Parallel Network Analysis

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My Topic

- What's new with me (Sandia National Laboratories) in

  Discrete math/algorithms ∩

  High-performance computing ∩

  Applications?

Why would we want to use a parallel algorithm for an application?

- When we have to:
  - Too Slow
  - Too Big
  - Too Inaccurate

- Application evolution
  - More constraints
  - Finer discretization
  - Larger instances

Overview

- Applications
  - Sensor network design/management
  - Analysis of large-scale (e.g. social) networks

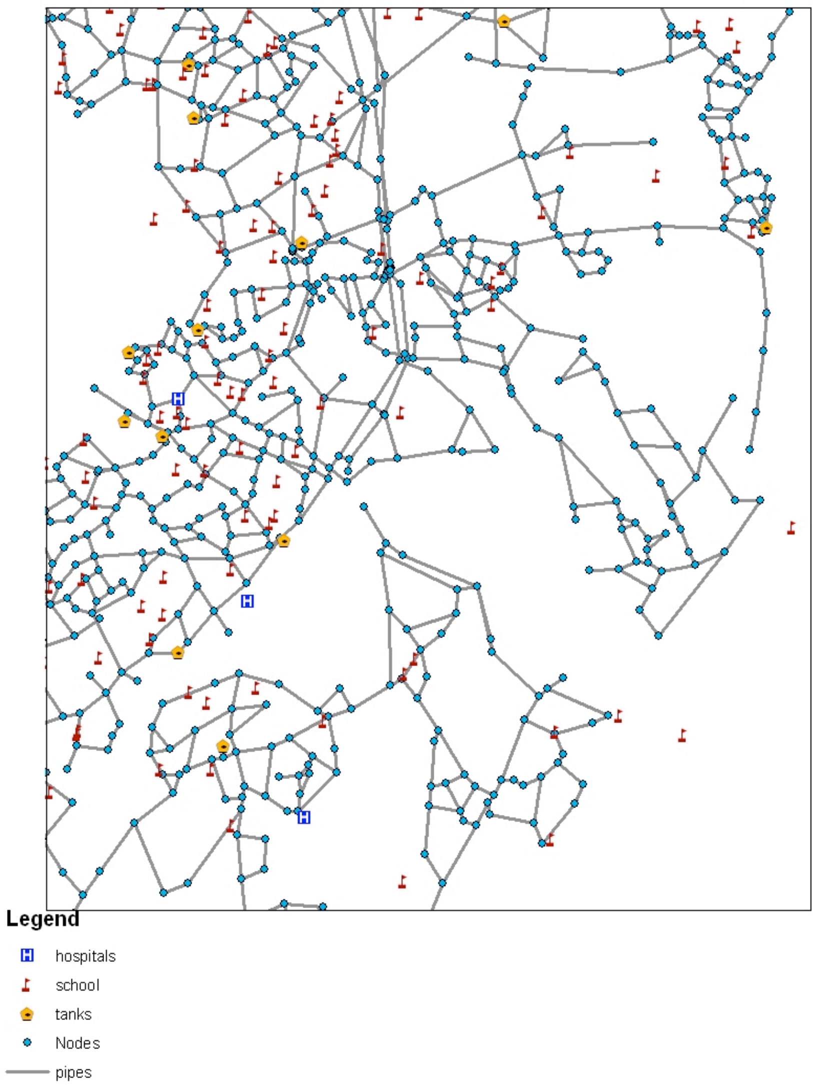
- Methods
  - Coarse-grained (“embarrassing” parallelism)
  - Unusual (for us) hardware/architecture
    - Old primitives
  - Memory reduction → parallel algorithms

A Sensor Placement Problem

Issue: Contamination released in a municipal water network

Goal: Place k sensors on network nodes as an early warning system

- Protect human populations
- Limit network remediation costs



Sponsored by the US Environmental Protection Agency (EPA) National Homeland Security Research Center (NHSRC)

One Water Sensor Placement Formulation

Given an enumerable set of events: (location, time) pairs

- Simulate the evolution of a contaminant plume
- For each event determine
  - Where event can be observed
  - Impact prior to that observation

- Assume first sensor witness of contamination signals general alarm

- Minimize average impact

There can be 100,000s to millions of scenarios.

- Parallelize the simulations and impact calculations

Obvious, but important: from weeks to hours using cluster

Finding an Approximate Solution

- For more complex versions, can express as an integer program
  - Linear objective, linear and integrality constraints
- Benchmarks heuristics
- A good approximate solution
  - Speeds search (enables pruning)
  - Allows early stop

- Use linear programming relaxation (drop integrality)
- LP optimum gives lower bound, fractional solution
- Goal: “round” to a real solution

- Trials completely independent

k of N selection

Given N variables with $0 < x_i^* < 1$ for $i = 1..N$ and $\sum_i x_i^* = k$

- Select k of the x_i^* such that probability of selecting i is reasonably related to x_i^*

In multiple applications, this selection is the main (only) decision

- Sensor placement
- Mobile sink scheduling for wireless sensor networks
- Picking a tail in robust optimization formulations
- Enforcing node degree in graph generation

Randomized Rounding

- Simplest form: treat $0 \leq x_i \leq 1$ as probability
- Set $y_i = 1$ with probability x_i and $y_i = 0$ otherwise
- If don't select exactly k, try again (and again…)
- But can use conditional Poisson sampling to efficiently sample from this "lucky" distribution (Chen, Dempster, Lui, 2004)
- Use dynamic programming to precompute conditional probabilities
- Decode a random toss to a feasible solution
- Selects uniformly over "lucky" tosses.

Rounding with One Cardinality Constraint

Doerr (2004), motivated by Srinivasen (2001)

Finds a randomized rounding y such that:

- $\Pr(y_i = 1) = x_i^*$
- $\sum_i y_i = k$ (respects cardinality constraint)

Simple (base) case

All x_i^* are 1/2.

Let X be the set of x_i^* with value 1/2.

|X| is even because $\sum_i x_i^* = k$ and k is integer

Pair elements of X: (x_i^*, x_j^*)

Set (y_i, y_j) = (1,0) or (0,1) each with probability 1/2.

$$(0,1) \xleftarrow{\frac{1}{2}} \left(\frac{1}{2}, \frac{1}{2}\right) \xrightarrow{\frac{1}{2}} (1,0)$$

General Case

- Do the base case for lowest order bit ℓ (most to right of binary point)

$$\left(x_i - 2^{-\ell}, x_j + 2^{-\ell}\right) \xleftarrow{\frac{1}{2}} \left(x_i, x_j\right) \xrightarrow{\frac{1}{2}} \left(x_i + 2^{-\ell}, x_j - 2^{-\ell}\right)$$

- After this operation, the rightmost bit is in place ℓ - 1.
- Iterate to compute y in O(n ℓ) time.
  - n = number of variables $0 < x_i^* < 1$
  - ℓ = lowest order bit of any of the x_i^*, maybe 1000
- Numerical issue: In (floating point) practice, $\sum_i x_i^* = k$ not an integer

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Cardinality-Constrained Rounding Summary

- Doerr
  - O(nL) time $L \approx 1000$ (multiprecision)
  - Paired total correlation, otherwise independent
  - $\Pr(y_i = 1) = x_i^*$

- Conditional Poisson sampling
  - O(k(n-k)) preprocessing (k < 100), then O(n) sampling
  - Pairwise independence
  - $p(y_i = 1) = p(\mathrm{y}_\mathrm{i} = 1 \mid k \text{ selected})$
  - 3 orders of magnitude faster
  - Any subset is possible

