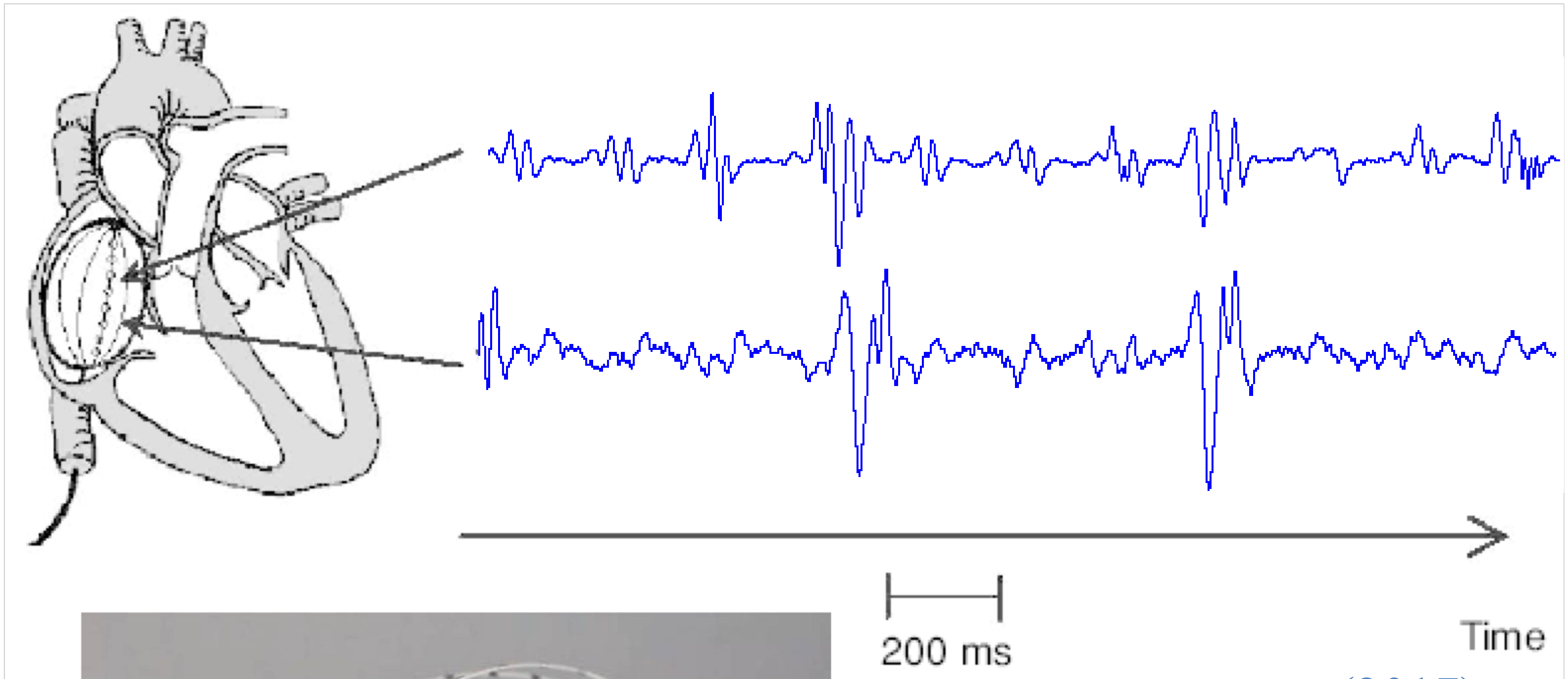
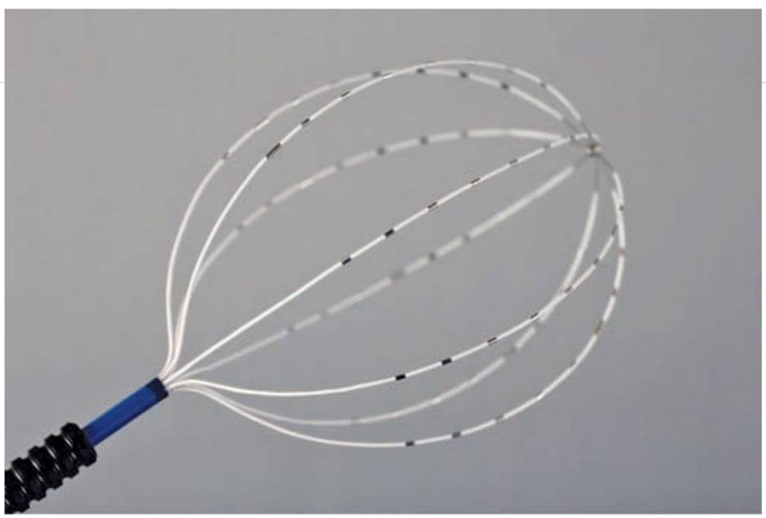
Reconstructing Rotor Dynamics from Sparse Noisy Data

Daniel Gurevich, Roman Grigoriev

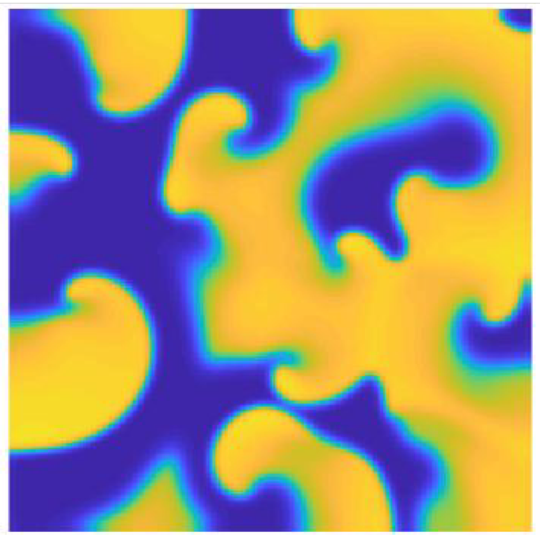
Rotor mapping for ablation therapy





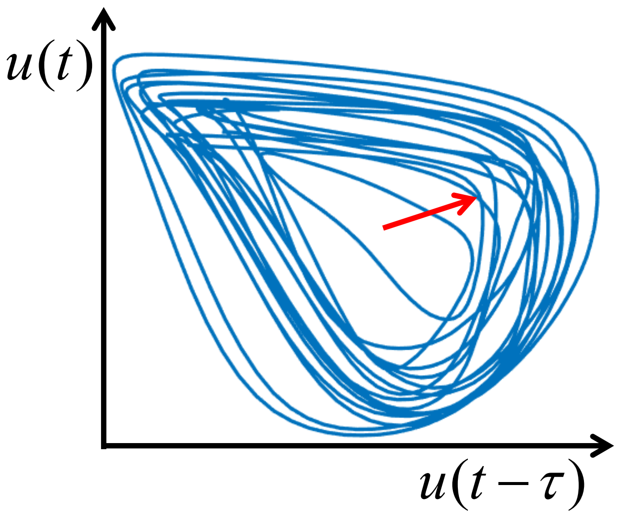
Ashikaga (2017)

