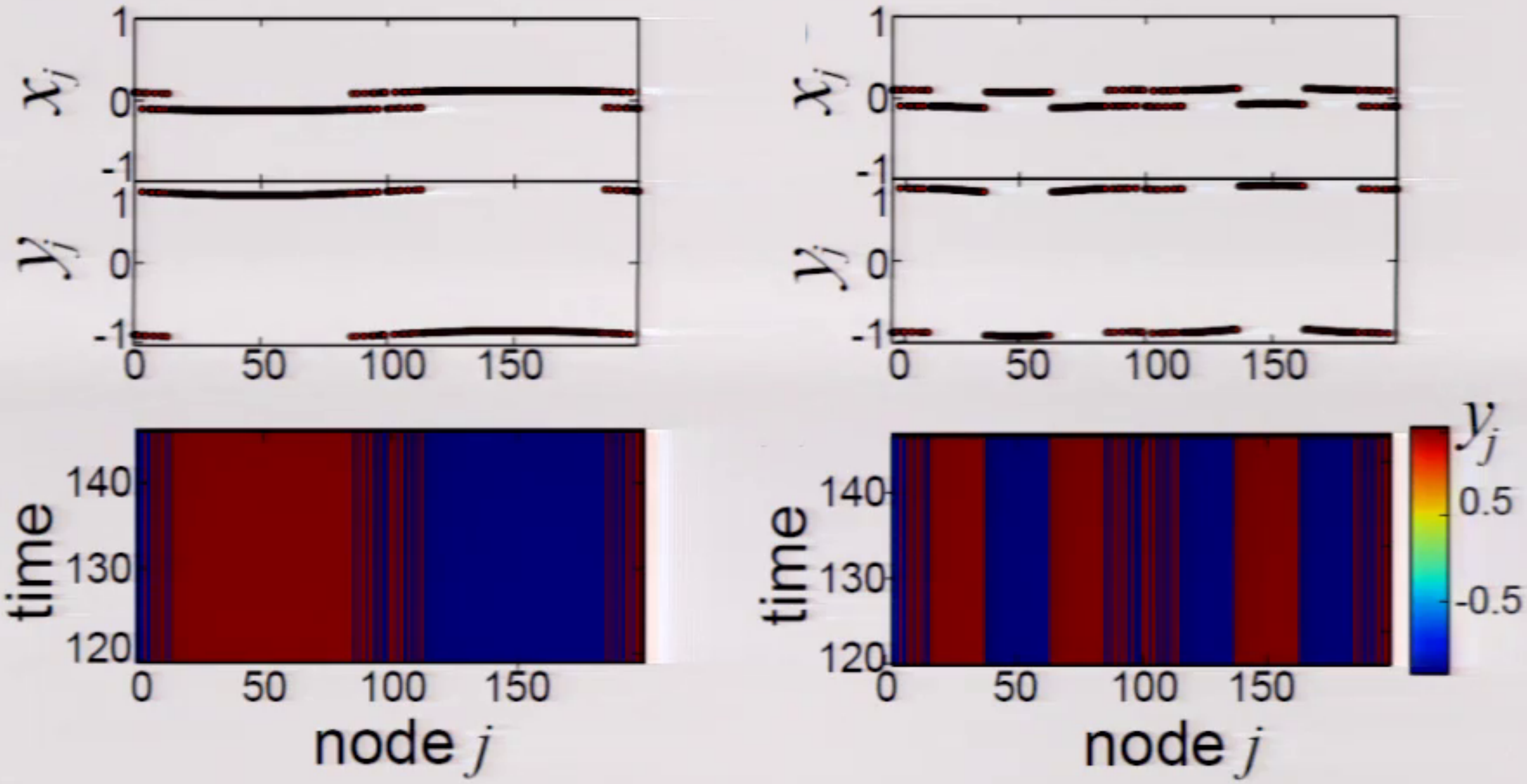
- Oscillatory behavior: **amplitude chimera** and **in-phase synchronized regime**
- Steady state: **chimera death** – inhomogeneous steady state




**1-CD: 1-cluster chimera death**
**3-CD: 3-cluster chimera death**
**N-CD: multicluster (>3) chimera death**

**AC: amplitude chimera**
**SYNC: in-phase synchronized oscillations**

# Chimera death: chimeras of steady states




Coexisting domains of spatially coherent and spatially incoherent oscillation death