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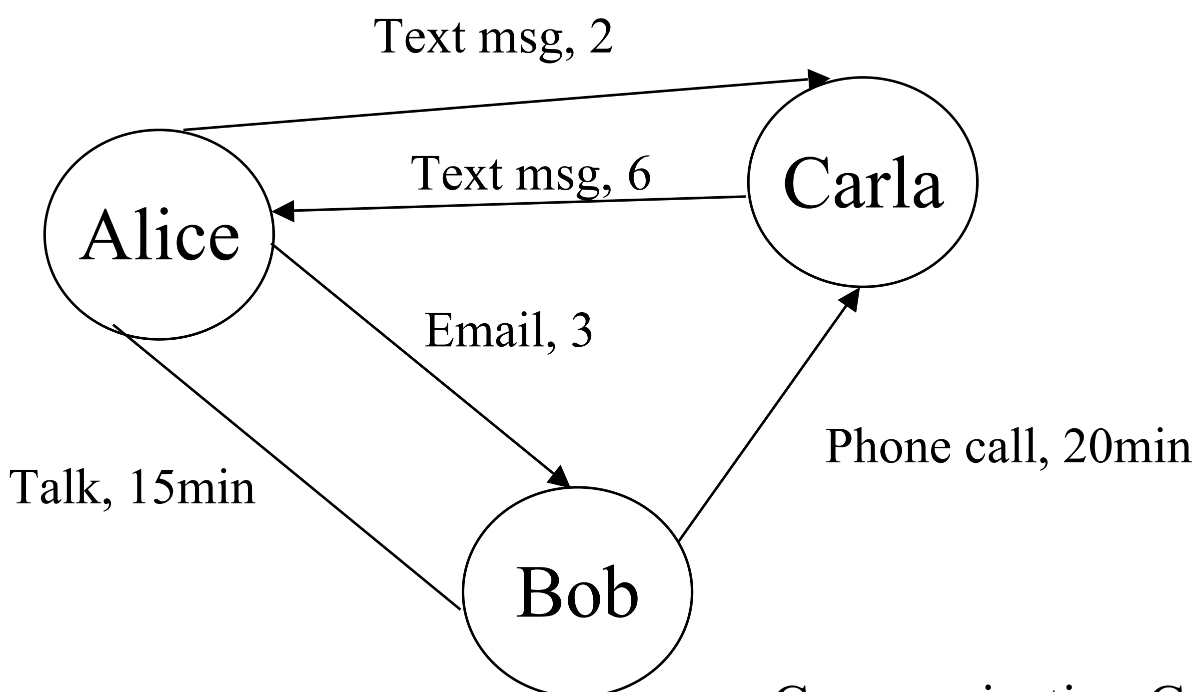
Embrace "Embarrassing" Parallelism

- Other recent uses
  - Integer programming pseudocost initialization
  - Feasibility pump integer programming heuristic
  - Progressive Hedging for stochastic programs
  - Constraint generation for scheduling mobile sinks in a wireless network

- Embarrassing parallelism increases the maximum feasible problem size
- Buys time to do the harder parallelization if necessary
- Using it can present other interesting algorithmic questions

Graph Analysis

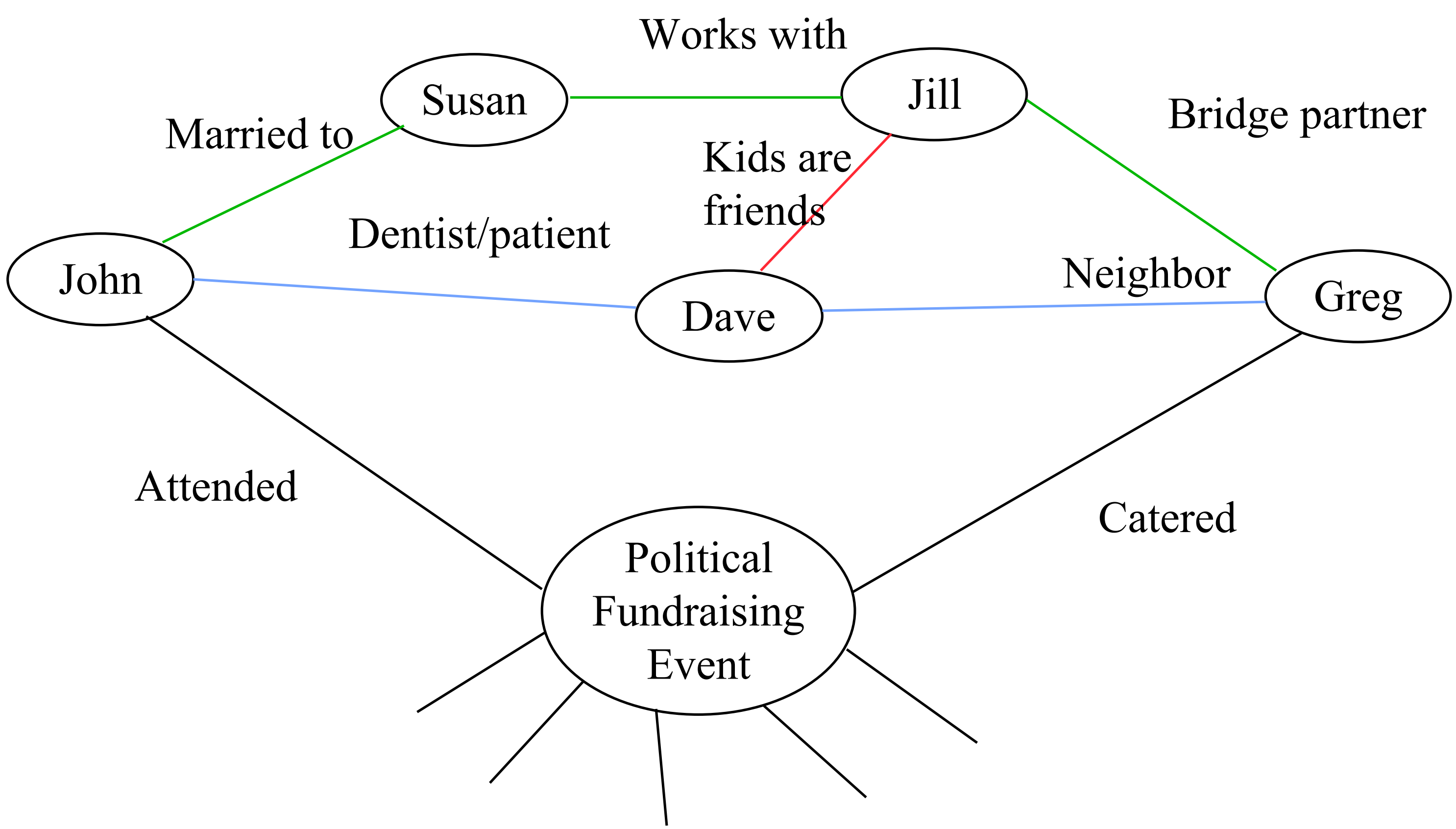
- Nodes (circles) represent entities
- Edges (lines) represent a relationship between a pair of entities
- Nodes and/or edges can have labels (names) and weights (values)