Model of atrial fibrillation

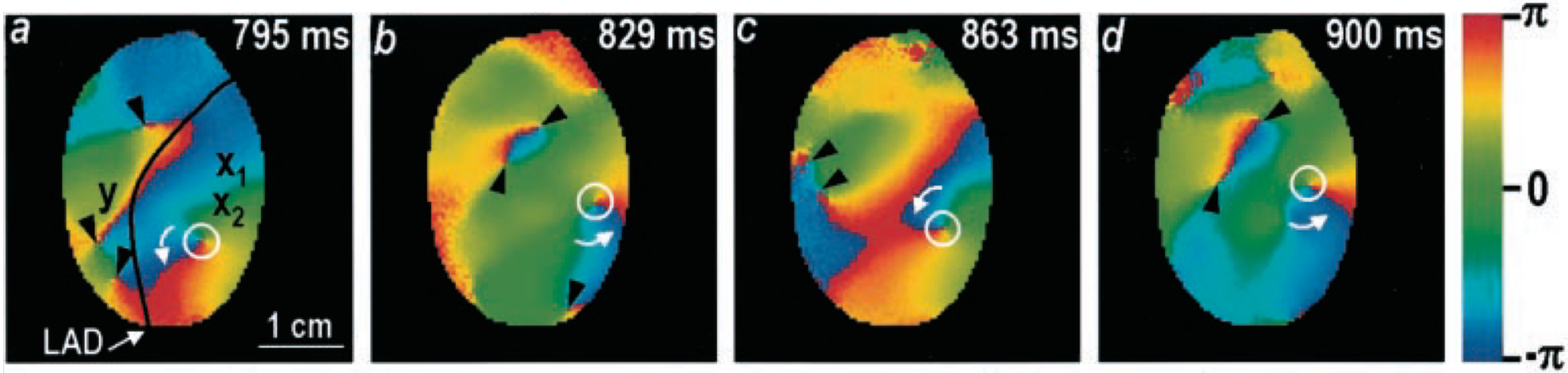


Karma model

Phase singularities

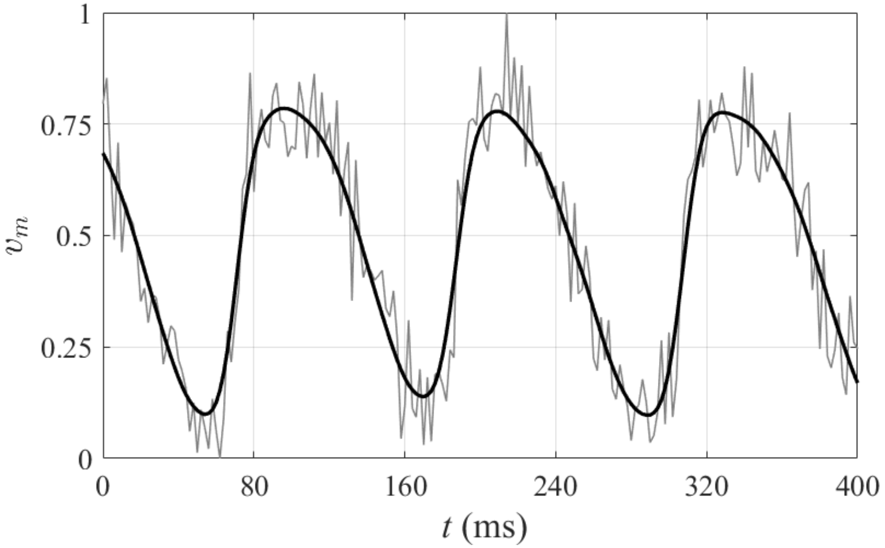


$$q = \frac{1}{2\pi}\oint d\varphi = \begin{cases} -1, & \text{PS (clockwise)} \\ 0, & \text{no PS} \\ +1, & \text{PS (counter-clockwise)} \end{cases}$$

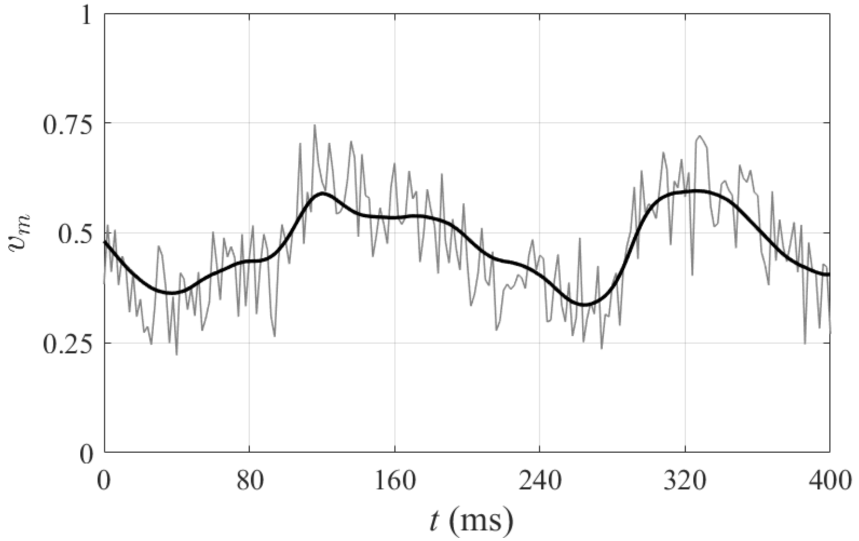


Wu et al. (2004)

