A flow-kick framework for exploring resilience

Snowbird, May 2017

Mary Lou Zeeman, Bowdoin College

A flow-kick framework for exploring resilience

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2013

Our Plan today

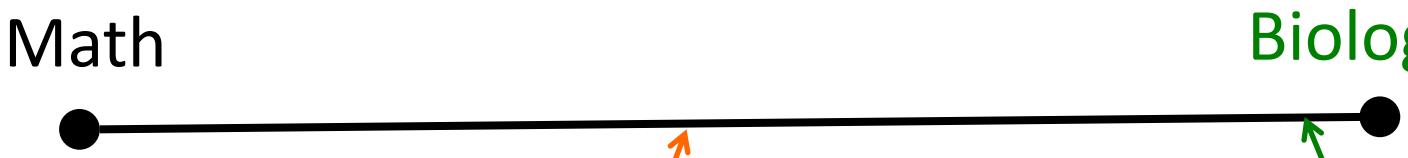
- Decision support
- Resilience
- Flow-kick systems

Bridge Crossing : Math Biology

Develop models
Design experiments

Commune with data
Work with biologists
What can be measured?
What can be controlled?
Sources of uncertainty?

Bridge Crossing : Math Biology



Abstract the ideas and structures

Oscillators
Excitability
Bifurcation
Networks
Stability/ Adaptability
Stochasticity

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Design experiments

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Work with biologists
What can be measured?
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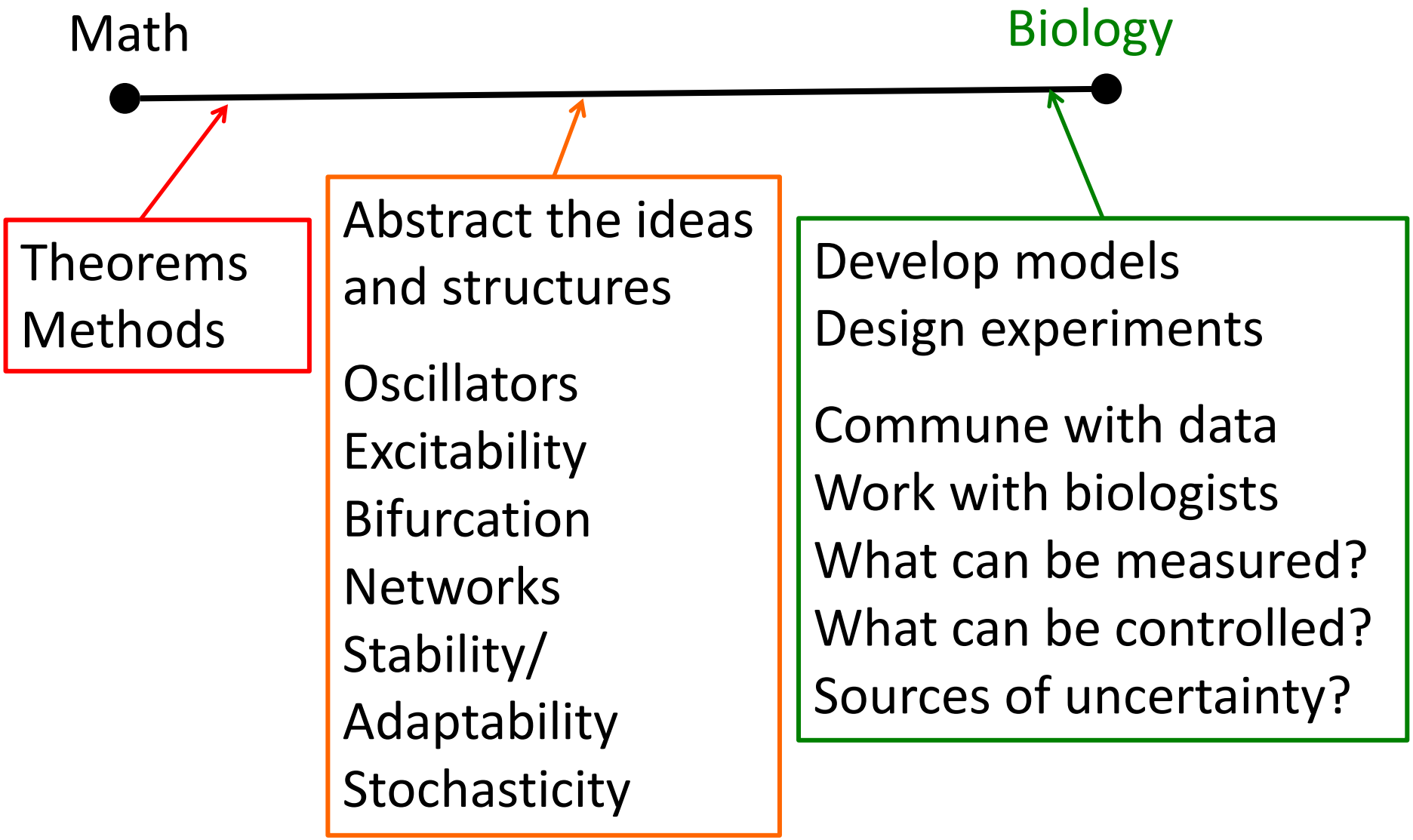

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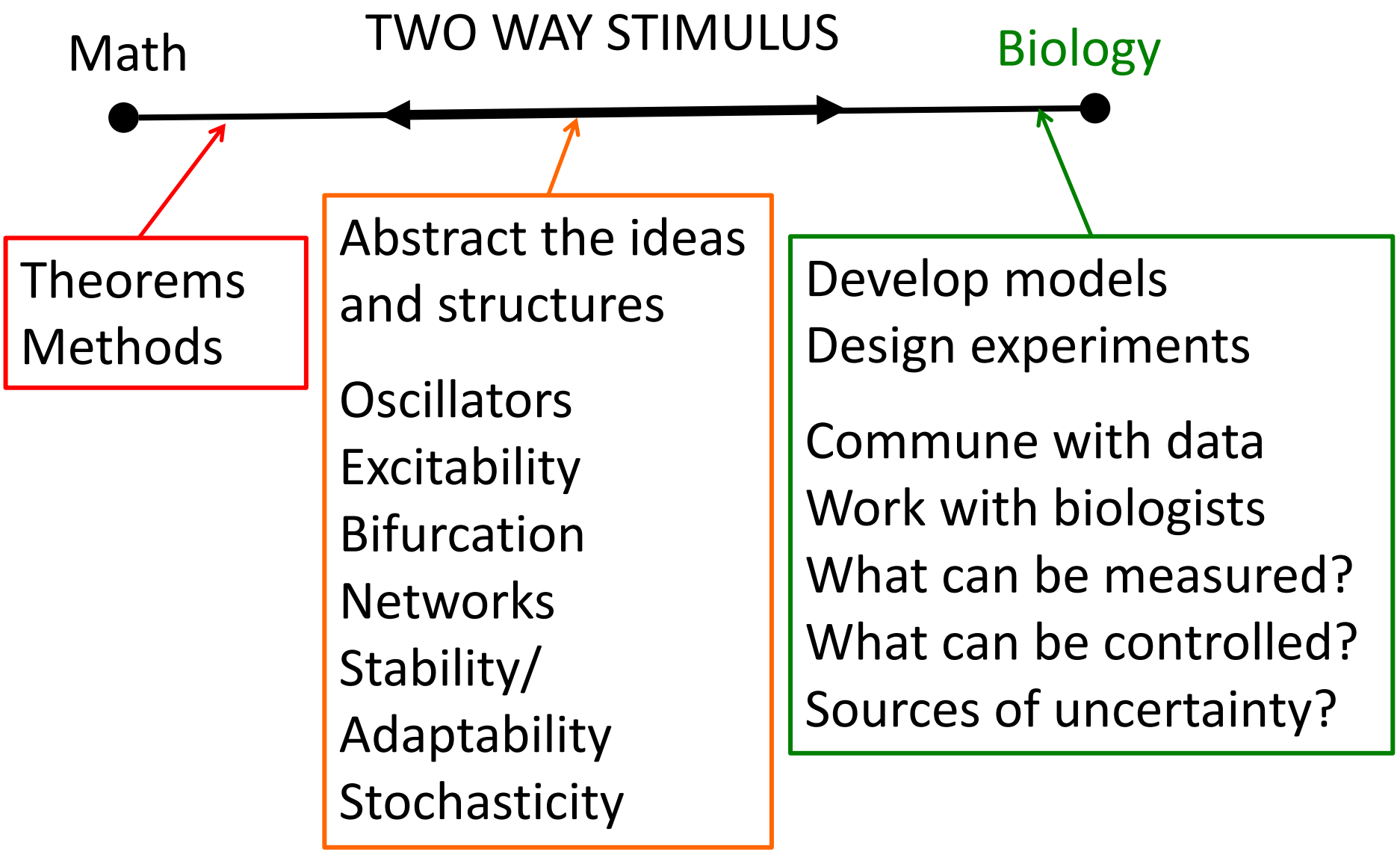
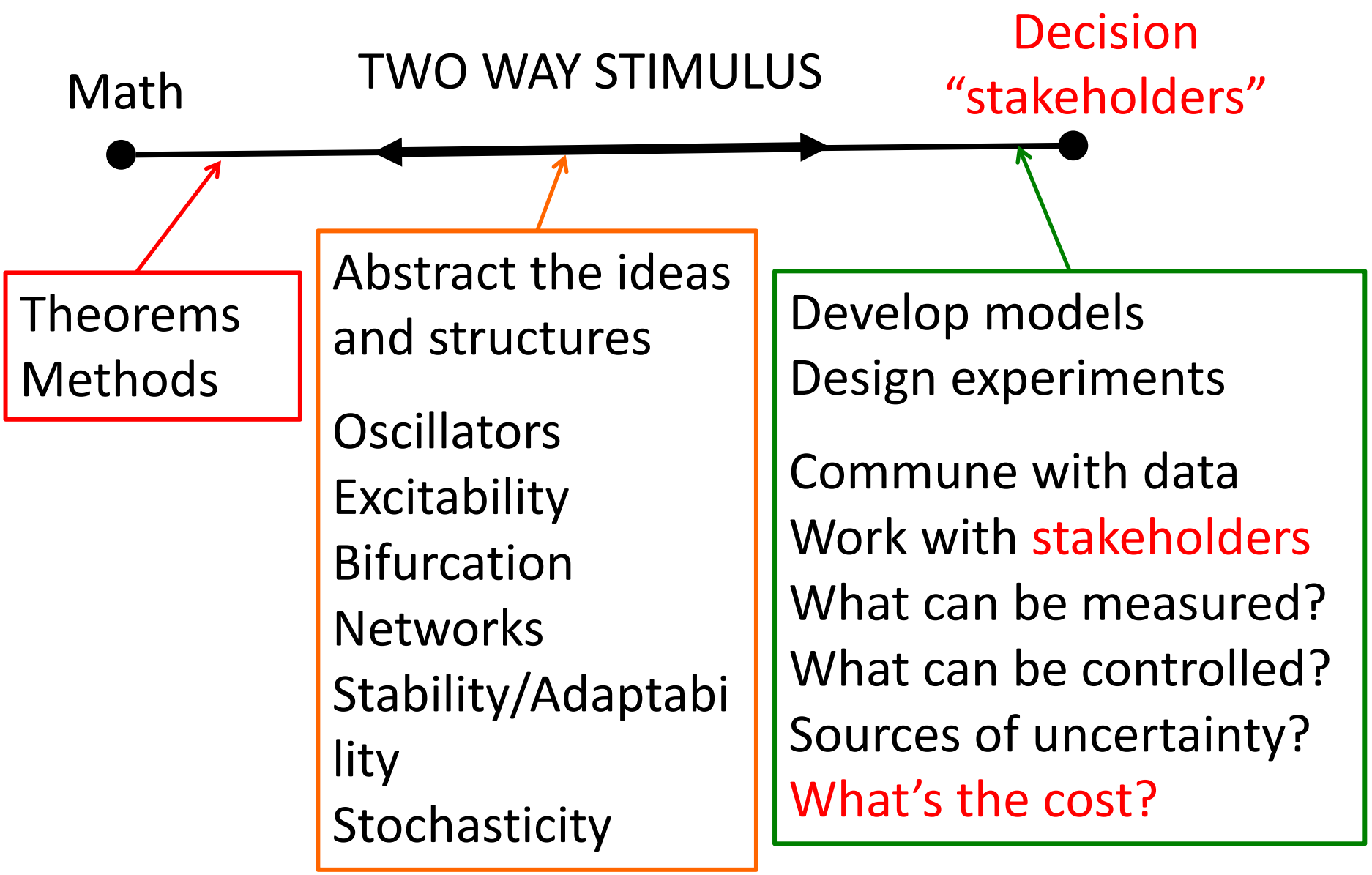


Bridge Crossing : Decision Support



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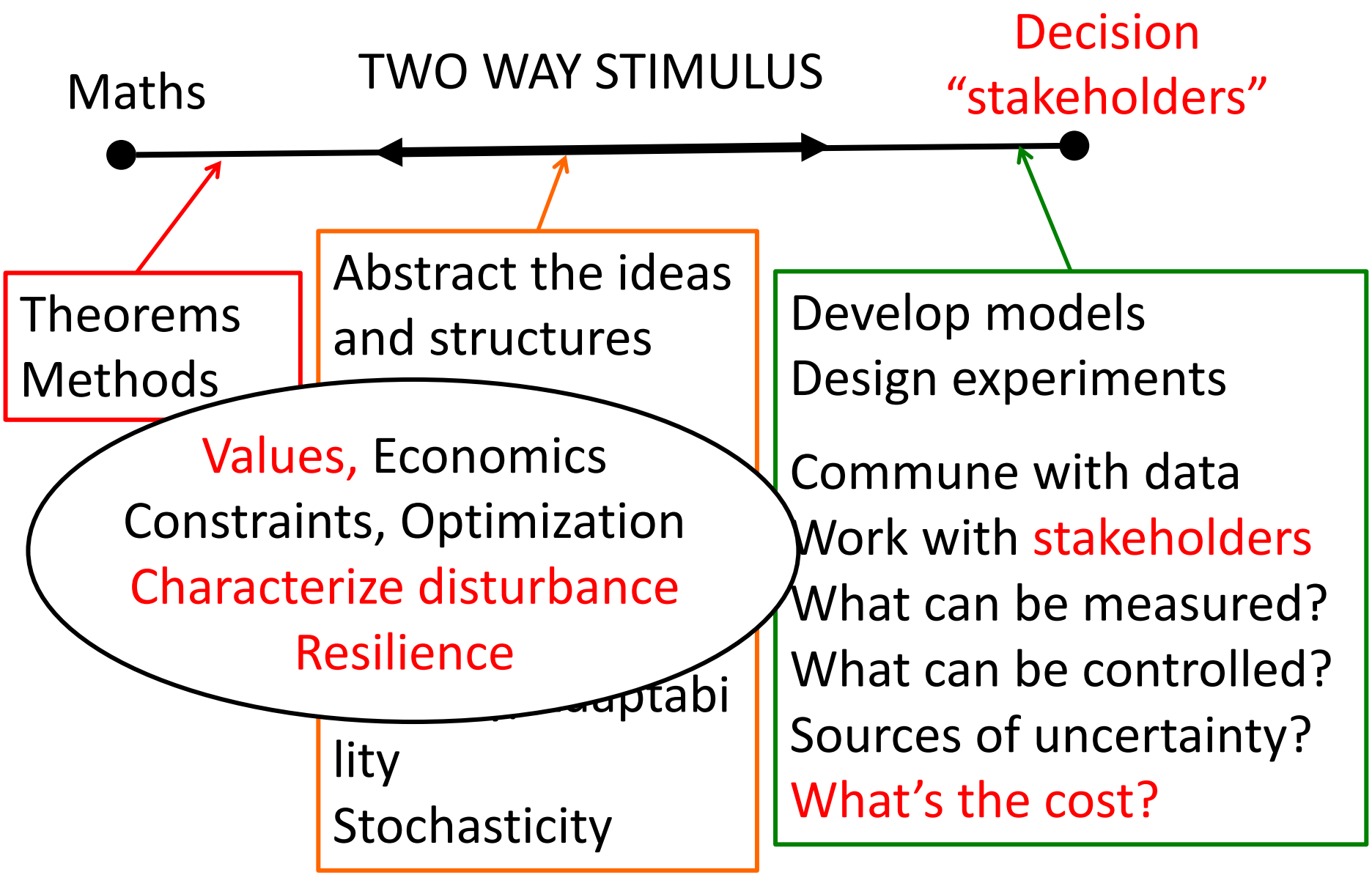
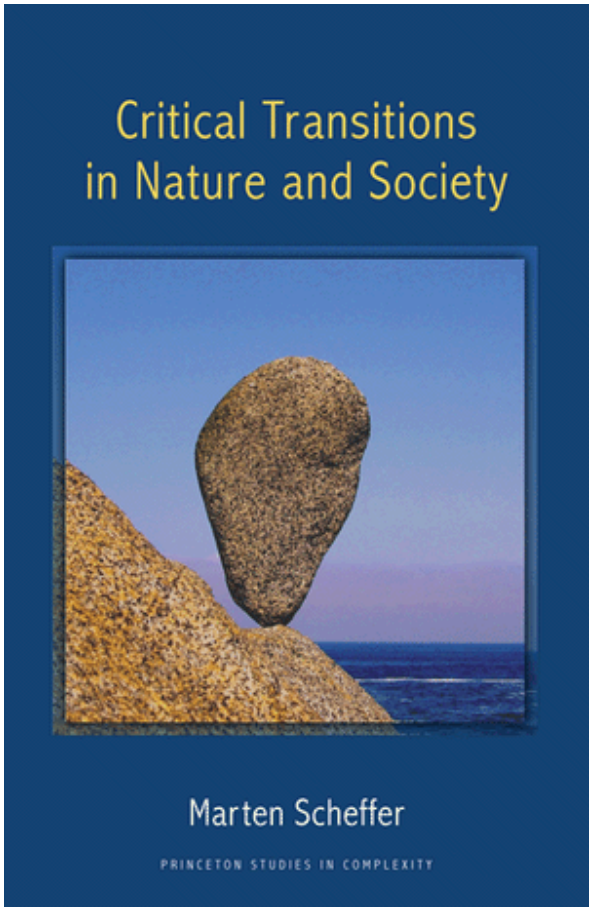
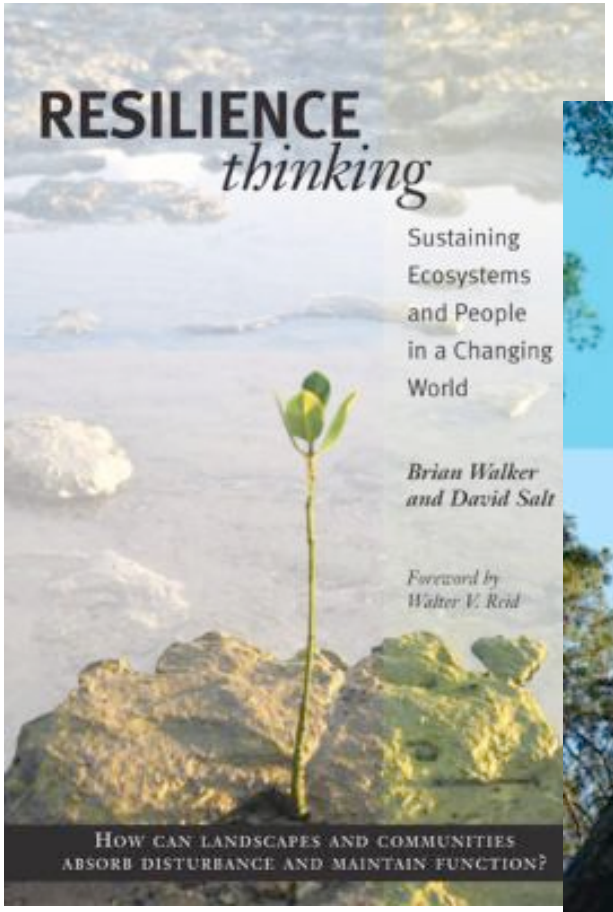


What is Resilience?



Marten Scheffer



Brian Walker & David Salt

“Resilience of what to what?”

Ecosystems (2001) 4: 765–781
DOI: 10.1007/s10021-001-0045-9

ECOSYSTEMS

MINIREVIEW

From Metaphor to Measurement: Resilience of What to What?

Steve Carpenter,[1]* Brian Walker,[2] J. Marty Anderies,[2] and Nick Abel[2]

[1]*Center for Limnology, 680 North Park Street, University of Wisconsin, Madison, Wisconsin 53706, USA; and* [2]*CSIRO Sustainable Ecosystems, GPO Box 284, Canberra, ACT, 2615 Australia*

“Resilience is the magnitude of disturbance that can be tolerated before a socioecological system moves to a different region of state space controlled by a different set of processes.”

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Alternative stable states

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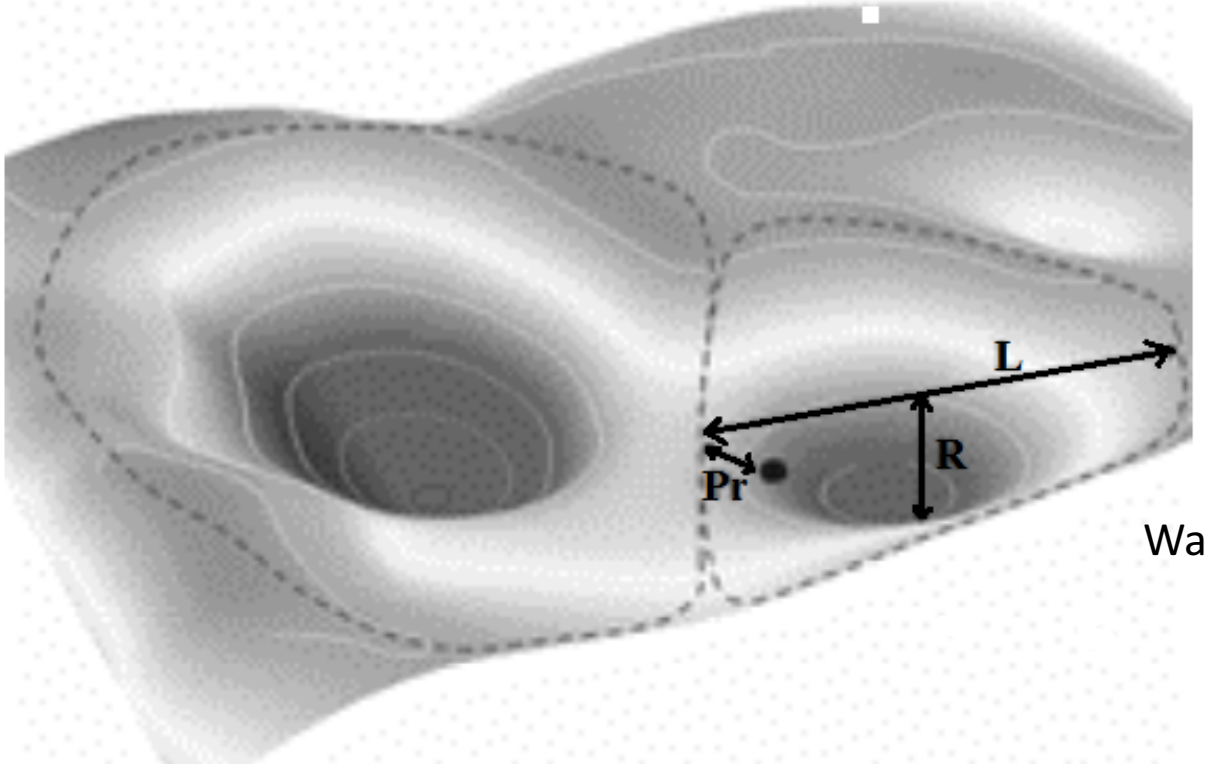

Walker et al. (2004)

Alternative stable states

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What is Resilience?

Slippery qualitative idea:

The ability of a system to absorb change and disturbance while maintaining its basic structure and function.

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The ability of a system to absorb change and disturbance while maintaining its basic structure and function.

Examples:

- A city “disturbed” by hurricane or other disaster.
- A building disturbed by earthquake,
- An ecosystem disturbed by water, fire, grazers, diseases, invasives, nutrients or weather extremes.
- Agriculture disturbed by pests, climate, economics.
- Fishery disturbed by harvesting.
- The climate system disturbed by life.
- Human medicine and psychology.

