Spatial aspects in vaccination

(with P. van den Driessche)

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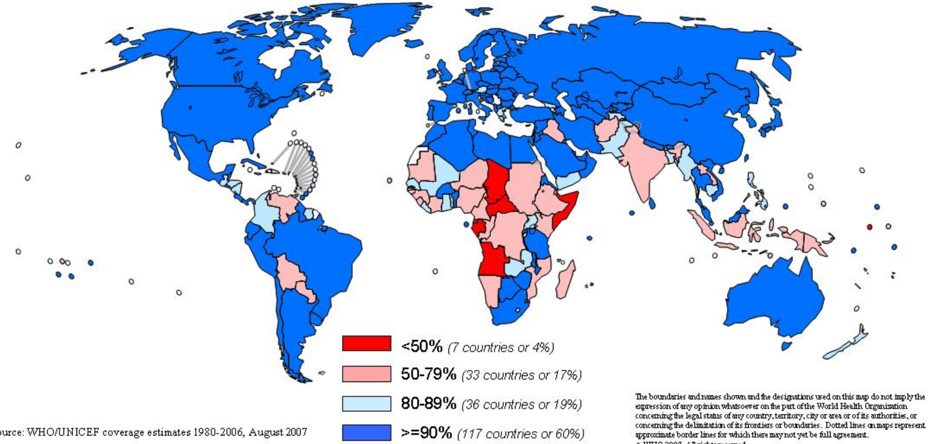
Pathogens have been moving around for a while

It first began, it is said, in the parts of Ethiopia above Egypt, and thence descended into Egypt and Libya and into most of the [Persian] King's country. Suddenly falling upon Athens, it first attacked the population in Piraeus [..] and afterwards appeared in the upper city, when the deaths became much more frequent.

Thucydides (c. 460 BC - c. 395 BC)
History of the Peloponnesian War
(plague of Athens of 430 BC)