Interpreting noisy data

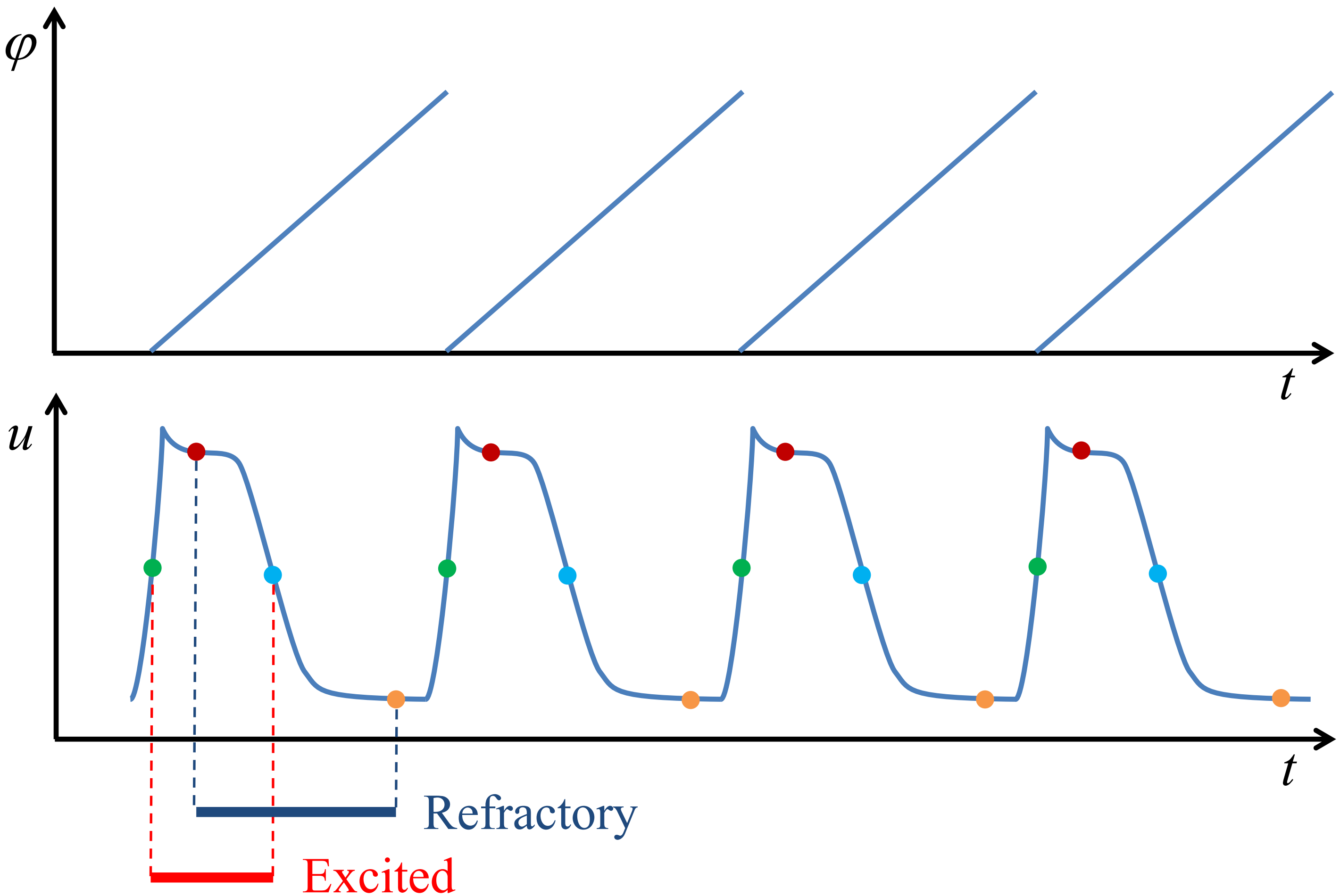


Voltage trace far from PS



Voltage trace near PS

Phase singularities & level sets



Topological description

- Complexity of the excitation pattern can be quantified by the number of phase singularities (PS)

Marcotte & Grigoriev (2017)

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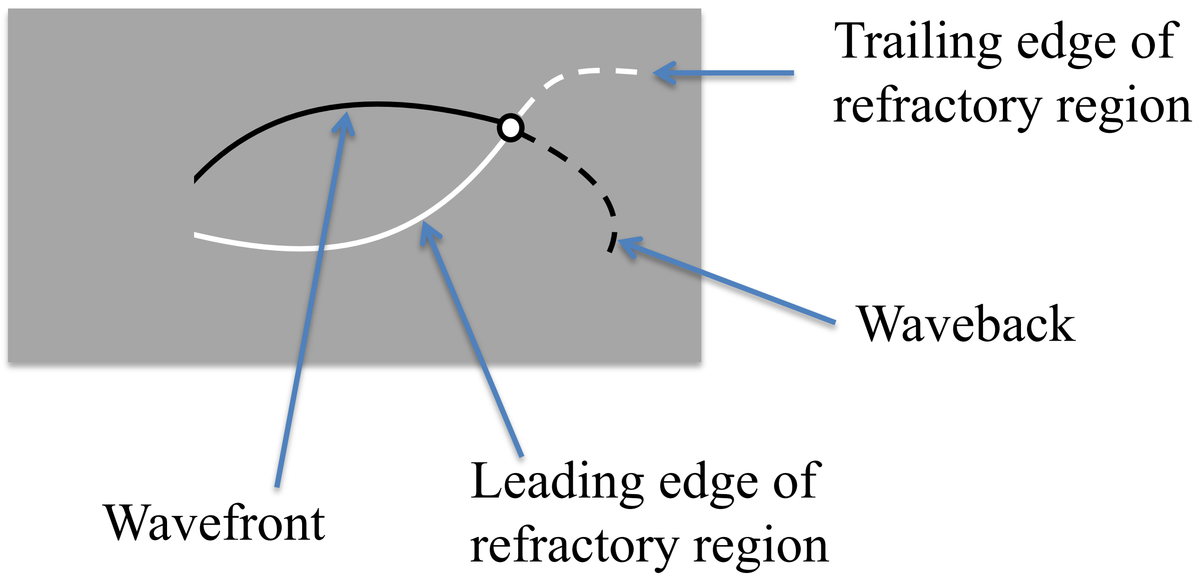