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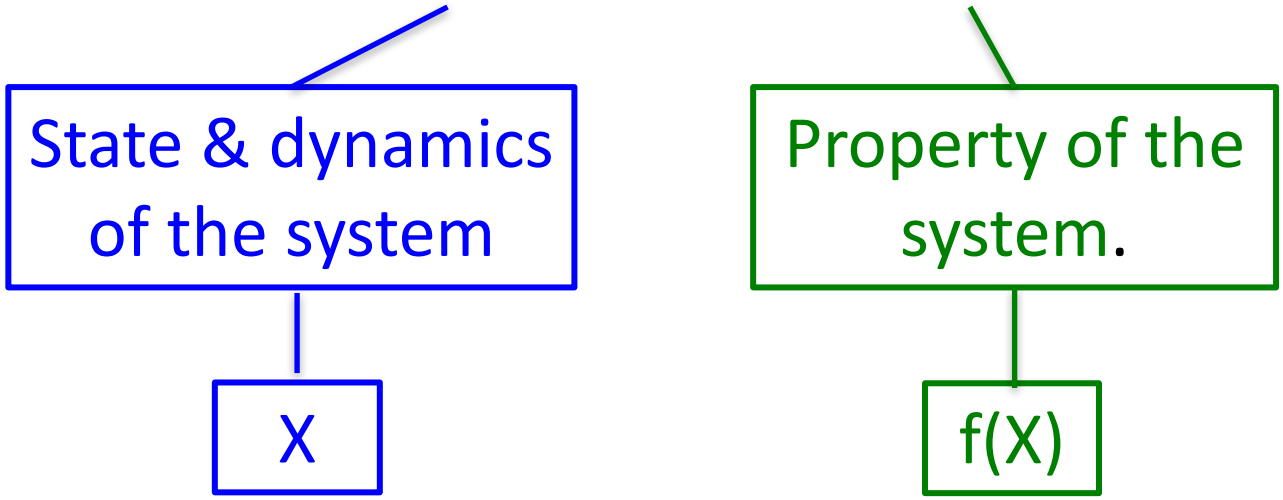

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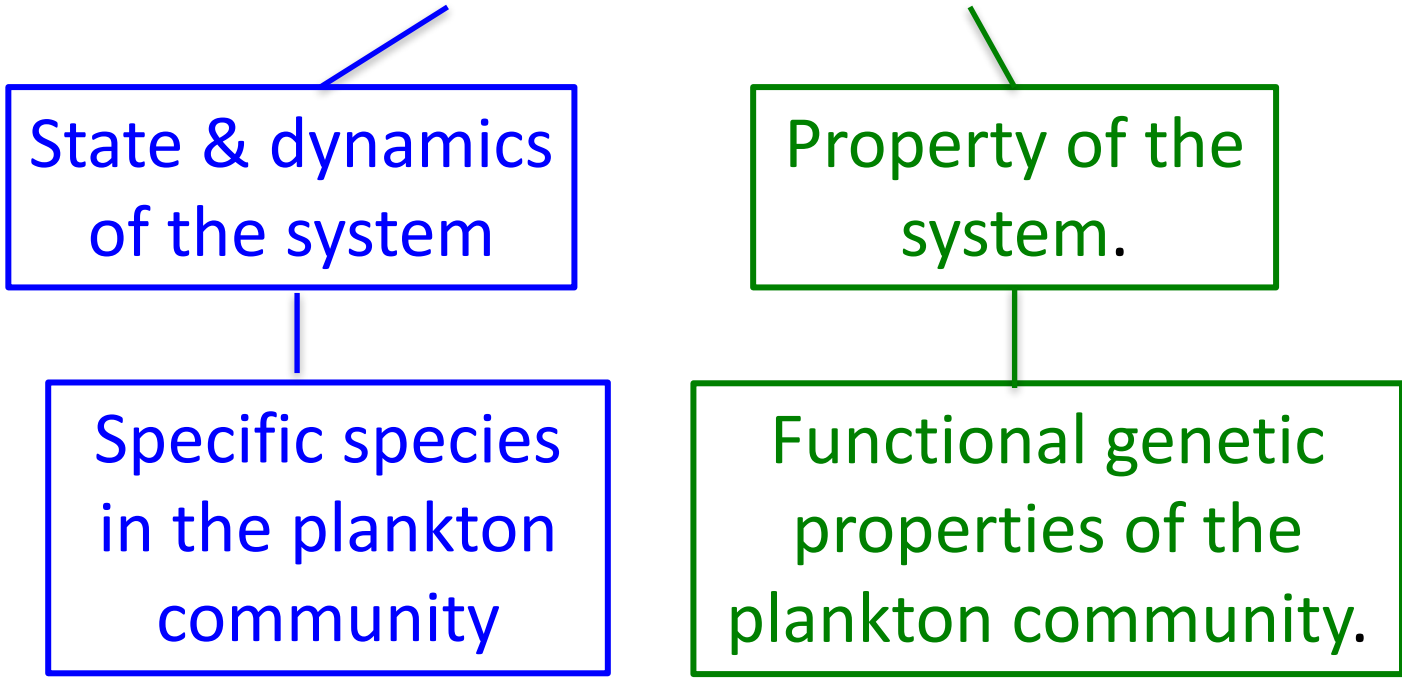

e.g. Ocean plankton community

Single large disturbance

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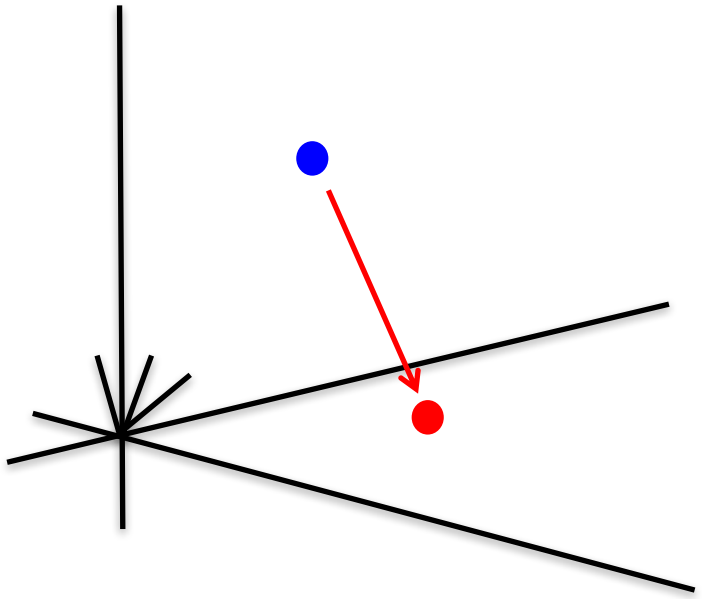
Example: city “disturbed” by hurricane or other disaster



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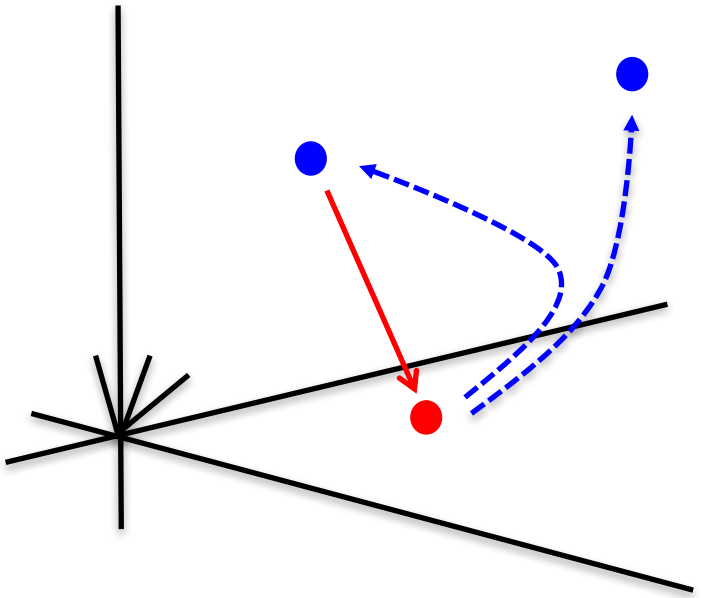
Building “disaster resilience” is about building mechanisms for rapid transient dynamics along ‘desirable’ route back to functional - or even ‘improved’ - city.

e.g. minimizing disease outbreaks, protecting water supply, etc.



Accumulating small disturbances

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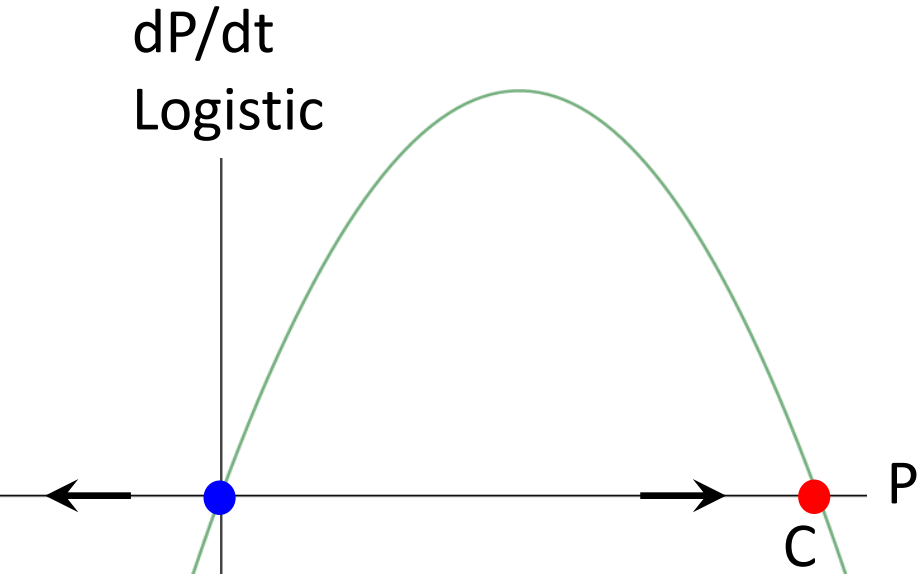
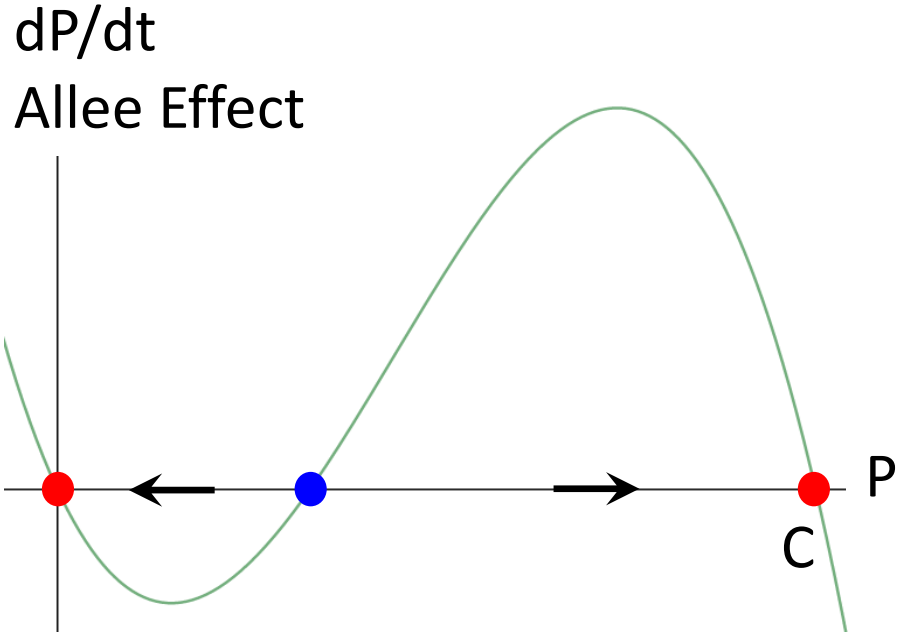
Example: Fishery subject to harvesting





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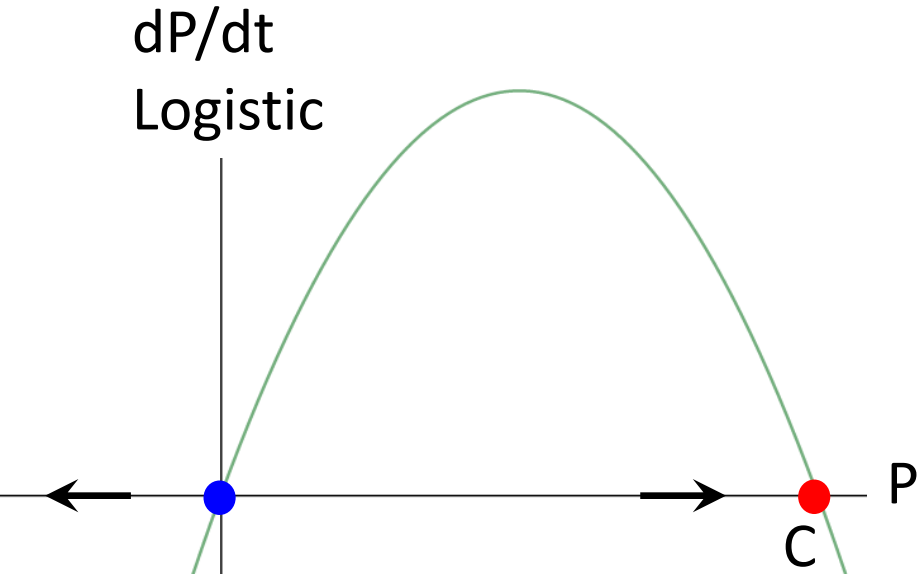

Population flows toward C

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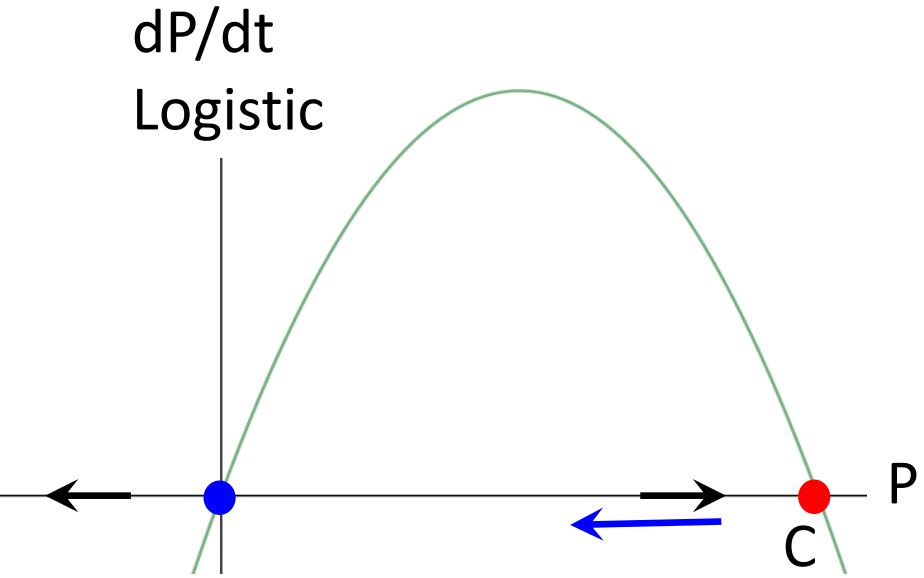

Population flows toward C

Harvest ‘kicks’ pop’n down

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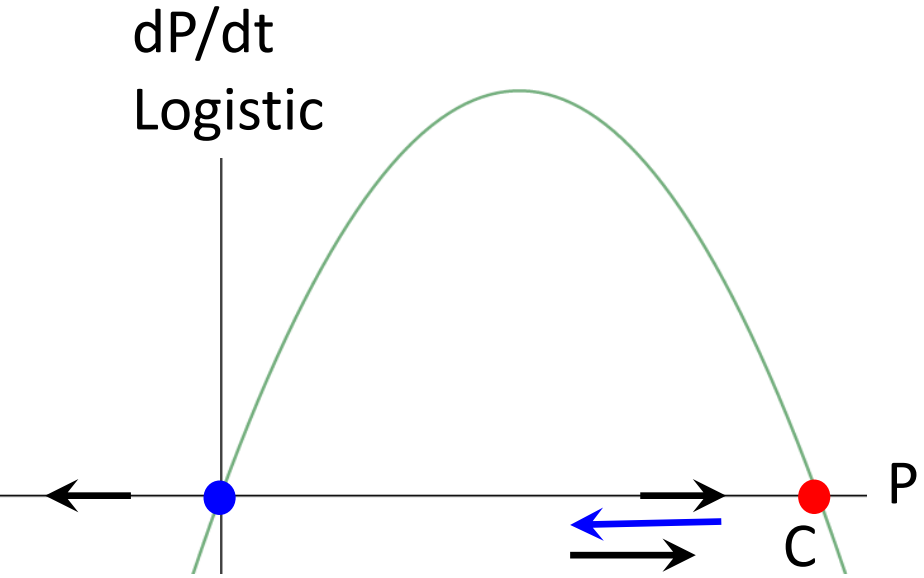

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Population flows for time τ

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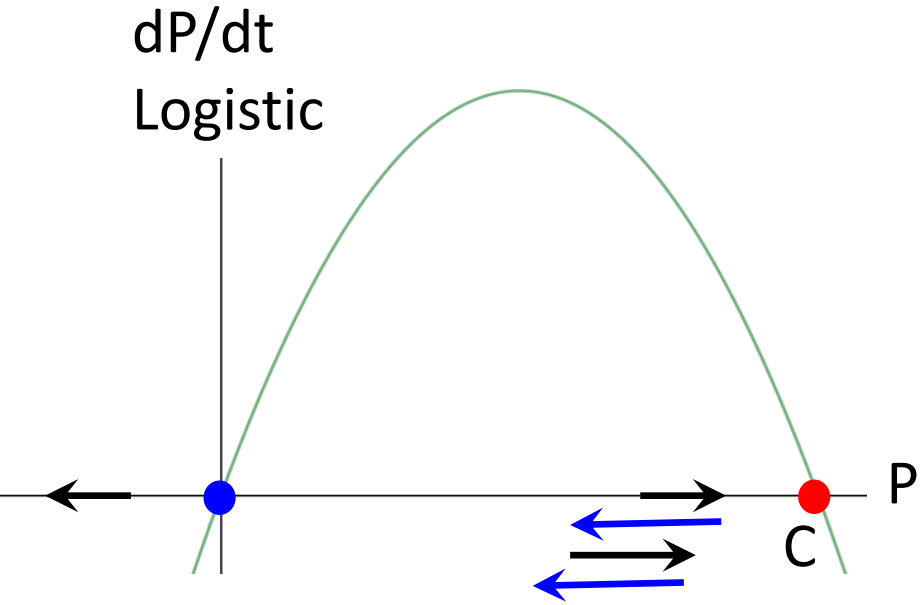

Population flows toward C

Harvest ‘kicks’ pop’n down

Population flows for time τ

Another harvesting kick

etc....

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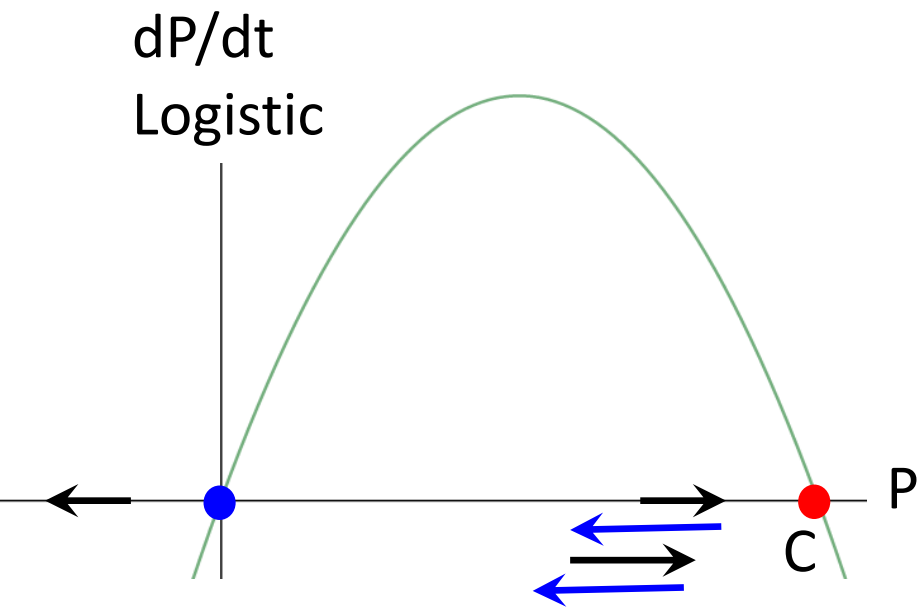

Is stock population resilient to this level of harvesting?

i.e. Do transient flow and kick balance?

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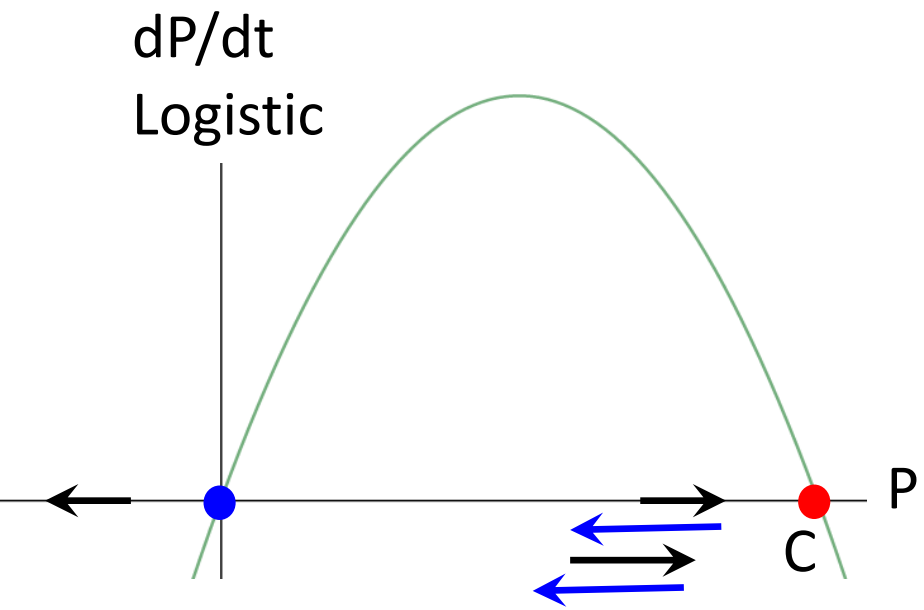

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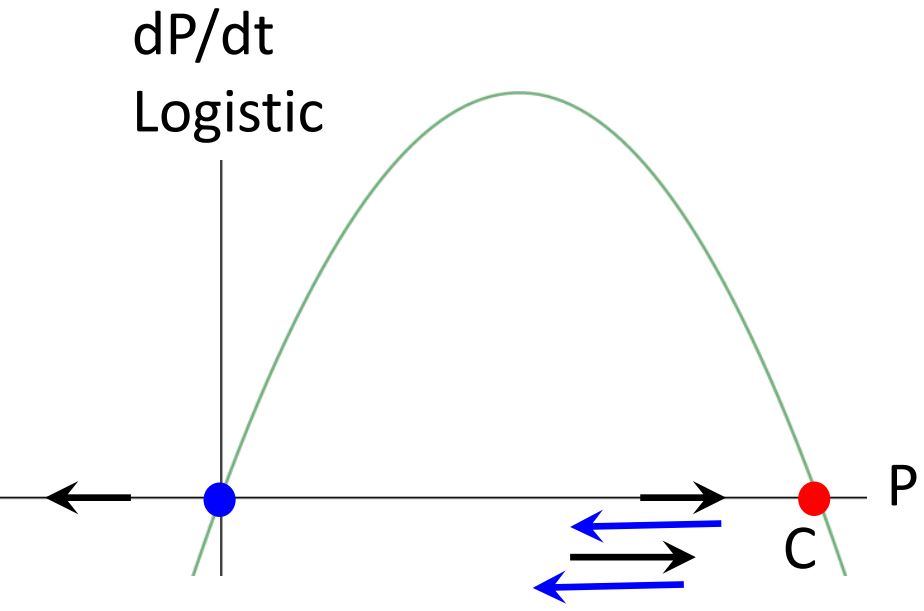

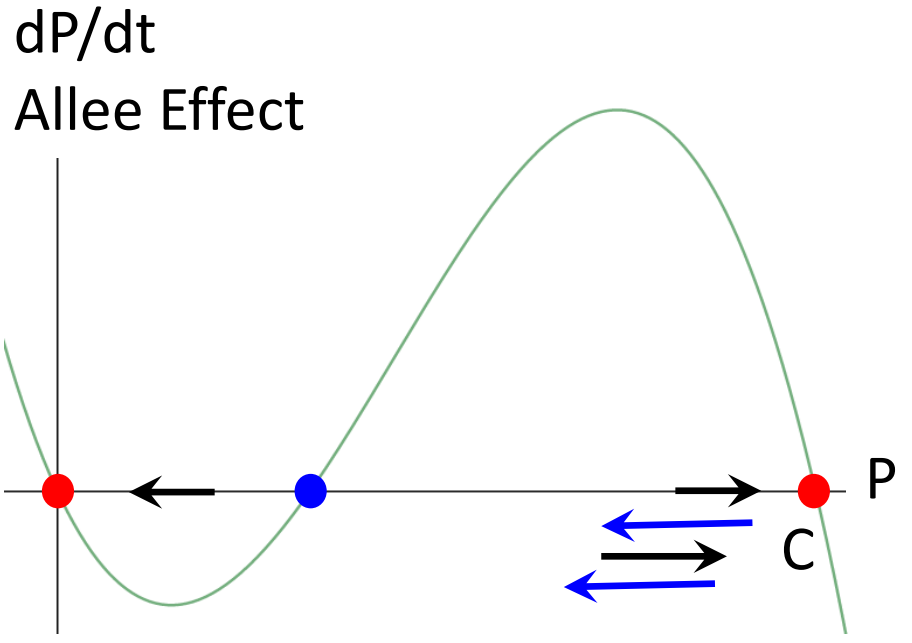

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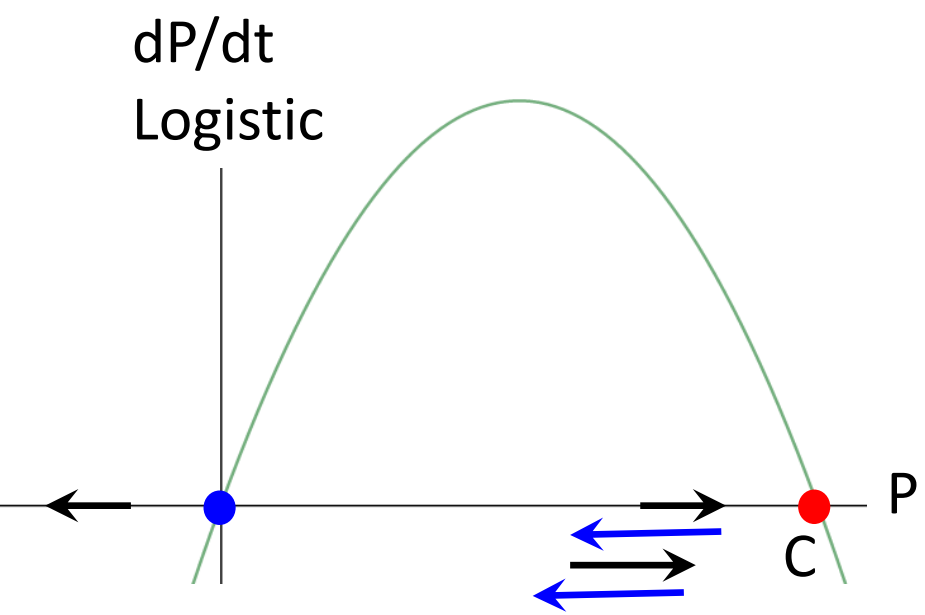

Social value:

If transient flow and kick balance, do we like where they balance?

e.g. bioeconomics, tragedy of the commons, etc

Accumulating small disturbances

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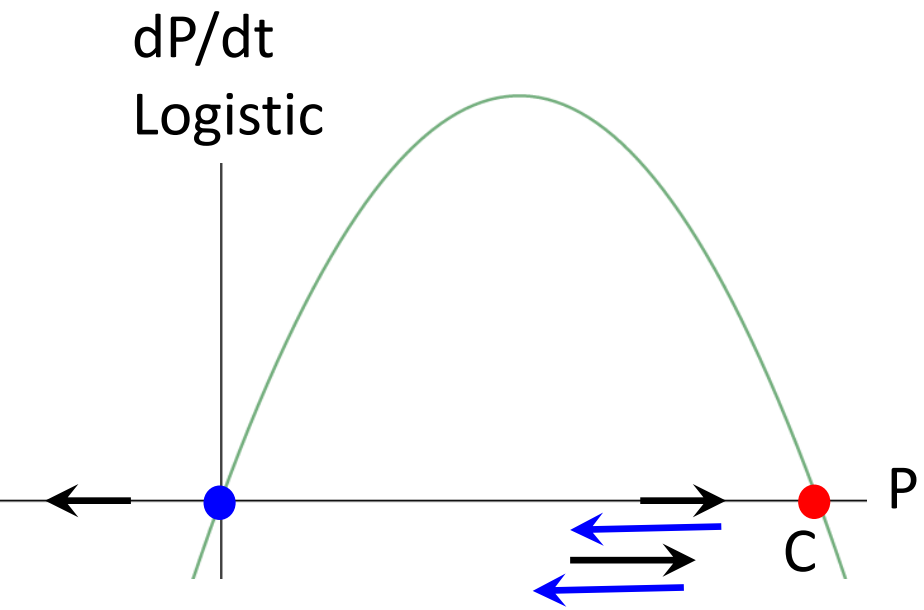
Example: Management of invasive species



Social value:

How do we limit invasion?

