Designing Aluminum Members

Randy Kissell, P.E.
Trinity Consultants

Learning Outcomes

- Know the major aluminum alloy groups and their uses
- Know the principal structural properties of aluminum
- Become proficient in designing aluminum structural members and connections

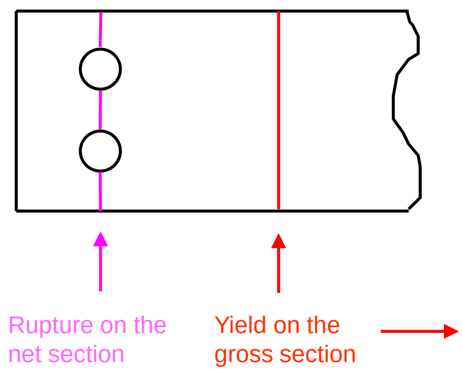
Course Outline

- 6 Tension members
- 7 Compression members
- 8 Flexural members
- 9 Members in shear or torsion

6. Tension Members

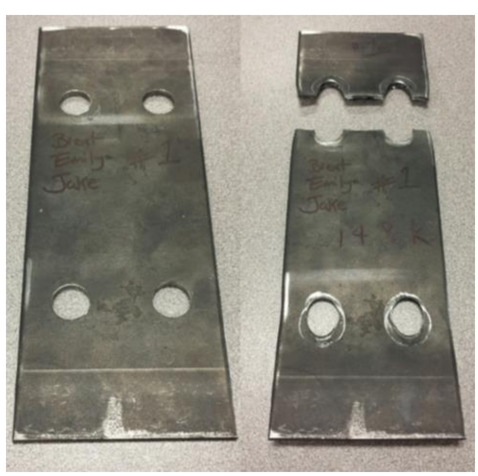
- SAS Chapter D covers axial tension
- Tensile limit state is reached at:
  - Rupture on the net section (Ω = 1.95)
  - Yield on the gross section (Ω = 1.65)
- Same criteria as in AISC for steel
- It's assumed that the net section exists only over a short portion of the member length, so yielding there won't cause much elongation

Net and Gross Sections



5

Rupture on the Net Section



6

Allowable Tension Stress Example

- 6061-T6 Extrusions:

  F_{ty} = 35 ksi, F_{tu} = 38 ksi
- Allowable stress on the gross section:

  $F/\Omega_y = F_{ty}/\Omega_y = 35/1.65 = 21.2$ ksi
- Allowable stress on the net section:

  $F/\Omega_u = F_{tu}/(\Omega_u k_t) = 38/[(1.95)(1.0)]$

  = 19.5 ksi

  Net section always governs

Allowable Tensile Stress Example

- 6063-T5 Extrusions:

  F_{ty} = 16 ksi, F_{tu} = 22 ksi
- Allowable stress on the gross section:

  $F/\Omega_y = F_{ty}/\Omega_y = 16/1.65 = 9.7$ ksi
- Allowable stress on the net section:

  $F/\Omega_u = F_{tu}/(\Omega_u k_t) = 22/[1.95)(1.0)]$

  = 11.3 ksi

  Gross or net section could govern

Tension Coefficient k_t

- k_t is a notch sensitivity factor
- For alloys in SAS, $k_t \geq 1$ only for :
  - 2014-T6, 2219-T87, 6005-T5, and 6105-T5, $k_t = 1.25$
  - 6066-T6 and 6070-T6, $k_t = 1.1$
- 6005A-T61 has same F_{ty} and F_{tu} as 6005-T5, but $k_t = 1.0$ for 6005A-T61

LRFD Tension Example

- 6061-T6 Extrusions:

  $F_{ty} = 35$ ksi, $F_{tu} = 38$ ksi
- LRFD design stress on the gross section:

  $\phi_y F = \phi_y F_{ty} = 0.90(35) = 31.5$ ksi
- LRFD design stress on the net section:

  $\phi_u F = \phi_u F_{tu} / k_t = 0.75(38)/(1.0) = 28.5$ ksi

  So just like ASD, net section governs.

Net Area

- SAS Section D.3.1
- Net area = gross area – (hole area)
- For staggered hole patterns

  net width = $w - \Sigma D_{he} + \Sigma s^2/4g$

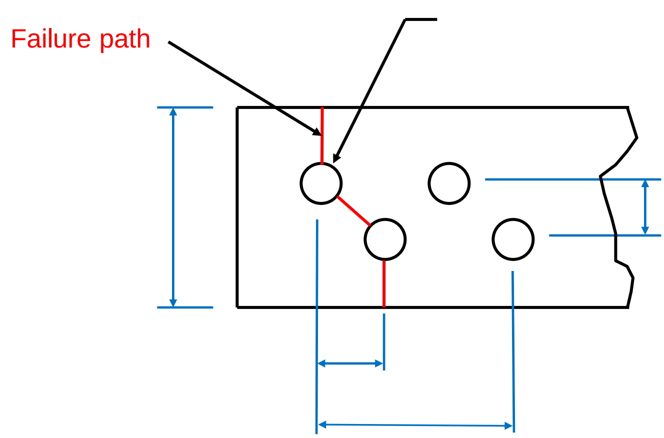
  where w = gross width

  D_{he} = hole effective diameter

  s = pitch (spacing ll to load)

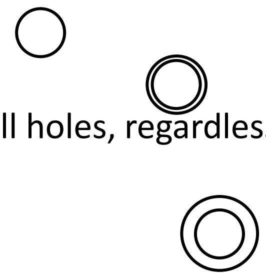
  g = gauge (spacing $\perp$ to load)

Net Width

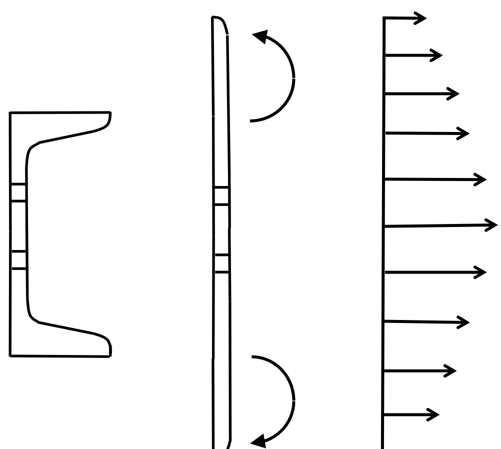


Hole Effective Diameter (D_{he})

- SAS uses, for D_h = nominal hole diameter:
  - For drilled holes, $D_{he} = D_h$
  - For punched holes, $D_{he} = D_h$ + 1/32”
- AISC Steel Spec uses $D_{he} = D_h$ + 1/16” for all holes, regardless of how they are fabricated



Shear Lag in a Channel



Effective Net Area in Tension A_e

- SAS Section D.3.2
- If all parts of x-section aren't connected to joint, full net area isn't effective in tension
- Example: Channel bolted through its web only (not flanges)
- SAS addresses angles, channels, tees, zees, rectangular tubes, and I beams