Immunization coverage with 3rd dose of polio vaccines in infants, 2006

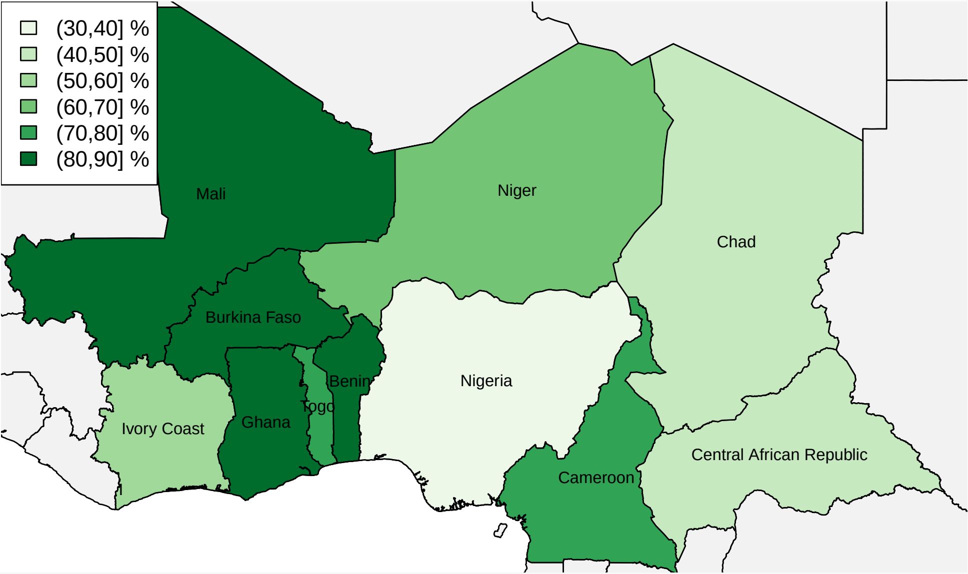


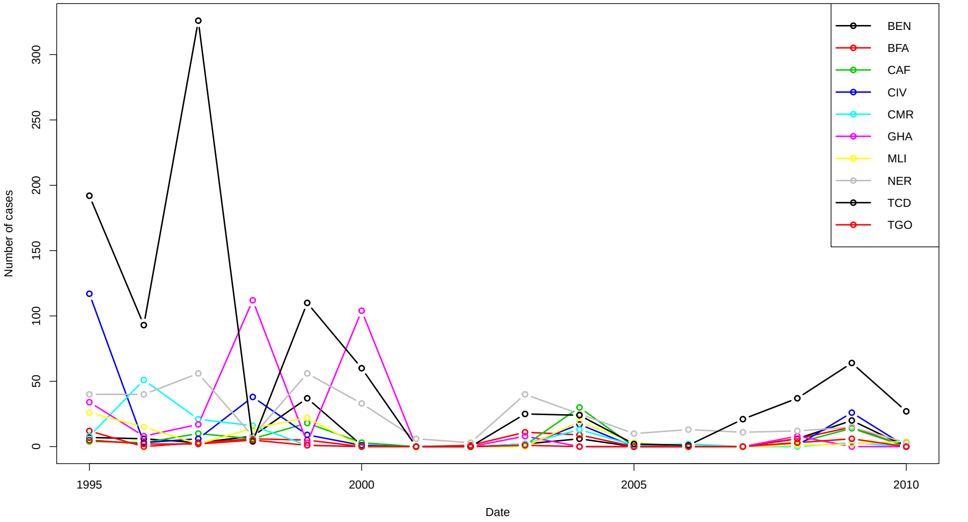
Source: WHO/UNICEF coverage estimates 1980-2006, August 2007

Date of slide: 23 August 2007

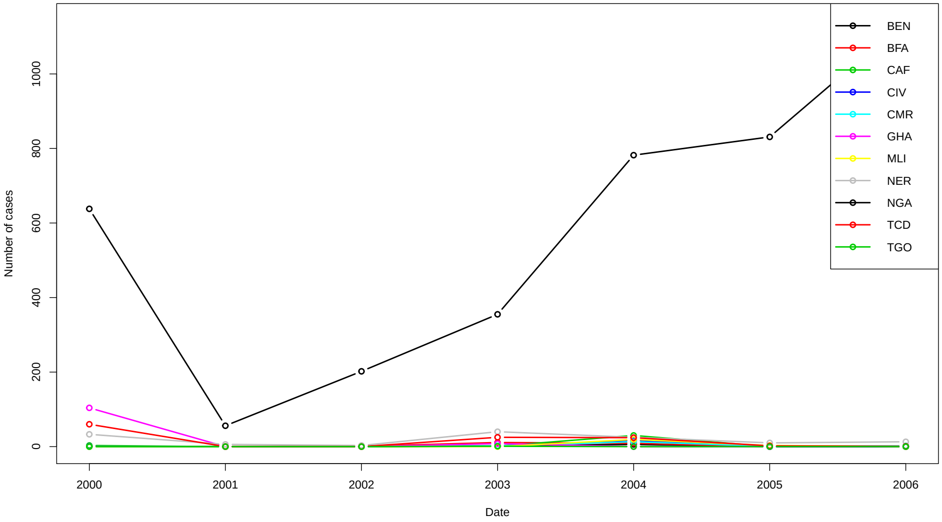
The boundaries and names shown and the designations used on this map do not imply the expression of any opinion whatsoever on the part of the World Health Organization concerning the legal status of any country, territory, city or area or of its authorities, or concerning the delimitation of its frontiers or boundaries. Dotted lines on maps represent approximate border lines for which there may not yet be full agreement.

World Health Organization


(30,40] %
(40,50] %
(50,60] %
(60,70] %
(70,80] %
(80,90] %
Mali
Niger
Chad
Burkina Faso
Benin
Nigeria
Togo
Ghana
Ivory Coast
Cameroon
Central African Republic



Number of cases
Date
BEN
BFA
CAF
CIV
CMR
GHA
MLI
NER
TCD
TGO
0
50
100
150
200
250
300
1995
2000
2005
2010