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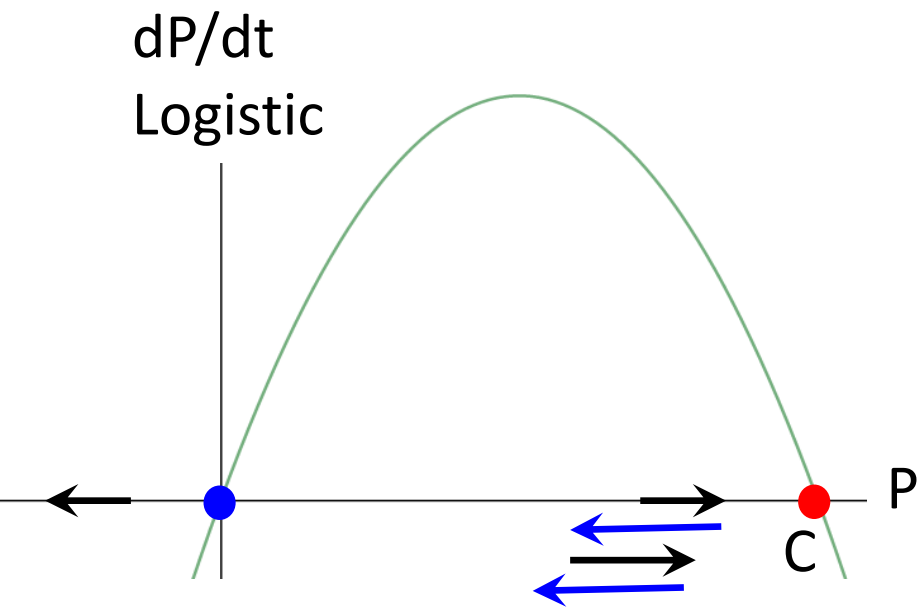


Social value:

How do we limit invasion?

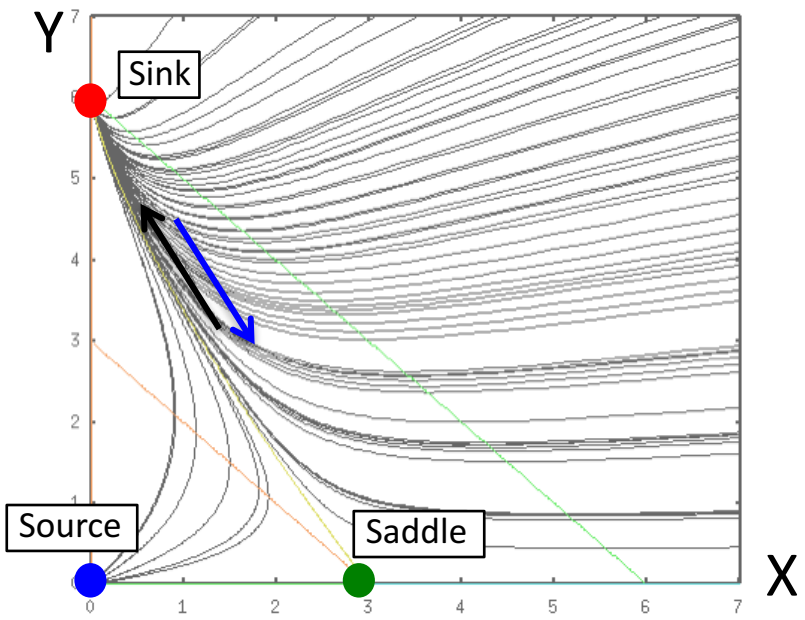
Suggests a different preferred balance between transient flow and kick than for fishery

Disturbance as the norm

Slippery qualitative idea:

The ability of a system to absorb change and disturbance while maintaining its basic structure and function.

Example: Ecosystem disturbed by fire, disease, grazers, weather...



e.g. Forest:

Without fire, Y outcompetes X

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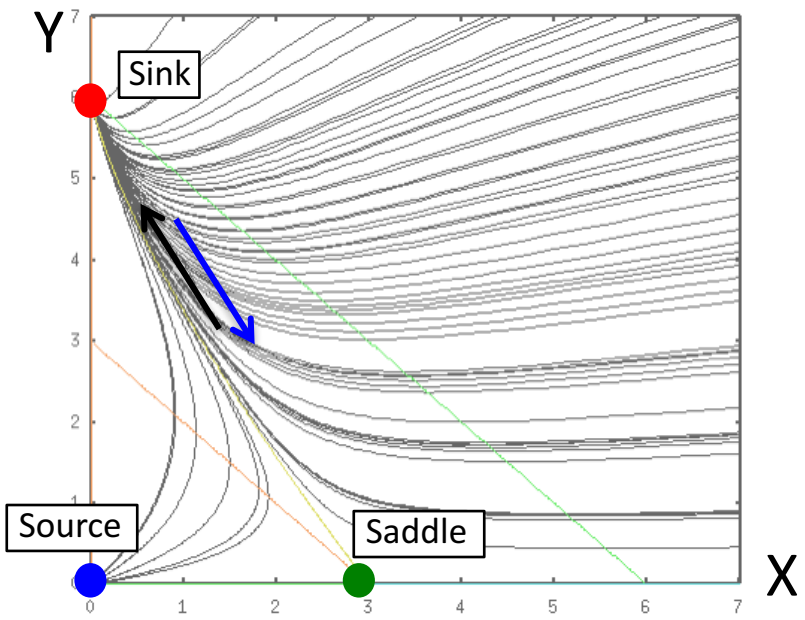

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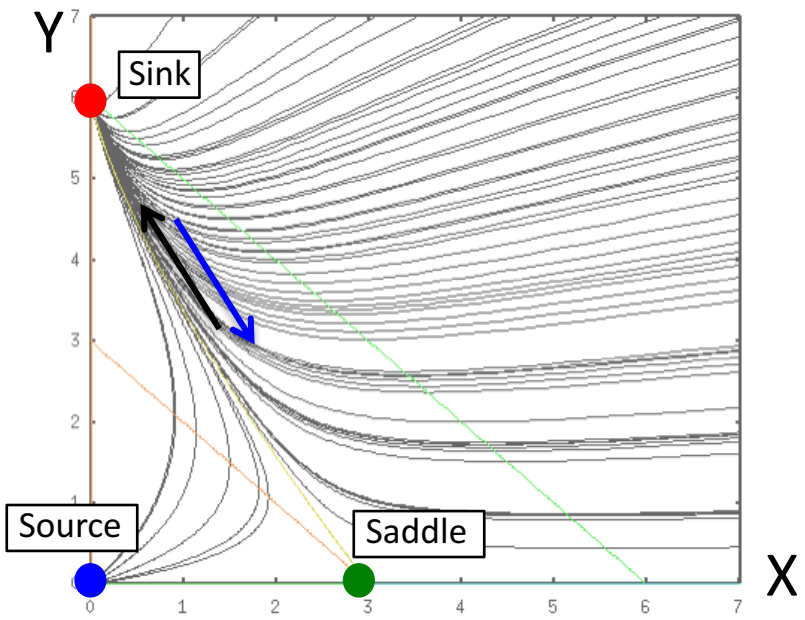

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Fire disturbance promotes biodiversity, via transient dynamics

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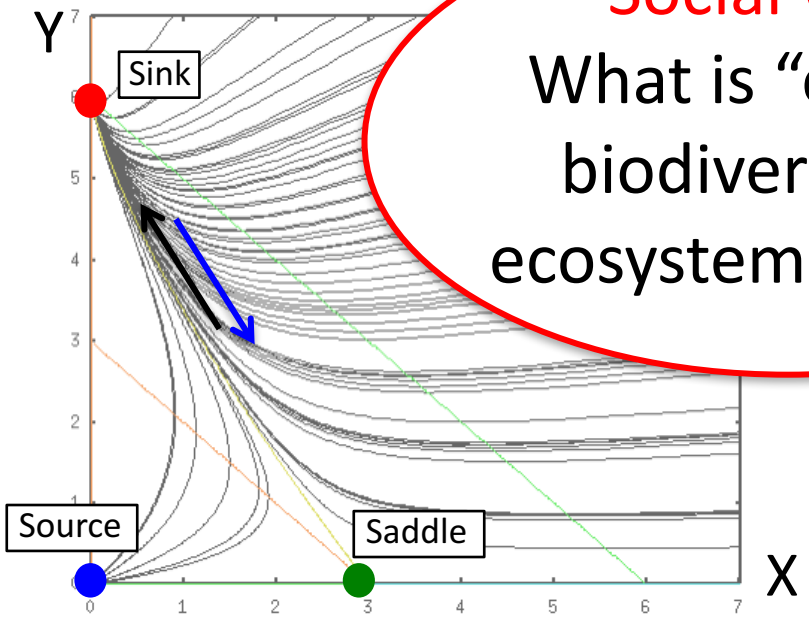
Social value:

What is "enough" biodiversity? or ecosystem services?



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Example: Dryland ecosystem disturbed by water.

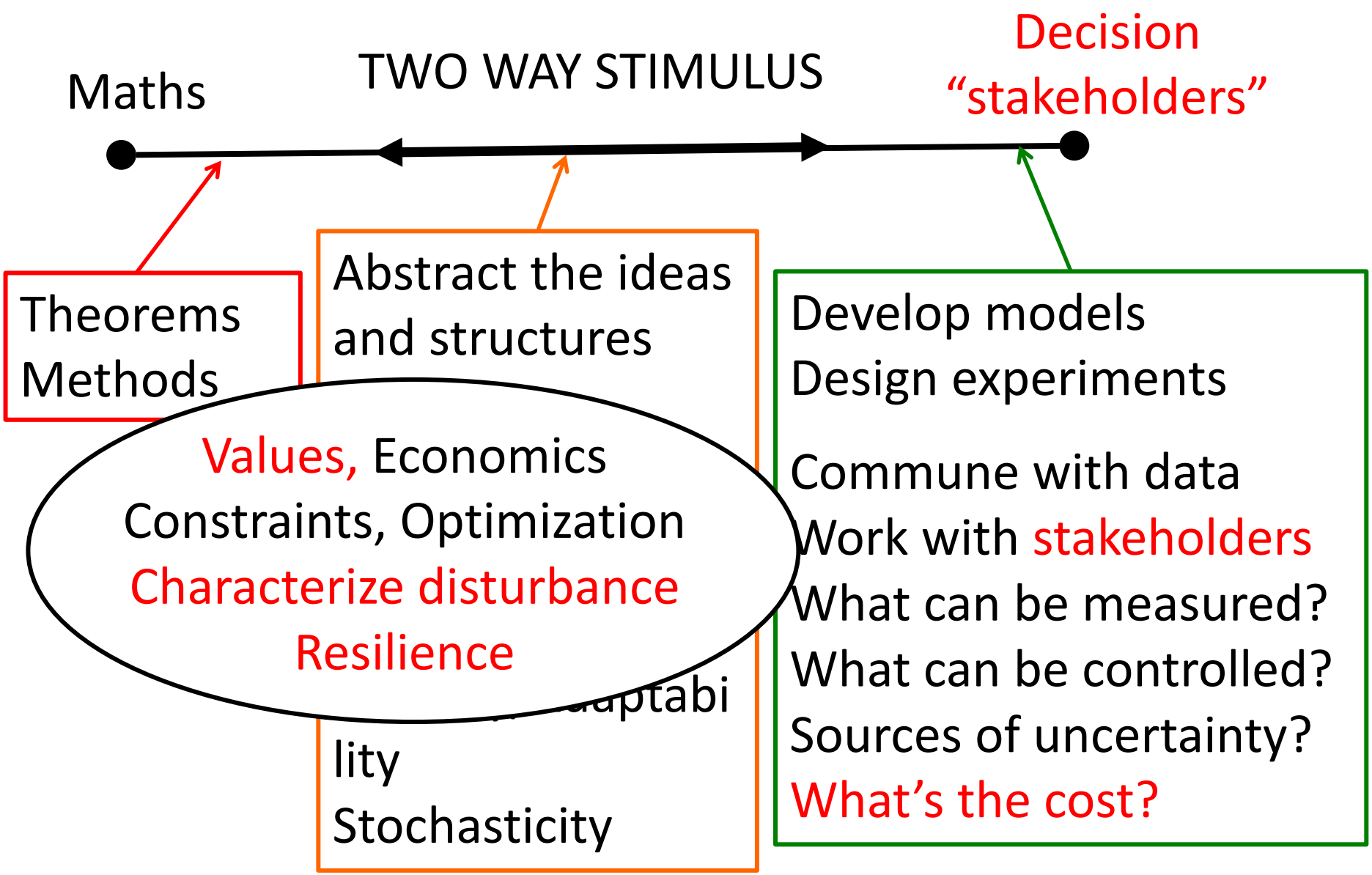
Makes up 40% of Earth's land

Home to 30% of human population

Social value: Can the land feed the people?

i.e. is there enough vegetation to support the people?

Bridge Crossing : Decision Support



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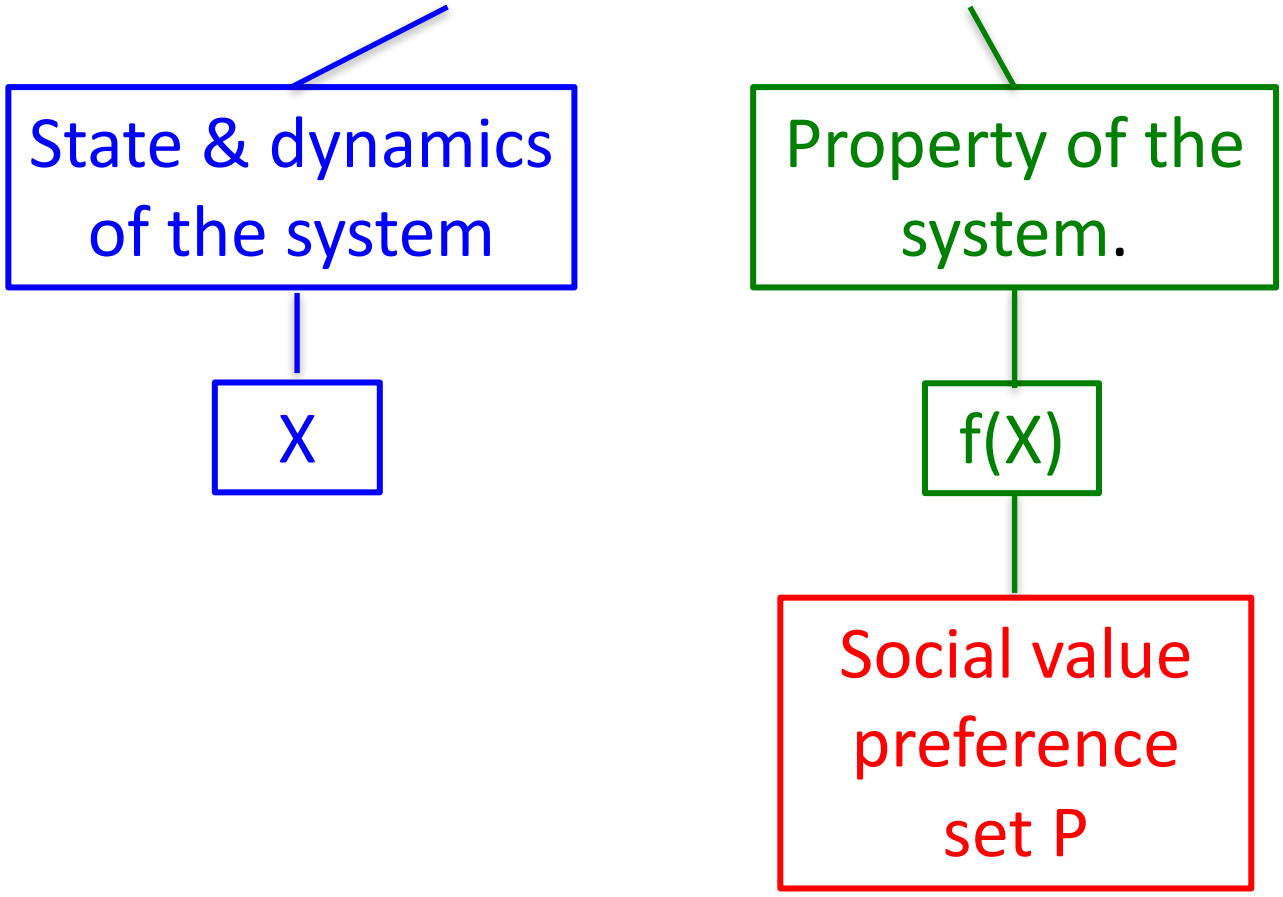

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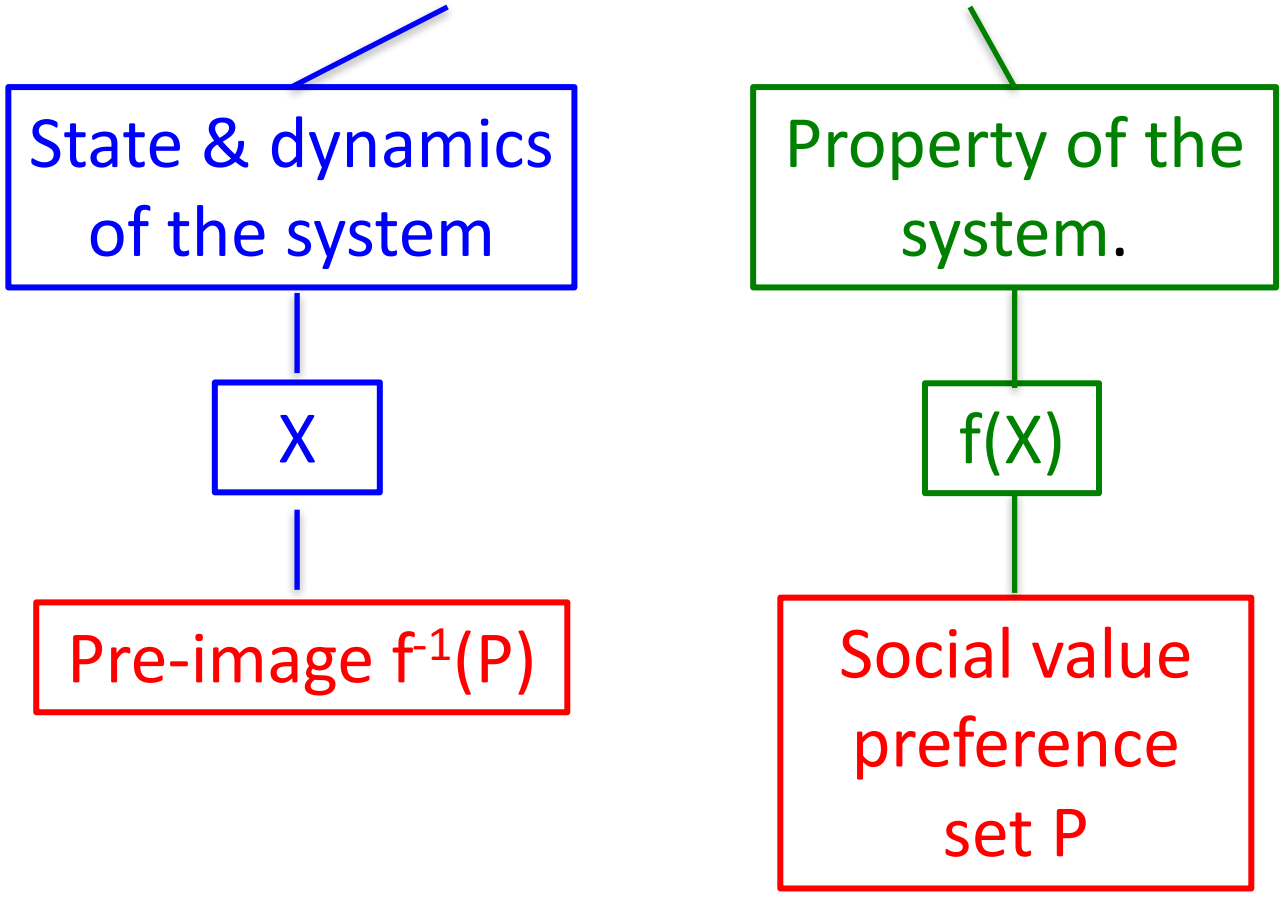

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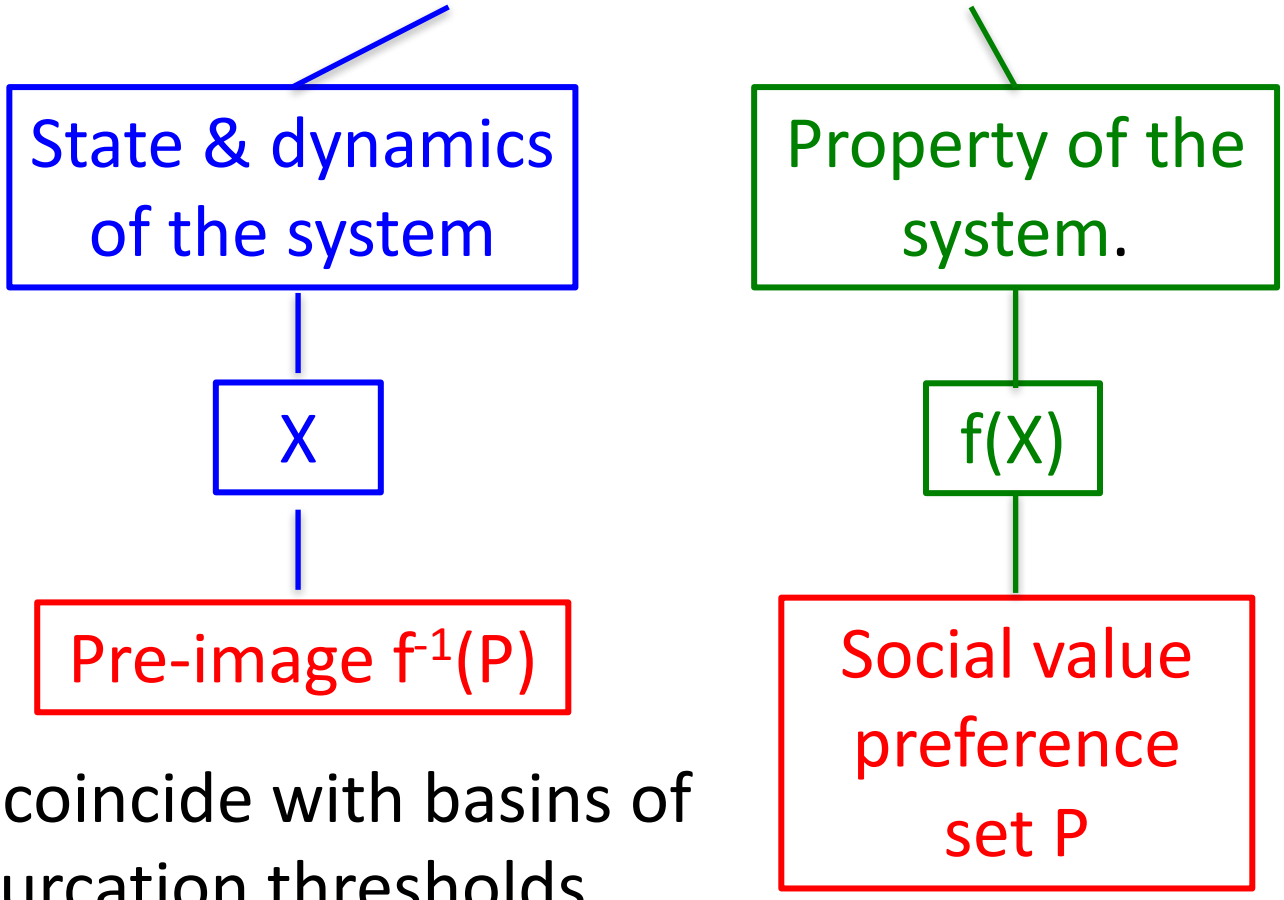