Communication Graph

A Semantic Relationship Graph

Every path between two points represents a potential relationship

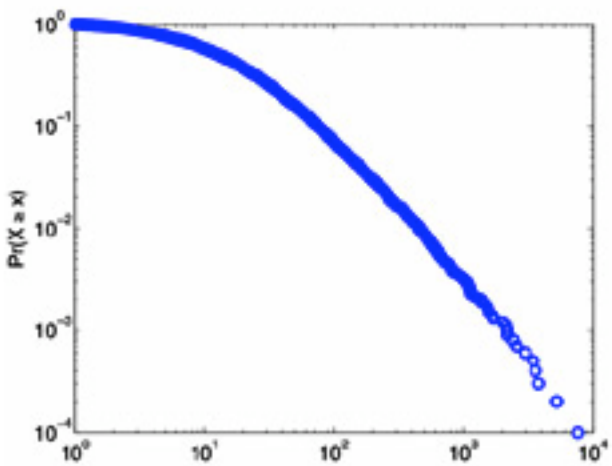


Analysis of Massive Graphs

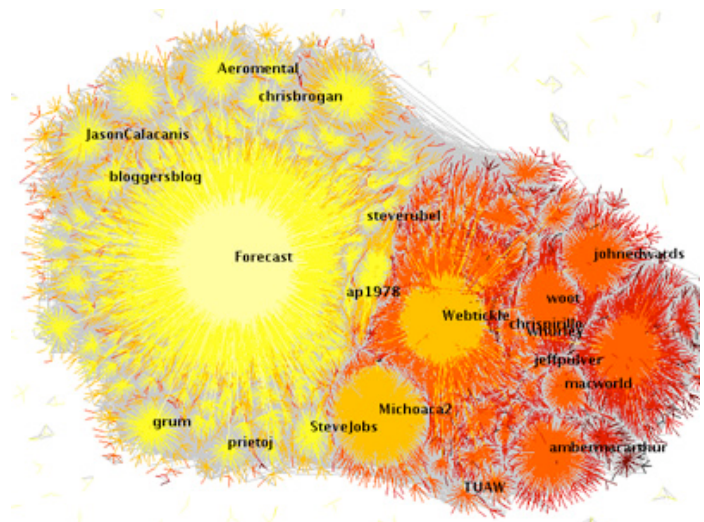
- Finding communities
  - Subgraphs where nodes are more connected to each other than to the rest of the graph
- Exploring relationships between individuals
- Finding patterns (normal/abnormal)

- Power law degree distribution common

Frequency (log scale)



[Clauset, 2007] Degree (log scale)



Twitter social network (|V|≈200M)

[Akshay Java, 2007]

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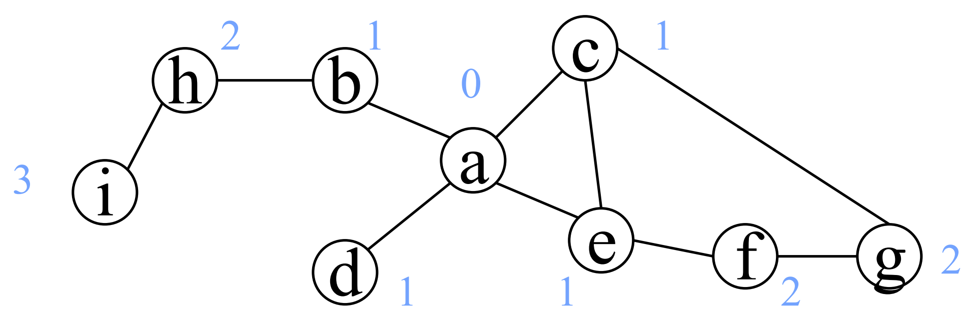
Graph Algorithms

- There are many good serial algorithms (powerful modeling tool)
  - Generally nodes gather information from neighbors, traversals
  - Large amount of communication relative to computation
  - Limited locality, unpredictable

This is hard for

- Distributed memory: how to partition the graph?
- SMP (shared): cache management



Breadth-First Search Levels

Caveat

There are lots of parallel architectures/systems, many new

- Distributed memory - tightly coupled or cluster
- Symmetric Memory Processors (SMP)
- Grid
- Cloud
- Multicore
- Graphical Processing Units (GPUs)
- Massive multithreading (XMT)

For any given application, one of these may be faster and/or give better performance/unit cost.

Example (Devine, Plimpton): matrix-vector multiplication on a distributed-memory machine (pagerank, some graph traversal)

Massive Multithreading: The Cray MTA-2

- Slow clock rate (220Mhz)
- 128 "streams" per processor
- Global address space
- Word-level synchronization
- Atomic increments
- Simple, serial-like programming model
- Advanced parallelizing compilers

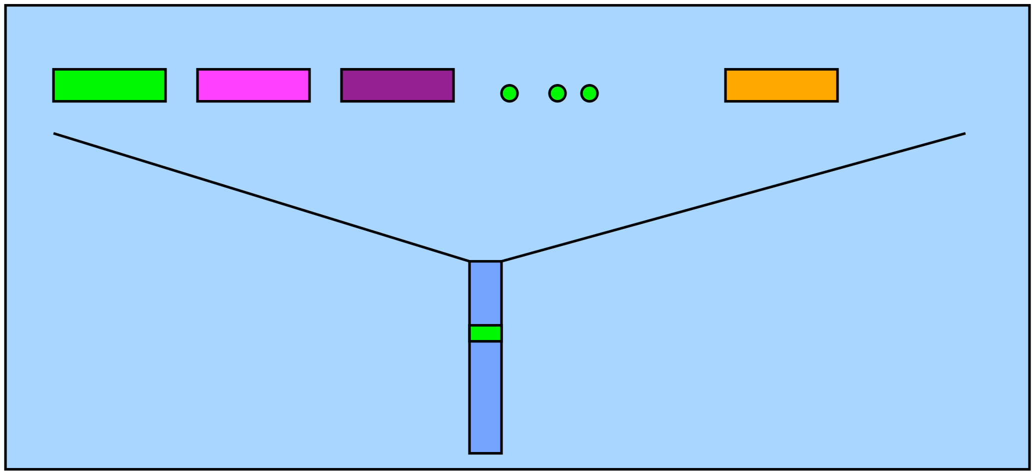
No Processor Cache

Latency Tolerant:

important for Graph Algorithms

Hashed Memory

Cray MTA Processor



• Each thread can have up to 8 memory refs in flight

• Round trip to memory ~150 cycles (MTA-2)

• New Cray XMT combines up to 8192 MTA proc. with Red Storm network

  • *Faster clock, but less network bandwidth*

  • *More memory (up to 128TB), but slower memory*

Additional Challenges

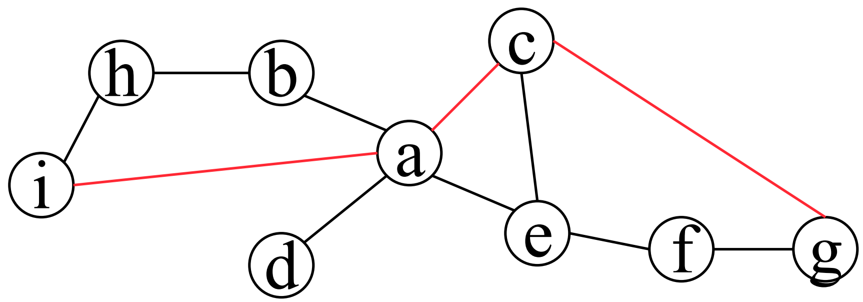
- Deep pipeline (21)
- One instruction per thread in the pipe

The good news: FAST context switches (one clock cycle)
The bad news: Context switch is mandatory every clock cycle