Number of cases
0
200
400
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2001
2002
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Date
BEN
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NGA
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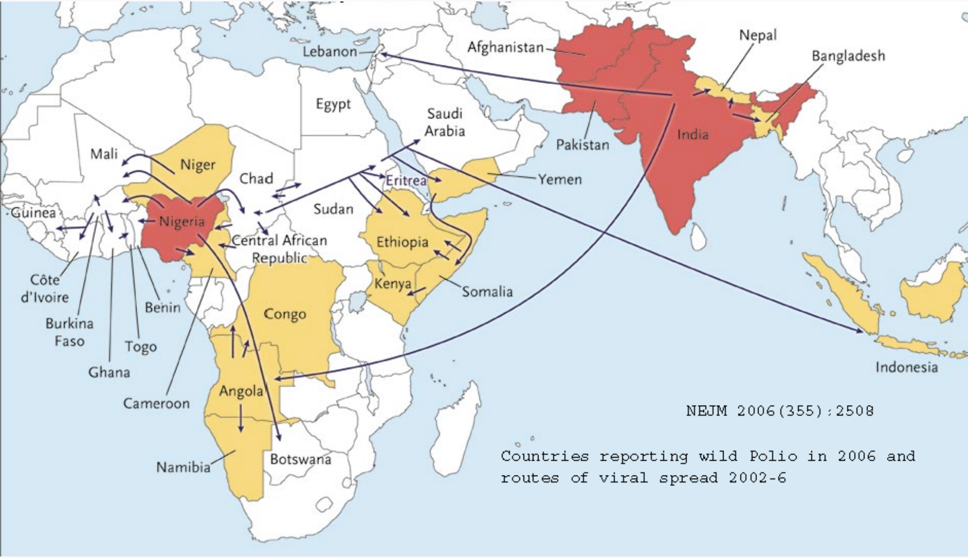
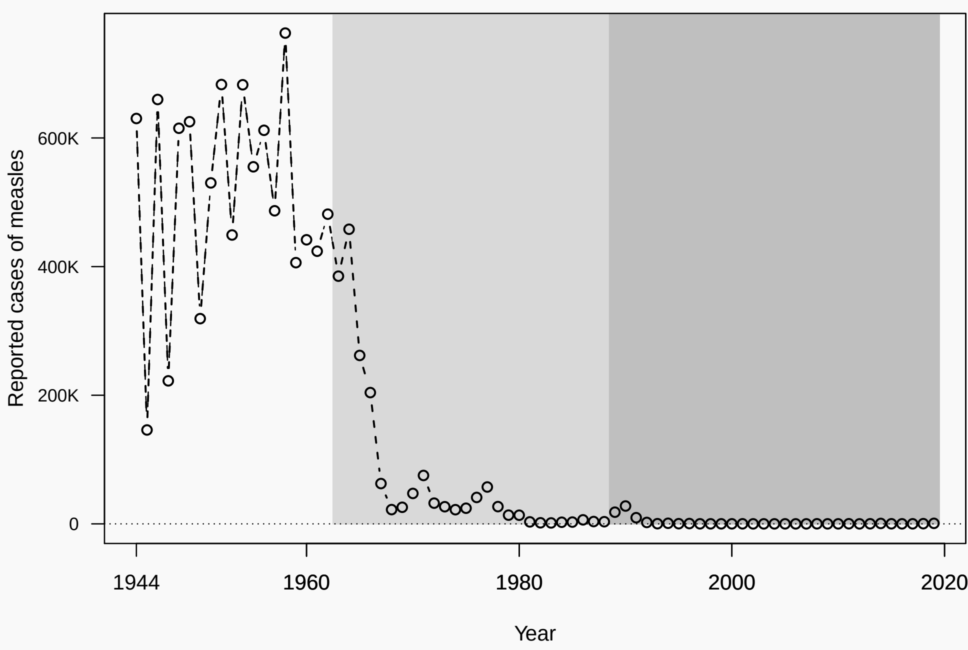
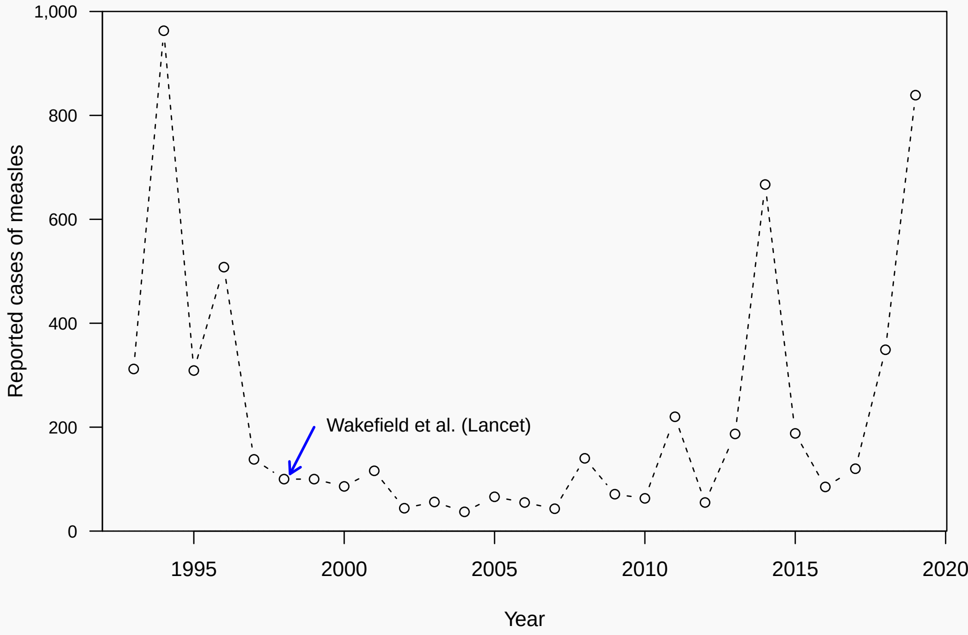