May or may not coincide with basins of attraction, or bifurcation thresholds

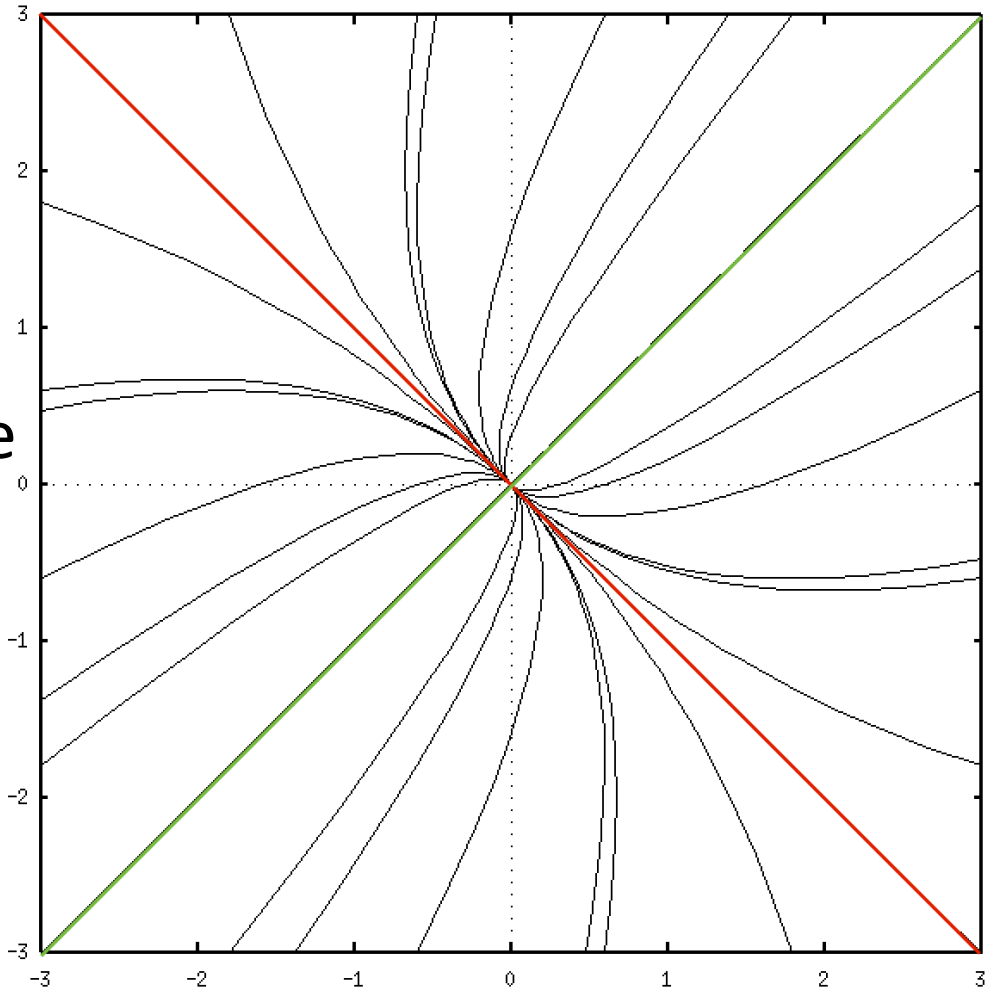
Flow-kick dynamics

Given:

System of ODE's $dx/dt=F(x)$

Flow $\varphi_t(x)$

Start with 2-D linear example for illustration



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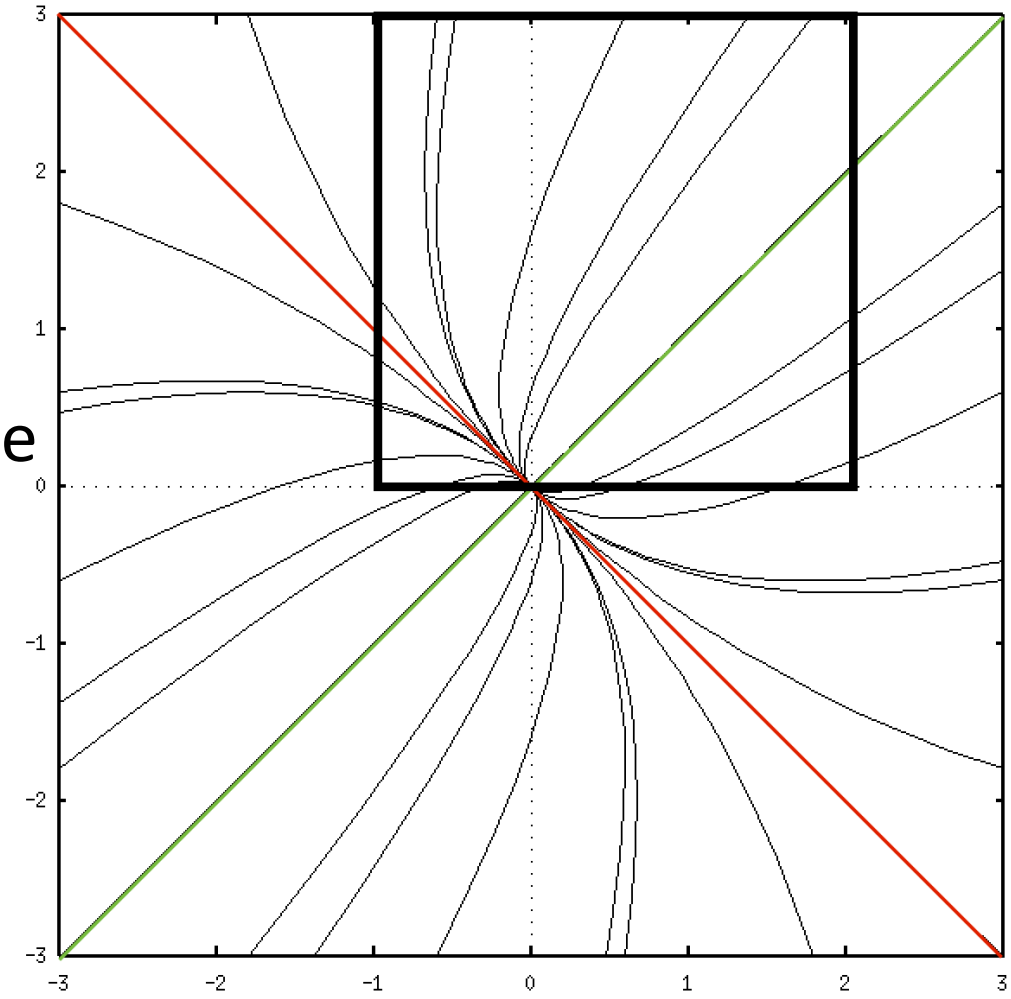


Flow-kick dynamics

Given:

System of ODE's $dx/dt=F(x)$

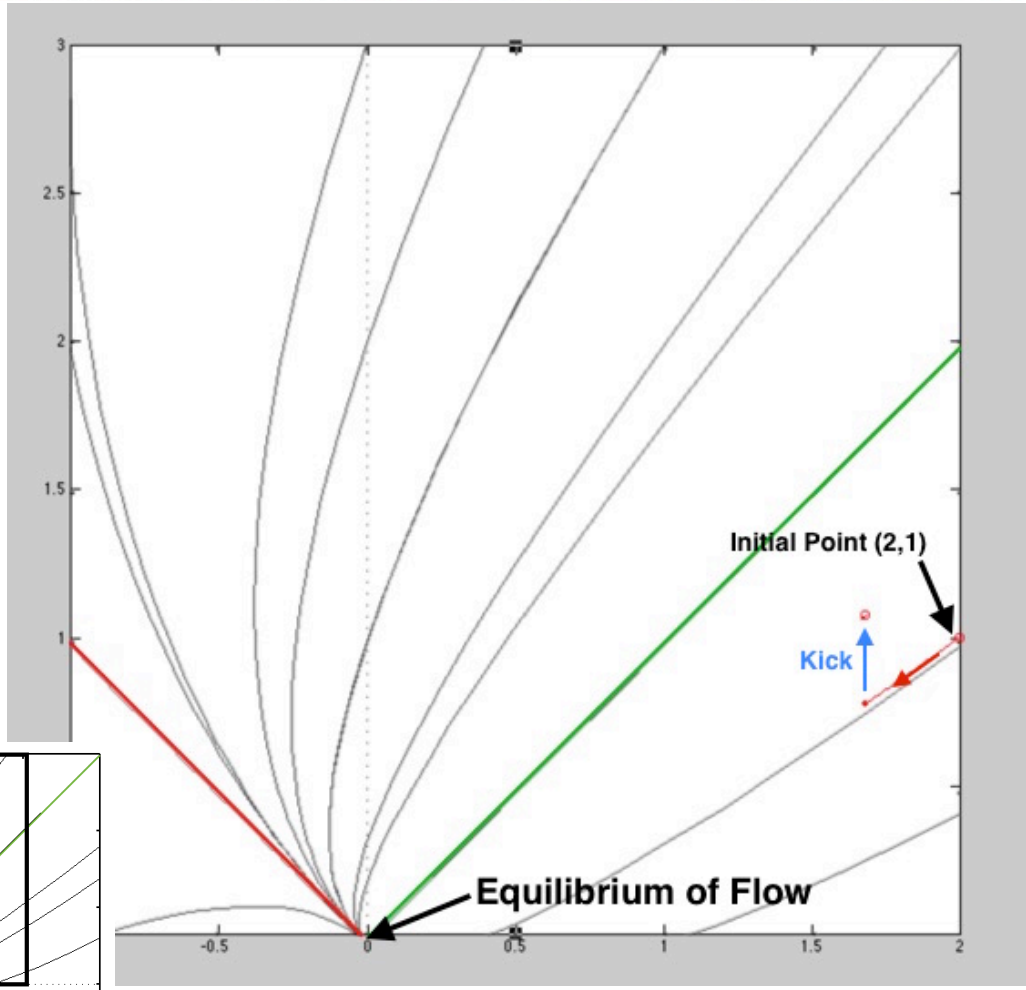
Flow $\varphi_t(x)$

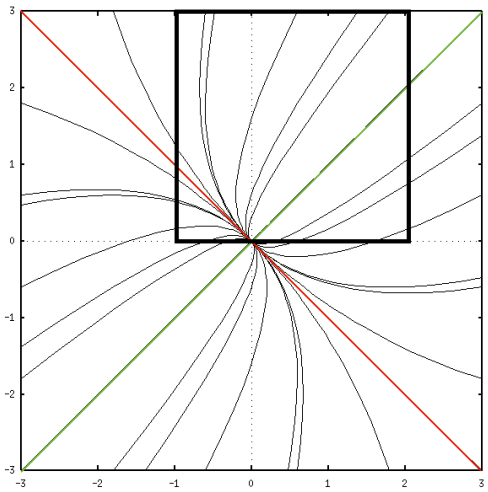
Flow time τ

Kick vector k

Define flow-kick map:

$G(x) = \varphi_\tau(x) + k$





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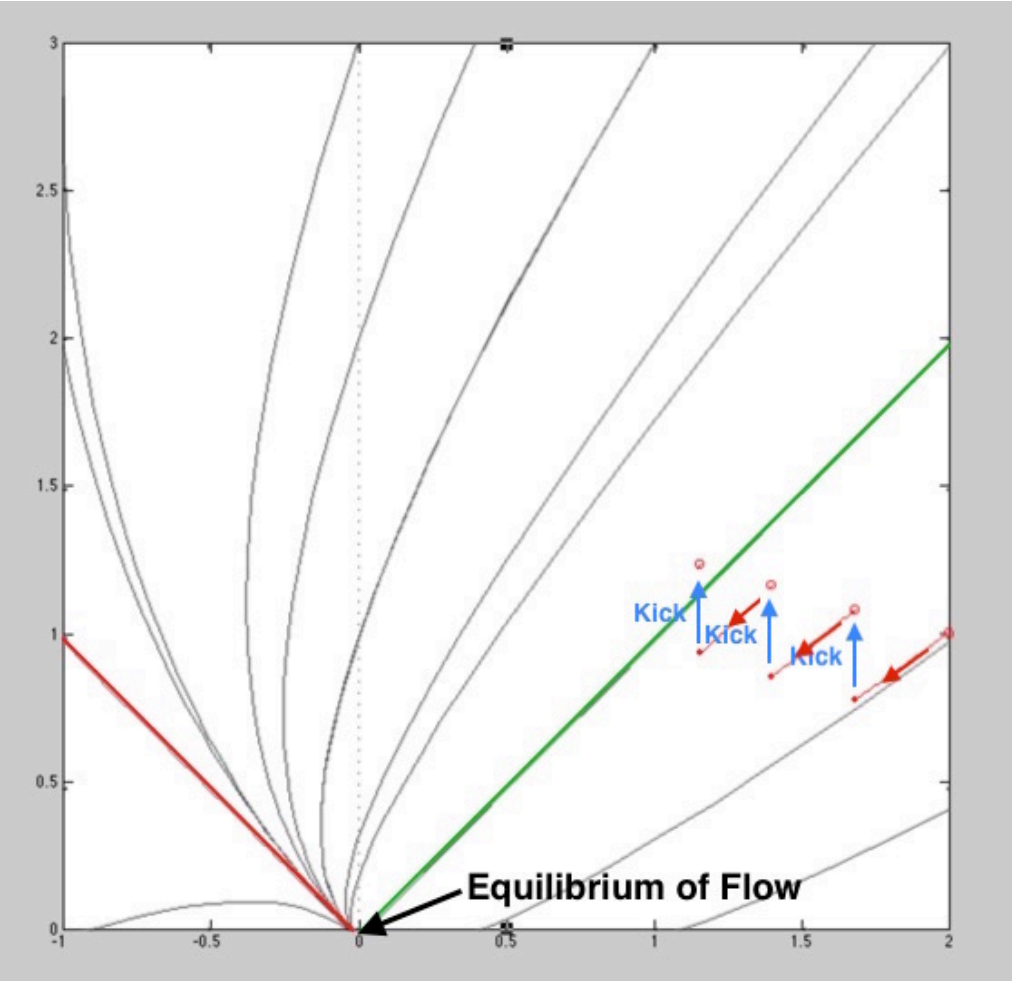

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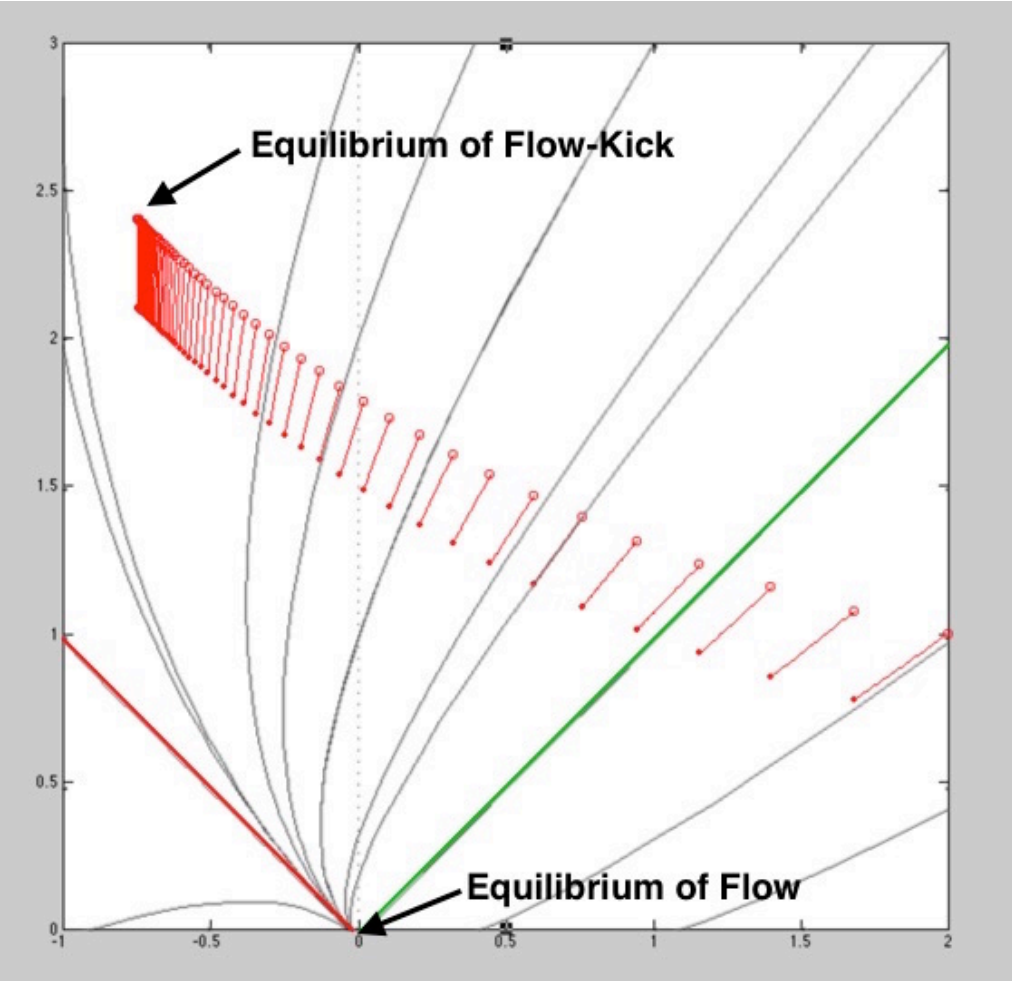

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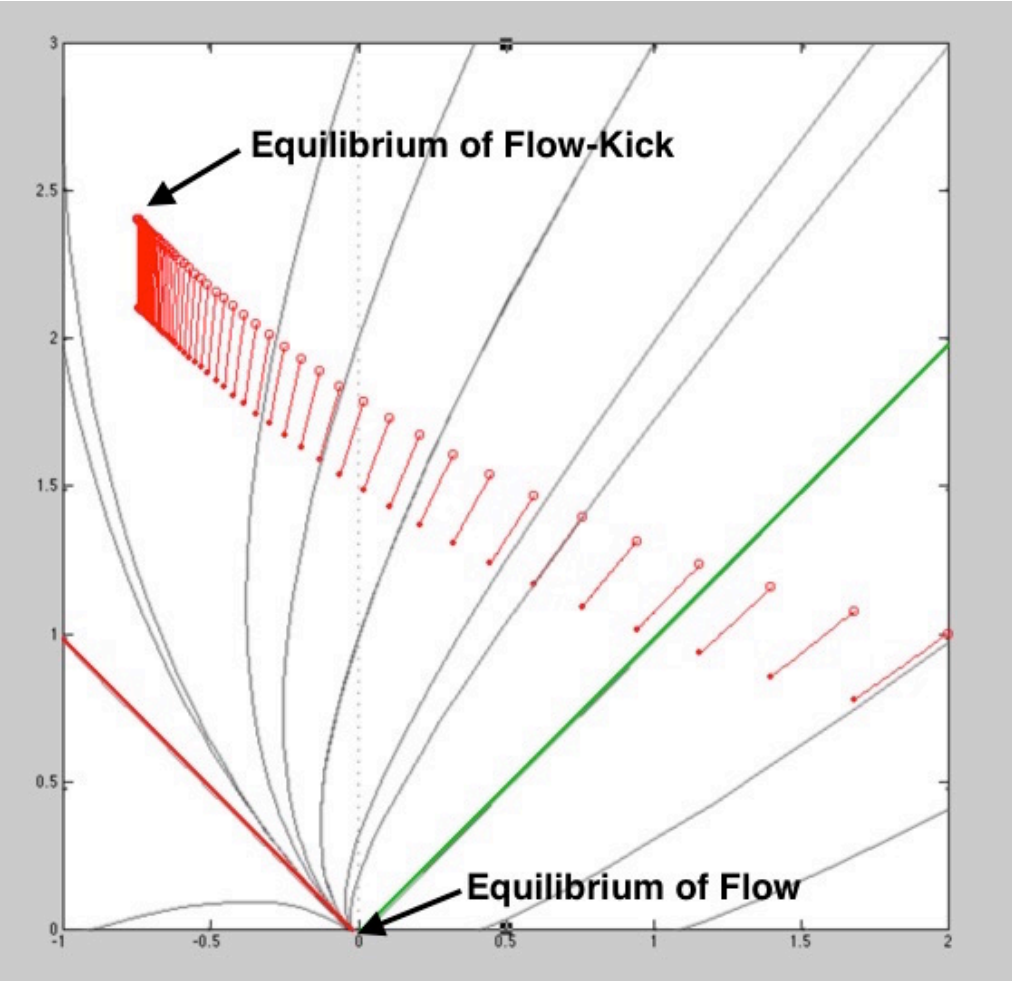

Brings focus to:

1) Transient dynamics

2) 'Value' we assign to new location in state space

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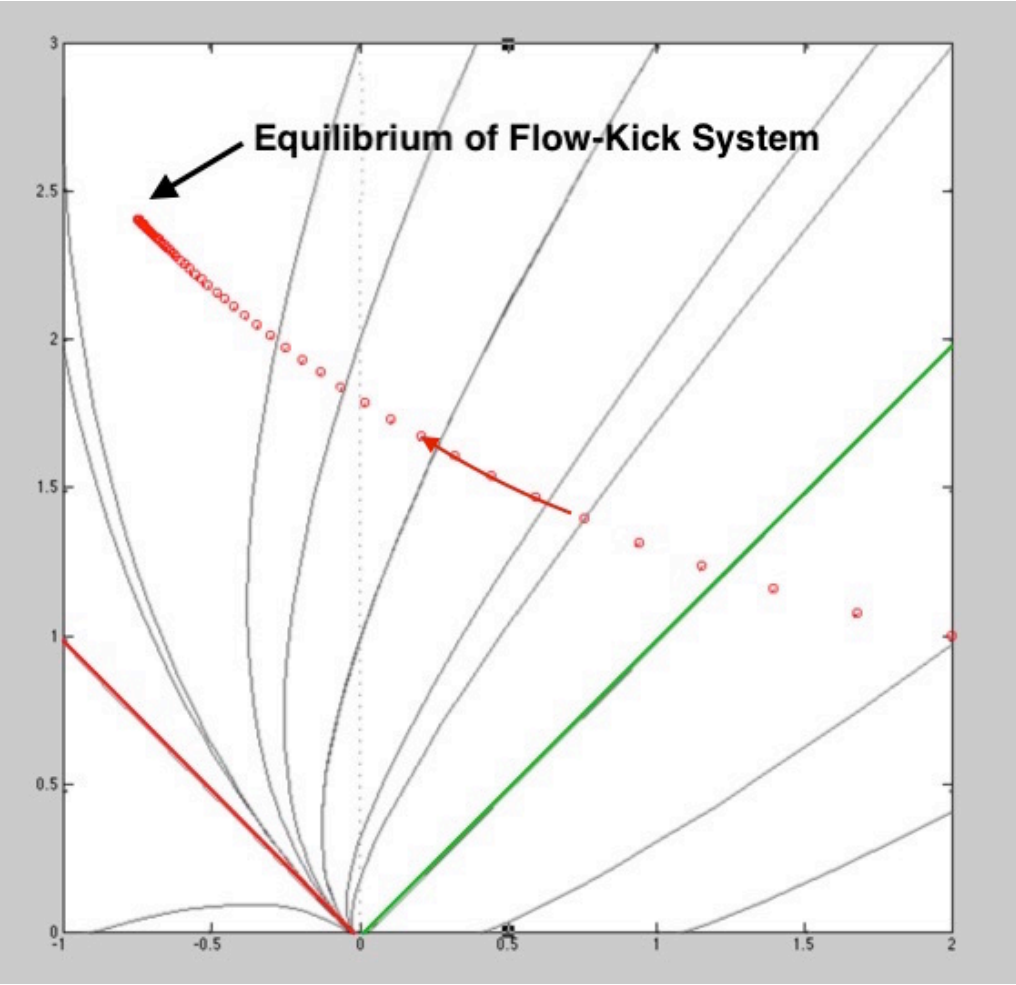
$G(x) = \varphi_\tau(x) + k$

Iterate...