- Example: 40 streams, each with 4 memory references in flight will tolerate latency
- One processor is approximately equal to a linux box if using perfectly

Unweighted S-T Connectivity

- Compute the minimum number of edges between two specific nodes

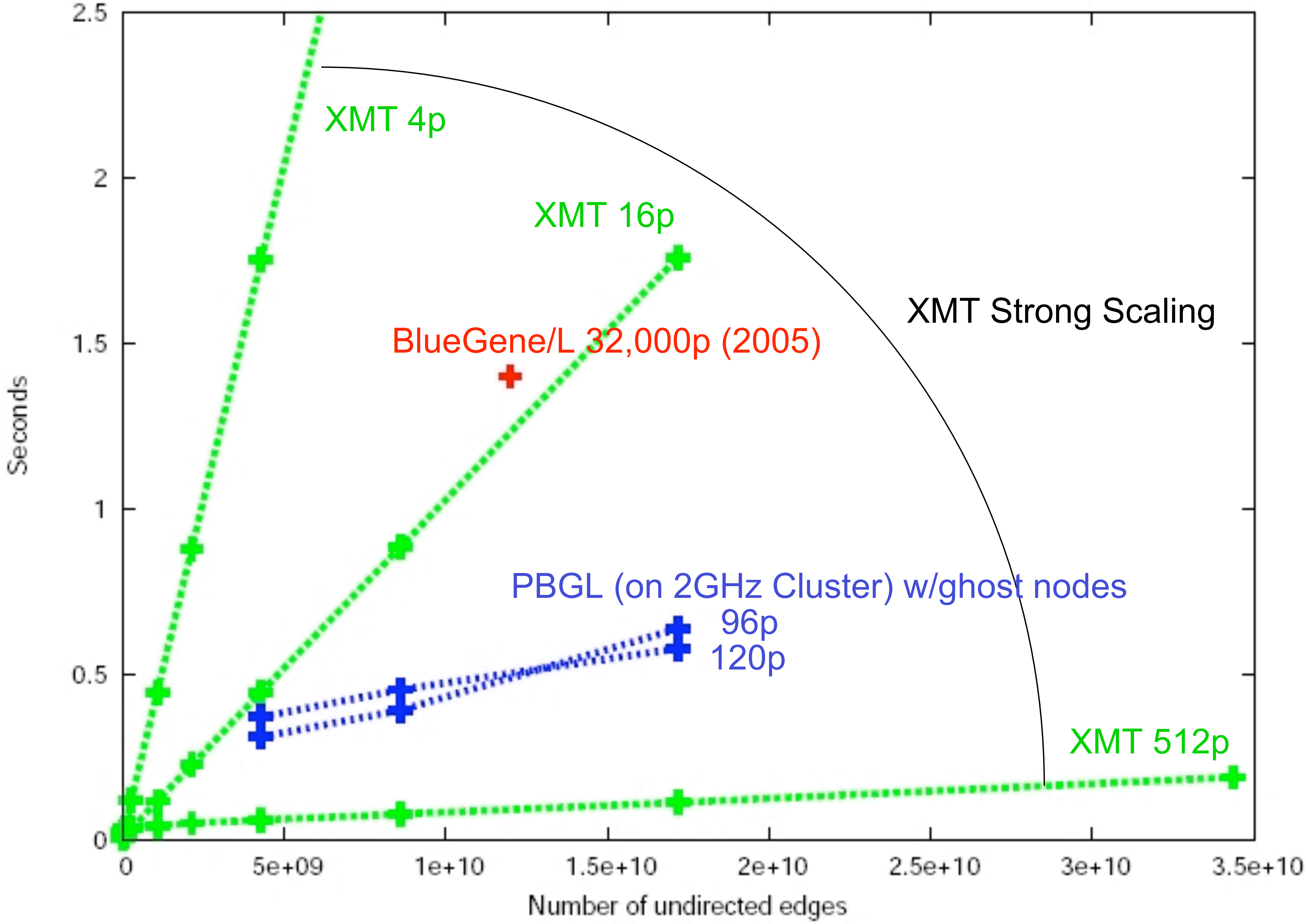


Shortest i-to-g path is length 3

BFS from both s and t till they meet

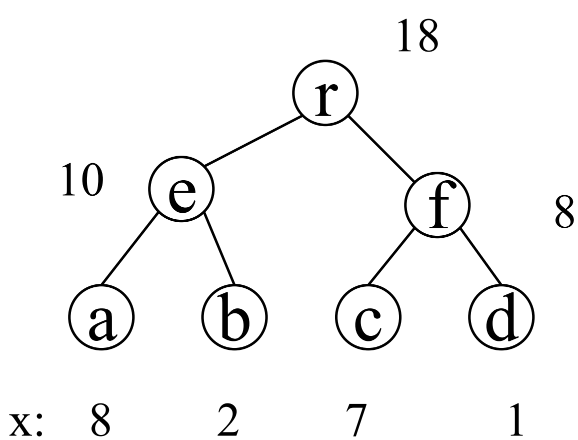
Computational example:

- Erdos-Renyi graphs
- Expected shortest path is constant sized
- 5 trials for each of 10 random s-t pairs


2.5
2
1.5
1
0.5
0
Seconds
0
5e+09
1e+10
1.5e+10
2e+10
2.5e+10
3e+10
3.5e+10
Number of undirected edges
XMT 4p
XMT 16p
BlueGene/L 32,000p (2005)
XMT Strong Scaling
PBGL (on 2GHz Cluster) w/ghost nodes
96p
120p
XMT 512p

Programming the XMT

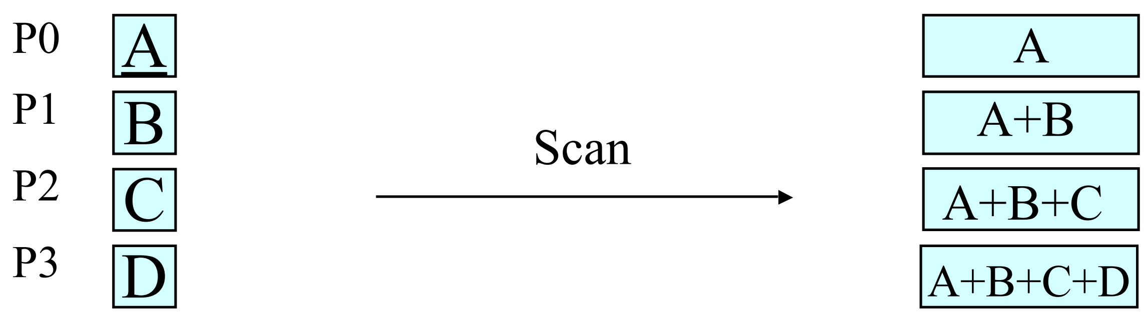
- Compiler directives
  - As with Cilk++, permission, not commands
- Negotiate with the compiler
- Multithreaded graph library (MTGL) encapsulates some primitives (e.g. BFS)
- Compiler recognizes a reduction:

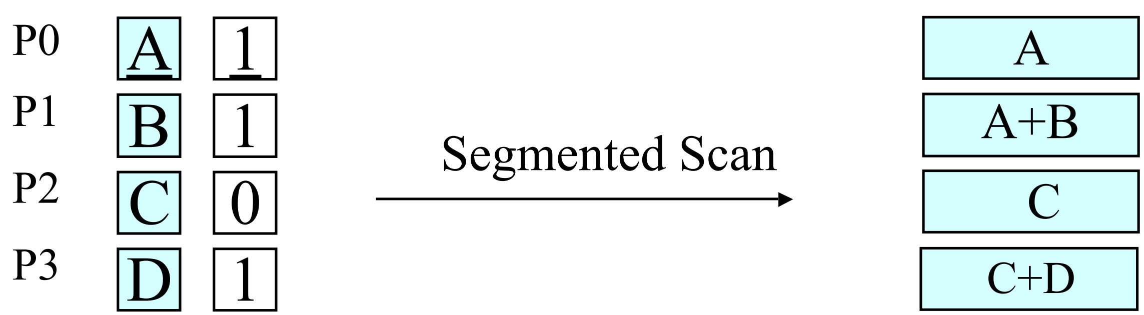



```
total := 0
For i := 1 to n
  total += x[i]
```

Parallel Prefix (Prefix-sum, Scan)

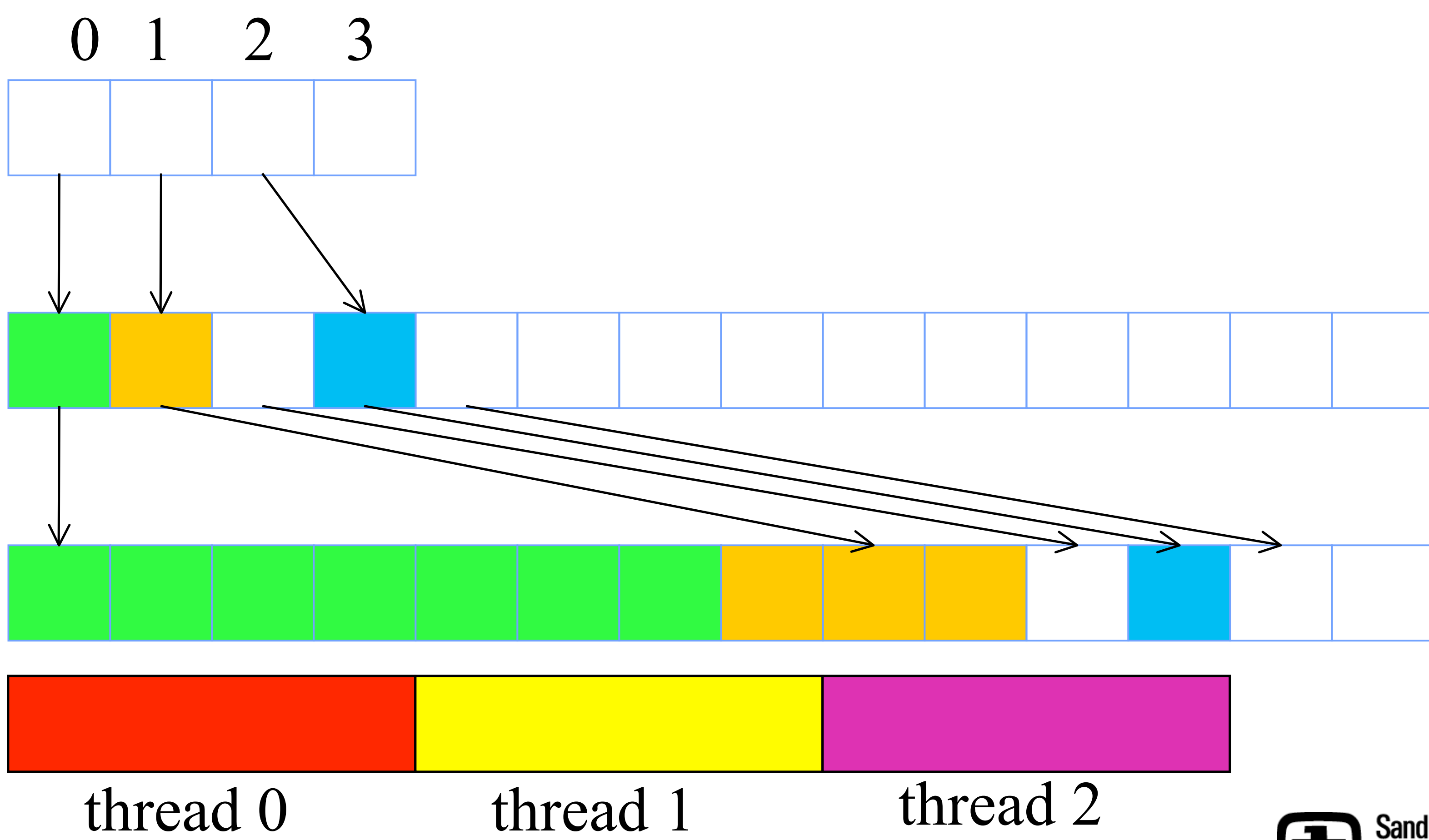
- Introduced by Blelloch [1990].
- “Sum” is binary associative (+,*,min,max, left-copy)
- Applications: lexically comparing strings of characters, adding multiprecision numbers, evaluating polynomials, sorting and searching.





Parallel prefix example in BFS

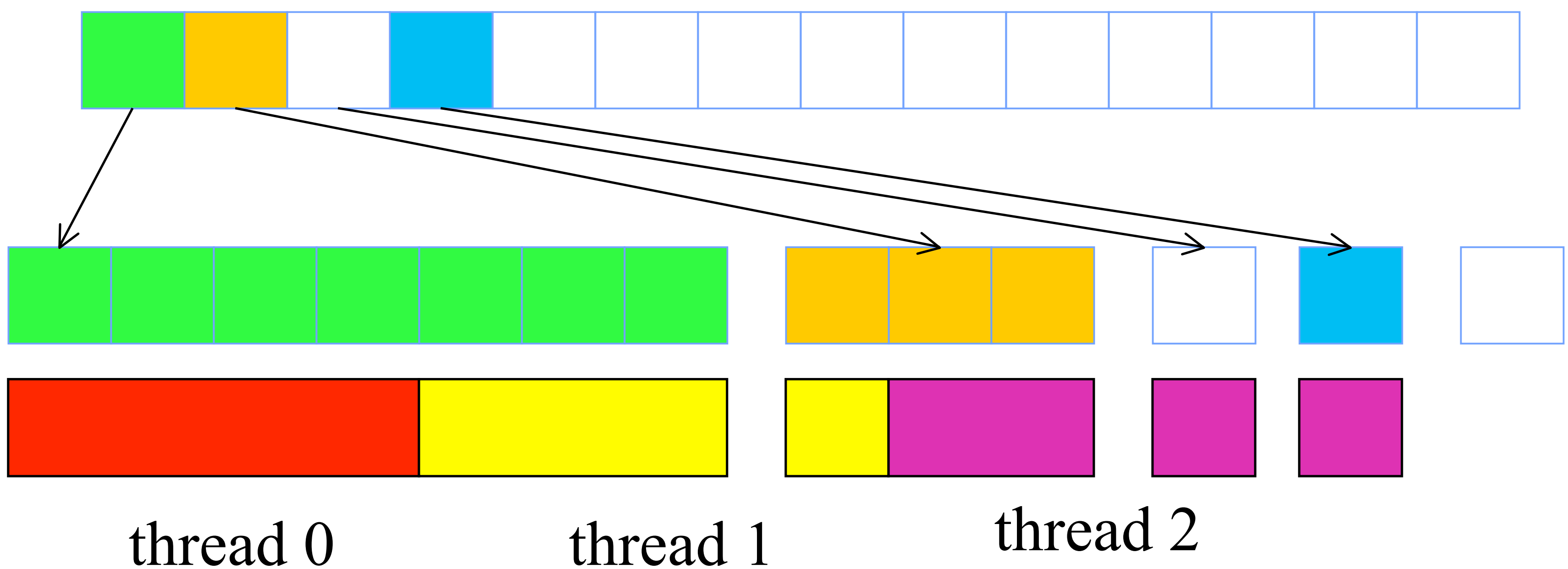
- Parallelize each level of a breadth-first search
- Create C chunks by equally dividing the neighborhood of the nodes currently in the queue



Parallel prefix example in BFS

- Total work 12. Each thread gets four (I + 1 to 4i).