- Each PS lies at the intersection of two level sets (e.g., $\partial_t u = 0$ and $\partial_t^2 u = 0$).



Marcotte & Grigoriev (2017)

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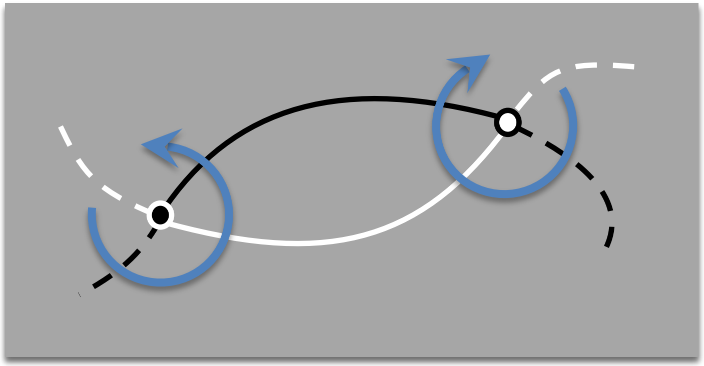
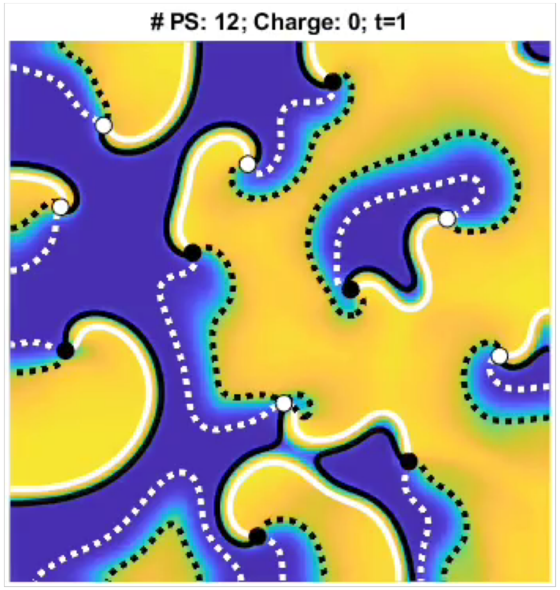
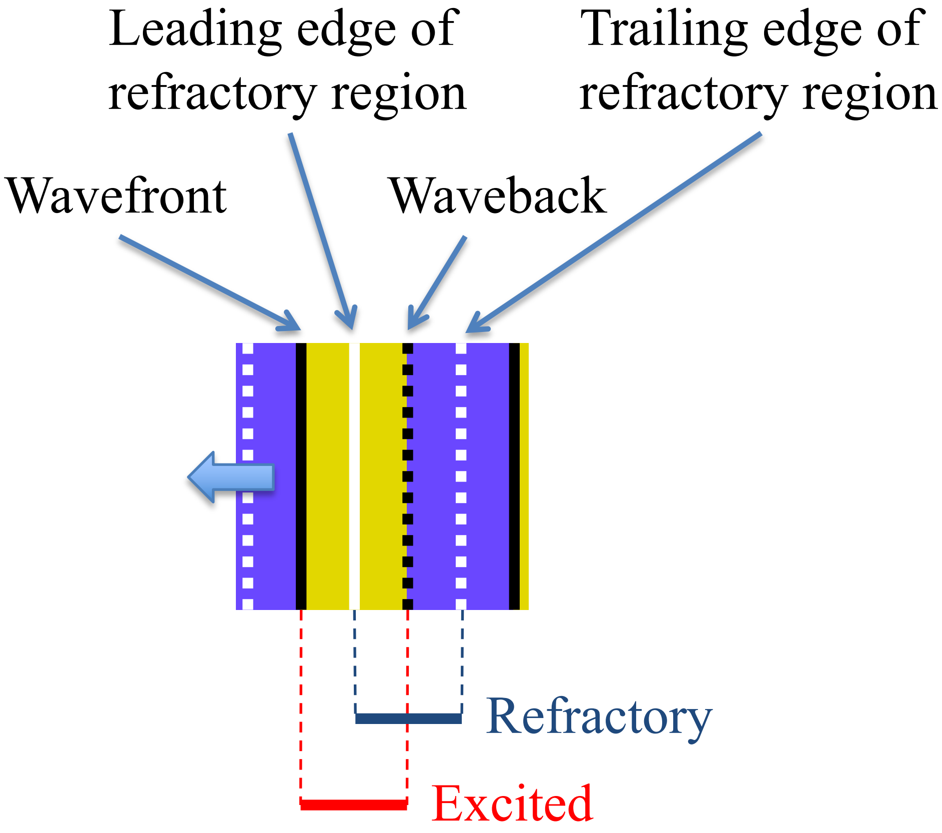