Brings focus to:

1) Transient dynamics

2) 'Value' we assign to new location in state space

Flow-kick dynamics

Given:

System of ODE's $dx/dt=F(x)$

Flow $\varphi_t(x)$

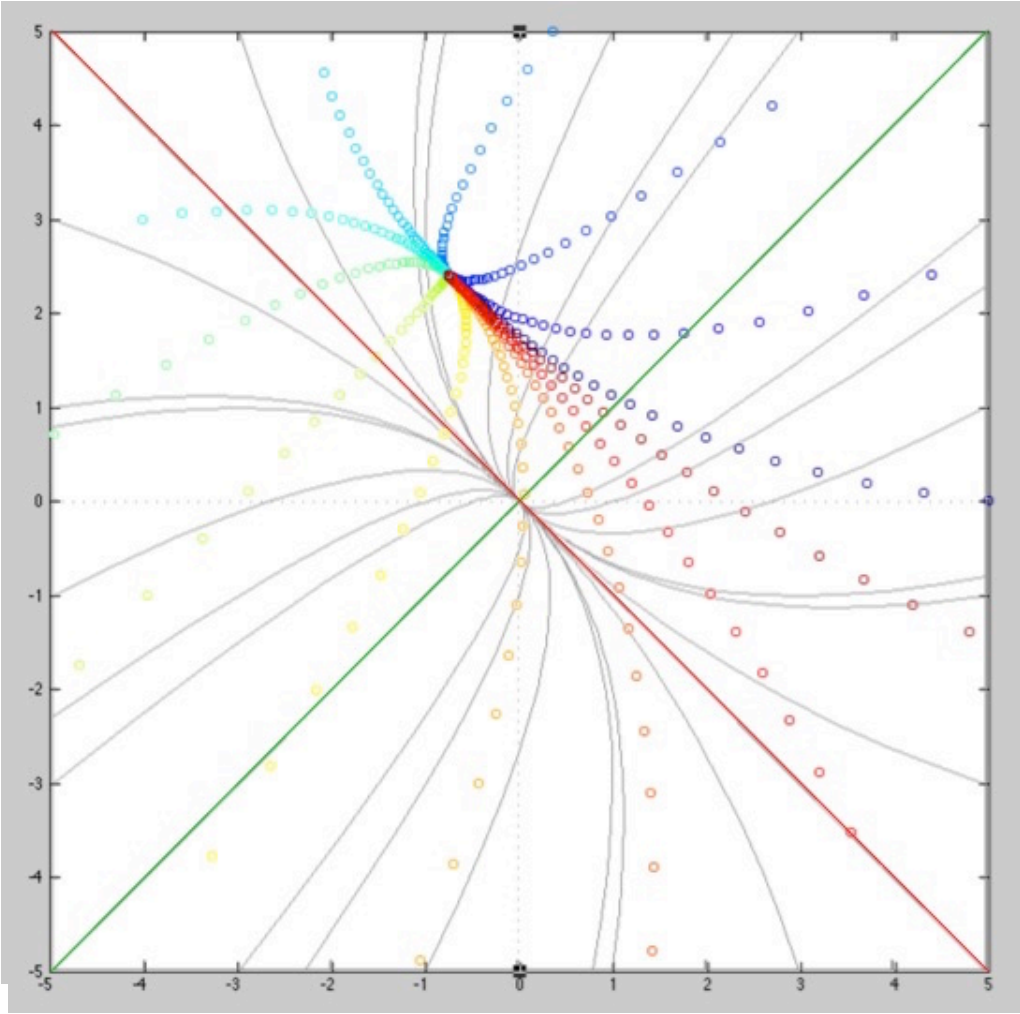
Flow time τ

Kick vector k

Define flow-kick map:

$G(x) = \varphi_\tau(x) + k$

Iterate...



Theorem: If F is linear

1) There is a unique flow-kick equilibrium.

Flow-kick dynamics

Given:

System of ODE's $dx/dt=F(x)$

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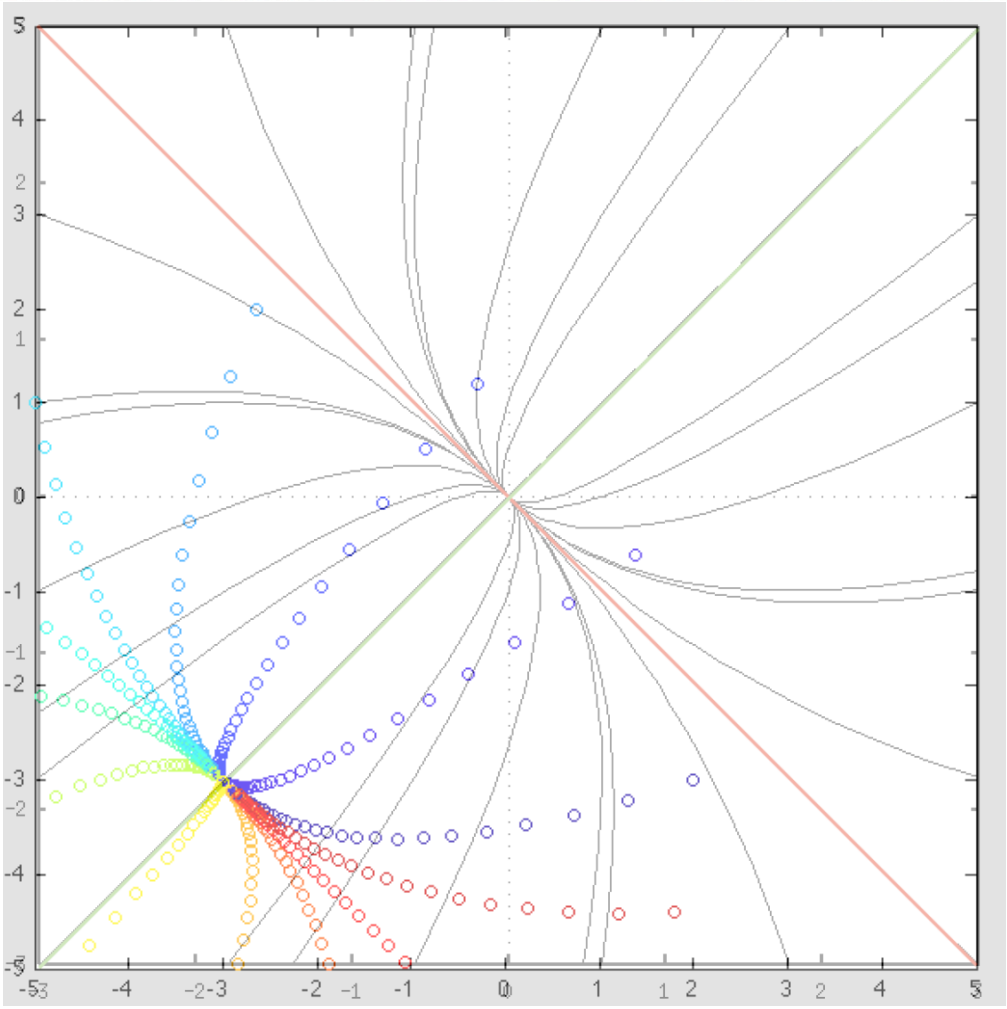

Theorem: If F is linear

1) There is a unique flow-kick equilibrium.

Choice of (τ, k) can put the flow-kick equilibrium anywhere.

Flow-kick dynamics

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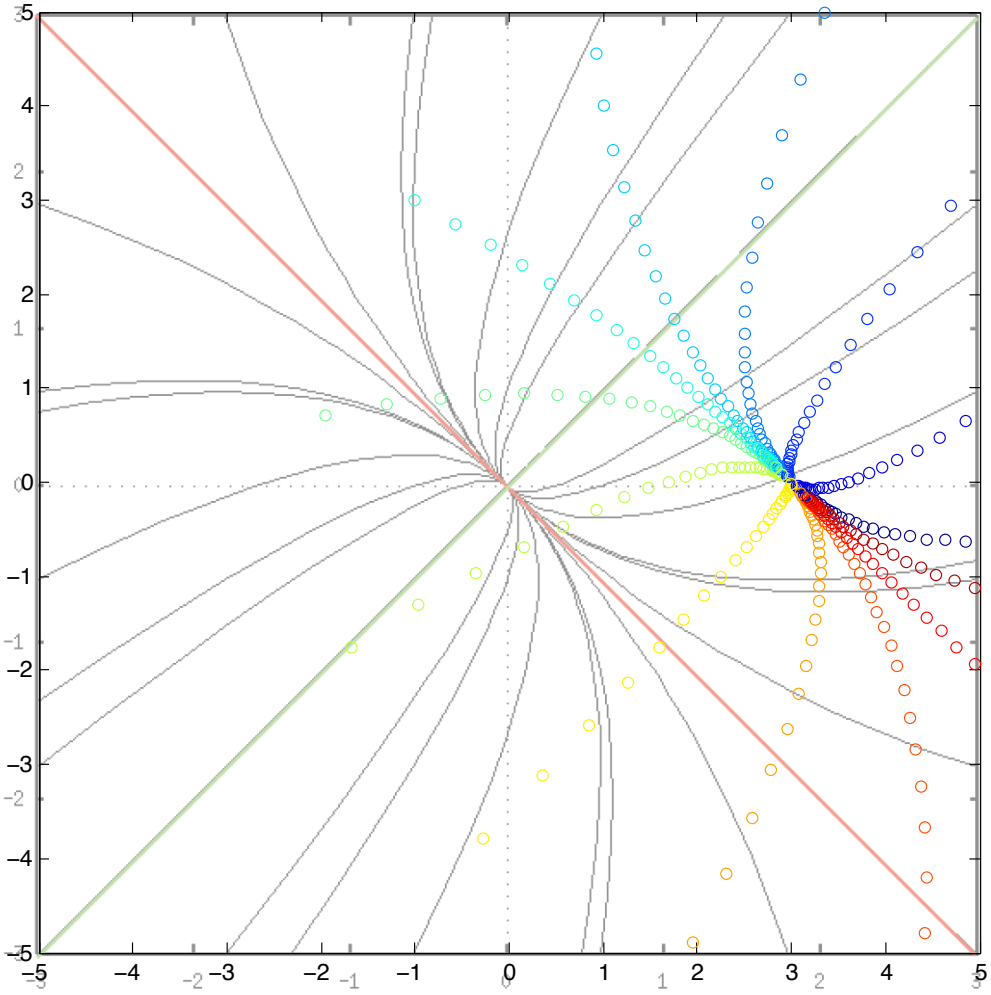
Iterate...



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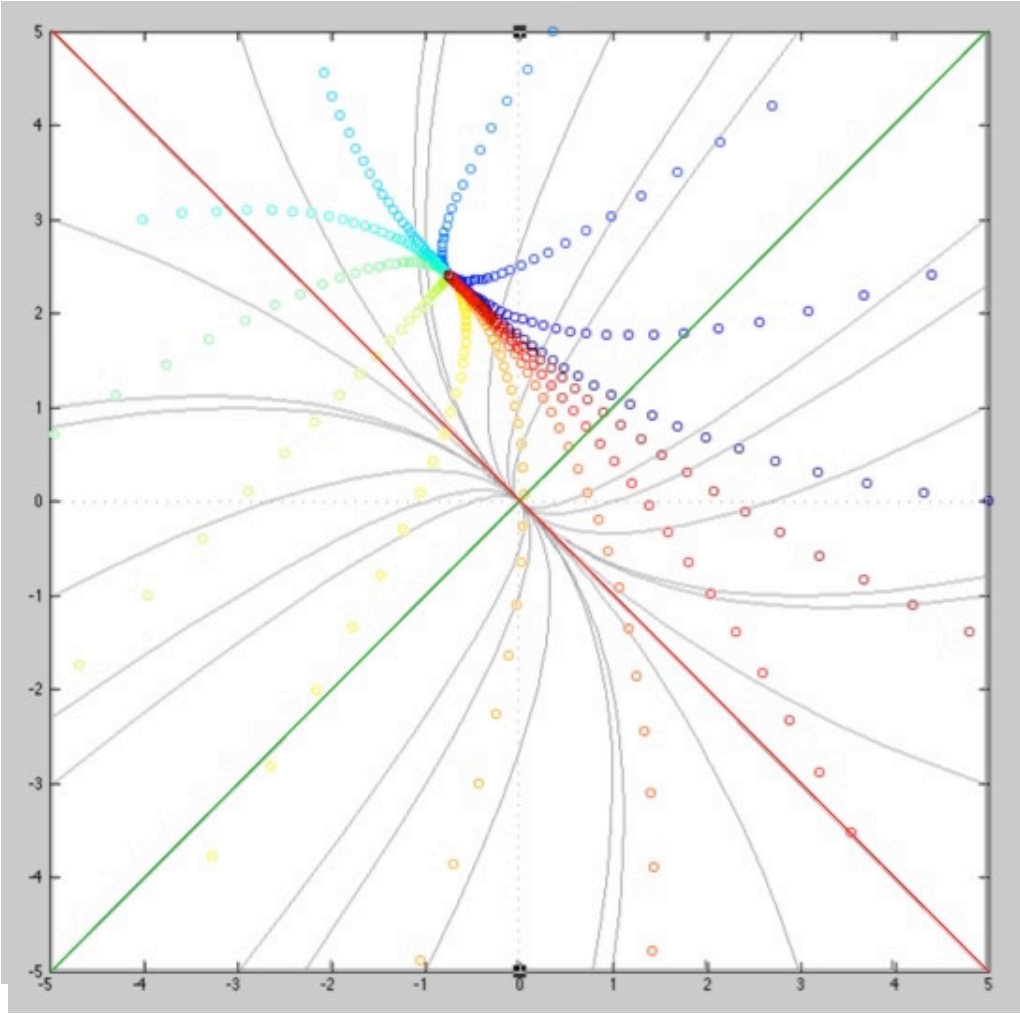
Iterate...



Theorem: If F is linear

1) There is a unique flow-kick equilibrium.

2) Flow dynamics are translated to the flow-kick equilibrium.

Linear flow-kick dynamics

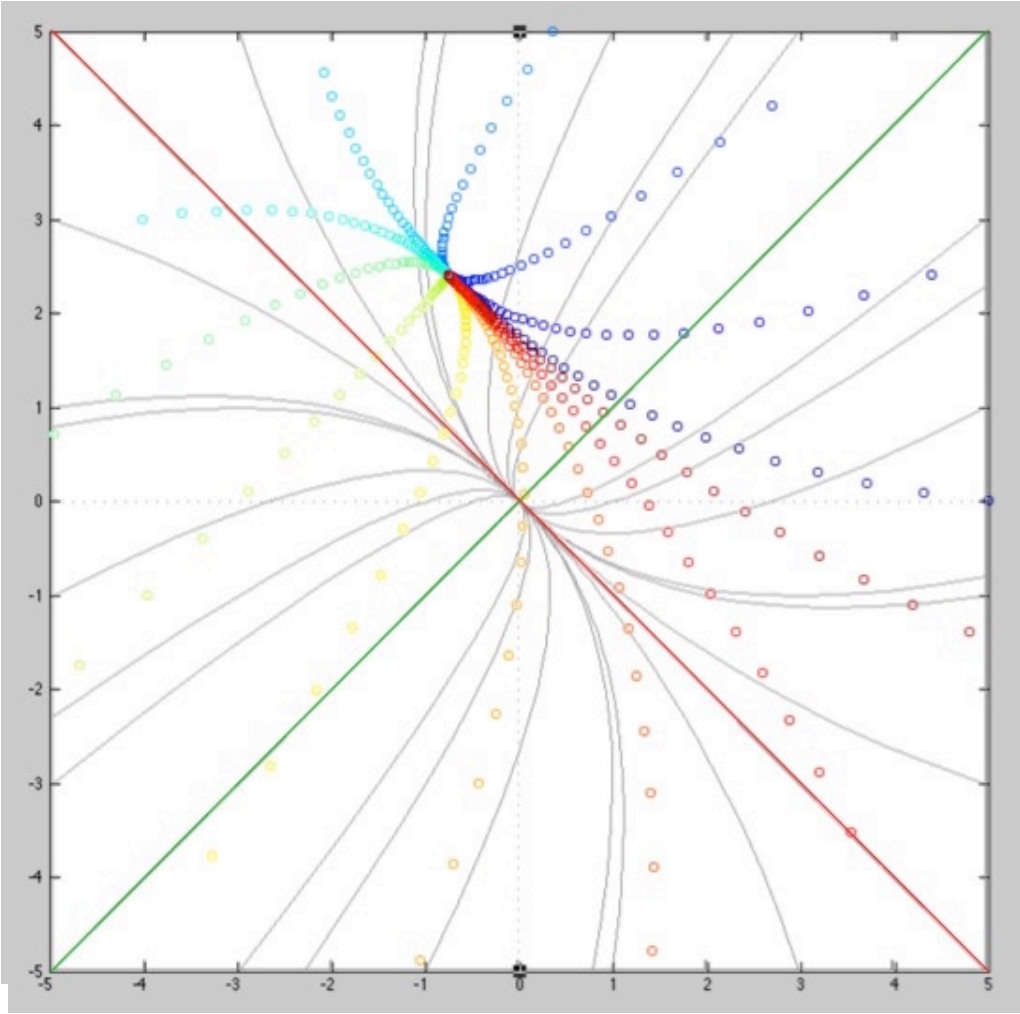
Given:

System of ODE's $dx/dt=F(x)$

Flow $\varphi_t(x)$

Flow time τ

Kick vector k



Proof of 2:

$G(x) = \varphi_\tau(x) + k$

$DG|_x = D\varphi_\tau|_x + Dk|_x$

$= D\varphi_\tau|_0$

Theorem: If F is linear

1) There is a unique flow-kick equilibrium.

2) Flow dynamics are translated to the flow-kick equilibrium.

Linear flow-kick dynamics

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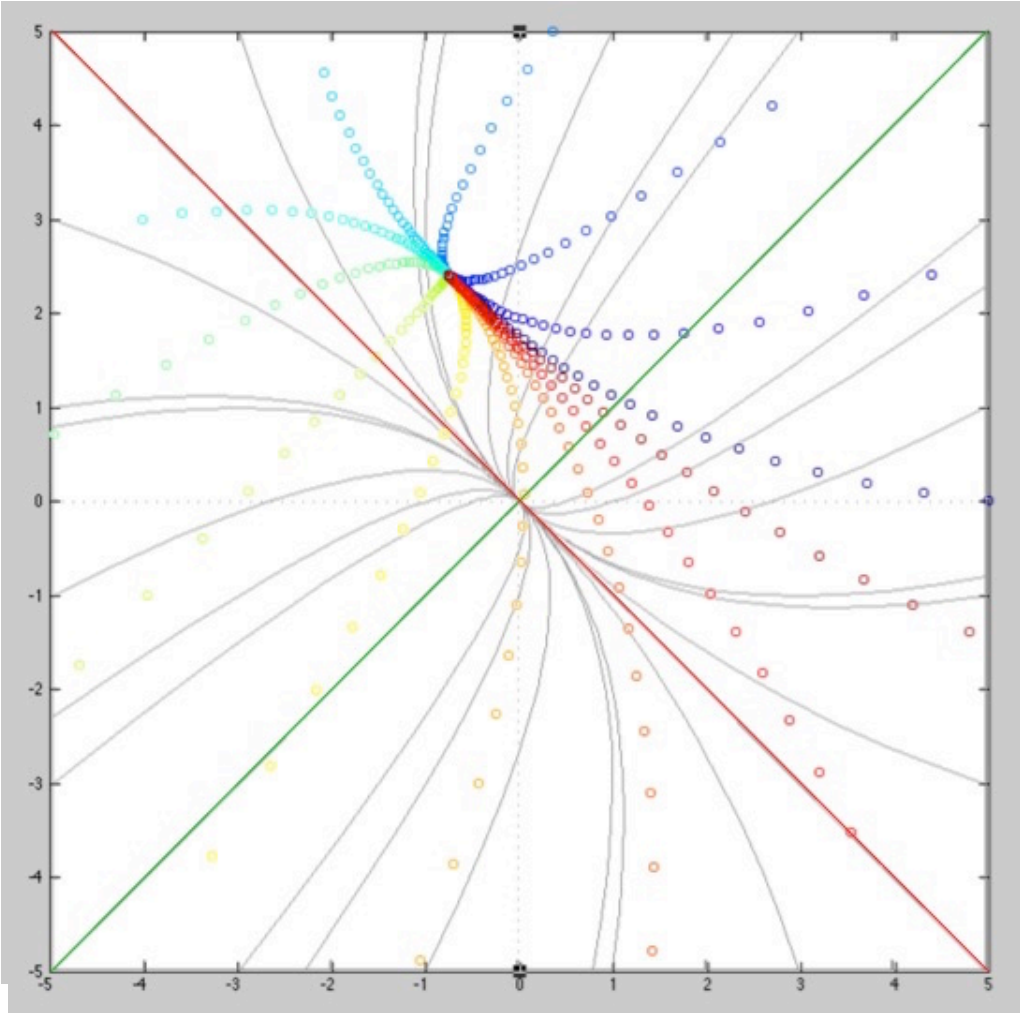


Proof of 2:

$G(x) = \varphi_\tau(x) + k$

$DG|_x = D\varphi_\tau|_x + Dk|_x$

$\neq D\varphi_\tau|_0$

Not true for nonlinear F

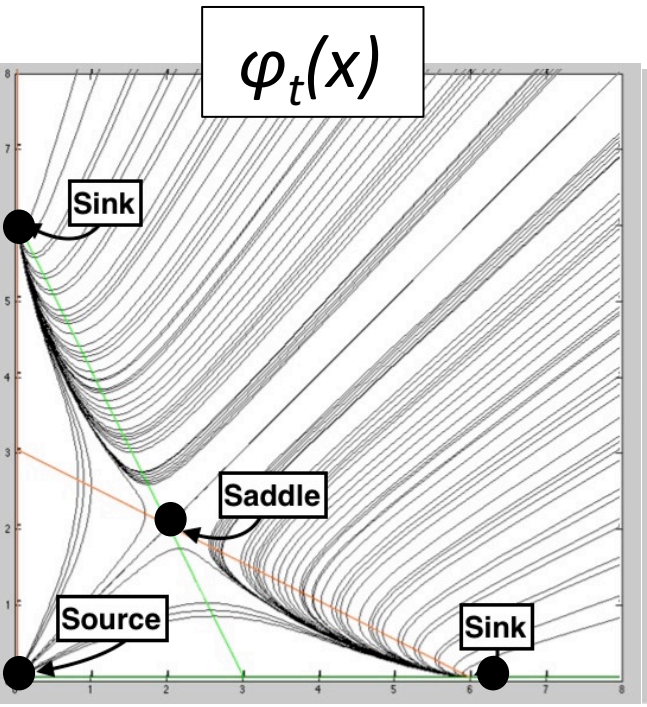
Theorem: If F i

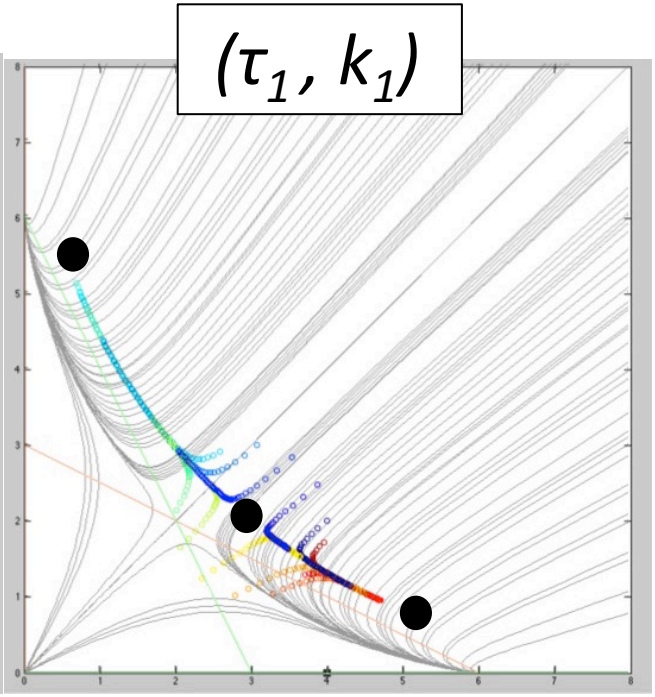
1) There is a unique flow-kick equilibrium.

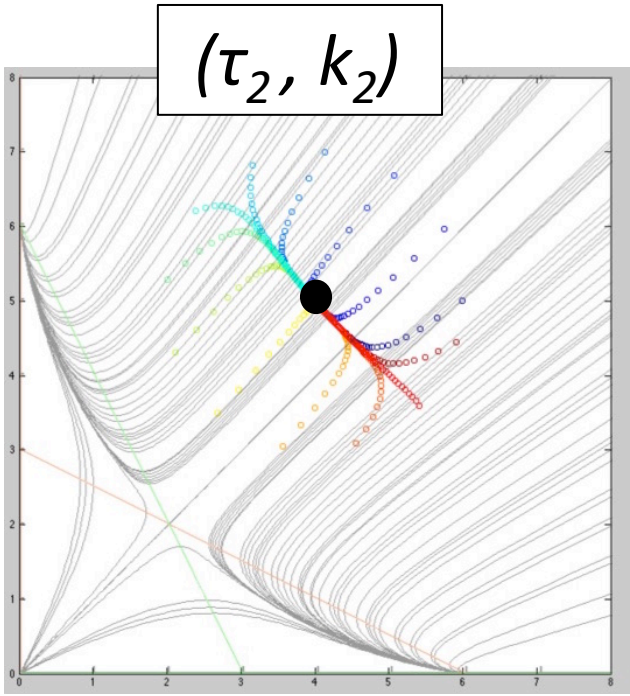
2) Flow dynamics are translated to the flow-kick equilibrium.