Effective Net Area A_e

- Effective net area = A_e

  $A_e = A_n(1 - \bar{x}/L_C)(1 - \bar{y}/L_C)$

  but no less than A_n of connected elements

  A_n = net area

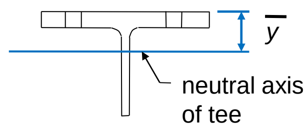
  $\bar{x}$ = eccentricity in x direction

  $\bar{y}$ = eccentricity in y direction

  L_C = length of connection in load direction

Effective Net Area Example

- For a tee bolted through its flange only:



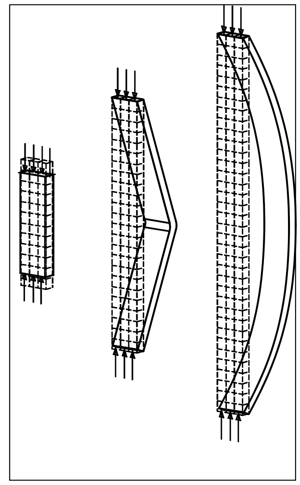
- Other examples are in ADM Part II D.3.2
- When only a single row of fasteners is used, L_c = 0 and A_e = A_n of connected elements only

7. Compression Members

- Column = axial compression member
- SAS Chapter E addresses columns
- Column strength is the least of:
  - Member buckling strength
  - Local buckling strength
  - Interaction between member buckling and local buckling strengths

Compressive Limit States

- Yielding (squashing)
- Inelastic buckling
- Elastic buckling



Elastic Buckling

- Elastic buckling stress = $F_e = 0.85\pi^2 E / \lambda^2$
- *E* is the only material property that elastic buckling strength depends on
- $\lambda = kL/r$ = largest slenderness ratio for buckling about any axis
- All other things equal, F_e for aluminum is 1/3 F_e for steel since $E_a = E_s /3$

Member Buckling

- 0.85 factor accounts for member out-of-straightness
- k = 1 for all members (see Section C.3)
- Allowable member buckling strengths really haven't changed from 2005 SAS:

$$\frac{\pi^2 E}{1.95(kL/r)^2} \approx \frac{0.85\pi^2 E}{1.65(kL/r)^2}$$

Inelastic Buckling

- Inelastic buckling strength = F_c = $(B_c - D_c\lambda)[0.85 + 0.15(C_c - \lambda)/(C_c - \lambda_1)]$
  - When $\lambda = C_c$, $F_c = 0.85\ \pi^2 E / C_c^2$
  - When $\lambda = \lambda_1$, $F_c = F_{cy}$

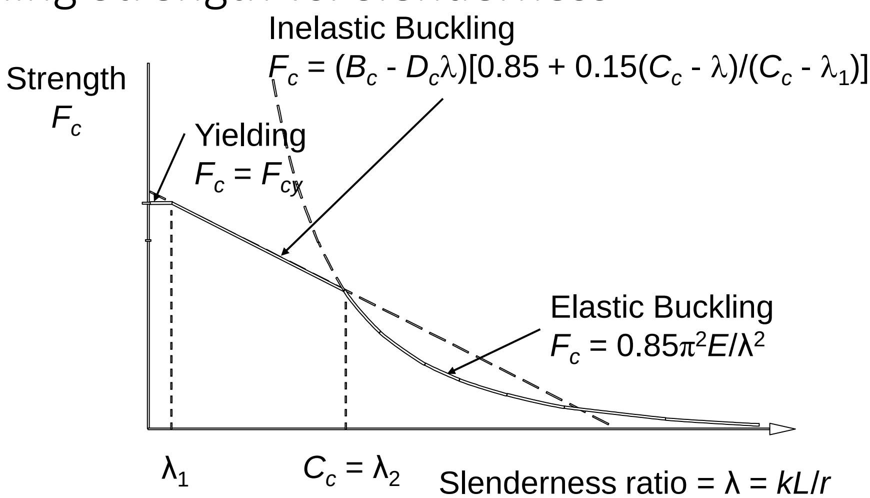
Inelastic Buckling Constants

- Inelastic buckling strength = F_c = $(B_c - D_c\lambda)[0.85 + 0.15(C_c - \lambda)/(C_c - \lambda_1)]$
- B_c (y intercept) and D_c (slope) are buckling constants that depend on F_{cy} and E
- Calculate them by SAS equations in:
  - Table B.4.1 for O, H, T1 thru T4 tempers
  - Table B.4.2 for T5 thru T9 tempers
- B_c , D_c , and C_c are tabulated in ADM Part VI Table 1-1 (unwelded) and 1-2 (welded)

Yielding

- Yield strength is simply $F_c = F_{cy}$
- Yielding depends only on material strength

Buckling Strength vs. Slenderness



Slenderness Limits λ_1, λ_2

- λ_1 is the slenderness for which yield strength = inelastic buckling strength
- λ_2 is the slenderness for which inelastic buckling strength= elastic buckling strength
- Slenderness ratios (kL/r) are not *limited* by λ_1 and λ_2; λ_1 and λ_2 are just the limits of applicability of compressive strength equations

6061-T6 Column Strength

- ADM Part VI, Table 2-19 gives allowable stresses based on SAS rules
- For $kL/r \leq 17.8$, $F_{cy}/\Omega = 21.2$ ksi
- For $17.8 < kL/r < 66$,

$F_c/\Omega = 25.2 – 0.232(kL/r) + 0.000465(kL/r)^2$

- For $kL/r \geq 66$, $F_c/\Omega = 51{,}350/(kL/r)^2$

Slenderness Limits Demonstrated

- For $kL/r = \lambda_2 = 66$:
  - Inelastic buckling allowable stress is

  $25.2 – 0.232(66) + 0.000465(66)^2 = 11.9$ ksi
  - Elastic buckling allowable stress is

  $51{,}350/(66)^2 = 11.8$ ksi ≈ 11.9 ksi
  - Difference is only due to round off in allowable stress expressions

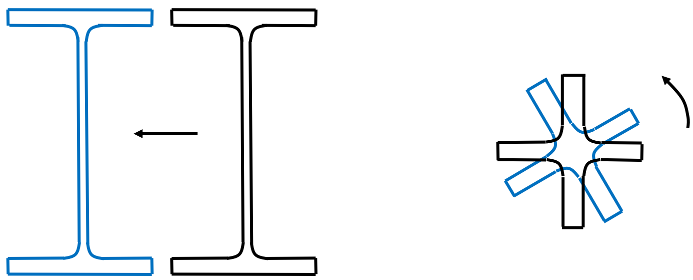
Column Example

- What's the allowable member buckling compressive stress for a column given:
  - 6061-T6
  - Pinned-end support conditions
  - Length = 95"
  - Shape is AA Std I 6 x 4.03
    r_x = 2.53", r_y = 0.95"
  - No bracing