# What is the impact of noise?

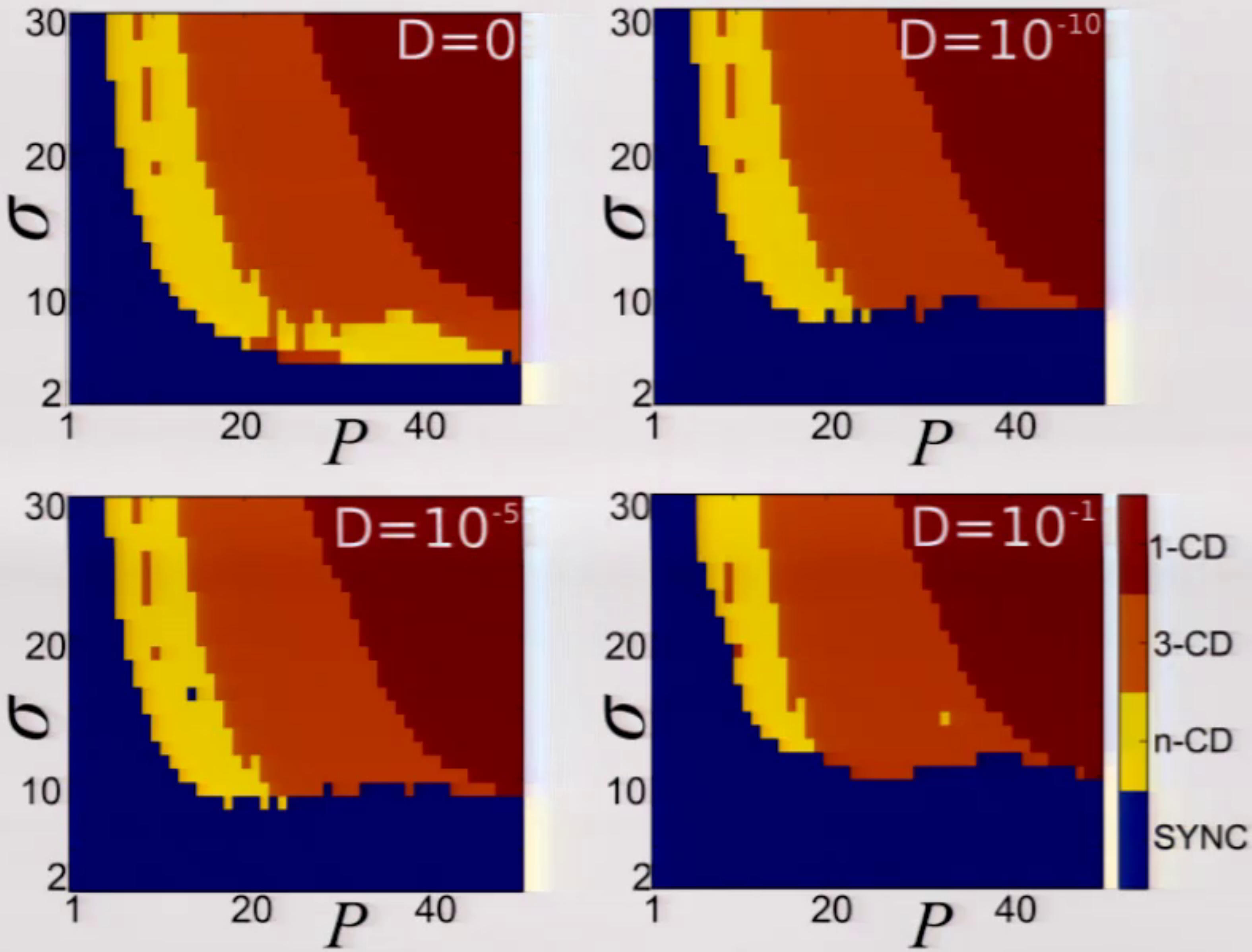
Network of Stuart-Landau oscillators

$$\dot{z}_j = f(z_j) + \frac{\sigma}{2P} \sum_{k=j-P}^{j+P} [Re(z_k) - Re(z_j)]$$