NEJM 2006(355):2508

Countries reporting wild Polio in 2006 and routes of viral spread 2002-6


Reported cases of measles
600K
400K
200K
0
1944
1960
1980
2000
2020
Year


1,000
800
600
400
200
0
Reported cases of measles
Wakefield et al. (Lancet)
1995
2000
2005
2010
2015
2020
Year

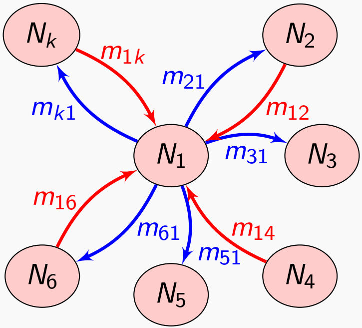
Quick intro to metapopulations

Split (continuous) space into $|\mathcal{P}|$ geographical locations (*patches*)

Each location contains **compartments**. Compartments are relatively homogeneous groups of individuals, e.g., susceptible humans, infected mosquitoes, etc.

Individuals in a compartment **may** move between patches and m_{cqp} is rate of movement of individuals from compartment $c \in \mathcal{C}$ from patch $p \in \mathcal{P}$ to patch $q \in \mathcal{P}$

Modelling movement



$$N_1' = \sum_{p \in \mathcal{P} \setminus \{1\}} m_{1p} N_p - N_1 \sum_{p \in \mathcal{P} \setminus \{1\}} m_{p1}$$

or

$$N_1' = \sum_{p \in \mathcal{P}} m_{1p} N_p \text{ assuming } m_{11} = - \sum_{p \in \mathcal{P} \setminus \{1\}} m_{p1}$$

In each patch, put a system describing the evolution of the number of individuals in each compartment present