- Each PS has a topological charge: $q = \mathrm{sign}(\hat{\mathbf{z}} \cdot \nabla u \times \nabla \partial_t u) = \pm 1$
- The net topological charge is conserved*: $\sum_i q_i = 0$
- Phase singularities can only be created/destroyed in pairs*

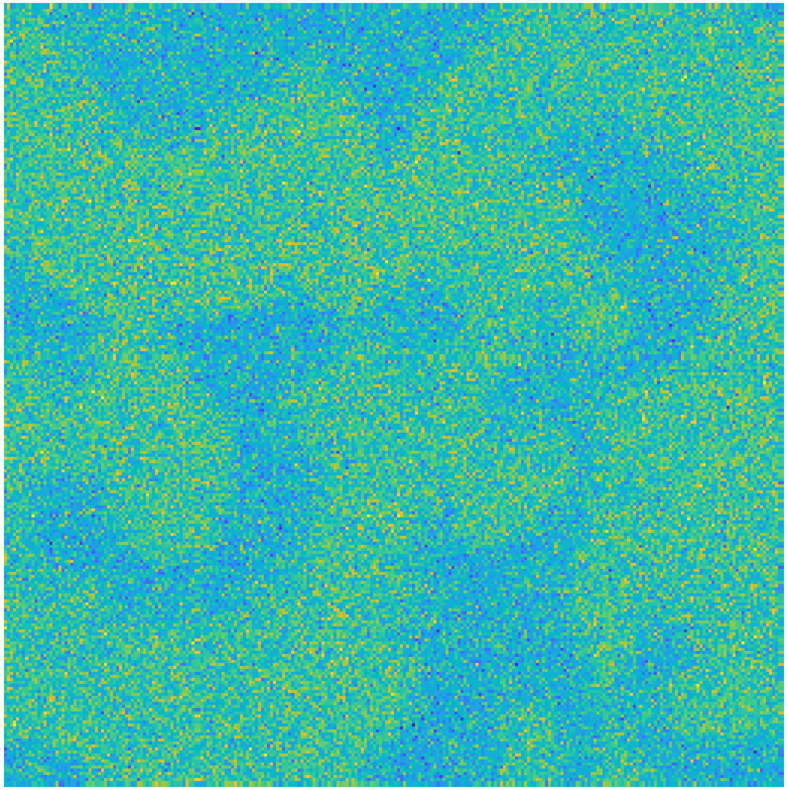
Practical implementation

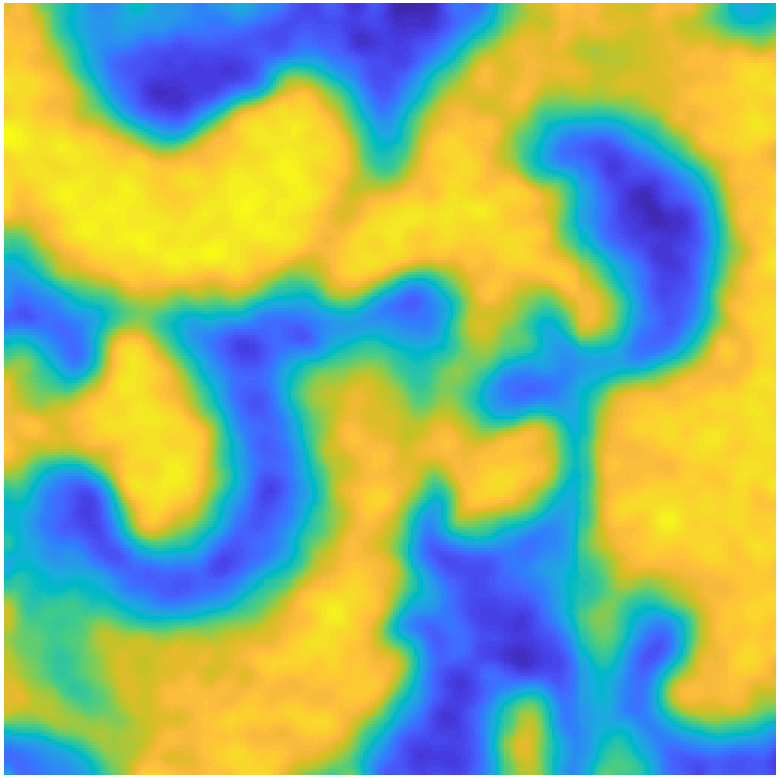


Karma model

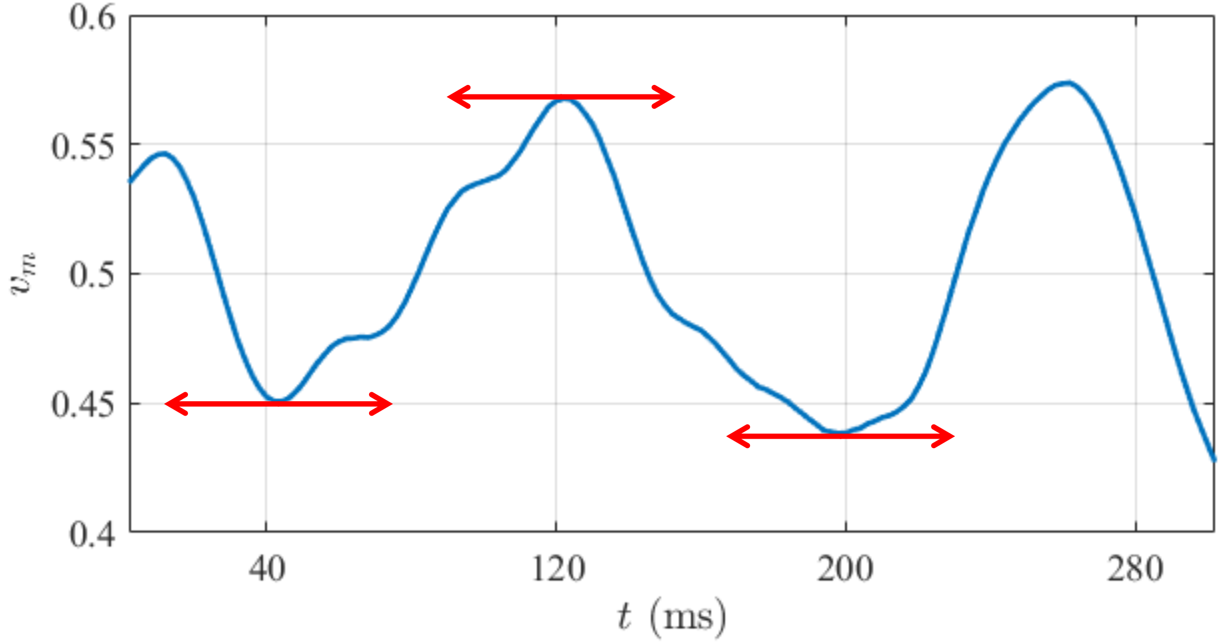


Step 1: Gaussian smoothing

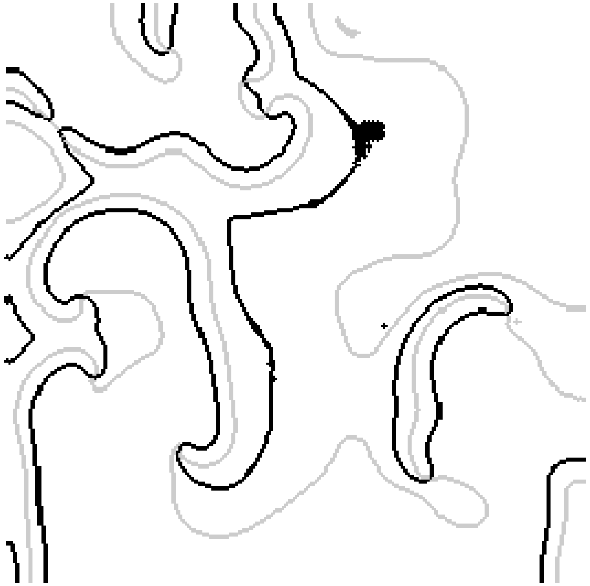




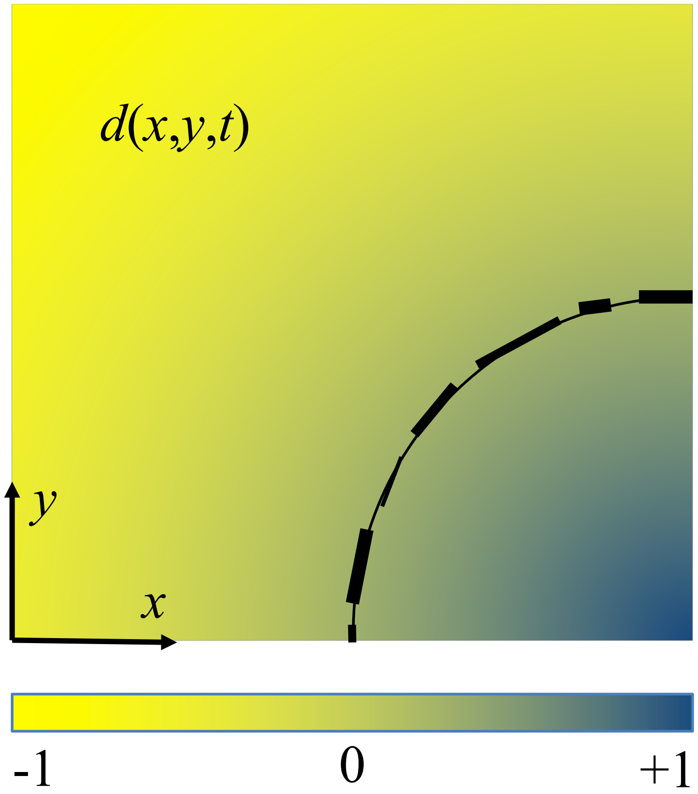
Step 2: robust "time derivative"