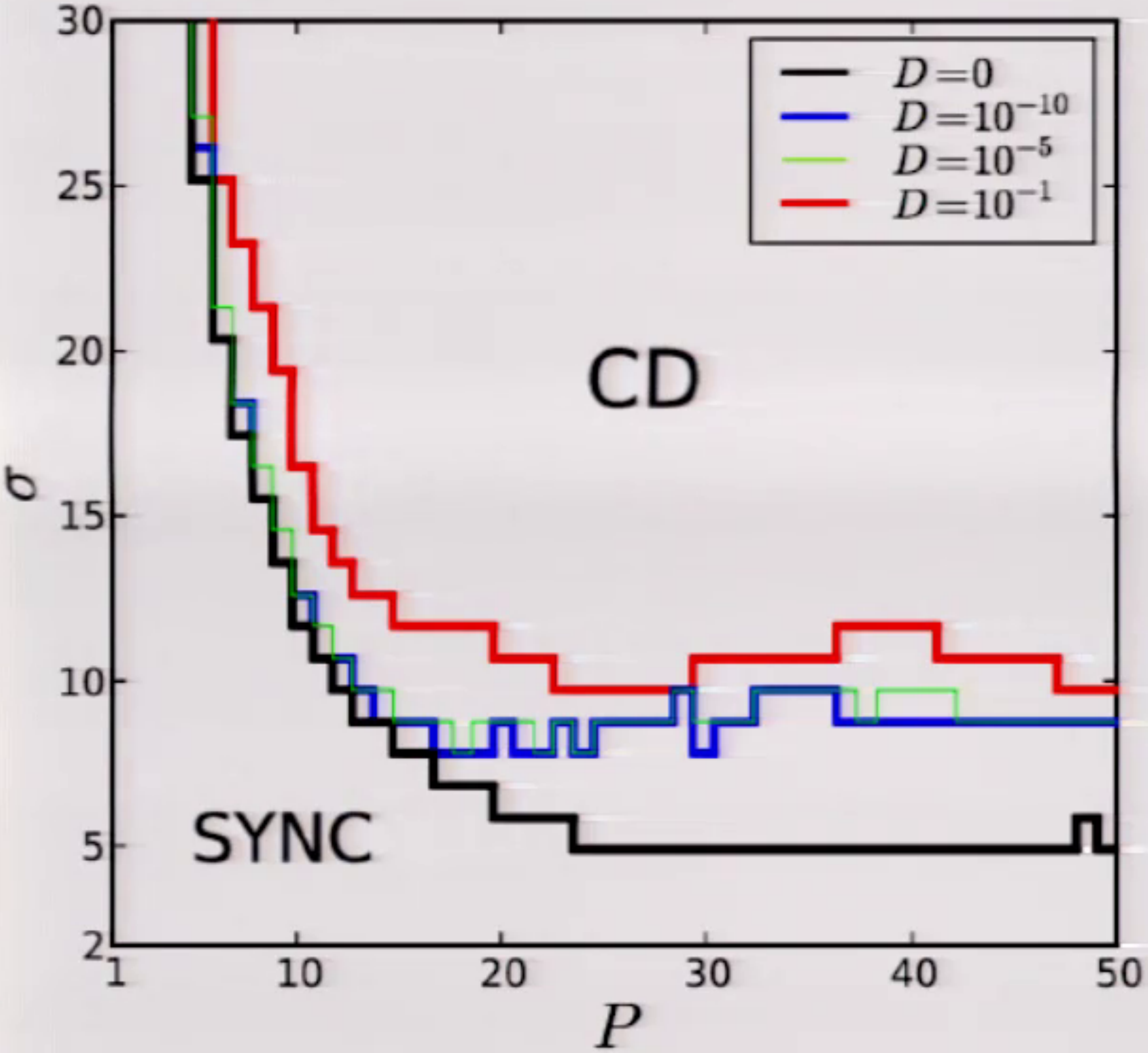
Network of Stuart-Landau oscillators with noise

$$\dot{z}_j = f(z_j) + \frac{\sigma}{2P} \sum_{k=j-P}^{j+P} [Re(z_k) - Re(z_j)] + \sqrt{2D}\,\xi_j(t)$$