Example: Lotka-Volterra competition

$G(x) = \varphi_\tau(x) + k$







Example: Lotka-Volterra competition

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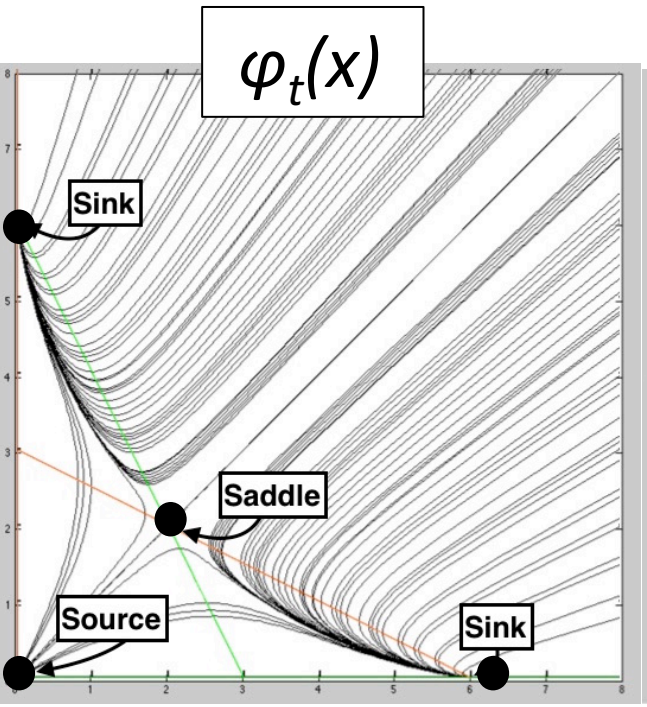
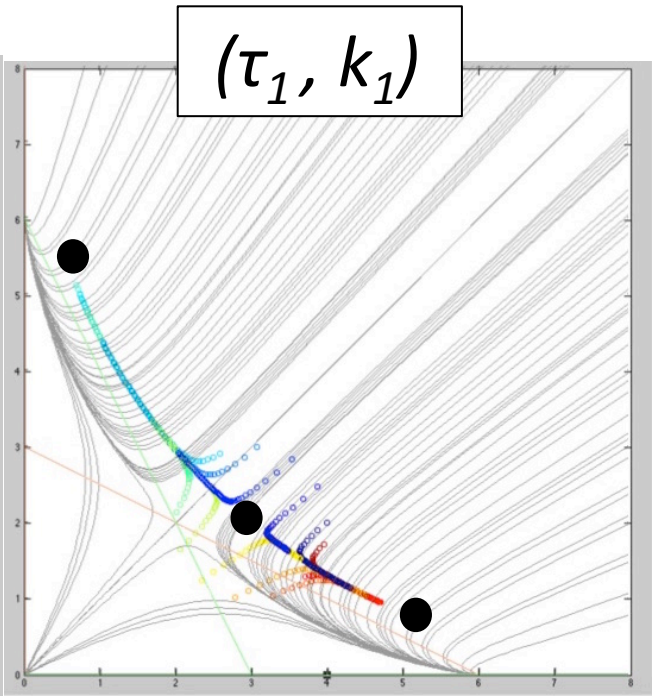
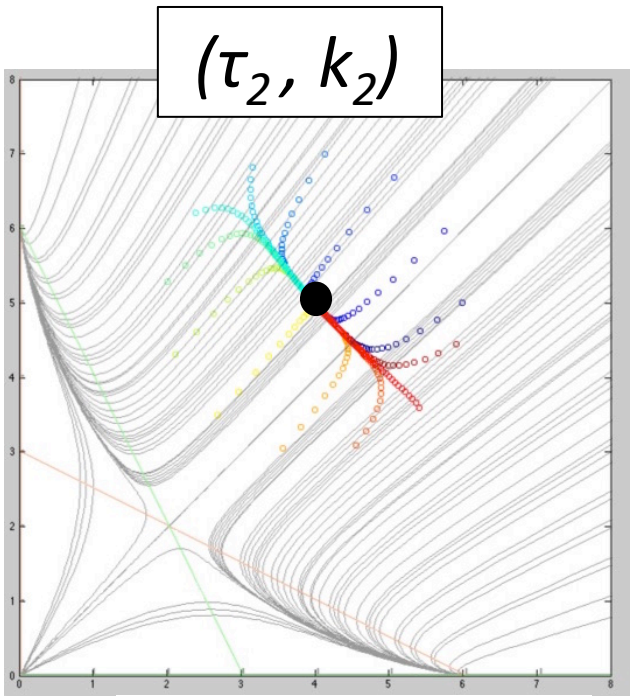
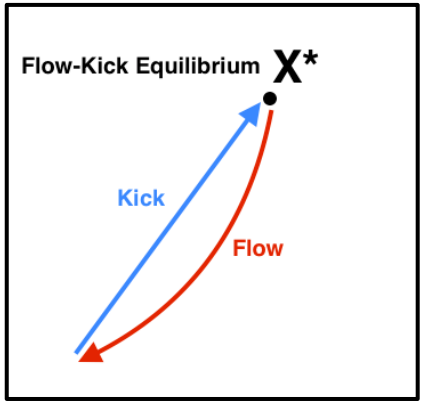






$DG|_x = D\varphi_\tau|_x$

= Closest linear approximation to φ_t *at* x



Example: Lotka-Volterra competition

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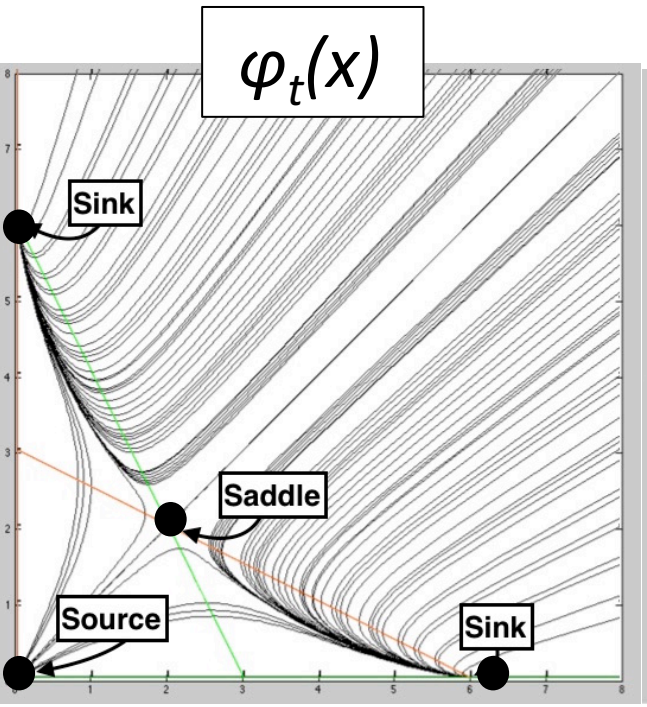
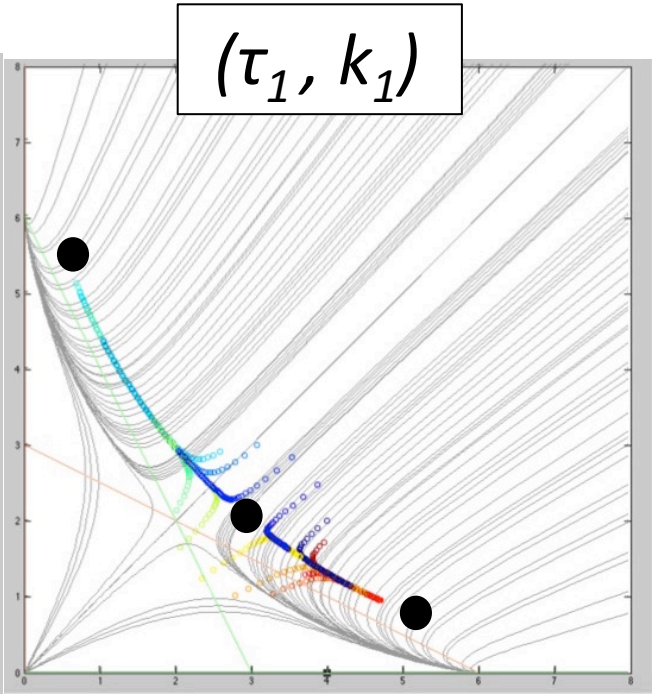
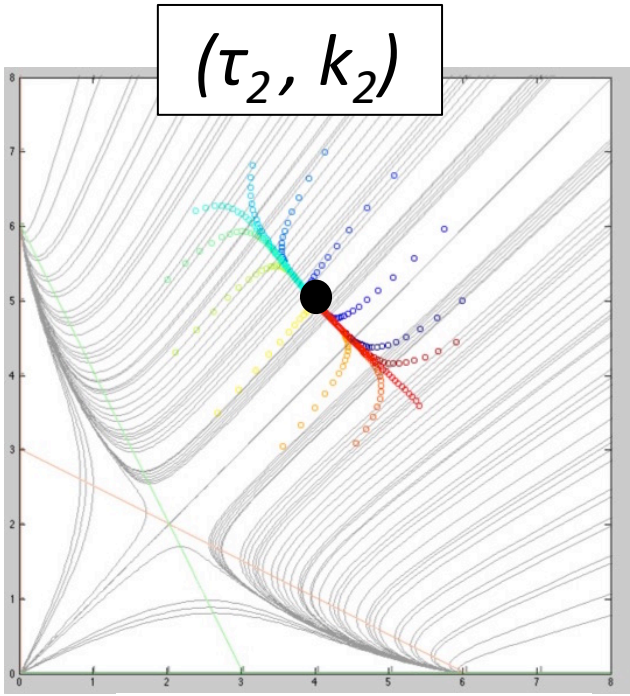




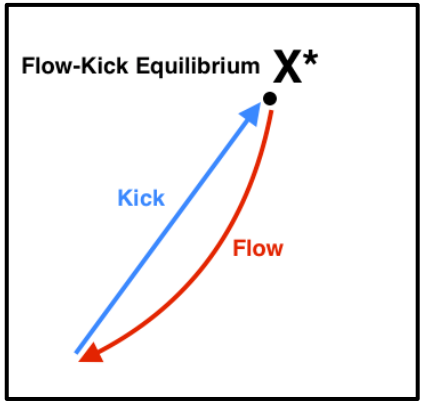


$DG|_x = D\varphi_\tau|_x$

$=$ Closest linear approximation to φ_t *at* x

Use variational equation to calculate $D\varphi_t|_x$

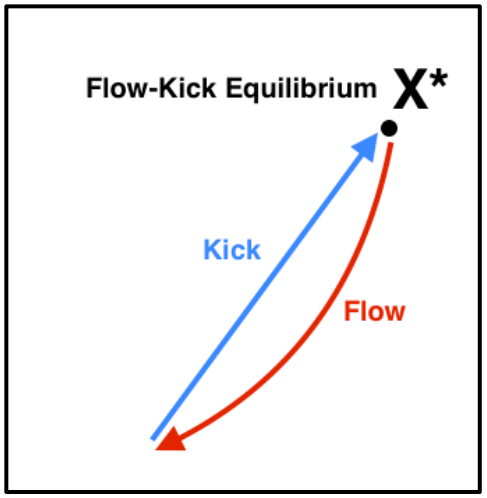
(Like calculating Liapunov exponents)



Linearising flow-kick dynamics

$G(x) = \varphi_\tau(x) + k$

$DG|_x = D\varphi_\tau|_x$



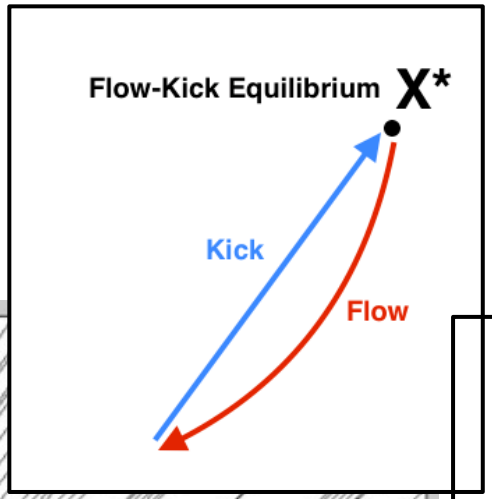
Variational equation:

Use $DF|_x$ along flow to calculate $D\varphi_\tau|_x$

Linearising flow-kick dynamics

$G(x) = \varphi_\tau(x) + k$

$DG|_x = D\varphi_\tau|_x$



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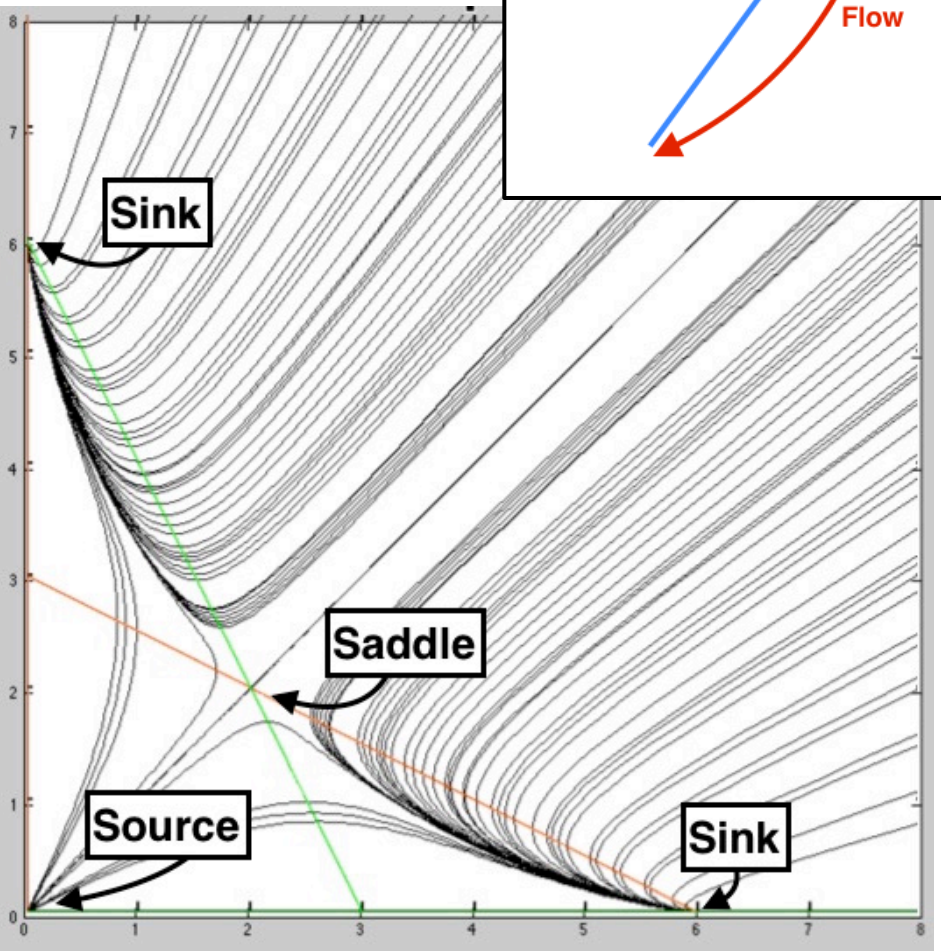
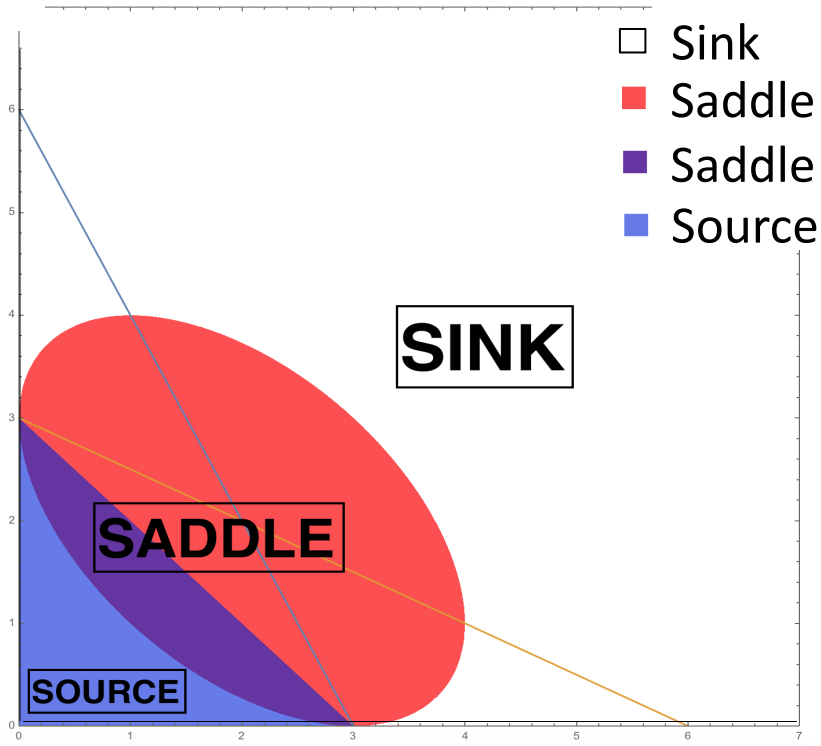


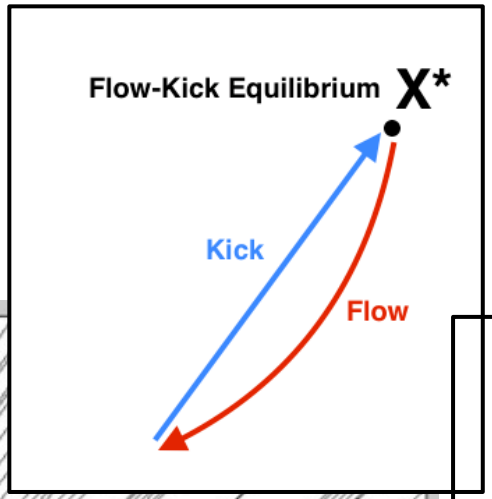
Dynamical classification of $DF|_x$



Linearising flow-kick dynamics

$G(x) = \varphi_\tau(x) + k$

$DG|_x = D\varphi_\tau|_x$



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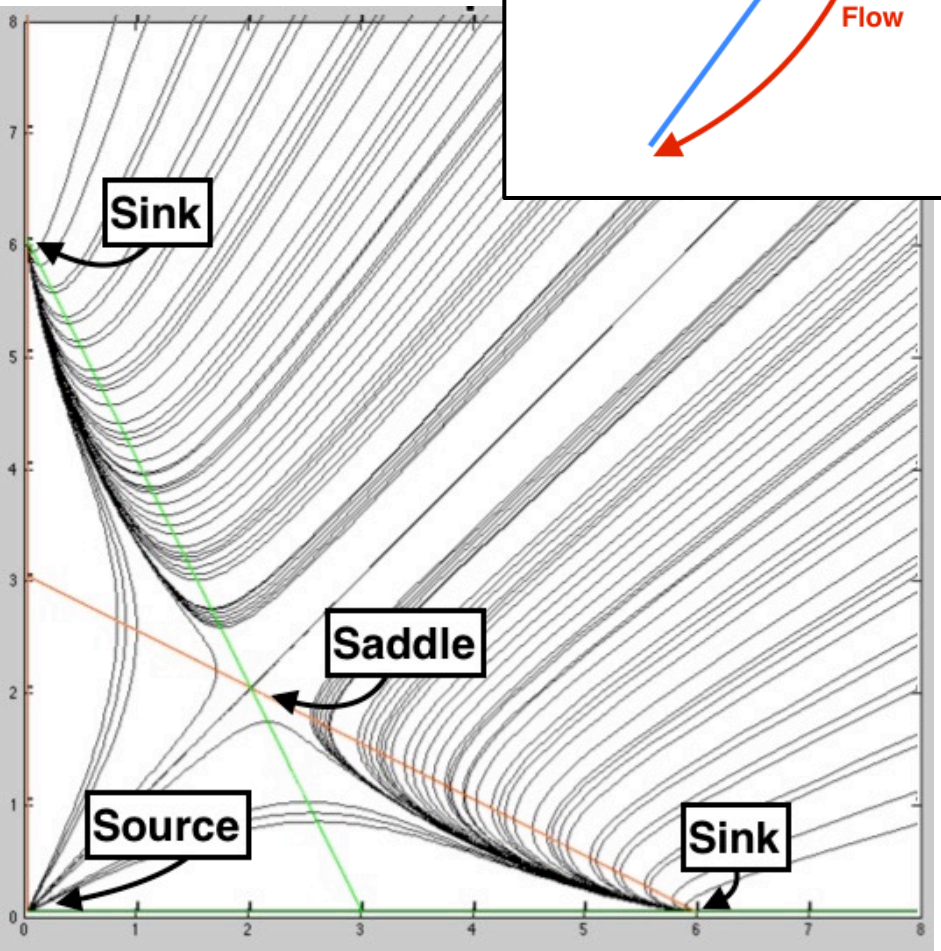
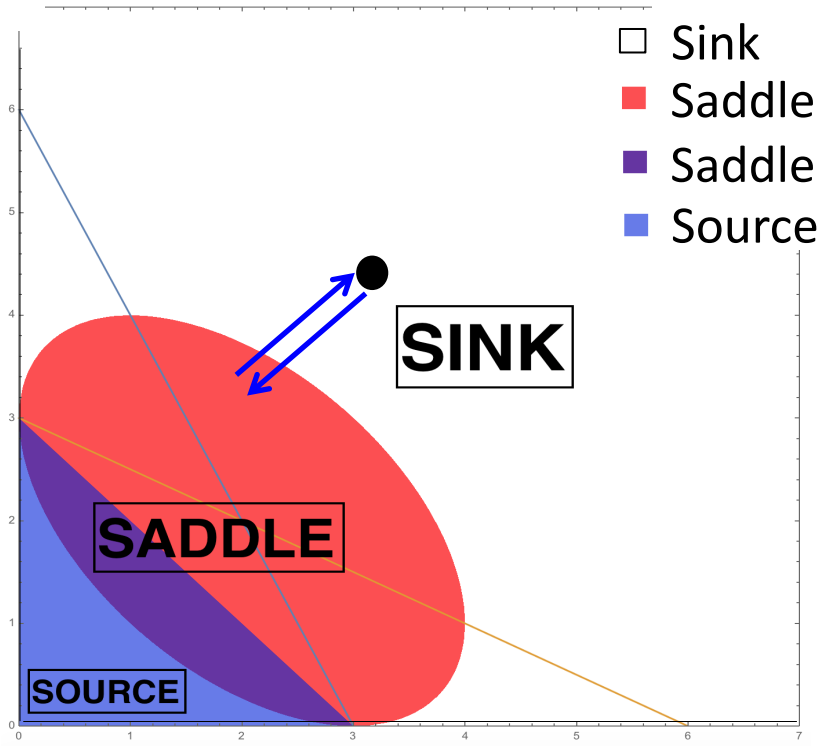
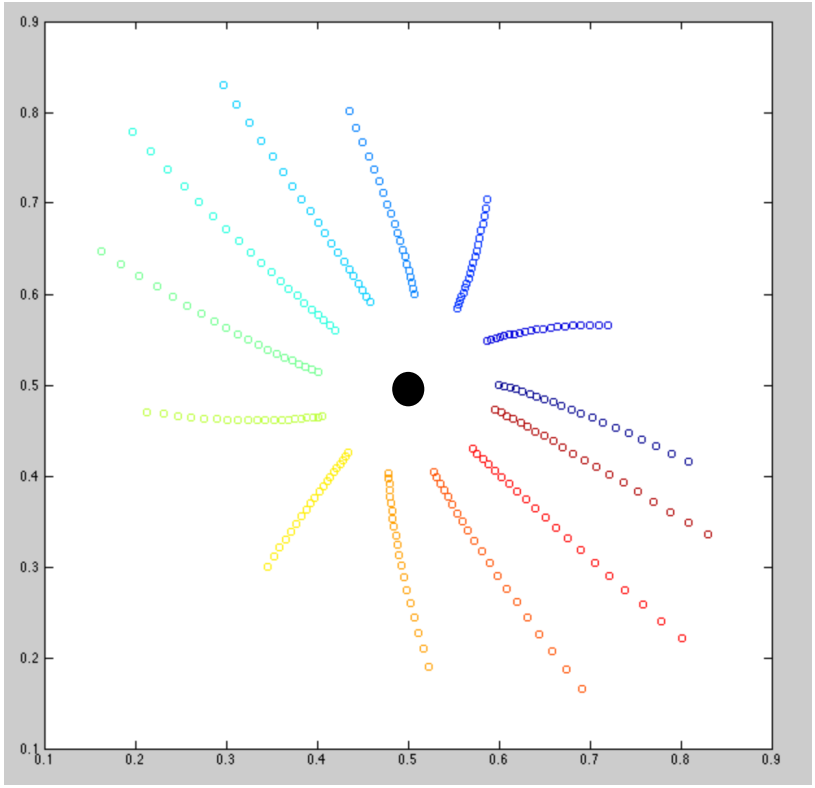
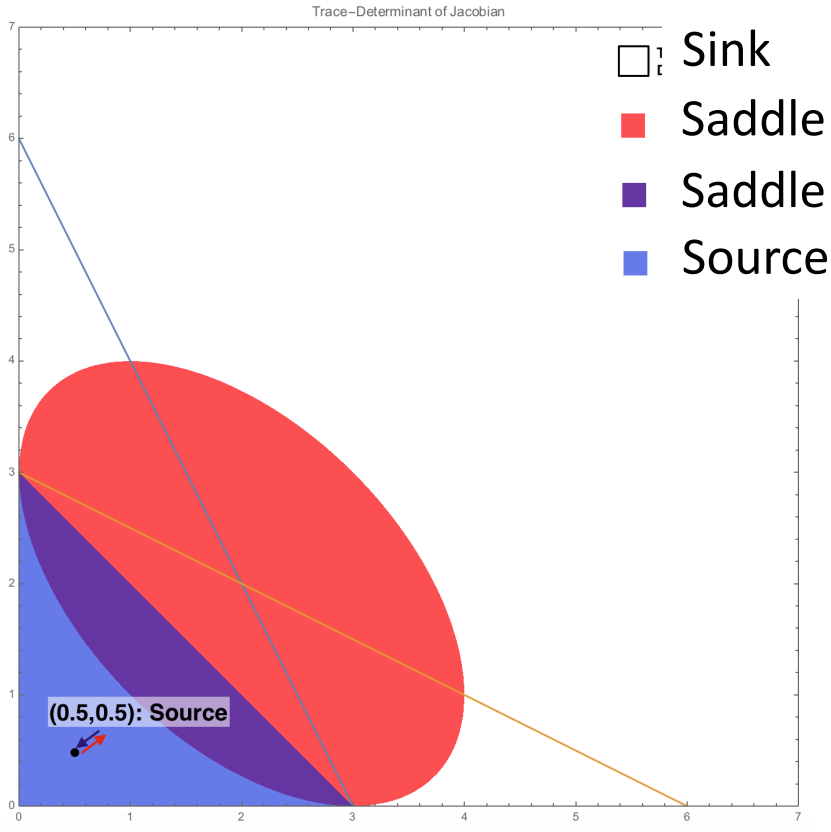