Column Example Answer

- Column will buckle about minor axis since slenderness ratio kL/r is larger there:
- kL/r = (1.0)(95")/0.95" = 100
- Since $kL/r = 100 > \lambda_2 = 66$ (buckling is elastic), so
  $F_c/\Omega = 51{,}350/(100)^2 = 5.1$ ksi
- We need to check local buckling, too

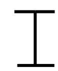
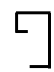
Flexural & Torsional Column Buckling



Types of Column Member Buckling

- Flexural (lateral movement)
- Torsional (twisting about longitudinal axis)
- Flexural-Torsional (combined effect)

Torsional and Flexural-Torsional Buckling

- SAS Section E.2.2 addresses
  - a) doubly symmetric sections 
  - b) singly symmetric sections 
  - c) unsymmetric sections 

Torsional and Flexural-Torsional Buckling

- Calculate torsional or flexural-torsional elastic buckling stress F_e using equations given for the above cases
- Use F_e to calculate the slenderness ratio $\lambda = \pi \sqrt{E/F_e}$
- Use λ in member buckling equations of E.2 to determine compressive strength

Torsional Buckling Example

- I 6 x 4.03 is doubly symmetric; E.2.2a gives the torsional buckling stress as:

$$F_e = \left(\frac{\pi^2 E C_w}{(k_z L_z)^2} + GJ \right) \frac{1}{I_x + I_y}$$

$$F_e = \left(\frac{\pi^2 (10{,}100)(25.3)}{(95)^2} + (3800)(0.0888) \right) \frac{1}{22.0 + 3.1}$$

$$F_e = 24.6 \text{ ksi}$$

Torsional Buckling Example

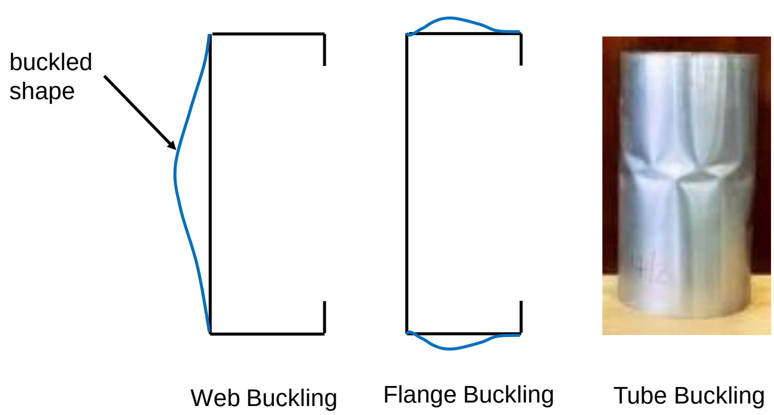
$$\lambda = \pi \sqrt{\frac{E}{F_e}} = \pi \sqrt{\frac{10{,}100}{24.6}} = 63.7 < 100$$

Because the torsional buckling slenderness (63.7) is less than the flexural buckling slenderness (100), the torsional buckling strength is greater than the flexural buckling strength, and does not govern.

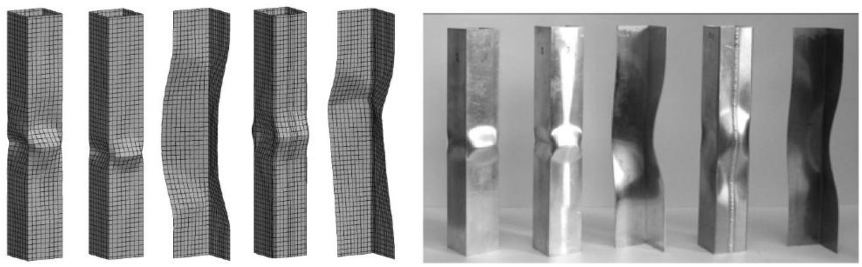
Local Buckling

- Local buckling is buckling of an element of a shape (i.e., a flange or web)
- Buckle length ≈ width of element
- If local buckling strength of all elements > yield strength, the shape is compact, and local buckling won't occur
- Since aluminum shapes vary widely (extrusions, cold-formed shapes), we can't assume aluminum shapes are compact

Local Buckling Examples



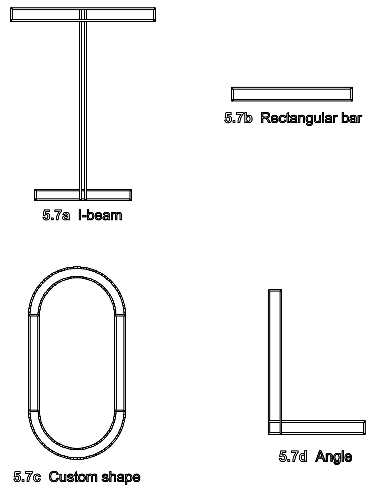
Local Buckling of a Tube



Elements of Shapes are Called:

- Element
- Flange or web
- Component
- (Plate)

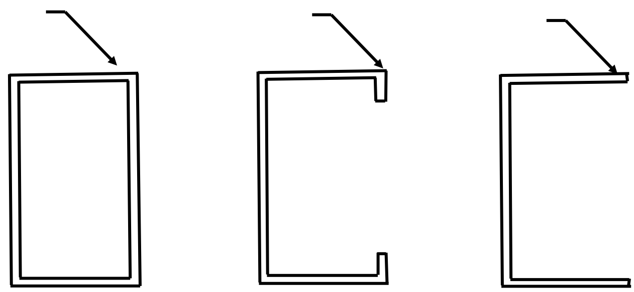
Dividing a Shape Into Elements



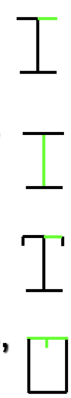
Elements of Shapes

- Cross sections can be subdivided into two types of elements:
  - Flat elements (slenderness = b/t)
  - Curved elements (slenderness = R_b/t)
- Longitudinal edges of elements can be:
  - Free
  - Connected to another element
  - Stiffened with a small element

Element Support Conditions



Elements in Uniform Compression Addressed by the SAS



- B.5.4.1 Flat element supported on one edge (flange of an I beam or channel)
- B.5.4.2 Flat element supported on both edges (web of I beam or channel)
- B.5.4.3 Flat element supported on one edge, other edge with stiffener
- B.5.4.4 Flat element supported on both edges, with an intermediate stiffener
- B.5.4.5 Curved element supported on both edges

Local Buckling Strengths

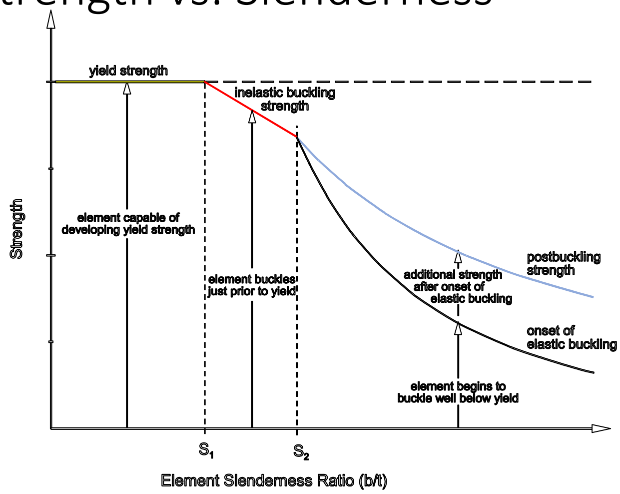
- Yielding $F_c = F_{cy}$
- Inelastic buckling $F_c = B_p - D_p\,(kb/t)$
- Elastic buckling $F_c = \pi^2 E\,/(kb/t)^2$