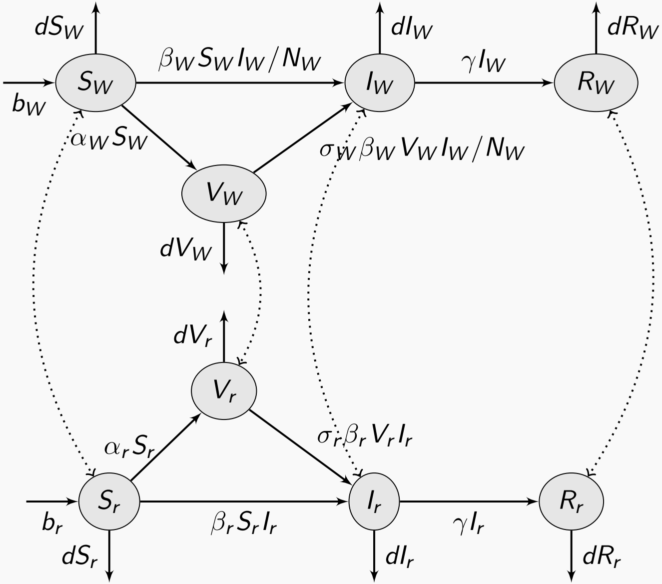
Assume **infected** (i) and **uninfected** (s) compartments $\mathcal{I}$ and $\mathcal{U}$ ($\mathcal{I} \cup \mathcal{U} = \mathcal{C}$). For all $k \in \mathcal{U}$, $\ell \in \mathcal{I}$ and $p \in \mathcal{P}$

$$
\begin{aligned}
s'_{kp} &= f_{kp}(S_p, I_p) + \sum_{q \in \mathcal{P}} m_{kpq} s_{kq} \\
i'_{\ell p} &= g_{\ell p}(S_p, I_p) + \sum_{q \in \mathcal{P}} m_{\ell pq} i_{\ell q}
\end{aligned}
$$

f and g describe interactions between compartments in a given location. Might involve more than S_p, I_p, but always local (p)

red terms describe movement of (individuals from) compartments between locations

MSc work of L. Wessel (now L. Malcolm)



Mathematical properties

Model inherits all the usual properties from [JA, CMcC & PvdD 2003]

No waning $\Rightarrow$ no backward bifurcation

Population converges patch-wise ($\Rightarrow$ 8D$\to$6D)

Reproduction numbers

Large city

$$\mathcal{R}_{vac}^{w} = \frac{\beta_w}{\gamma + d} \, \frac{d + \sigma_w \alpha_w}{d + \alpha_w}$$

Rural area

$$\mathcal{R}_{vac}^{r} = \frac{\beta_r}{\gamma + d} \, \frac{d + \sigma_r \alpha_r}{d + \alpha_r} \, \frac{b_r}{d}$$

Coupled system

$$\mathcal{R}_{vac} = \frac{(\gamma + d + m_{wr})\mathcal{R}_{vac}^{w} + (\gamma + d + m_{rw})\mathcal{R}_{vac}^{r} + \sqrt{\Delta}}{2(\gamma + d + m_{wr} + m_{rw})}$$

where Δ is some ugly expression

Proposition 1

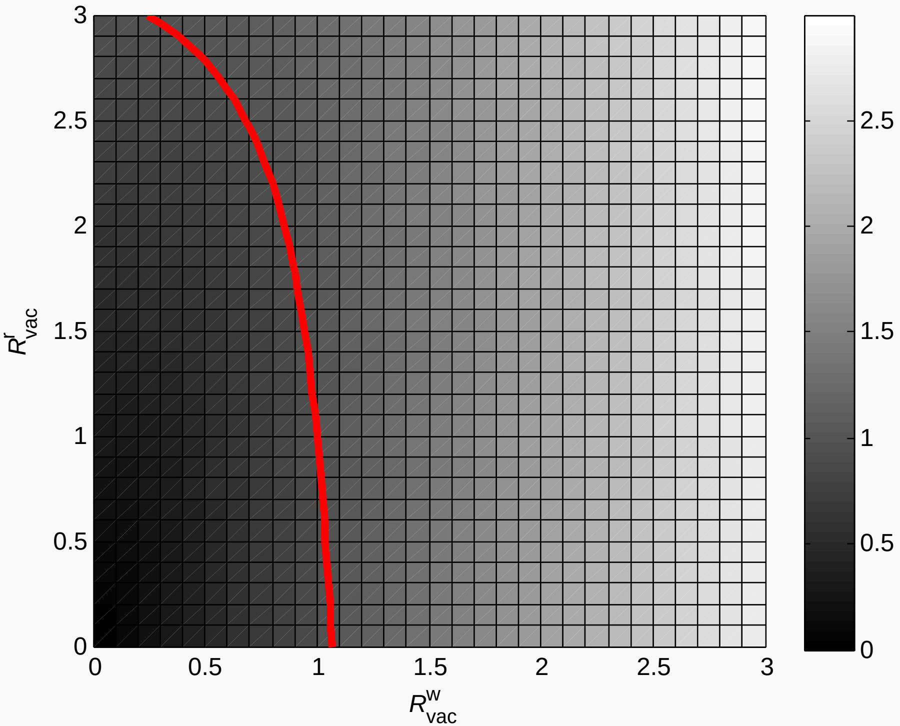
$$\min\left(\mathcal{R}_{vac}^{w}, \mathcal{R}_{vac}^{r}\right) \leq \mathcal{R}_{vac} \leq \max\left(\mathcal{R}_{vac}^{w}, \mathcal{R}_{vac}^{r}\right)$$