7	3	1	1	degree
7	10	11	12	cumulative degree



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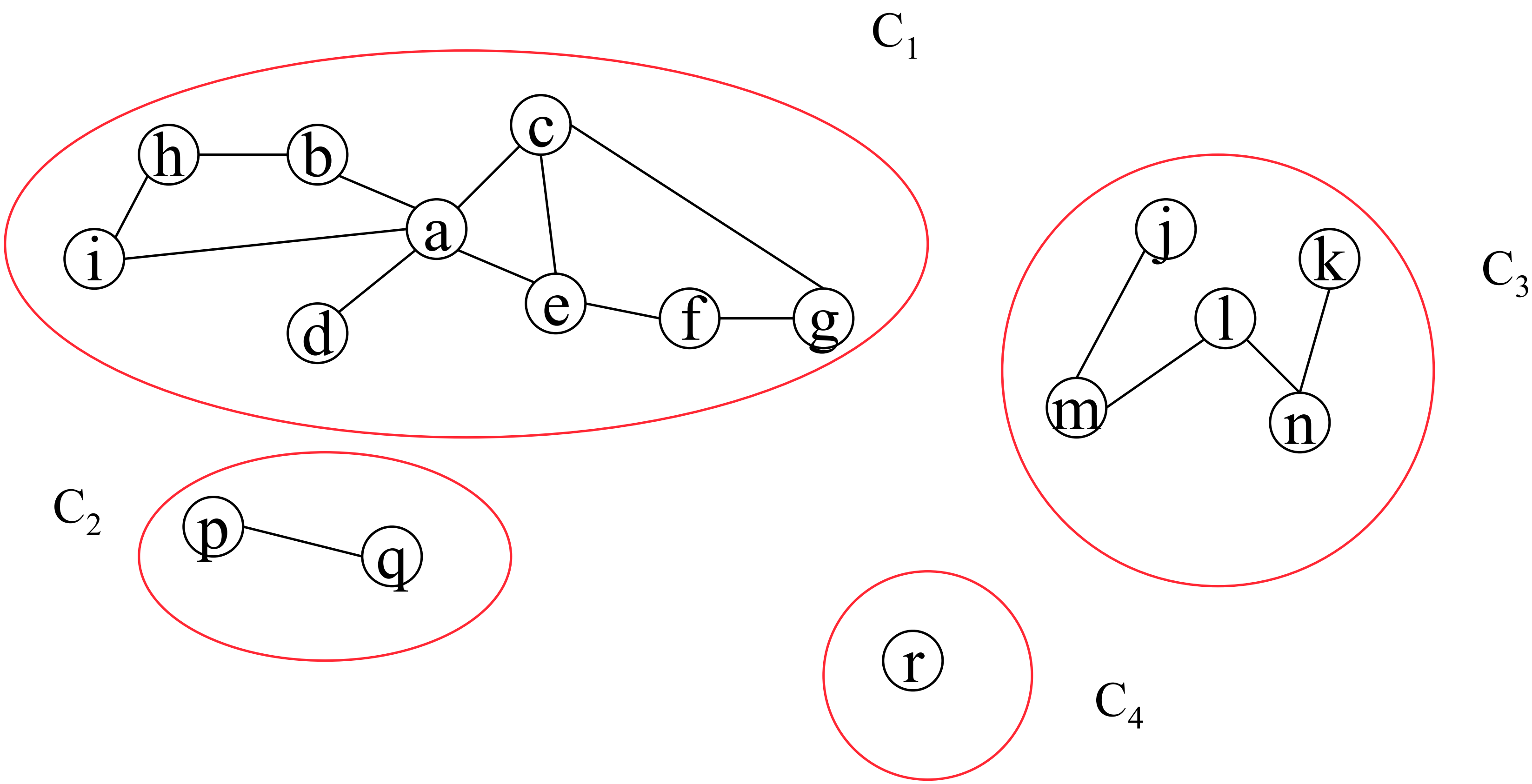
Connected Components Problem

- Give each node a label
- Two nodes have the same label if there is a path between them



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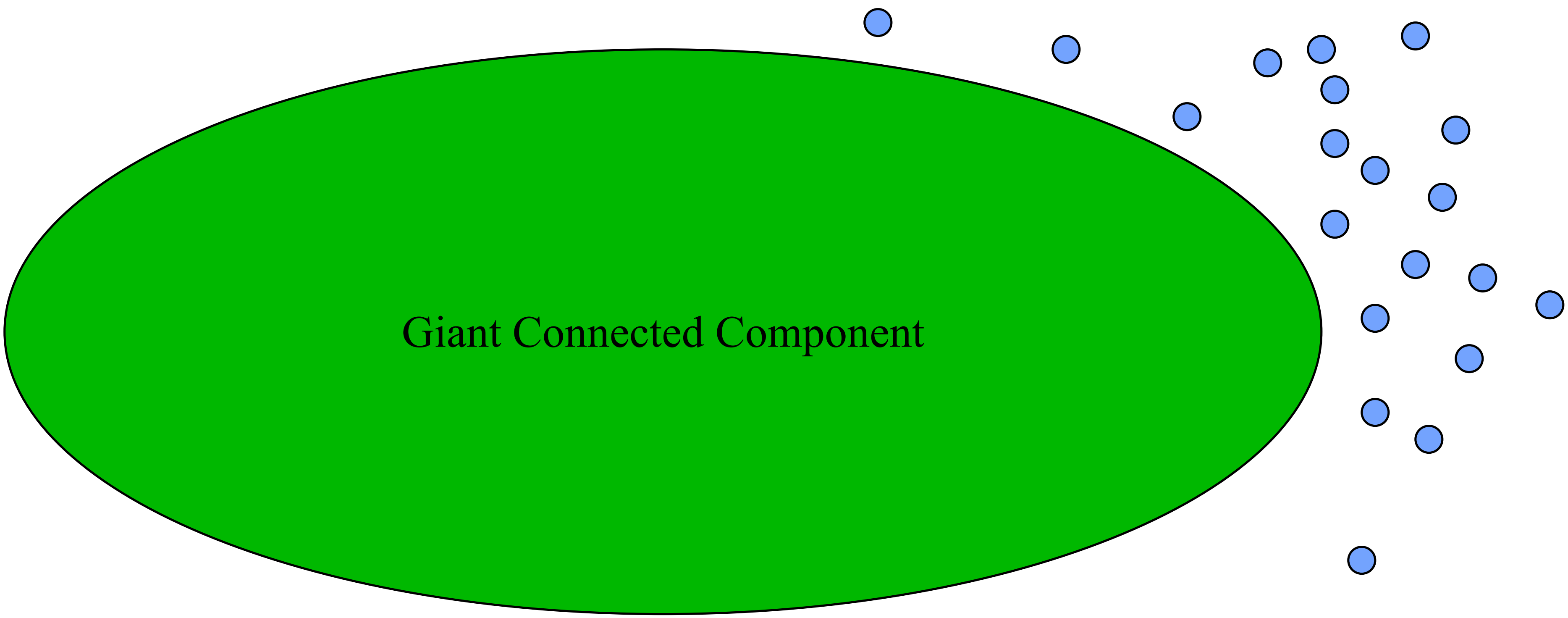
Connected Components on XMT (Power Law)