  $F_c = k_2\,(B_p E)^{1/2}/\,(kb/t)$
- k = edge support factor
  - k = 5.0 for elements supported on 1 edge
  - k = 1.6 for elements supported on both edges
- k_2 = postbuckling factor ≈ 2 (Table B.4.3)

Postbuckling Strength

- Only elements of shapes have postbuckling strength – members do not
- Postbuckling strength is not recognized by SAS for all types of elements
- After buckling elastically, some elements are capable of supporting more load
- If the appearance of buckling is unacceptable, don't include postbuckling

Element Strength vs. Slenderness



6061-T6 Column Flange Strength

- Yielding $\quad F_c/\Omega = F_{cy}/\Omega$
  - $\lambda_1 = 6.7 \quad F_c/\Omega = 35/1.65 = 21.2$ ksi
- Inelastic buckling $\quad F_c/\Omega = [B_p - D_p(5.0b/t)]/\Omega$
  - $\lambda_2 = 12 \quad F_c/\Omega = 27.3 - 0.910b/t$
- Elastic buckling $\quad F_c/\Omega = [\pi^2 E/(5.0b/t)^2]/\Omega$
  - $\lambda_2 = 10.5 \quad F_c/\Omega = 2417/(b/t)^2$
- Postbuckling $\quad F_c/\Omega = [k_2(B_pE)^{1/2}/(5.0b/t)]/\Omega$
  - $F_c/\Omega = 186/(b/t)$

Column Local Buckling - Flange

- Shape is AA Std I 6 x 4.03:

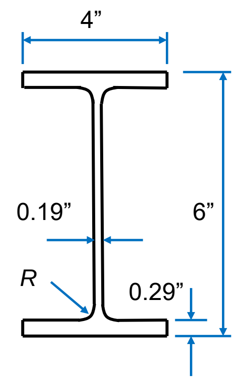
  flange slenderness = b/t

  b/t = (4” – 0.19”)/2/0.29” = 6.6

  λ_1 = 6.7 > 6.6 so

  F_c/Ω = 21.2 ksi

  You can deduct the flange-web fillet radius from b if $R \le 4t$, or conservatively neglect it



Column Local Buckling - Web

- Shape is AA Std I 6 x 4.03:

  web slenderness = b/t

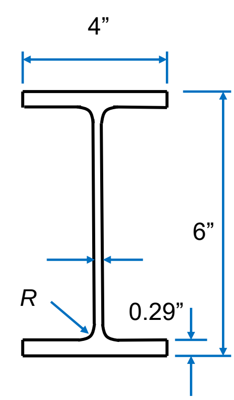
  b/t = [6” – 2(0.29”)]/0.19” = 28.5

  λ_1 = 20.8 < 28.5 < 33 = λ_2, so

  $F_c/\Omega = 27.3 - 0.291b/t =$

  F_c/Ω = 27.3 – 0.291(28.5) = 19.0

  You can deduct the flange-web fillet radius from b if $R \le 4t$, or conservatively neglect it

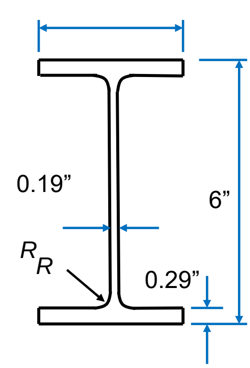


Local Buckling Strength

- Methods for local buckling strength:
  - Conservative, but easy approach: Use the least of local buckling strengths of the shape's elements, or
  - More accurate, but more work : Use the weighted average (SAS Section E.3.1) of the local buckling strengths
  - Direct strength method

Weighted Average Allowable Compressive Stress of I 6 x 4.03

- F_{cf}/Ω = 21.2 ksi
- F_{cw}/Ω = 19.0 ksi
- A_f = 2(4")(0.29")
  = 2.32 in^2
- A_w = (6" – 2(0.29"))(0.19")
  = 1.03 in^2
- $F_{ca}/\Omega = \dfrac{21.2(2.32) + 19.0(1.03)}{(2.32 + 1.03)}$
  = 20.5 ksi



Local Buckling/Member Buckling Interaction

- If the elastic local buckling stress < member buckling stress, member buckling stress must be reduced (SAS Section E.4)
- Reduced member buckling stress is F_{rc}:

  $F_{rc} = (F_c)^{1/3}(F_e)^{2/3}$

  where F_c = elastic member buckling stress

  F_e = elastic local buckling stress

  This only governs if elements are very slender and postbuckling strength is used

53

Local/Member Buckling Interaction Example

- Flange elastic buckling stress F_{ef}

$$F_{ef} = \frac{\pi^2 E}{(5.0b/t)^2} = \frac{\pi^2(10{,}100)}{(5.0(6.6))^2} = 91.5 \text{ ksi}$$

- Web elastic buckling stress F_{ew}

$$F_{ew} = \frac{\pi^2 E}{(1.6b/t)^2} = \frac{\pi^2(10{,}100)}{(1.6(28.5))^2} = 47.9 \text{ ksi}$$

Local/Member Buckling Interaction Example

- Member buckling stress F_c

$$F_c = \frac{0.85\pi^2 E}{(kL/r)^2} = \frac{0.85\pi^2 (10{,}100)}{(100)^2} = 8.5\text{ ksi}$$

- Since F_e = 47.9 ksi > F_c = 8.5 ksi, the member buckling strength need not be reduced for interaction between local and member buckling

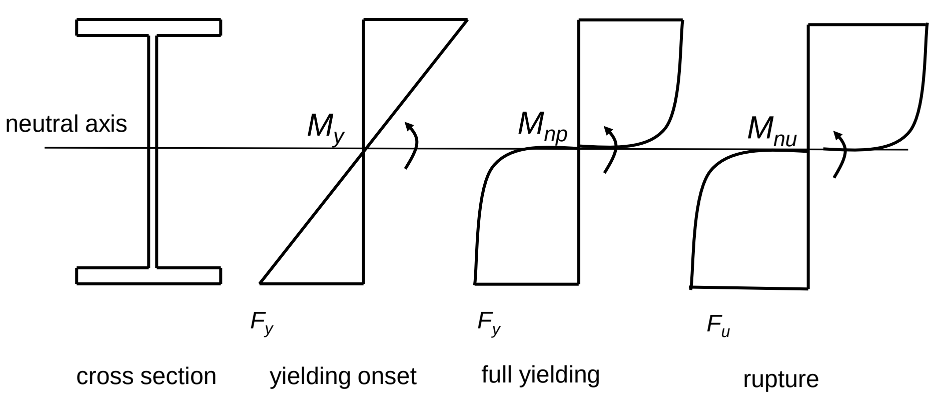
Column Design Summary

- Column strength is the least of:
  - Member buckling strength
  - Local buckling strength
  - Interaction between member and local buckling strengths
- $P_n = F_c A_g$