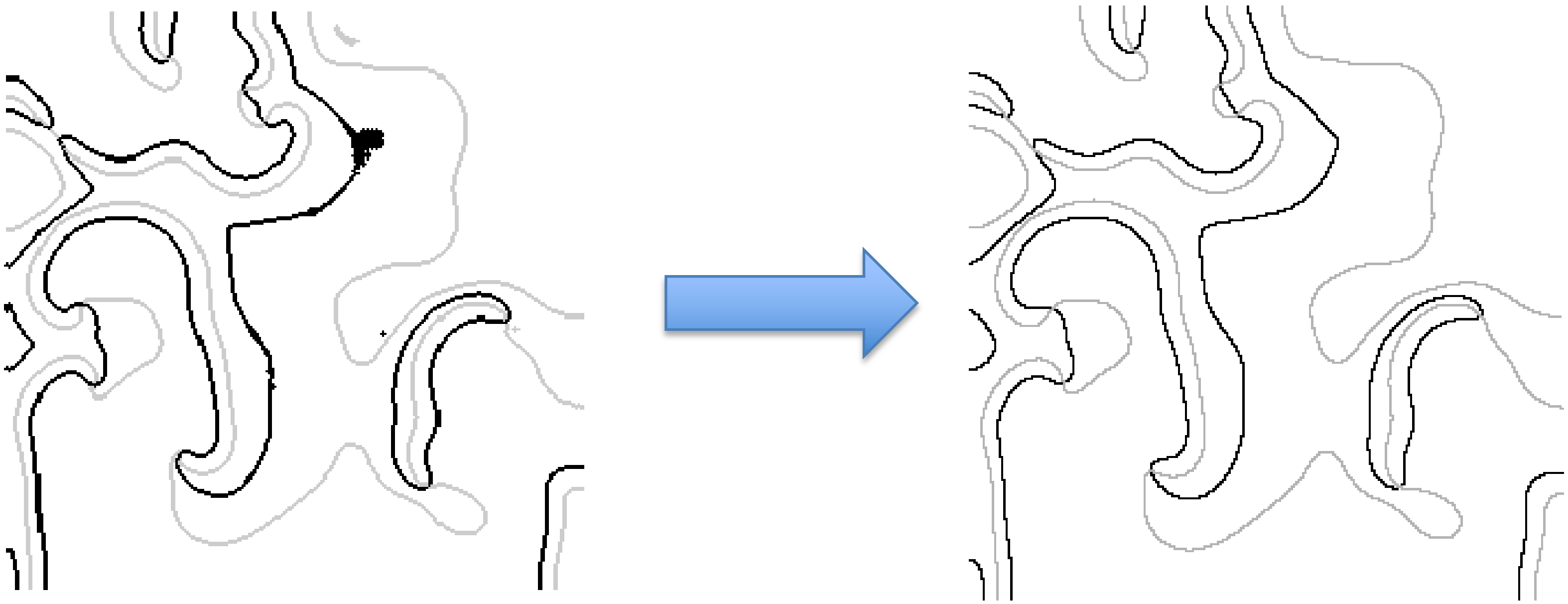
Step 3: coarse level sets



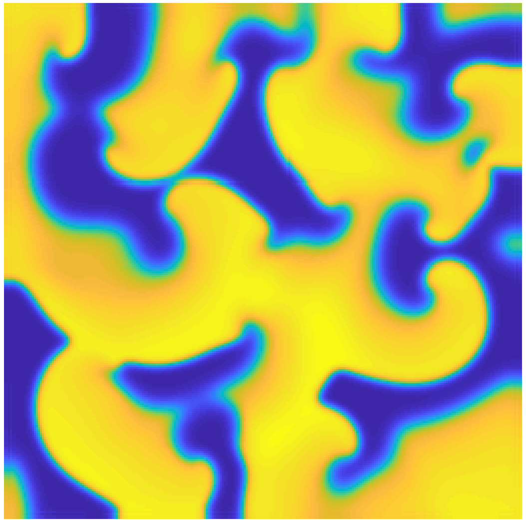
Step 4: signed distance function

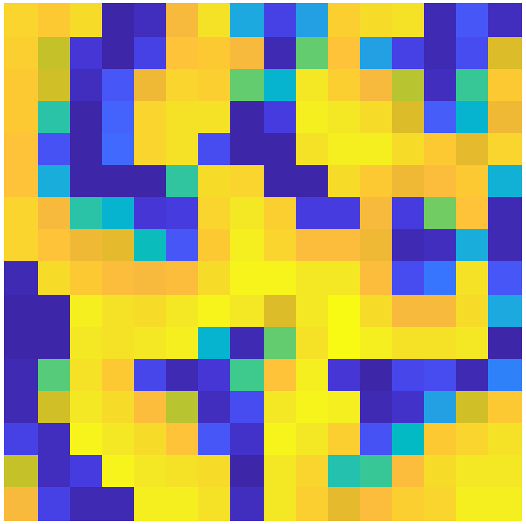


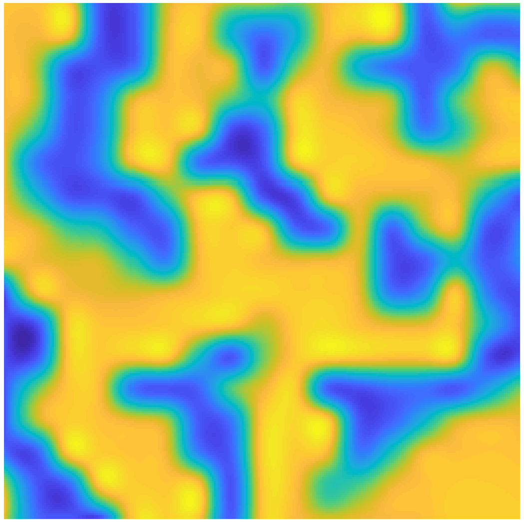
Step 5: smooth level sets



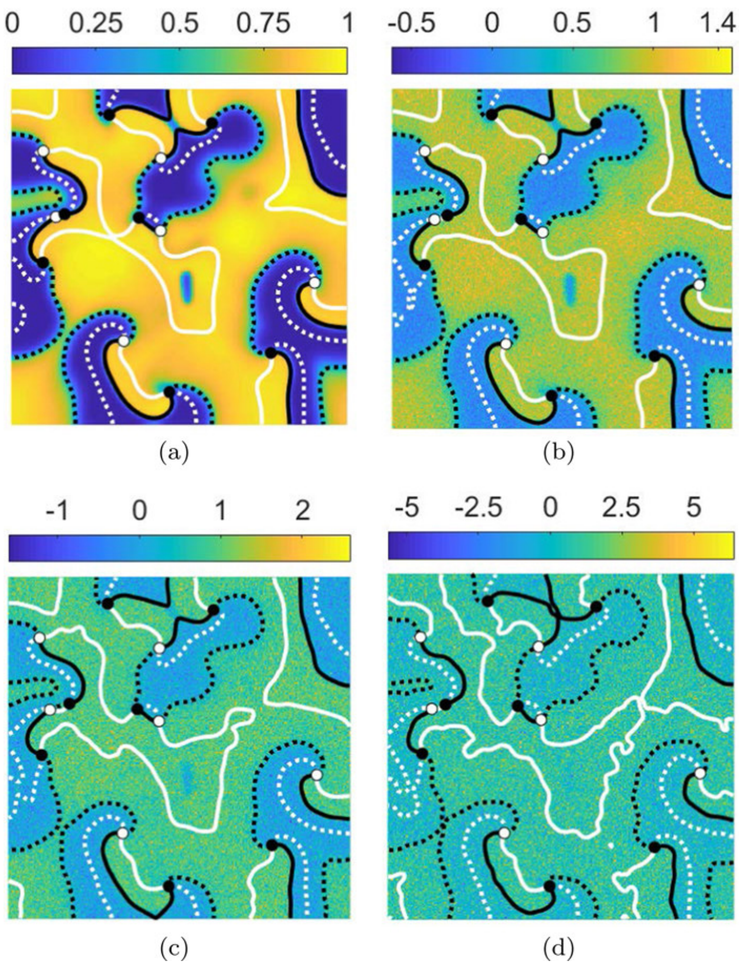
Dealing with sparsity



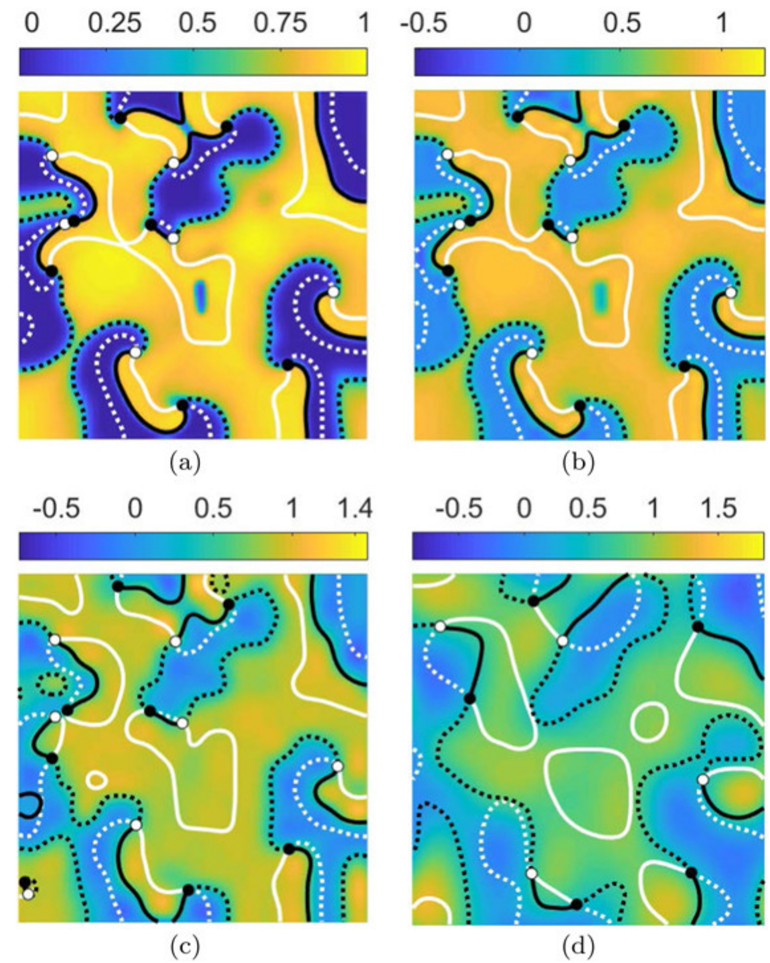




Effect of noise and sparsity



Full resolution, noise level η=0, 0.1, 0.3, 1



No noise, spatial grid 256x256, 32x32, 16x16, 8x8

Gurevich & Grigoriev (2019)