- Do a parallel BFS from the node of largest degree
  - Will likely label the largest (great) component



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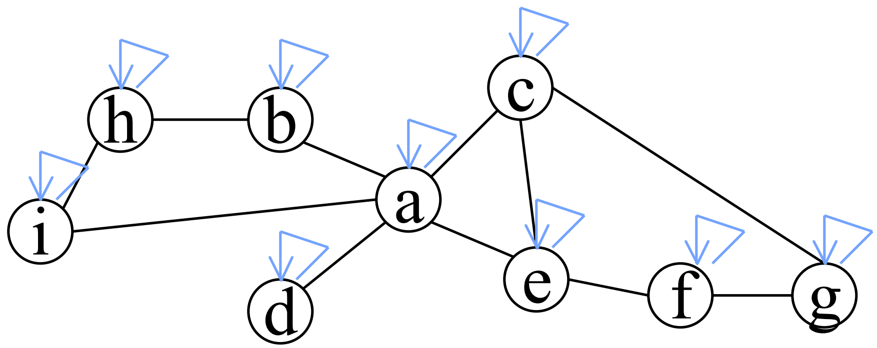
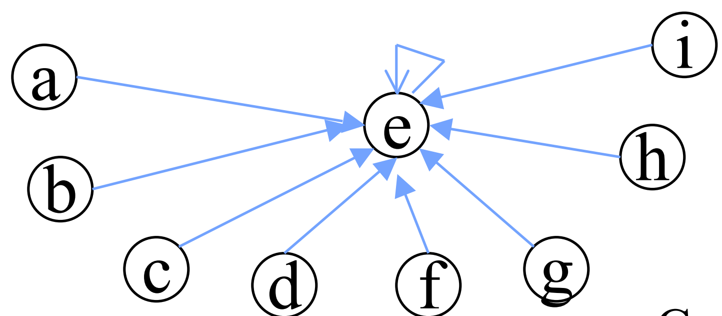
Connected Components on XMT (Power Law)

- Clean up with Shiloach-Vishkin PRAM algorithm
- Hook with edges, pointer jumping to create a star



Start as own leader (label)



Component small.
No significant hot spot

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Community Detection

- We are applying unconstrained facility location to finding communities on the XMT [See Jon Berry (MS75, 10:30) for a bit more]

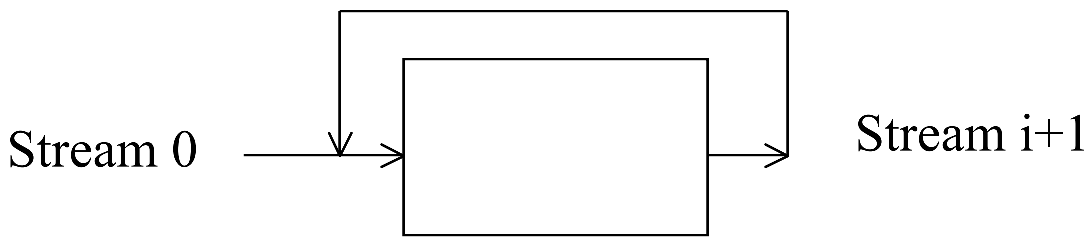
- Motivated by EPA water sensor network problem
  - EPA wanted low-memory algorithms (run on PC)
  - Sensor placement problem is p-median (facility location)
  - We adapted code from COIN-OR for unconstrained facility location

Low-Memory to Parallel

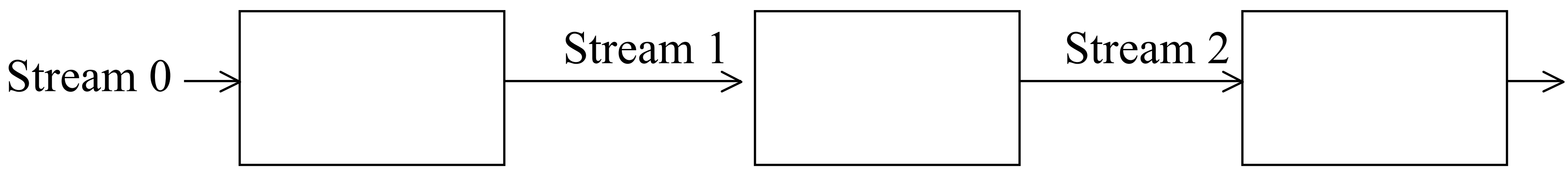
- Streaming algorithms
- Answer a question as the data set streams by
  - Use much less local space than the stream size
- Example: Watch a permutation of 1..n (n known) with one number missing. You have space for one number. Determine the missing number.
- Answer: store the sum of the numbers you have seen.

W-Stream

- Read a stream, write a stream for another pass



- Unroll for a parallel machine that keeps the streams in flight/use:



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Connected Components, W-Stream

- Input: edges of finite graph in a stream (v_1, v_2), …, (v_i, v_j)
- Output: (edge, label) pairs [label will be a vertex name, star]

Stream has two parts:

A: edges between partial components

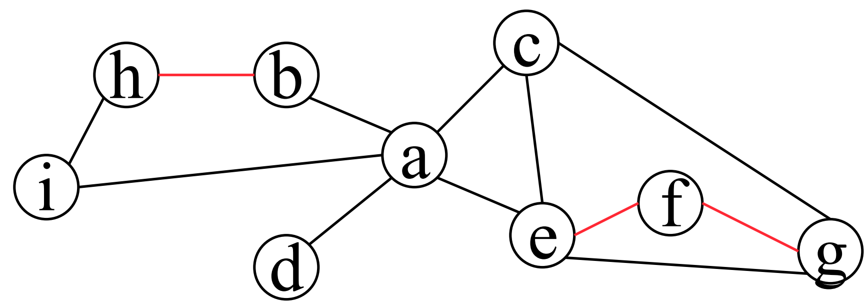
– Initially the graph edges

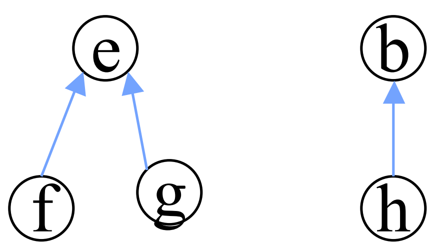
B: (node, label) pairs

– Initially (v_i, v_i) implicitly

W-Stream Connected Components

- For each processor (stage), accept edges from the A stream and compute connected components (stars) until memory full