Dynamical classification of $DF|_x$



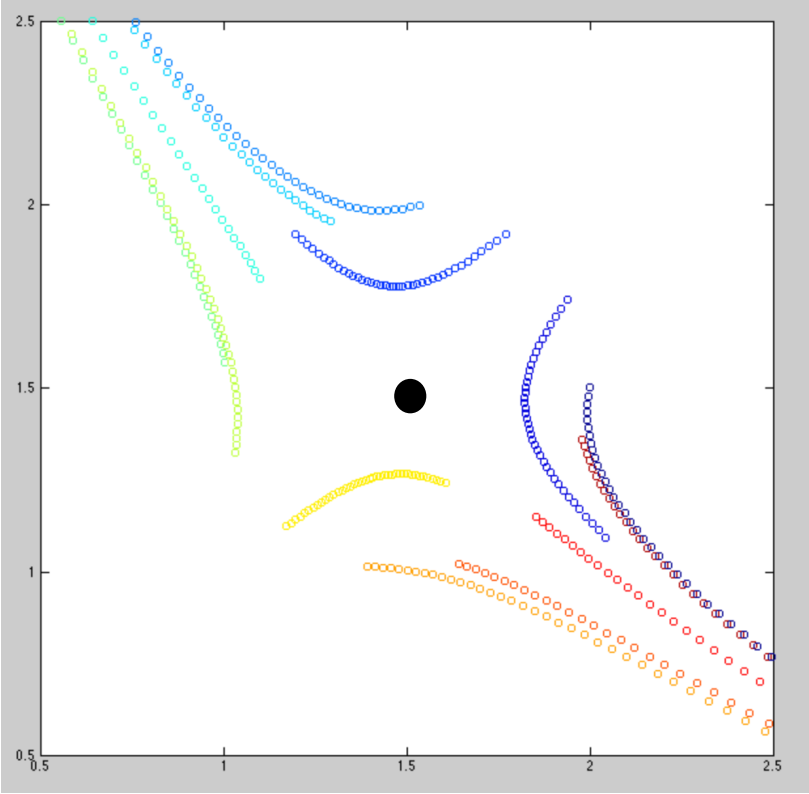
Flow-kick dynamics

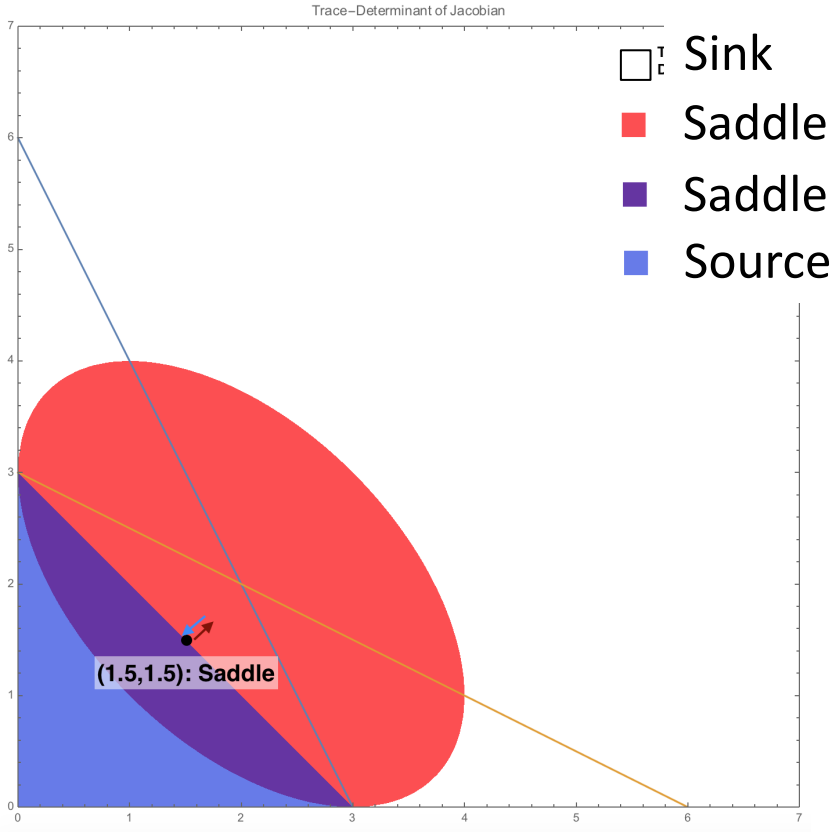




- Flow time $\tau = 0.5$
- Kick $k = (-0.0345, -0.0345)$
- **Source at $\mathbf{X}^* = (\mathbf{0.5}, \mathbf{0.5})$**

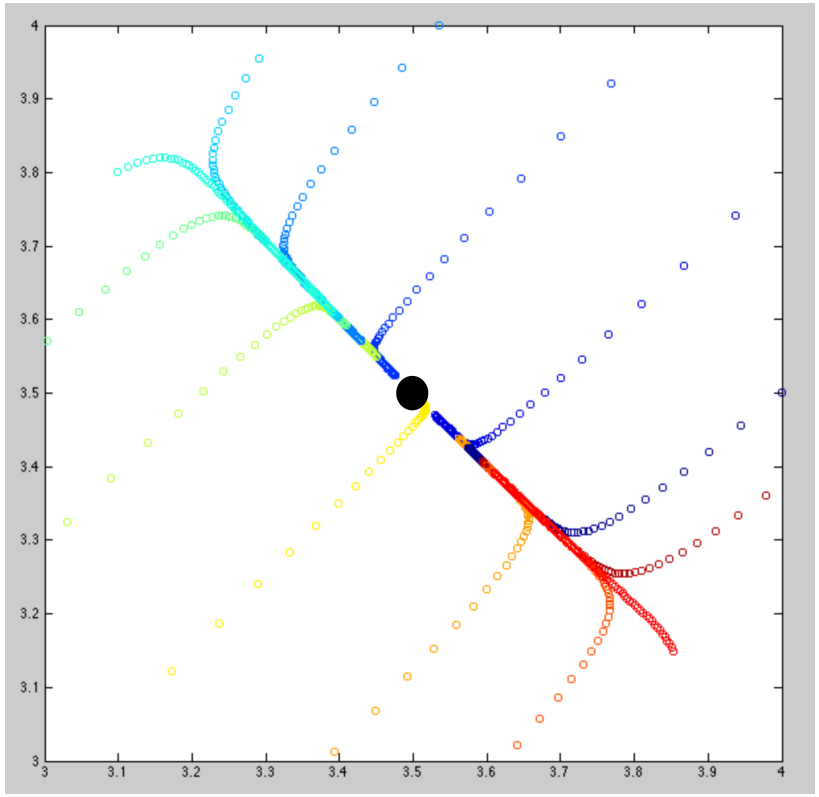
Flow-kick dynamics

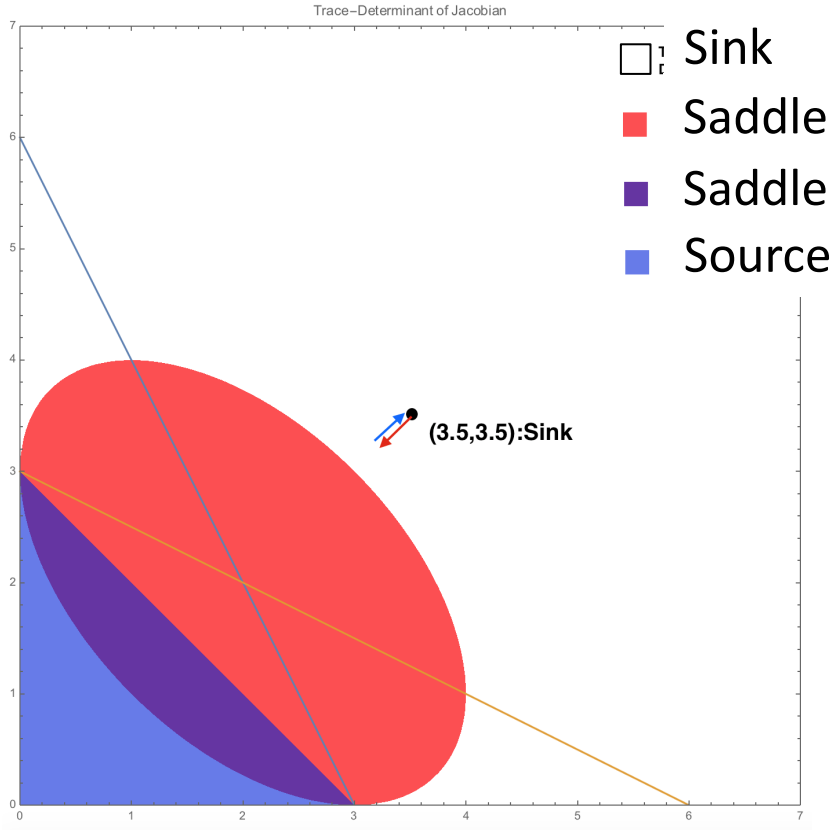




- Flow time $\tau = 0.5$
- Kick $k = (-0.0330, -0.0330)$
- **Saddle at $\mathbf{X}^* = (1.5, 1.5)$**

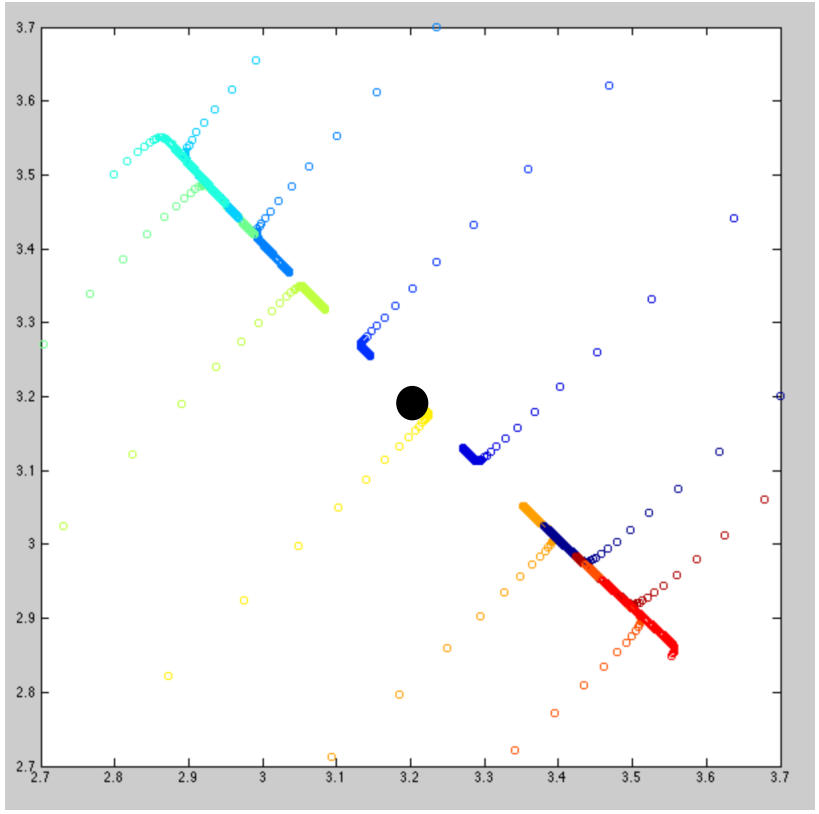
Flow-kick dynamics

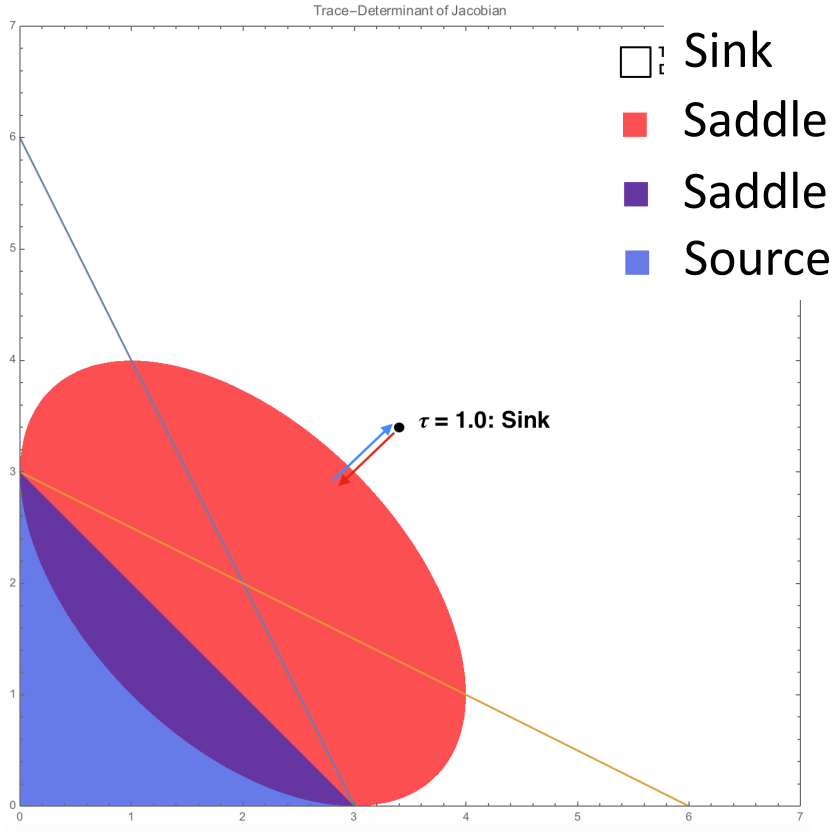




- Flow time $\tau = 0.5$
- Kick $k = (0.2122, 0.2122)$
- **Sink at $\mathbf{X}^* = (\mathbf{3.5}, \mathbf{3.5})$**

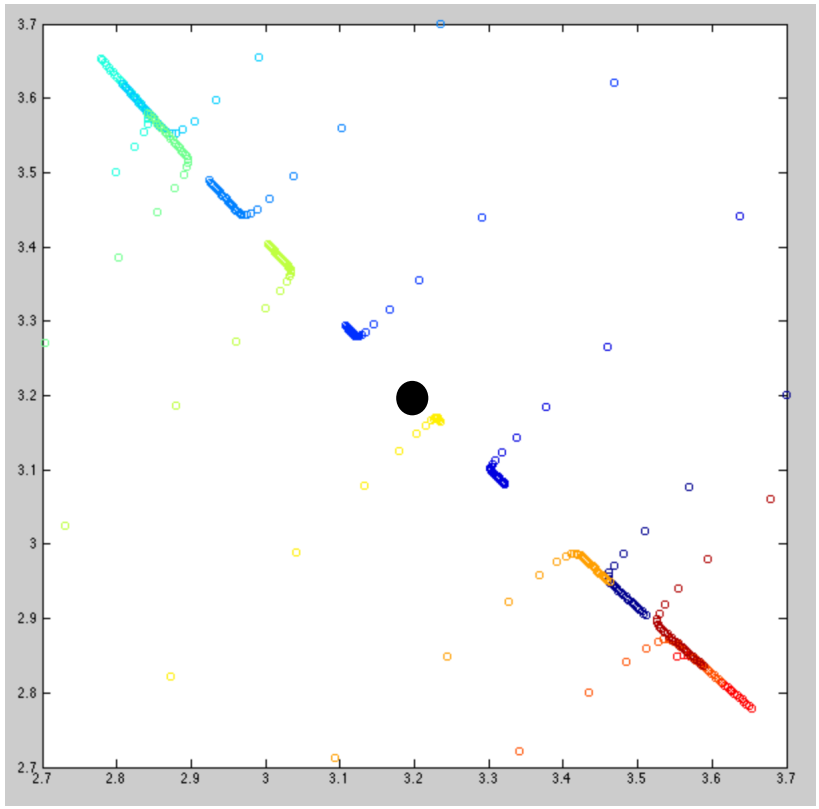
Flow-kick dynamics

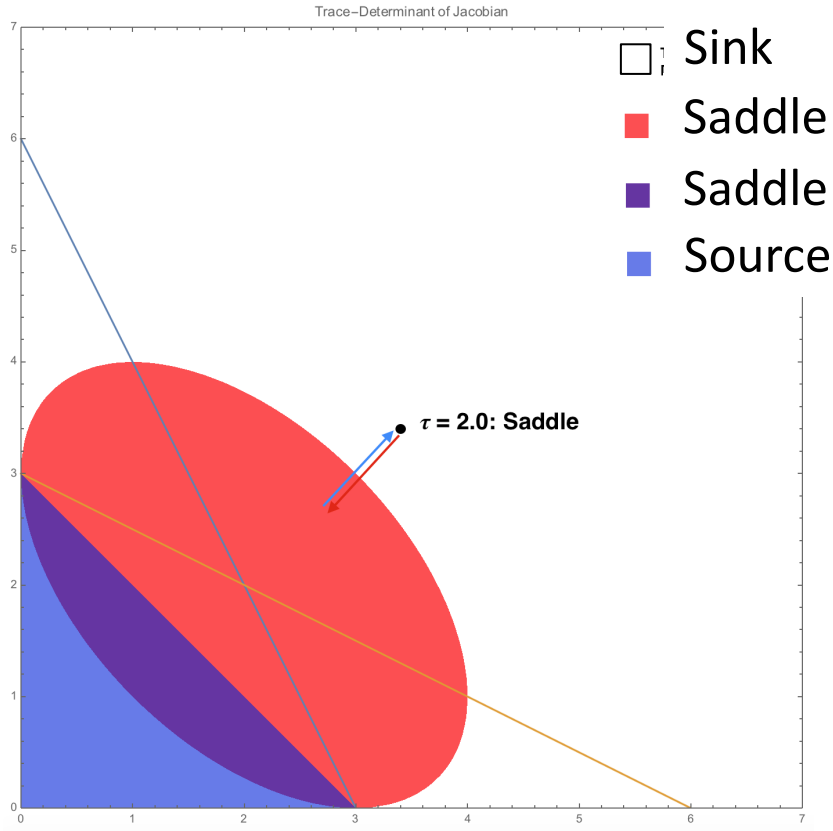




- Flow time $\tau = 1$
- Kick $k = (0.288, 0.288)$
- **Sink at $\mathbf{X}^* = (3.2, 3.2)$**

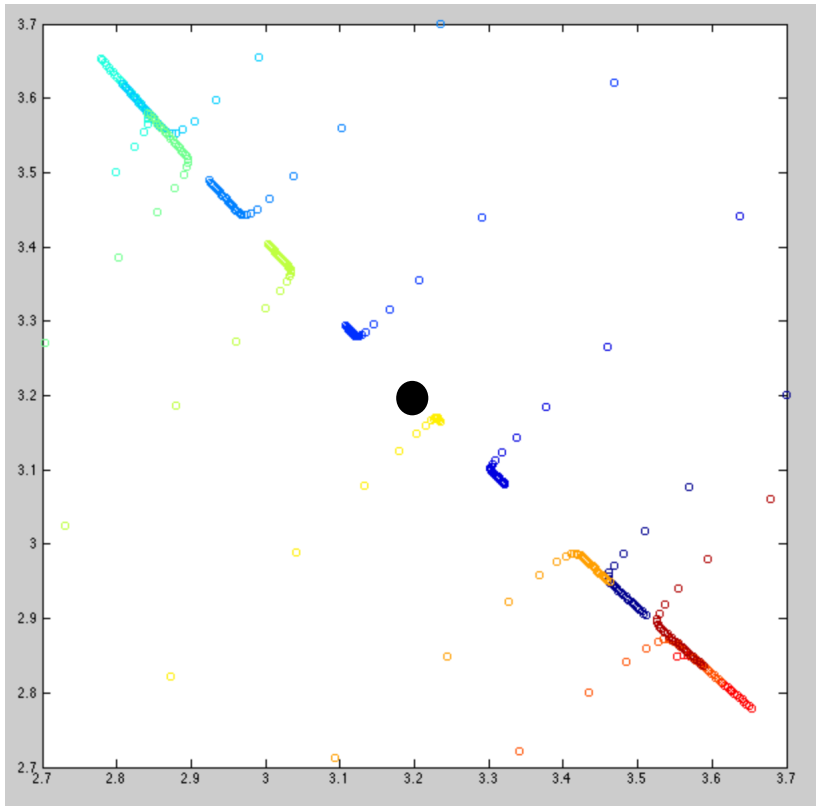
Flow-kick dynamics

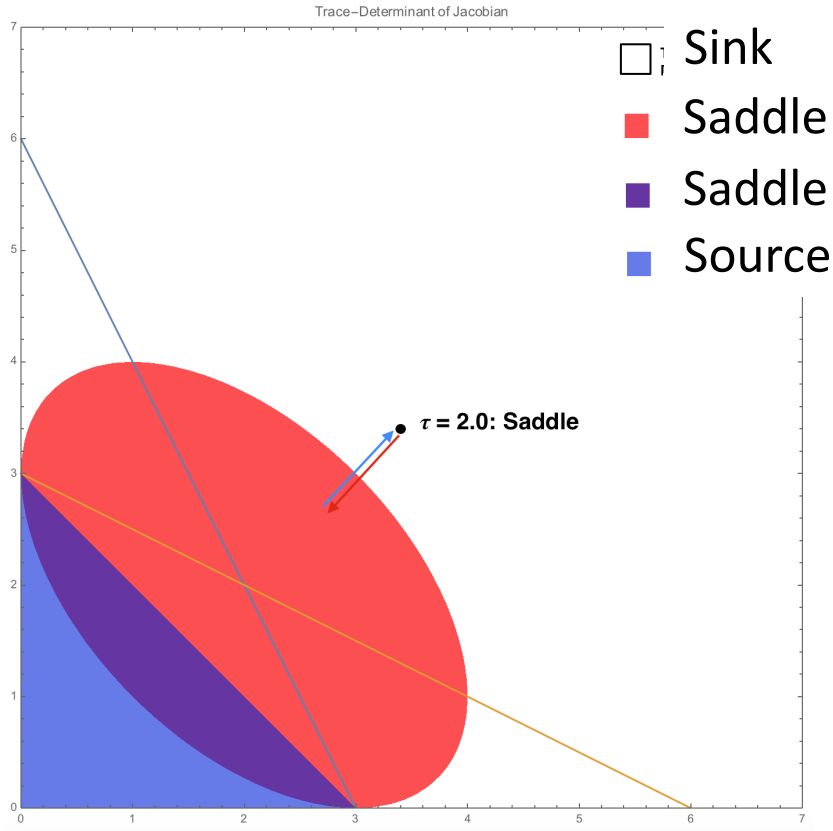




- Flow time $\tau = 2$
- Kick $k = (0.491, 0.491)$
- **Saddle at $\mathbf{X}^* = (3.2, 3.2)$**

Flow-kick dynamics



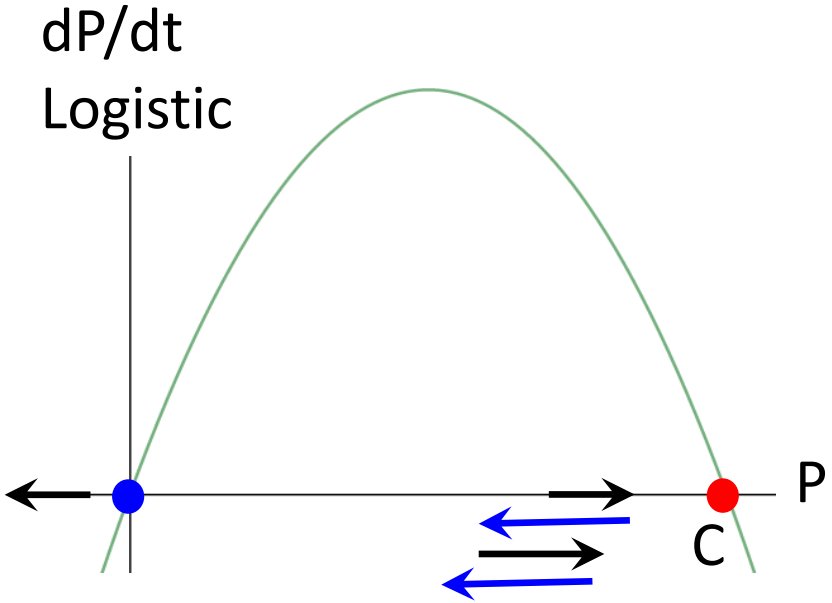


- Flow time $\tau = 2$
- Kick $k = (0.491, 0.491)$
- **Saddle at $\mathbf{X}^* = (3.2, 3.2)$**

Begs a bifurcation analysis

Returning to 1-D

Example: Fishery subject to harvesting



Same ideas as 2-d:

Choice of (τ, k) can put a flow-kick equilibrium anywhere in basin.