8. Flexural Members

- Beam = flexural member
- Beam strength limit states are:
  - F.2 Yielding $\Omega = 1.65$
  - F.2 Rupture $\Omega = 1.95$
  - F.3 Local buckling $\Omega = 1.65$
  - F.4 Member buckling (LTB) $\Omega = 1.65$

Yielding and Rupture in Beams

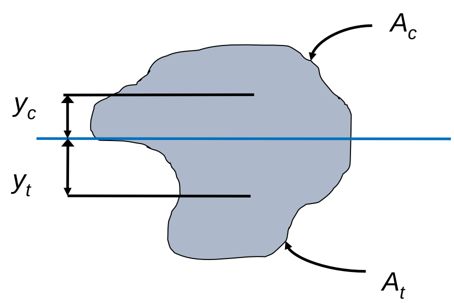


Plastic Modulus Z

- When both sides of n.a. are fully yielded, $F_{ty} A_t = F_{cy} A_c$
- Use F_{cy} for both F_{cy} and F_{ty} to determine Z
- So $A_c = A_t = A/2$

  $Z = A_t y_t + A_c y_c$

  $Z = A(y_t + y_c)/2$



Yielding in a Beam

Beam Yielding Strength M_{np}

- For wrought products M_{np} shall not exceed
  - ZF_{cy} , $1.5S_t F_{ty}$, $1.5S_c F_{cy}$
- For cast products M_{np} shall not exceed
  - $S_t F_{ty}$, $S_c F_{cy}$
- Before 2015 SAS used only part of the plastic modulus *Z* for yield strength
- 1.5*S* limit on *Z* is to prevent yielding at service loads (AISC limit is 1.6*S*)

Beam Yielding Strength M_{np}

- Example I 12 x 14.3:
  - S = 52.89 in^3 elastic section modulus
  - Z = 58.36 in^3 plastic section modulus
  - $Z = 2[5.38^2(0.31)/2 + 7(0.62)(5.38 + 0.62/2)]$

    web ↗ flange ↗
  - Z/S = 1.10 = shape factor < 1.5
  - M_{np} = (58.36 in^3)(35 k/in^2) = 2043 in-k
  - M_{np}/Ω= (2043 in-k)/1.65 = 1238 in-k

Beam Rupture Strength M_{nu}

- For wrought products and cast products $M_{nu} = Z F_{tu} / k_t$
- Before 2015 SAS used only part of the plastic modulus Z for rupture strength
- For I 12 x 14.3,

  M_{nu} = (58.36 in^3)(38 k/in^2)/1 = 2218 in-k

  M_{nu}/Ω = (2218 in-k)/1.95 = 1137 in-k

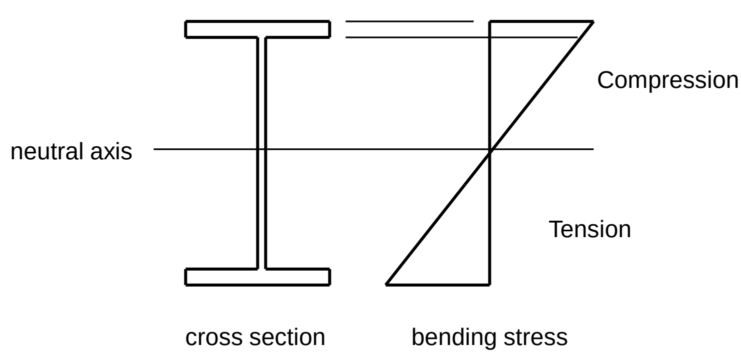
Local Buckling Flexural Strength

- Determine by one of these methods:
  - F.3.1 Weighted average
  - F.3.2 Direct strength
  - F.3.3 Limiting element

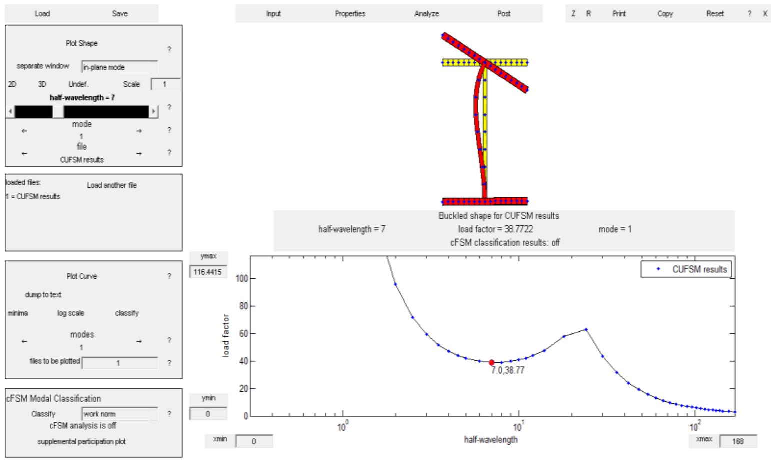
Local Buckling of Beam Elements

- Beam elements in uniform compression (flanges) are just like column elements in uniform compression (see B.5.4)

Beam Flange Stress



Local Buckling of a Flange



Local Buckling - Flange

- Shape is AA Std I 12 x 14.3:

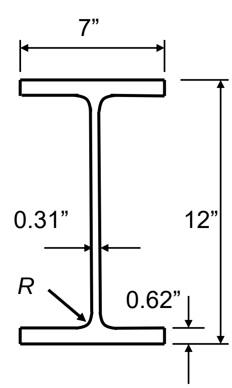
  flange slenderness = b/t

  b/t = (7" – 0.31")/2/0.62" = 5.4

  5.4 < 6.7= λ_1 so

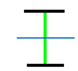
  F_c/Ω = 21.2 ksi

  You can deduct the flange-web fillet radius from b if $R \leq 4t$, or conservatively neglect it

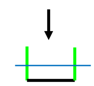


Beam Elements in SAS –Elements in Flexure (Webs)

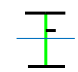
B.5.5.1 Flat element - both edges supported (web of I beam or channel)



B.5.5.2 Flat element - compression edge free, tension edge supported

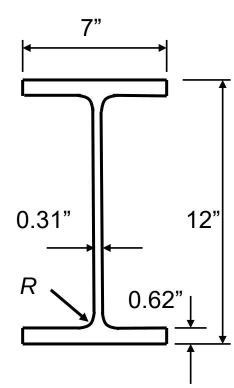


B.5.5.3 Flat element with a longitudinal stiffener – both edges supported (see B.5.5.3 for stiffener requirements