# Impact of noise

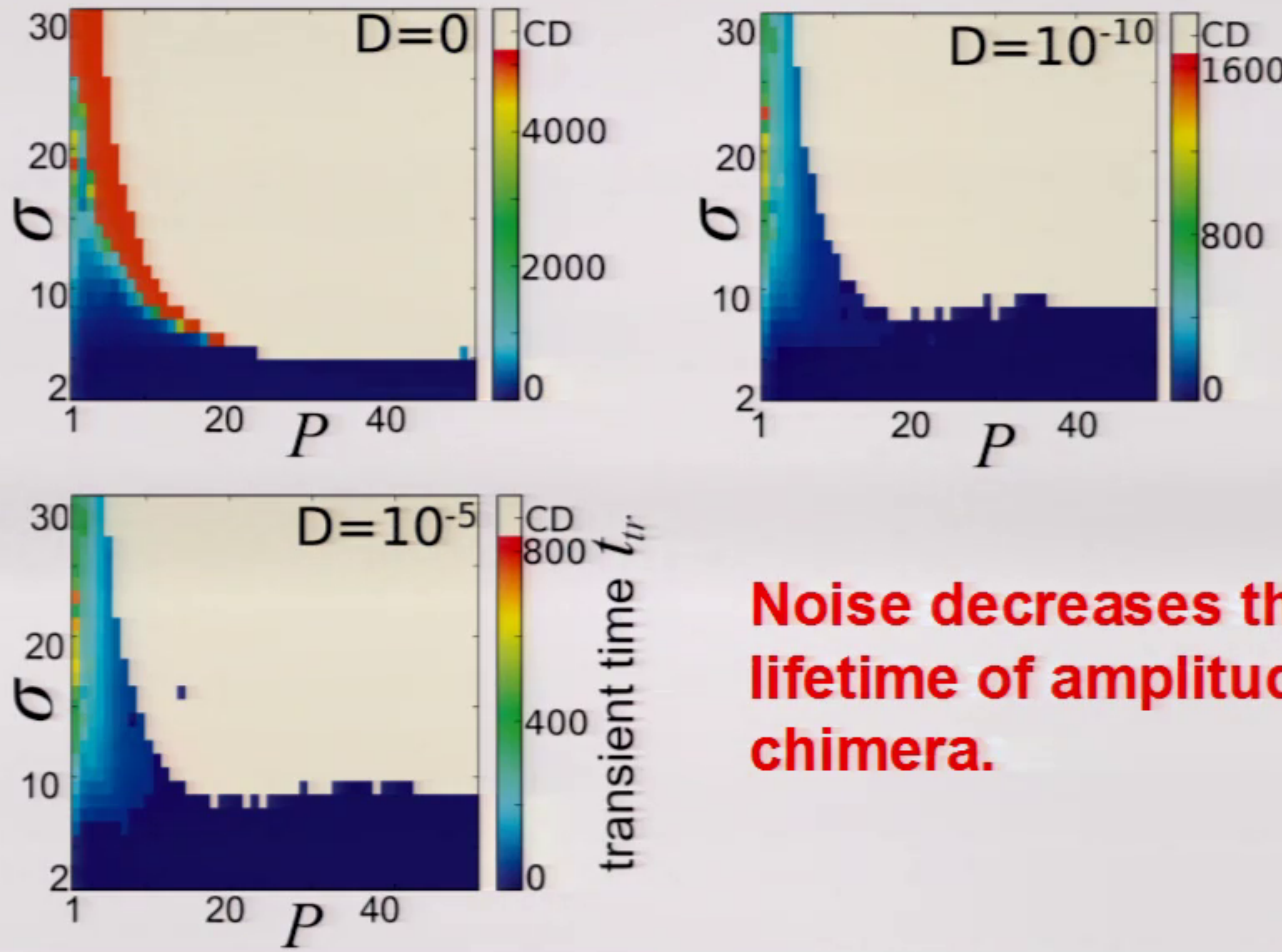


# Impact of noise




**The stronger the noise, the larger the oscillatory regime (SYNC).**

# Impact of noise




**Noise decreases the lifetime of amplitude chimera.**

# What is the impact of time delay?

Network of Stuart-Landau oscillators

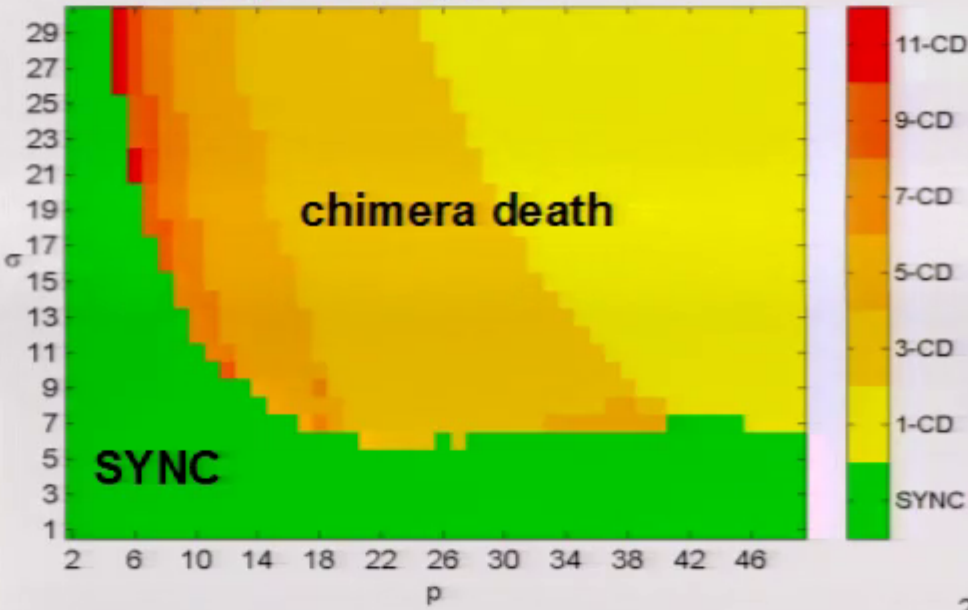