dP/dt=F(P) with flow $\varphi_t(P)$

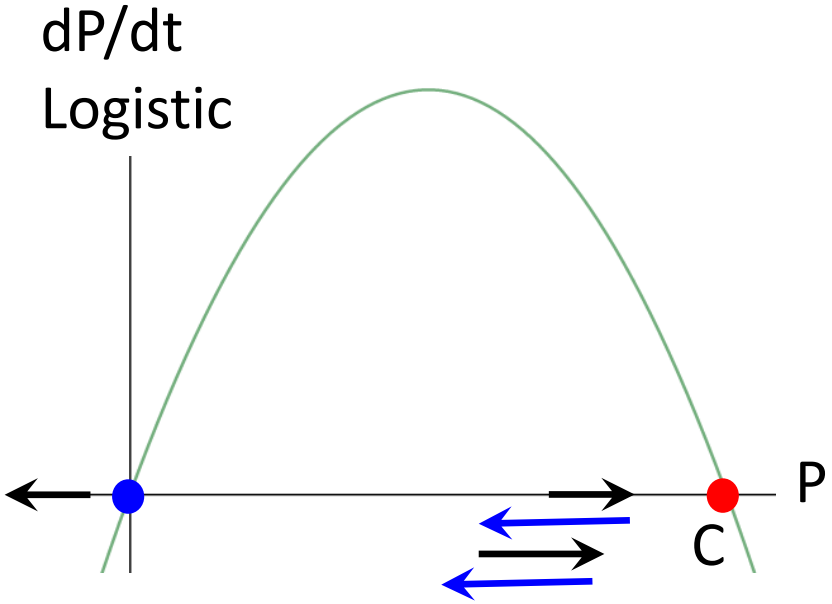
Flow time τ, kick k

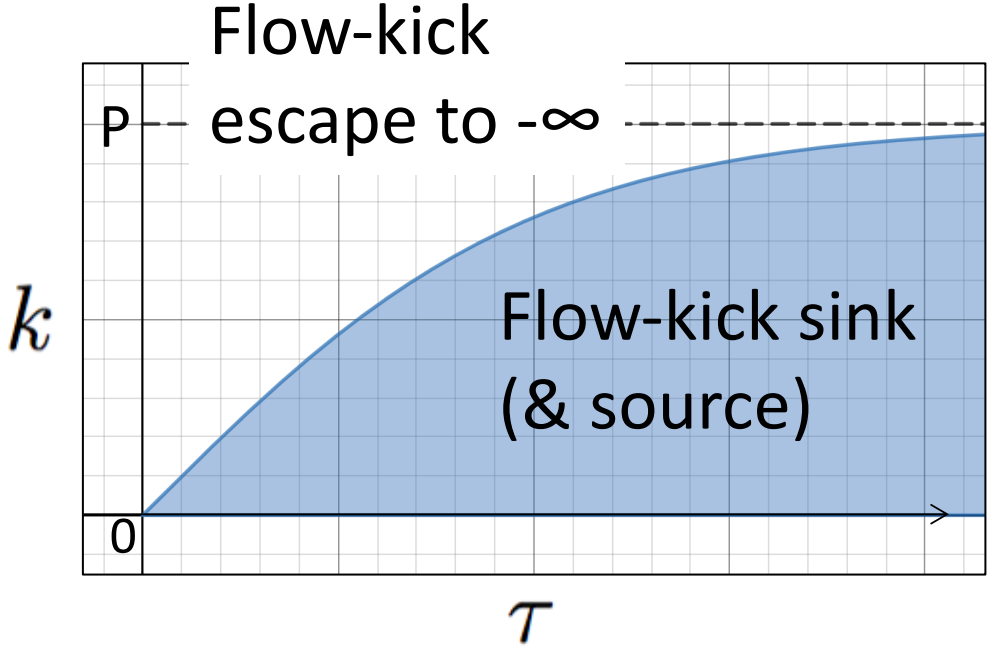
Flow-kick map:

$G(P) = \varphi_t(P) + k$

Resilience boundary

Example: Fishery subject to harvesting





dP/dt=F(P) with flow $\varphi_t(P)$

Flow time τ, kick k

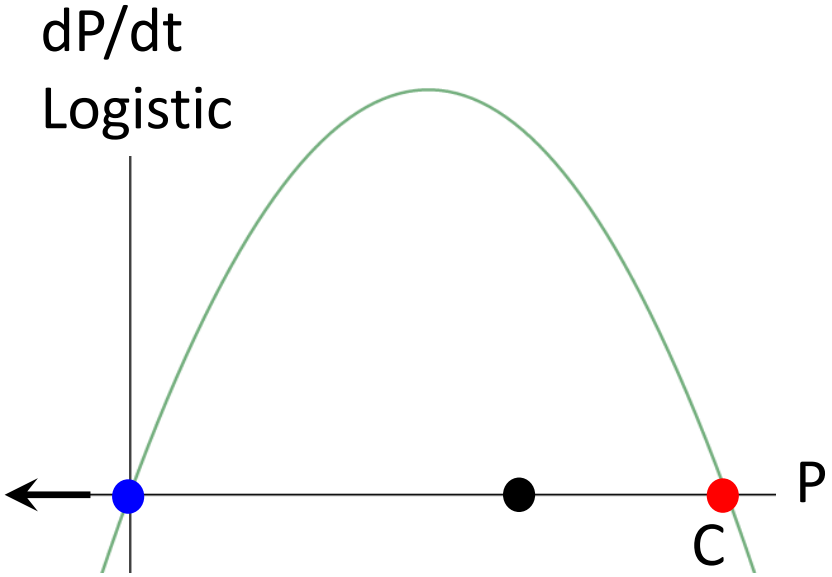
Flow-kick map:

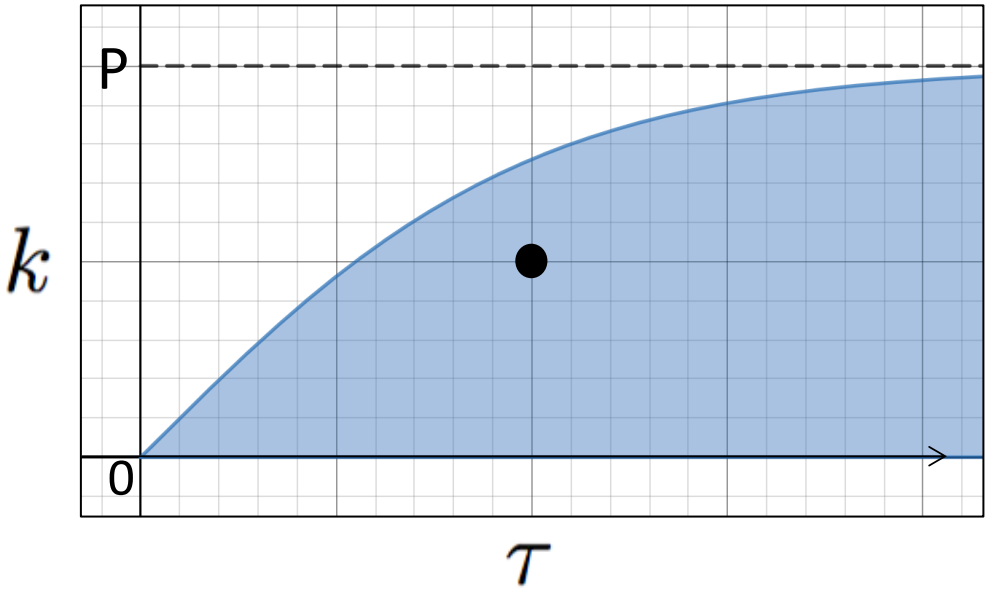
$G(P) = \varphi_t(P) + k$

Resilience boundary
= flow-kick saddle-node
bifurcation curve

Single harvesting strategy

Example: Fishery subject to harvesting





dP/dt=F(P) with flow $\varphi_t(P)$

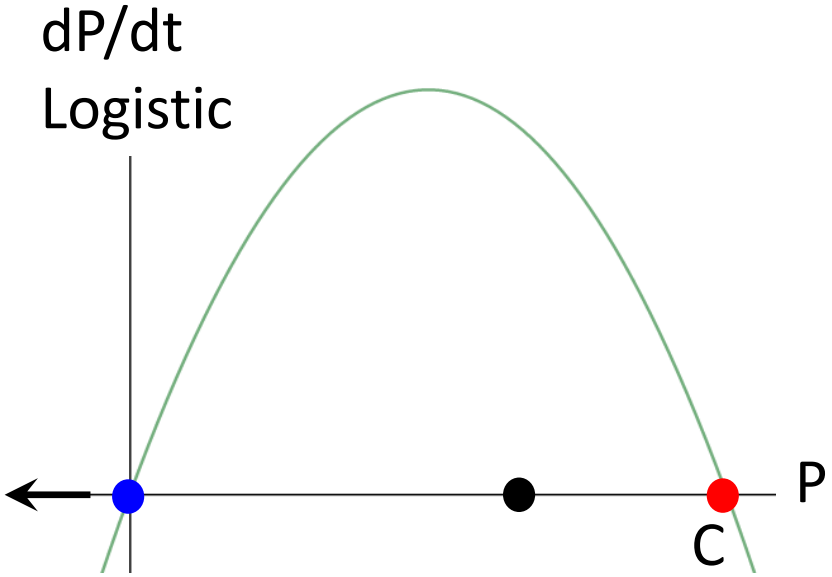
Flow time τ, kick k

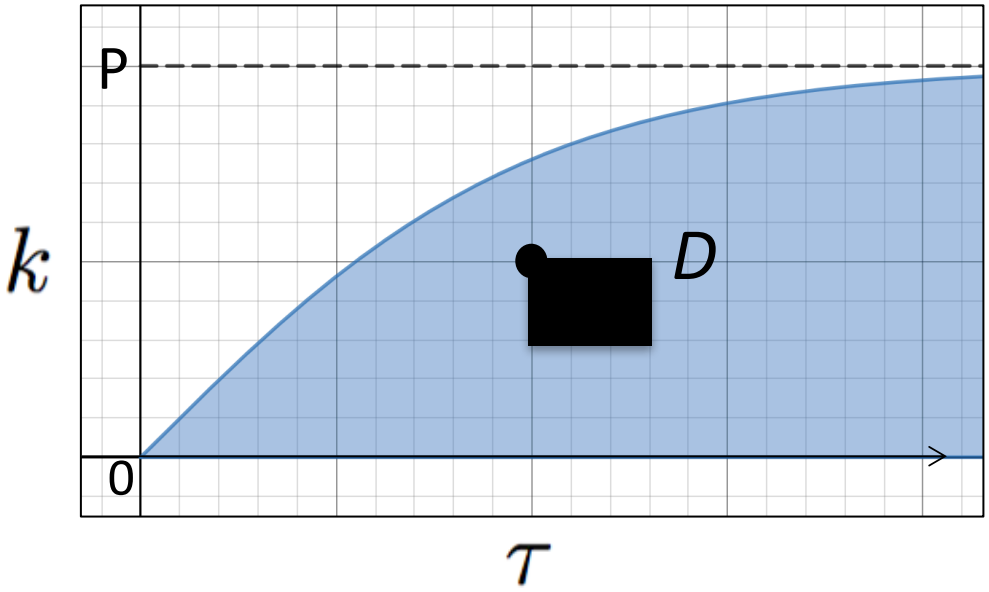
Flow-kick map:

$G(P) = \varphi_t(P) + k$

Stochastic harvesting strategy

Example: Fishery subject to harvesting





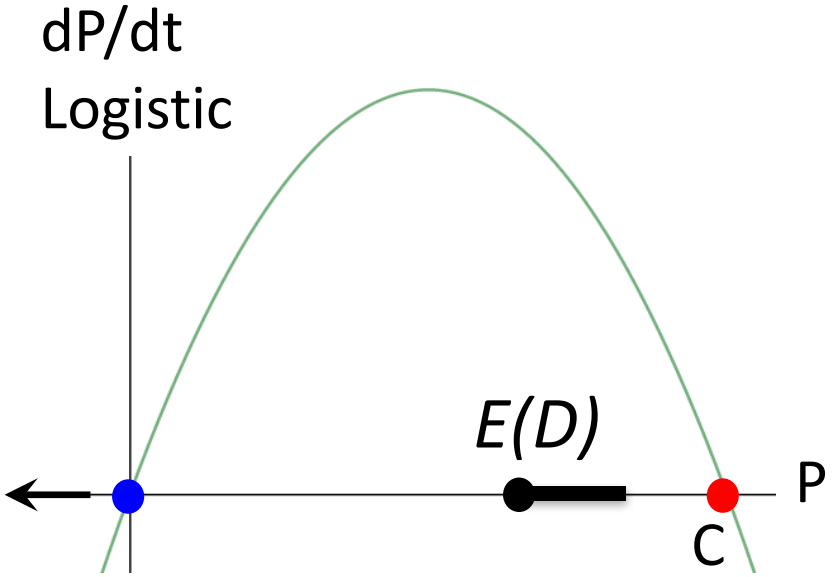
dP/dt=F(P) with flow $\varphi_t(P)$
Flow time τ, kick k

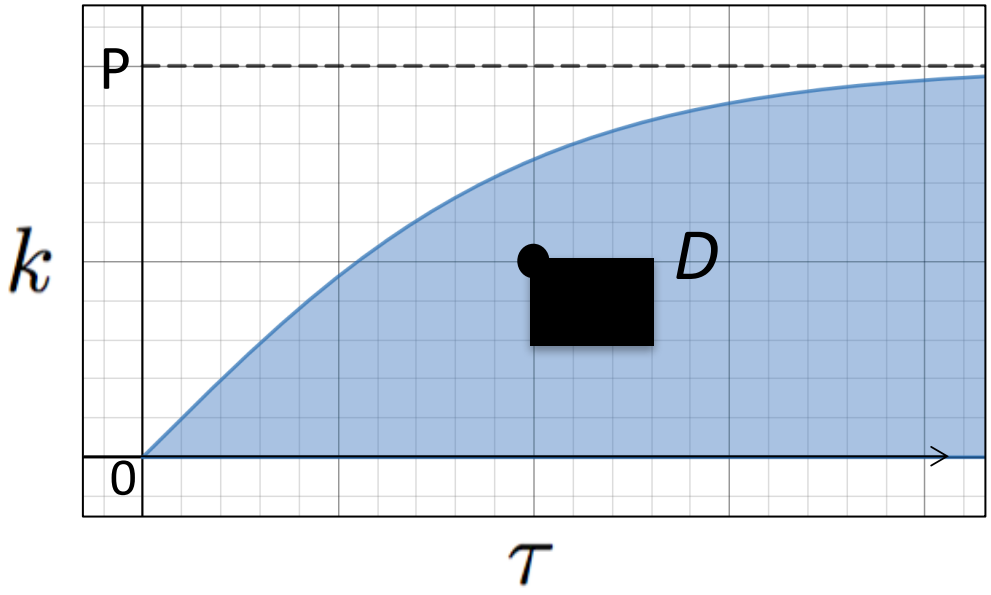
Flow-kick map:
$G(P) = \varphi_t(P) + k$

What if τ and k are chosen stochastically from a bounded domain, D?

Stochastic harvesting strategy

Example: Fishery subject to harvesting



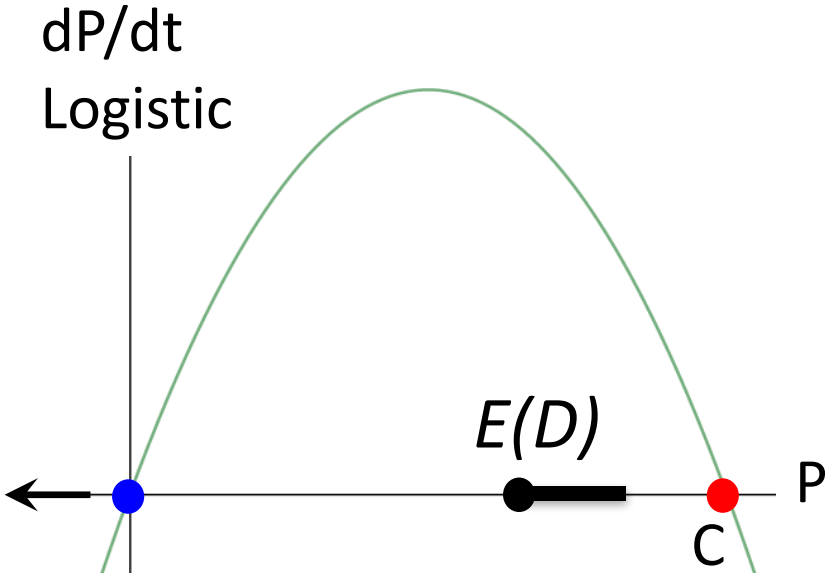


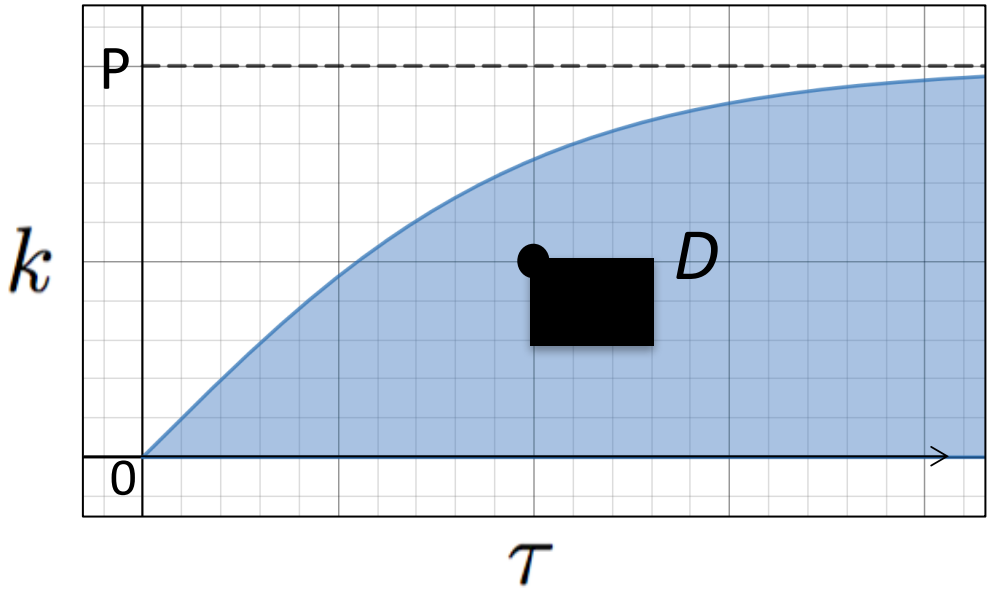
Set *E(D)* of flow-kick equilibria for fixed (τ,k) strategies from *D*

What if τ and k are chosen stochastically from a bounded domain, *D*?

Stochastic harvesting strategy

Example: Fishery subject to harvesting





Set *E(D)* of flow-kick equilibria for fixed (τ,k) strategies from *D*

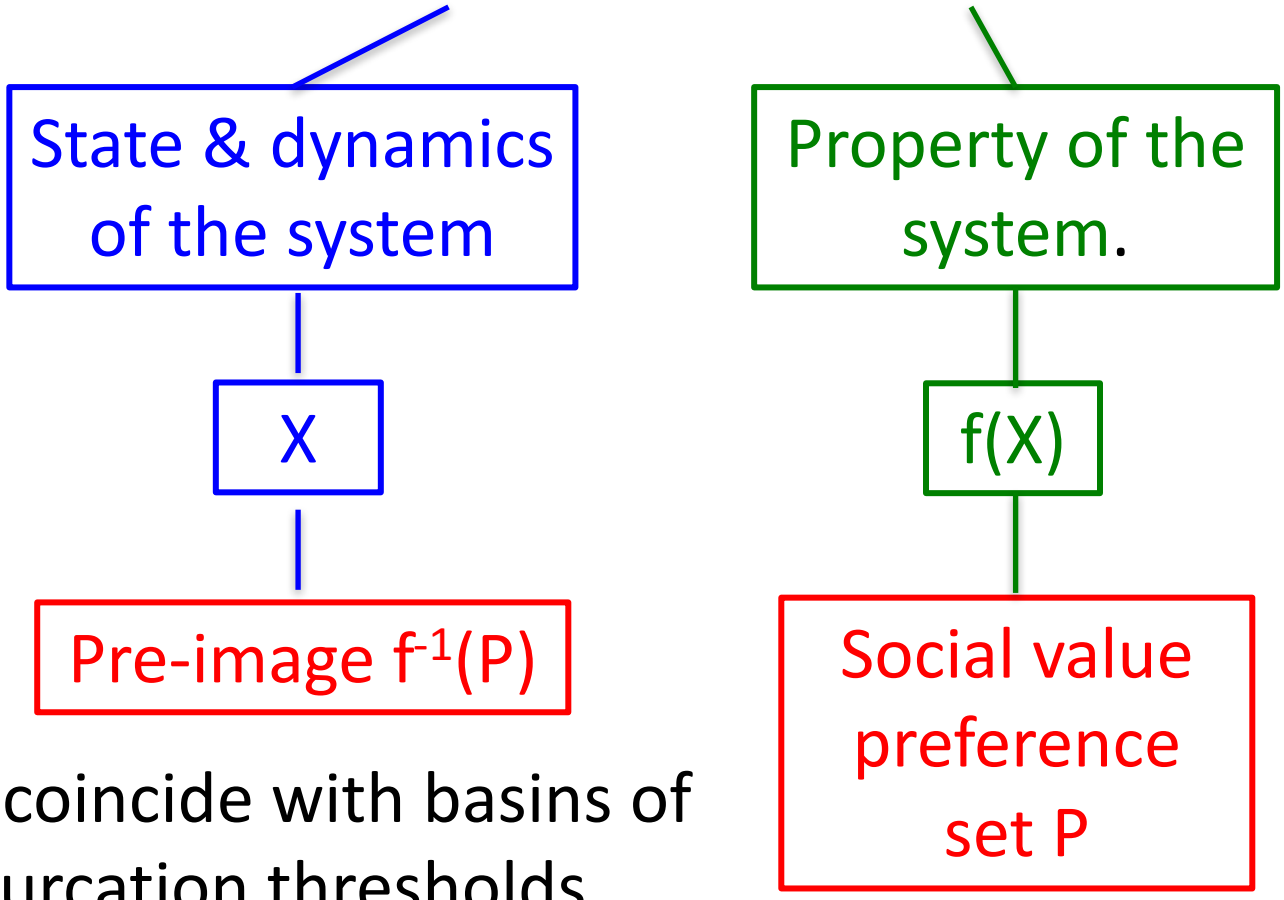
Stochastic flow-kick trajectory approaches *E(D)*

What if τ and k are chosen stochastically from a bounded domain, *D*?

What is Resilience?

Slippery qualitative idea:

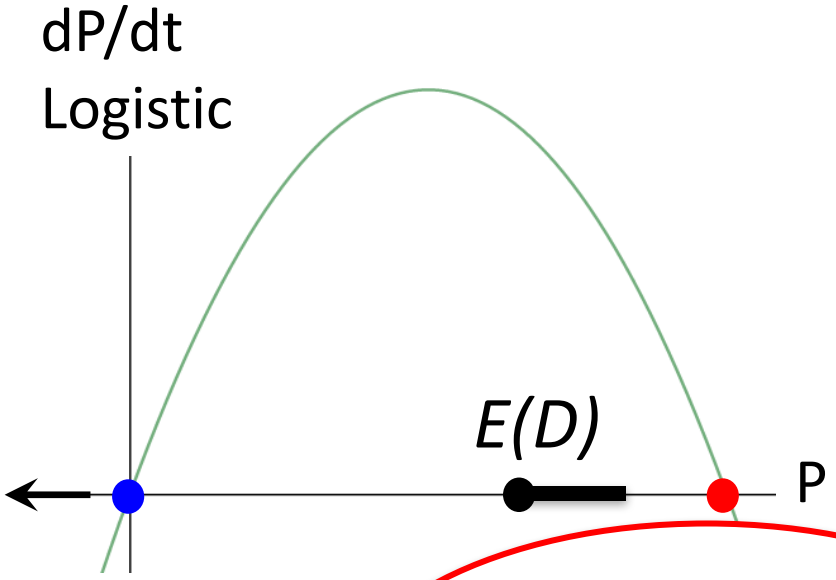
The ability of a system to absorb change and disturbance while maintaining its basic structure and function.

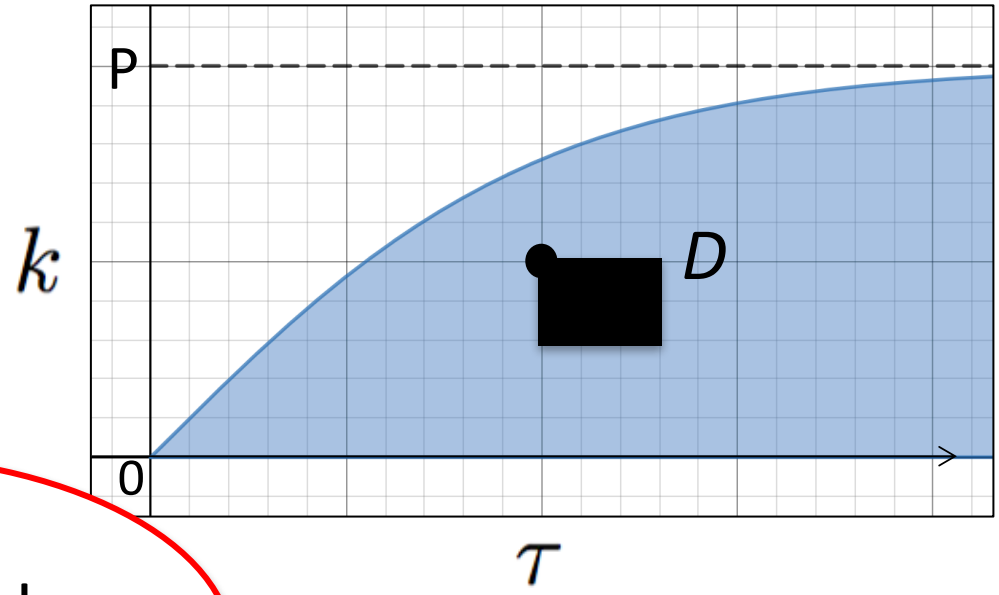


May or may not coincide with basins of attraction, or bifurcation thresholds

Stochastic harvesting strategy

Example: Fishery subject to harvesting





Set *E(D)* of

equilibria

strategies fr

Not so simple in 2-d…

Stochastic flow-kick trajectory approaches *E(D)*

What if τ and k are chosen stochastically from a bounded domain, D?

A flow-kick framework for exploring resilience

THANK YOU!

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