Effect of noise and sparsity

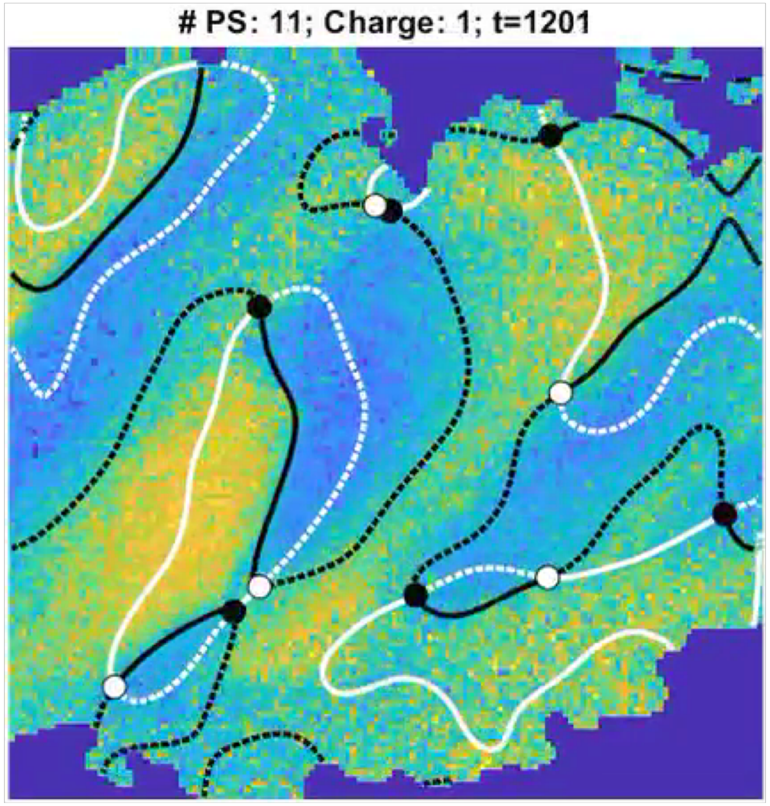
	256x256	64x64	32x32	16x16	8x8
$\eta=0$	0.995	0.995	0.994	0.955	0.255
$\eta=0.1$	0.993	0.994	0.992	0.957	0.308
$\eta=0.3$	0.988	0.988	0.985	0.954	0.357
$\eta=1$	0.990	0.974	0.849	0.695	...

Accuracy of PS detection (fraction matched)

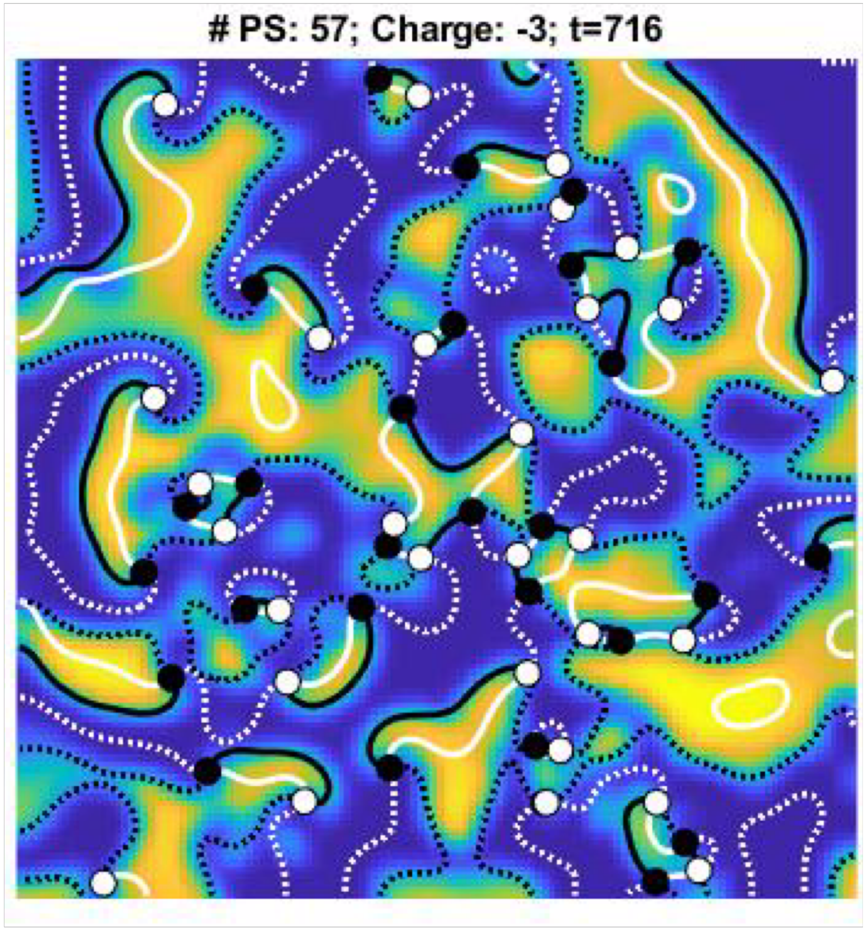
	256x256	64x64	32x32	16x16	8x8
$\eta=0$	1.1	1.1	1.4	4.8	9.9
$\eta=0.1$	1.2	1.3	1.6	4.6	9.6
$\eta=0.3$	1.7	1.8	2.3	4.9	9.4
$\eta=1$	2.3	3.1	4.5	6.9	...

Precision of PS detection (in fine grid units)

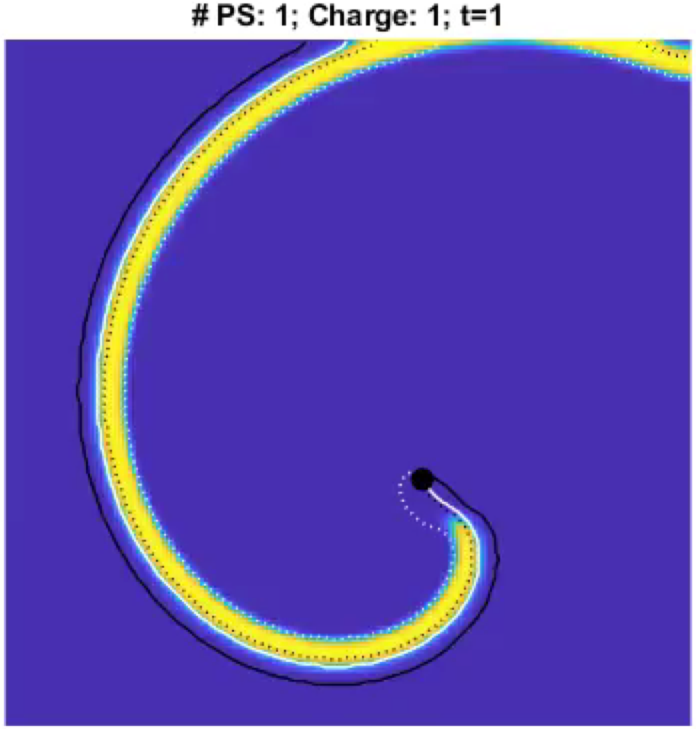
Ventricular fibrillation (pig)



More spirals? (Fenton-Karma model)