Setting up numerics

Disease parameters roughly representing measles

Transport parameters for Manitoba road network [JA & SP 2015]

City	Pop.	km $\to$ YWG	Trips day^{-1}
Winnipeg	663,617	-	-
Portage la Prairie	12,996	85	4,115
Morden	7,812	130	1,630
Peguis First Nations	2,609	184	650



3
2.5
2
1.5
1
0.5
0
R^{f}_{vac}
0
0.5
1
1.5
2
2.5
3
R^{w}_{vac}
2.5
2
1.5
1
0.5
0

Final considerations about this model

Two interesting little issues encountered when doing numerics:

1. How does one get coverage?

$$V^*/N^*$$

   Remember that V^* depends on $\mathcal{R}_{vac}$

2. How does one get # cases averted? Compare

$$I^*|_{\text{vacc}} \text{ with } I^*|_{\text{no vacc}}$$

Resident-traveller models

In previous model, when a person moves, they become a member of the patch they move to

Evaluation of public health policy in a globalised context: given "my" residents travel, what is the effect of "my" policy, both locally and globally?

Need to keep track of individuals, not only where they are but where they come from

$\Longrightarrow$ resident-traveller models

Framework of resident-traveller models

Suppose $|\mathcal{P}|$ locations

Let $p, q \in \mathcal{P}$ and denote

$$N_{pq}$$

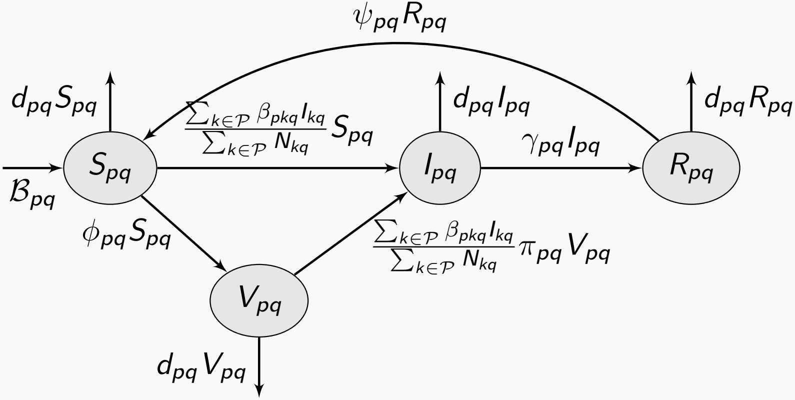
the number of individuals **resident** of patch p currently **travelling to** patch q

Dynamics:

$$N'_{pq} = \mathcal{B}_{pq} - d_{pq} N_{pq} + \sum_{k \in \mathcal{P}} m_{pqk} N_{pk}$$

where m_{pqk} rate of movement of individuals residents of p from patch k to patch q (and typically, $\mathcal{B}_{pq} = 0 \; \forall p \neq q$)