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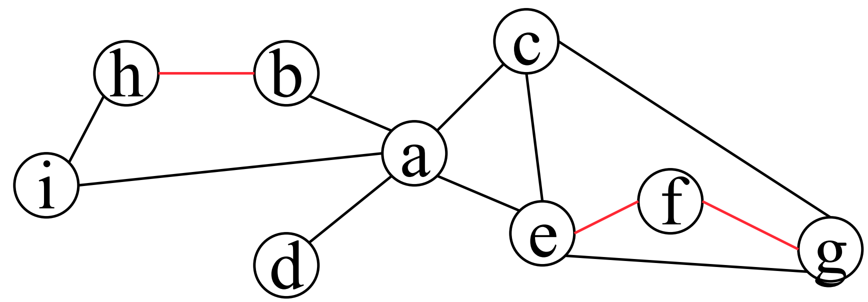
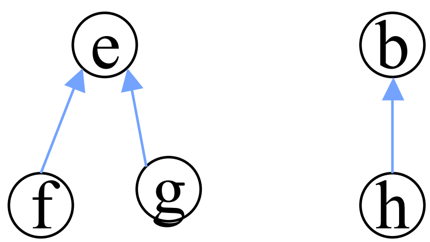
W-Stream Connected Components

For rest of A stage

- Map known nodes to labels
- Drop intracomponent edges

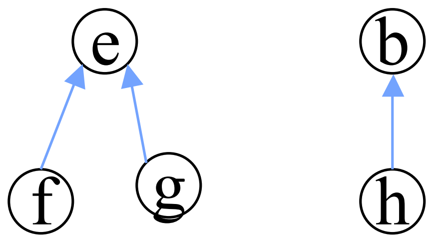




A Output: (i,b) for (i,h)
(c,e) for (c,g)
drop (e,g)
rest unchanged

Connected Components W-Stream

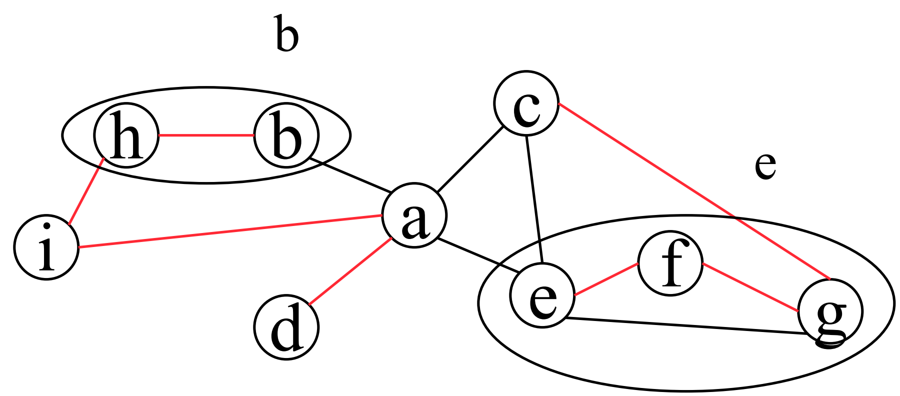
- At first stage, after last end, output A/B boundary marker and list the components:

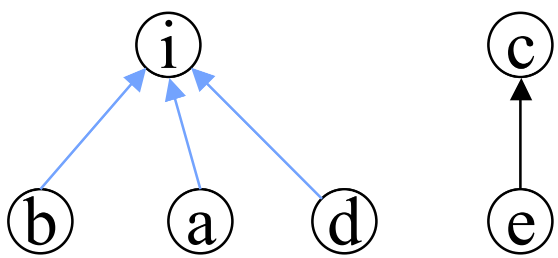


Output: (f,e), (g,e), (e,e), (h,b)

Connected Components W-stream

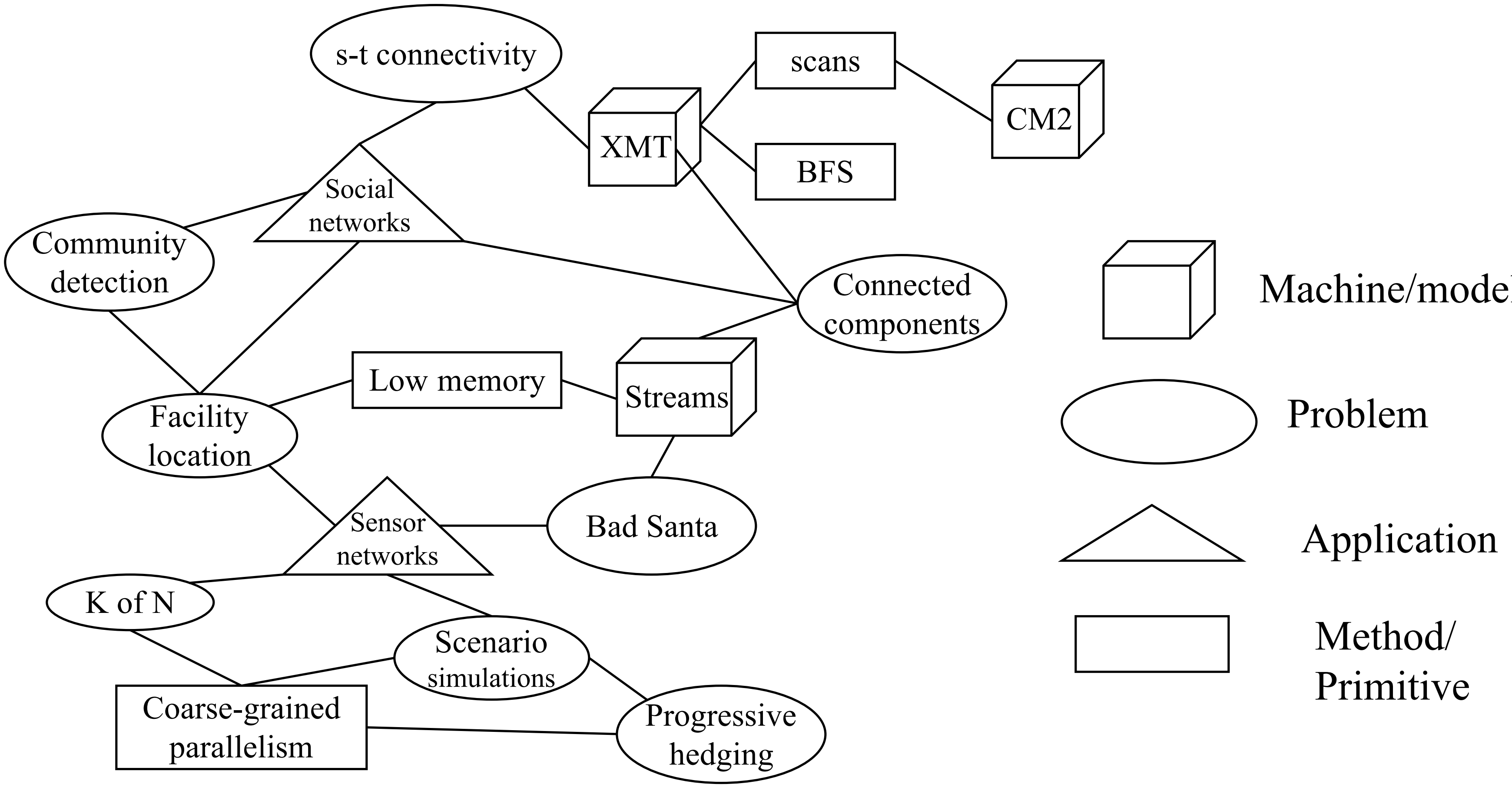
- Repeat A phase
- On B update labels as necessary





Output:
A: (i,c) from (a,c) and (a,e)
B: (f,c) for (f,e)
(g,c) for (g,e)
(e,c) for (e,e)
(h,i) for (h,b)
After marker: (b,i), (a,i), (d,i) (i,i) (c,c)
Note: e was done in phase A

The interconnection of concepts



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