$$\dot{z}_j = f(z_j) + \frac{\sigma}{2P} \sum_{k=j-P}^{j+P} \left[ Re(z_k) - Re(z_j) \right]$$

Network of Stuart-Landau oscillators with delay

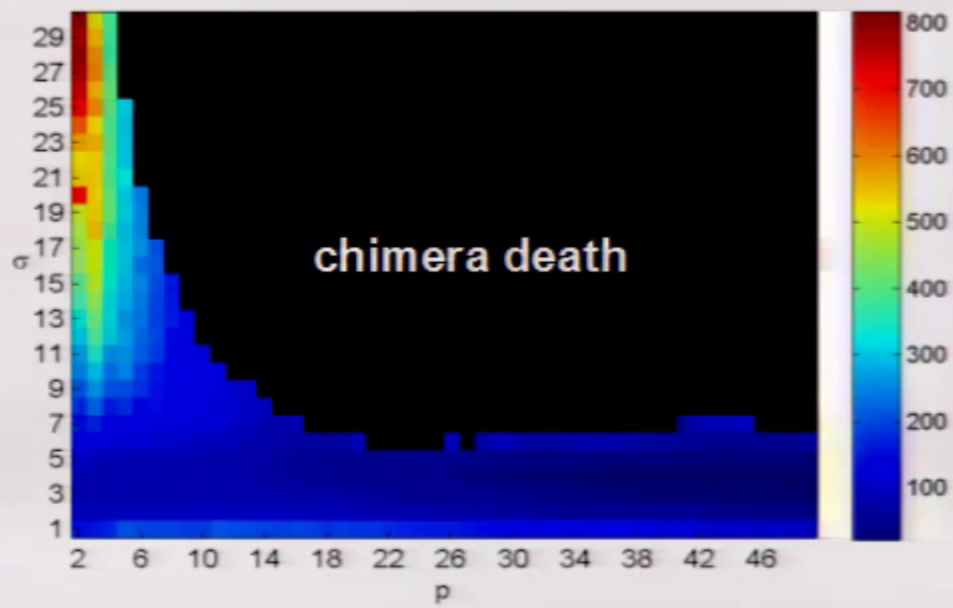
$$\dot{z}_j = f(z_j) + \frac{\sigma}{2P} \sum_{k=j-P}^{j+P} \left[ Re\big(z_k(t-\boldsymbol{\tau})\big) - Re\big(z_j(t)\big) \right]$$