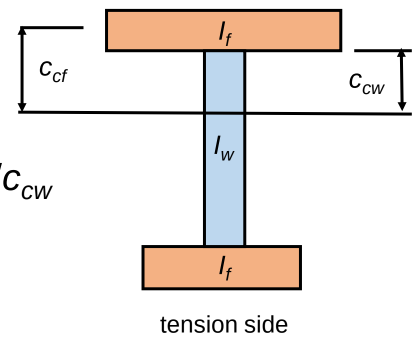
Local Buckling - Web

- Shape is AA Std I 12 x 14.3:
  - web slenderness = b/t
  - b/t = [12” – 2(0.62”)]/0.31” = 34.7
  - $\lambda_1 = 33.1 < 34.7 < 77 = \lambda_2$, so
  - $F_c/\Omega = 40.5 - 0.262b/t =$
  - $F_c/\Omega = 40.5 - 0.262(28.5) = 31.4$
  - You can deduct the flange-web fillet radius from b if $R \leq 4t$, or conservatively neglect it



Weighted Average Bending Strength (F.3.1)

$$M_{nLB} = F_{cf}\, I_f / c_{cf} + F_{cw}\, I_w / c_{cw}$$



Weighted Average Example

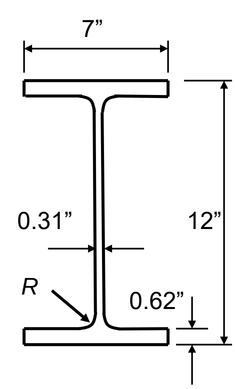
- For I 12 x 14.3,

  $M_{nLB} /\Omega = F_{cf}\, I_f /\Omega c_{cf} + F_{cw}\, I_w /\Omega c_{cw}$

  = (21.2)(281.3)/5.69 +

  (31.4)(32.18)/5.38

  = 1236 in-k



Direct Strength Method (F.3.2)

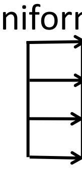
- Determine elastic local buckling stress F_e (one way is finite strip method, like for members in axial compression)
- Determine slenderness

  ratio λ for the shape $\lambda = \pi \sqrt{E/F_e}$

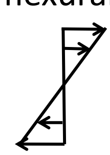
  Use F.3.2 to determine the local buckling strength of the shape

Limiting Element Method (F.3.3)

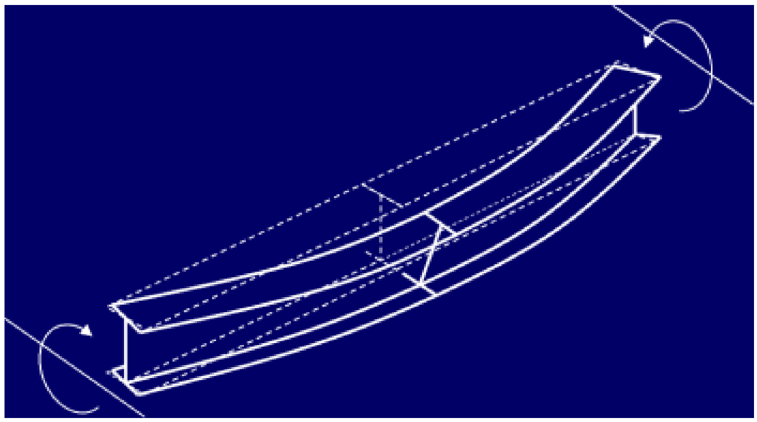
- Stress in each element shall not exceed the local buckling strength of that element
- Determine F_{LB} using B.5.4.1 thru B.5.4.4 for elements in uniform compression



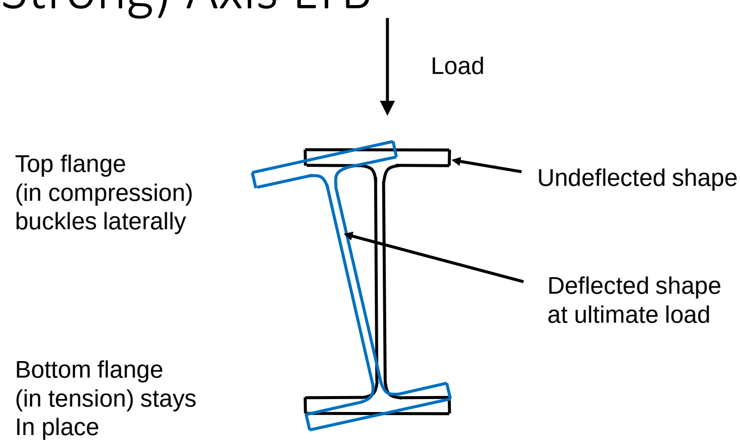
- Determine F_{LB} using B.5.5.1 thru B.5.5.4 for elements in flexural compression



Lateral-Torsional Buckling (LTB)



Major (Strong) Axis LTB



LTB = Lateral-torsional buckling

6061-T6 LTB Strength

- Inelastic buckling $\lambda < C_c = 66$

  $M_{nmb}/\Omega = [M_{np}(1 - \lambda/C_c) + \pi^2 E\lambda S_c/C_c^3]/\Omega$

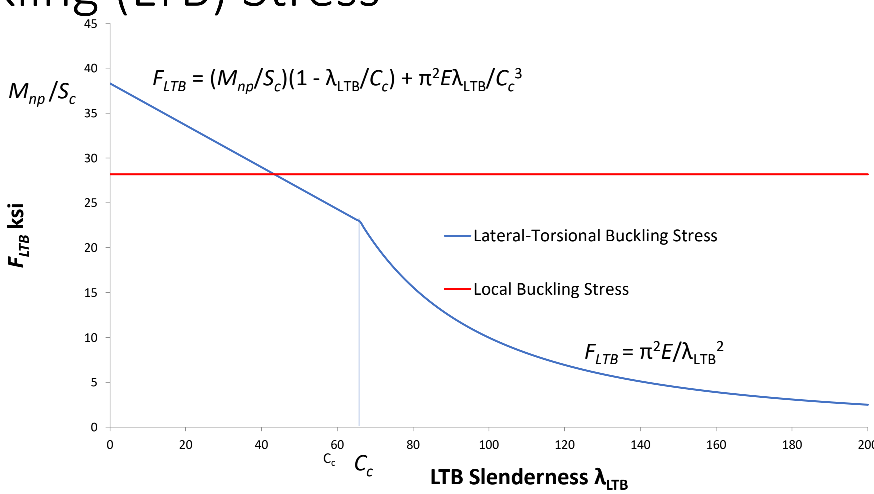
  $M_{nmb}/\Omega = M_{np}/1.65 - \lambda(M_{np}/109 - 0.210S_c)$

- Elastic buckling $\lambda \geq C_c = 66$

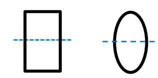
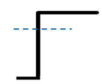
  $M_{nmb}/\Omega = \pi^2 ES_c/(\Omega\lambda^2)$