Model in a resident-traveller context



$\mathcal{B}_{pq} = 0$ if $p \neq q$

Model in a resident-traveller context

$p, q \in \mathcal{P}$,

$$S'_{pq} = \mathcal{B}_{pq} + \psi_{pq} R_{pq} - \left(d_{pq} + \phi_{pq} + \frac{\sum\limits_{k \in \mathcal{P}} \beta_{pkq} I_{kq}}{\sum\limits_{k \in \mathcal{P}} N_{kq}} \right) S_{pq} + \sum_{k \in \mathcal{P}} m^S_{pqk} S_{pk}$$

$$I'_{pq} = (S_{pq} + \pi_{pq} V_{pq}) \frac{\sum_{k \in \mathcal{P}} \beta_{pkq} I_{kq}}{\sum_{k \in \mathcal{P}} N_{kq}} - (d_{pq} + \gamma_{pq}) I_{pq} + \sum_{k \in \mathcal{P}} m^I_{pqk} I_{pk}$$

$$R'_{pq} = \gamma_{pq} I_{pq} - (d_{pq} + \psi_{pq}) R_{pq} + \sum_{k \in \mathcal{P}} m^R_{pqk} R_{pk}$$

$$V'_{pq} = \phi_{pq} S_{pq} - \left(d_{pq} + \pi_{pq} \frac{\sum\limits_{k \in \mathcal{P}} \beta_{pkq} I_{kq}}{\sum\limits_{k \in \mathcal{P}} N_{kp}} \right) V_{pq} + \sum_{k \in \mathcal{P}} m^V_{pqk} V_{pk}$$

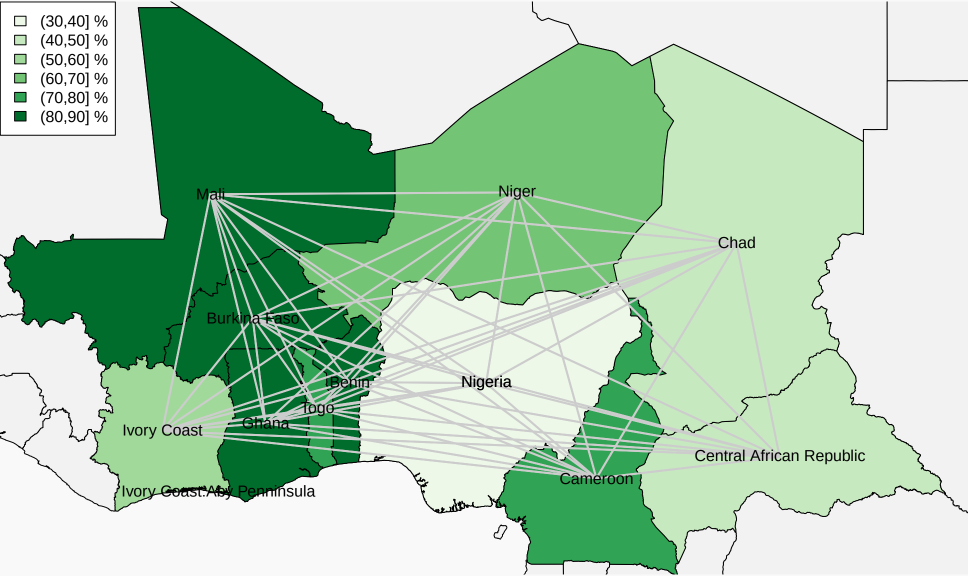
Done / Not done

Done: all the classic math stuff

Finishing up: numerics on polio situation

Model written with PvdD during 1st CDM Summer School on Math Epi in Banff .. 2004

Country	ISO code	Life expectancy	Population
Benin	BEN	60.9	11,175,692
Burkina Faso	BFA	60.4	19,193,382
Cent. Afr. Rep.	CAF	52.2	4,659,080
Cote d'Ivoire	CIV	53.6	24,294,750
Cameroon	CMR	58.1	24,053,727
Ghana	GHA	62.7	28,833,629
Mali	MLI	58.0	18,541,980
Niger	NER	60.1	21,477,348
Nigeria	NGA	53.4	190,886,311
Chad	TCD	52.9	14,899,994
Togo	TGO	60.2	7,797,694



(30,40] %
(40,50] %
(50,60] %
(60,70] %
(70,80] %
(80,90] %
Mali
Niger
Chad
Burkina Faso
Benin
Nigeria
Togo
Ghana
Ivory Coast
Central African Republic
Cameroon
Ivory Coast.Aby Penninsula

Thank you!