# Lifetime of amplitude chimeras: no time delay

## Map of regimes




## Transient time

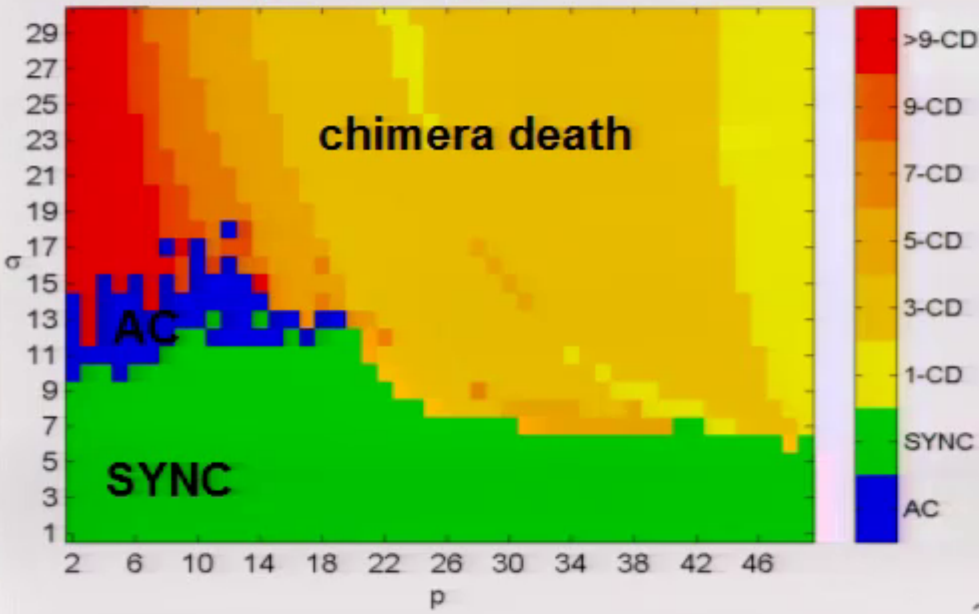



**No amplitude chimeras beyond t=900**

Integration time t=5000

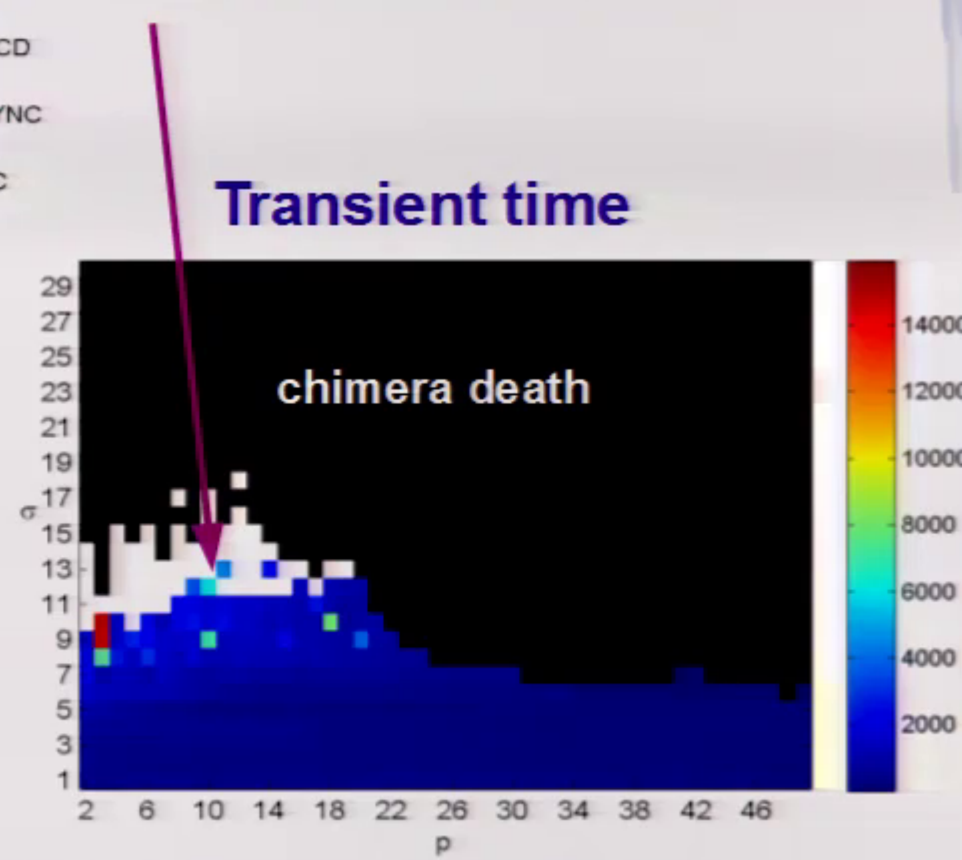
# Impact of time delay ($\tau=\pi$)