  $M_{nmb}/\Omega = 60{,}400S_c/\lambda^2$

Lateral-Torsional Buckling (LTB) Stress



Slenderness Ratio λ for LTB

Section	Shape	λ	Example
F.4.2.1	sym about bending axis	$\frac{L_b}{r_{ye}\sqrt{C_b}}$	
F.4.2.3	closed shape	$2.3\sqrt{\frac{S_x L_b}{C_b\sqrt{I_y J}}}$	
F.4.2.4	rectangular bar	$\frac{2.3}{t}\sqrt{\frac{dL_b}{C_b}}$	
F.4.2.5	any shape	$\pi\sqrt{\frac{ES_c}{C_b M_e}}$	

Unbraced Length for Beams L_b

- Slenderness ratio depends on unbraced beam length L_b
- L_b = length between bracing points or between a brace point and the free end of a cantilever beam. Braces:
  - restrain the compression flange against lateral movement, or
  - restrain the cross section against twisting
- Appendix 6 addresses brace design

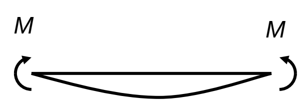
Bending Coefficient C_b

- C_b accounts for moment variation along the beam. For doubly symmetric sections:
- $C_b = \dfrac{4M_{max}}{(M^2_{max} + 4M^2_A + 7M^2_B + 4M^2_C)^{0.5}}$

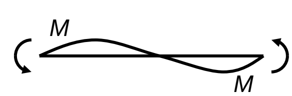
where M_A = moment at ¼ point
M_B = moment at midpoint
M_C = moment at ¾ point

C_b Examples

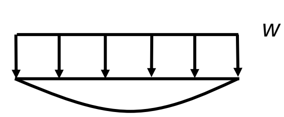
- C_b = 1.0 (min)



- C_b = 2.3



- C_b = 1.13



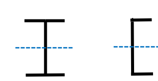
- It's always conservative to use C_b = 1; max C_b = 3.0

r_{ye} for Shapes Symmetric About the Bending Axis

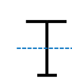
- F.4.2.1 allows using $1.2r_y$ or $r_y d/(2r_x)$ for r_{ye}
  - That's easy, but conservative in neglecting torsional strength, and unconservative if load is applied toward shear center
- It's worth determining r_{ye} using more precise equations given in F.4.2.1
  - That's more work, but more accurate

Calculating r_{ye}

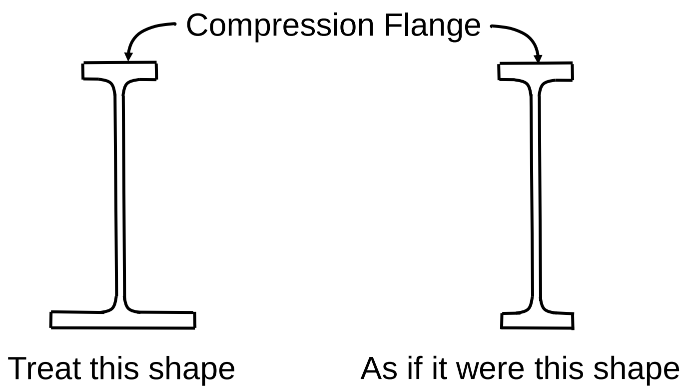
- F.4.2.1 Shapes symmetric about the bending axis: uses equations based on I_y, C_w, S_x, J, L_b, and where the load is applied relative to the beam's neutral axis



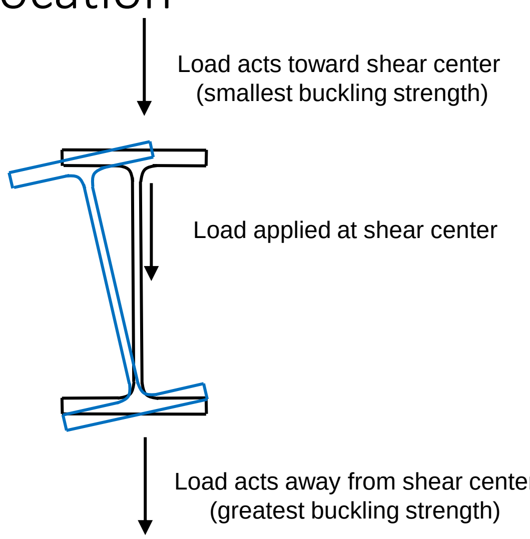
- F.4.2.2 Singly symmetric shapes unsymmetric about the bending axis
  - If $I_{yc} \leq I_{yt}$, you can transform the tension flange to look like the compression flange and use F.4.2.1



Singly Symmetric Beam Unsymmetric About Bending Axis



Transverse Load Location



r_{ye} for Shapes Symmetric about the Bending Axis

- Load applied toward shear center

$$r_{ye} = \sqrt{\frac{I_y}{S_x}\left[-\frac{d}{4} + \sqrt{\frac{d^2}{16} + \frac{C_w}{I_y} + 0.038\frac{JL_b^2}{I_y}}\right]}$$

- Load applied at shear center, or no load

$$r_{ye} = \sqrt{\frac{I_y}{S_x}\sqrt{\frac{C_w}{I_y} + 0.038\frac{JL_b^2}{I_y}}}$$

- Load applied away from shear center

$$r_{ye} = \sqrt{\frac{I_y}{S_x}\left[+\frac{d}{4} + \sqrt{\frac{d^2}{16} + \frac{C_w}{I_y} + 0.038\frac{JL_b^2}{I_y}}\right]}$$

LTB Example

- What's the allowable LTB moment for a beam given:
  - 6061-T6
  - Length = 86"
  - Shape is AA Standard I 12 x 14.3, r_y = 1.71, I_y = 35.48, S_x = 52.89, C_w = 1148, J = 1.26
  - No bracing between beam ends
  - Transverse load applied toward shear center

LTB Example Answer

- r_{ye} for slenderness ratio L_b/r_{ye} is

$$r_{ye} = \sqrt{\frac{I_y}{S_x}\left[-\frac{d}{4} + \sqrt{\frac{d}{16}^2 + \frac{C_w}{I_y} + 0.038\frac{JL_b}{I_y}^2}\,\right]} = 1.67$$

$\lambda = L_b/r_{ye}$ = 86”/1.67” = 51.5

- Since $L_b/r_y = 51.5 < 66 = C_c$,

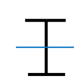
$M_{nLTB}/\Omega = M_{np}/1.65 - \lambda(M_{np}/109 - 0.210S_c)$

= 2043/1.65 – 51.5(2043/109 –0.210(52.89))

= 845 in-k

Open Section LTB Strength

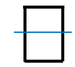
- Open section beam (e.g., I beam) resists lateral buckling mostly by warping strength; LTB strength is given by F.4.2.1 for shapes sym about the bending axis



- F.4.2.1 includes torsion strength (which increases as L_b increases) *and* warping strength if you don’t use the approximation $1.2r_y$ or $r_y d/(2r_x)$ for r_{ye}

Closed Section LTB Strength

- Closed section beam (e.g., rectangular tube) resists lateral buckling by torsion strength; LTB strength is given by F.4.2.3