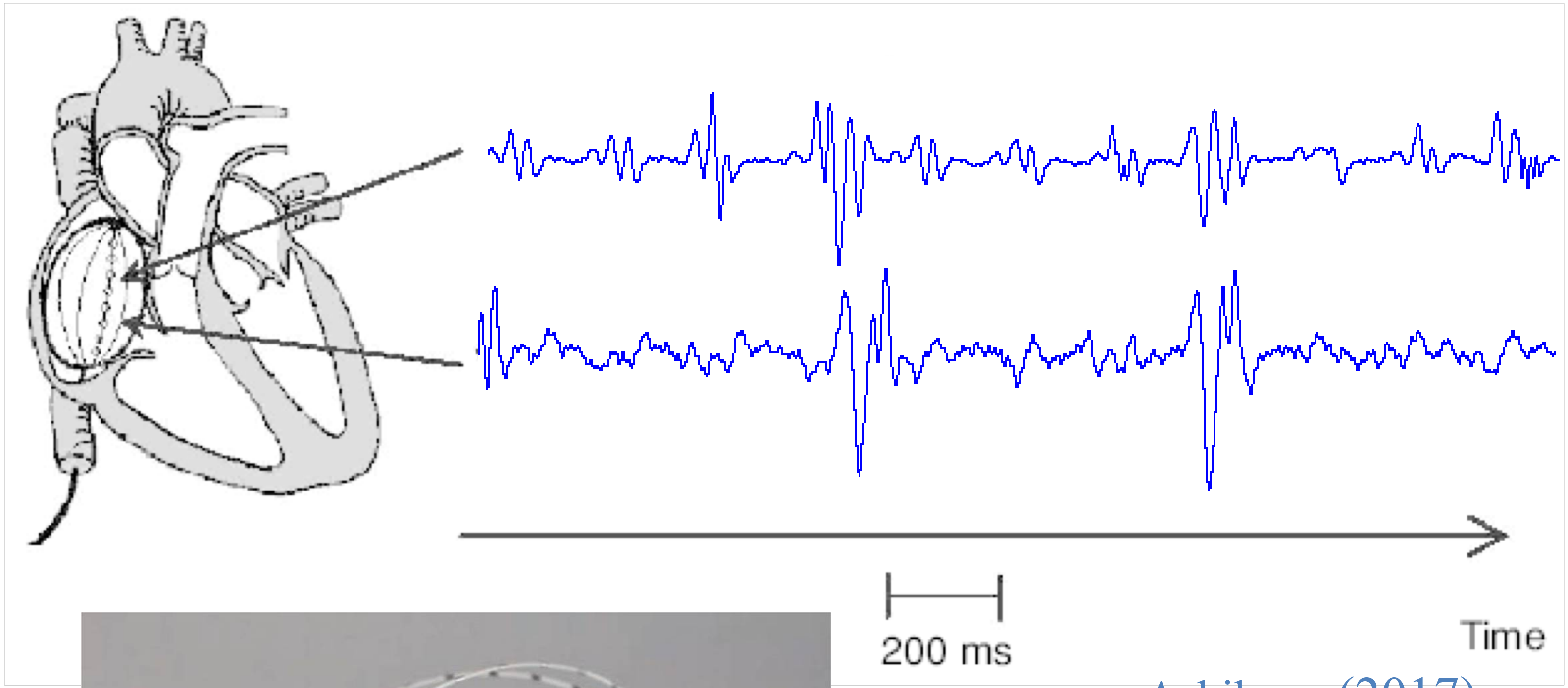
Sparse spiral (Barkley model)

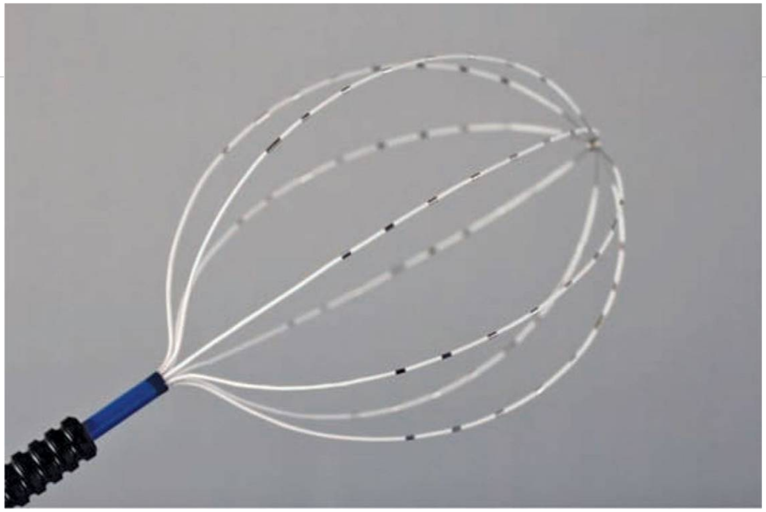


Summary

- Developed, implemented, and validated novel topological analysis of excitable systems
- This method is far more robust than existing techniques and directly applicable to numerical models and optical mapping recordings
- Can find and track many nonstationary PSs simultaneously and handle challenging edge cases
- The new approach promises to provide new insight into dynamical mechanisms underlying fibrillation

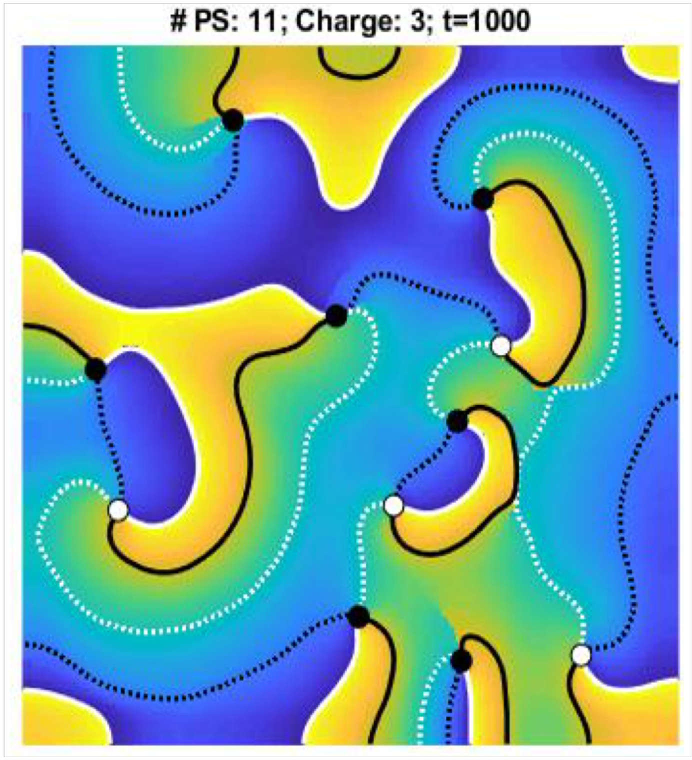
Rotor mapping for ablation therapy



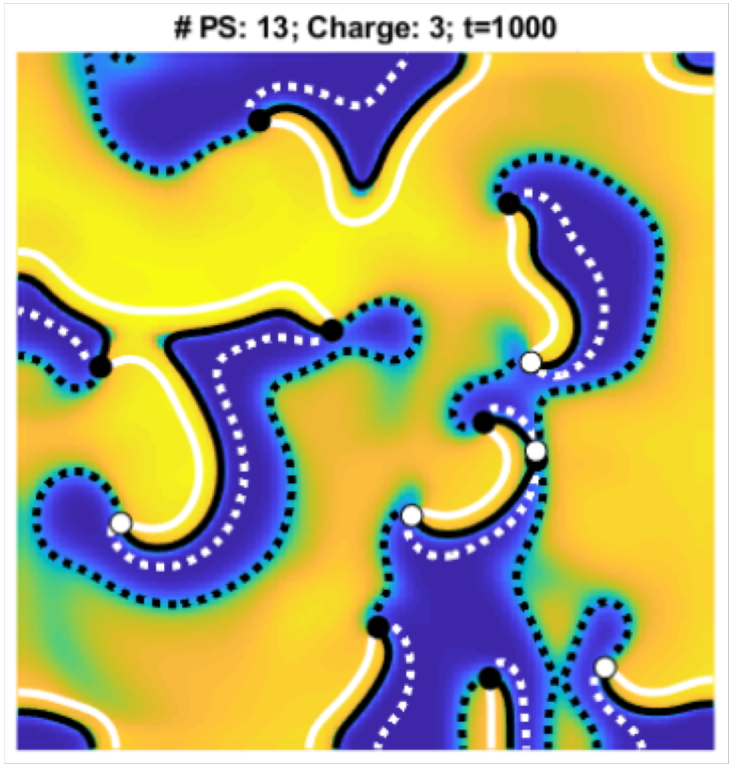


Ashikaga (2017)

Phase singularities & level sets



Level sets of phase



Level sets of voltage

Thank you!