## Map of regimes




Amplitude chimera lifetime t=20000

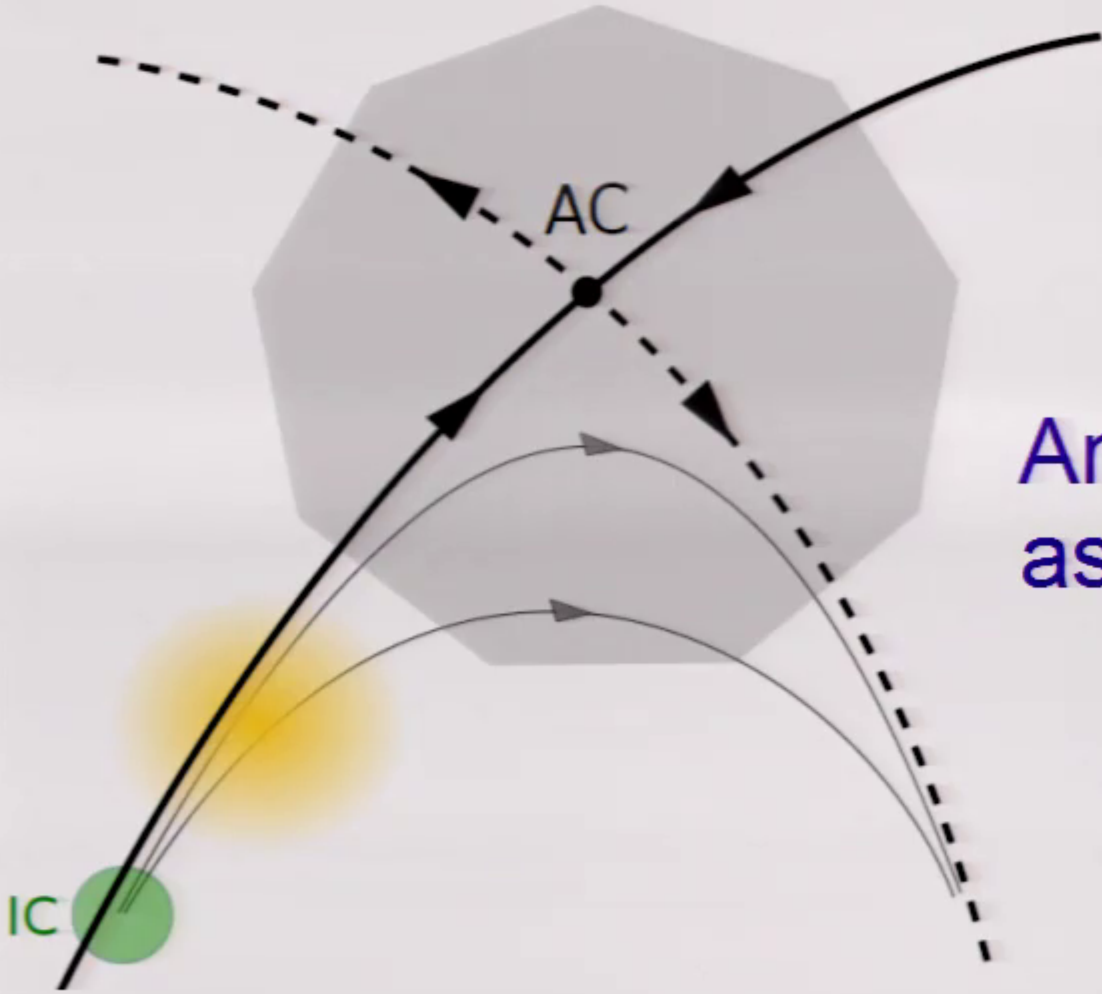
## Transient time




**Time delay enlarges the lifetime of amplitude chimeras!**

Integration time t=20000

# Open questions




Amplitude chimera as a saddle

# Open questions

Constructive role of noise?