- F.4.2.3 includes torsion strength only, not warping strength. If C_w << $0.038JL_b^2$, this isn't overly conservative
- F.4.2.3 assumes shape is sym about bending axis & load acts at shear center; usually this isn't very unconservative

Any Shape LTB Strength

- F.4.2.5 gives LTB for any shape; eq F.4-9:

λ = , $\frac{L_b}{r_{ye}\sqrt{C_b}}$ $r_{ye} = \sqrt{\frac{I_y}{S_x}\left[U + \sqrt{U^2 + \frac{C_w}{I_y} + 0.038\frac{JL_b^2}{I_y}}\right]}$

- $U = C_1 g_o + C_2 \beta/2$, where
- g_o = distance from load application to s.c.
- β = coefficient of monosymmetry
- C_1 and C_2 depend on loading; ≈ 0.5

I 12 x 14.3 Available Strengths

- Yielding M_{np}/Ω = 1238 in-k
- Rupture M_{nu}/Ω = 1137 in-k
- Local buckling M_{nLB}/Ω = 1236 in-k
- LTB M_{nLTB}/Ω = 845 in-k

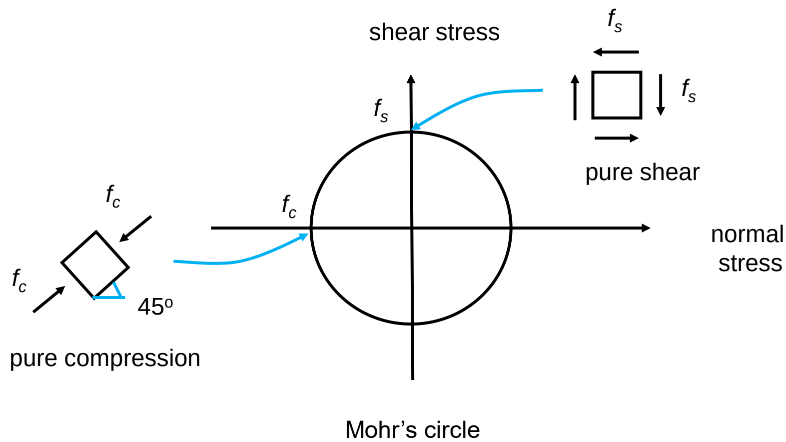
The available flexural strength is the least of these:

M_n/Ω = 845 in-k

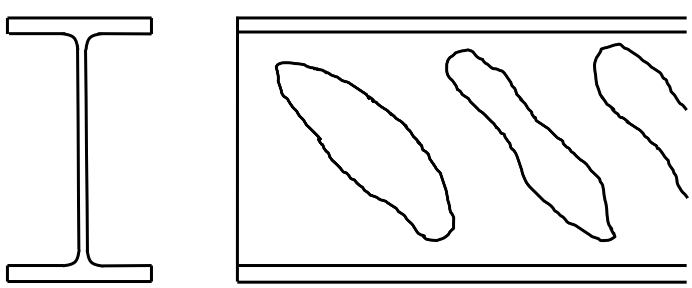
9. Members in Shear or Torsion

- Shear is addressed in SAS Chapter G
- Torsion is addressed in SAS Section H.2
- Safety factors:
  - Rupture (Ω = 1.95, new in 2015)
  - Yield and buckling (Ω = 1.65)

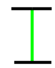
Shear Buckling

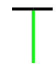


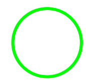
Web Shear Buckling

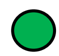


Elements in Shear in SAS

G.2 Flat element supported on both edges 

G.3 Flat element supported on one edge 

G.4 Pipes and round or oval tubes 

G.5 Rods 

6061-T6 Web Shear Strength

- Yielding $F_s/\Omega = F_{sy}/\Omega$

  $\lambda_1 = 35$ $F_s/\Omega = 0.6(35)/1.65 = 12.7$ ksi
- Inelastic buckling

  $F_s/\Omega = [B_s - D_s(1.25b/t)]/\Omega$

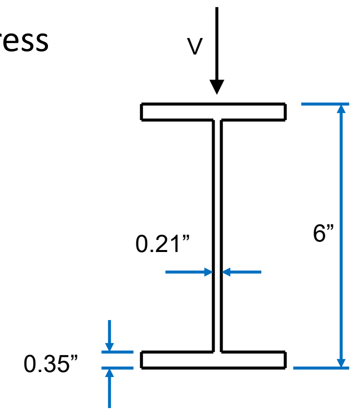
  $\lambda_2 = 63$ $F_s/\Omega = 16.5 - 0.107b/t$
- Elastic buckling

  $F_s/\Omega = [\pi^2 E/(1.25b/t)^2]/\Omega$

  $F_s/\Omega = 38{,}700/(b/t)^2$

Web Shear Example

- What's the allowable shear stress for a flat web given:
  - 6061-T6
  - Shape is AA Std I 6 x 4.69
    d = 6", t_f = 0.35", t_w = 0.21"

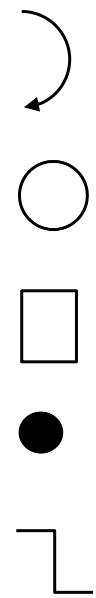


Web Shear Example Answer

- Web height is $b = d - 2t_f$ = 6" – 2(0.35")
  b = 5.3"
- Web slenderness ratio is
  b/t_w = 5.3"/0.21" = 25.2 < λ_1 = 35
- For yield F_{sy}/Ω = 0.6(35)/1.65 = 12.7 ksi
- For rupture F_{su}/Ω = 0.6(38)/1.95 = 11.7 ksi
- $V/\Omega = (dt_w)F_s/\Omega$ = (6")(0.21")(11.7 k/in^2)
  = 14.7 k

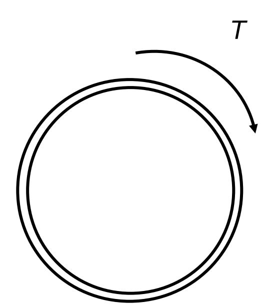
Torsion

- H.2.1 Pipes and
  Round or Oval Tubes
- H.2.2 Rectangular Tubes
- H.2.3 Rods
- H.2.4 Open Shapes



Torsion in a Round Tube

- 5050-H34 tube, F_{cy} = 0.9(20) = 18 ksi
- 10" diameter x 0.050" thick
- 96" long
- Determine the allowable
  shear stress F_s/Ω



Torsion Example

- Section H.2.1, slenderness λ:
- $\lambda = 2.9(R/t)^{5/8}(L/R)^{1/4}$
- $\lambda = 2.9(5''/0.05'')^{5/8}(96''/5'')^{1/4}$
- $\lambda = 108 \leq 108 = \lambda_2$
- So $F_s/\Omega = 10.0 - 0.061(108) = 3.4$ ksi

Thank You

- Please contact me with questions
  - rkissell@trinityconsultants.com
  - office: 919-493-8952; cell: 919-636-0072