# Chimeras in nature?

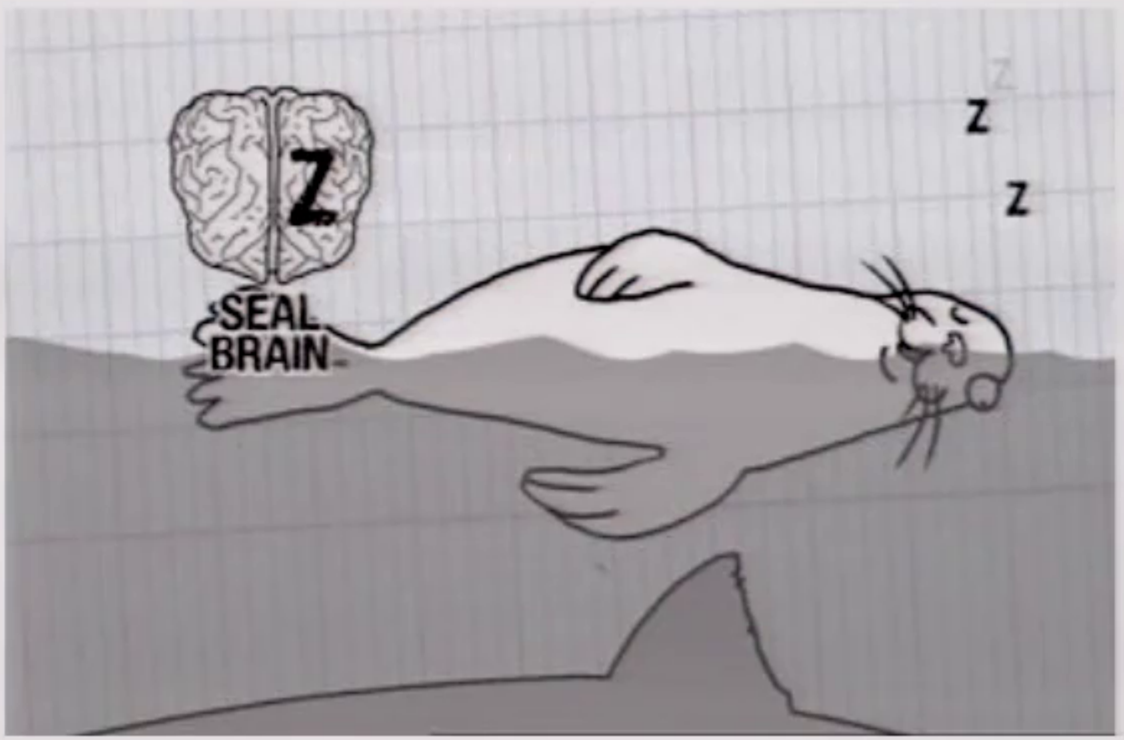
Unihemispheric sleep: one half of the brain is highly synchronized (sleeping) while the other half remains desynchronized (awake).

- dolphins and seals: escaping from predators and surfacing for air while sleeping

# Chimeras in nature?

Unihemispheric sleep: one half of the brain is highly synchronized (sleeping) while the other half remains desynchronized (awake).

- dolphins and seals: escaping from predators and surfacing for air while sleeping



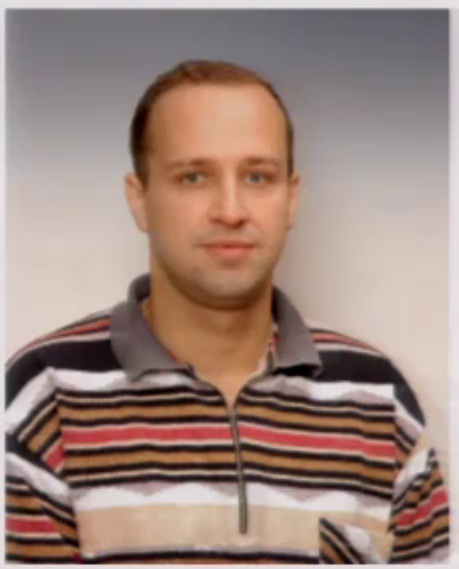
# Thanks to my collaborators

Marie Kapeller

Sarah Loos

Julien Siebert

A. Gjurchinovski



E. Schöll