Society for Industrial and Applied Mathematics

Physical, Numerical and Computational Challenges in Modelling Oceans for Climate

SIAM CSE 2019, Spokane Washington

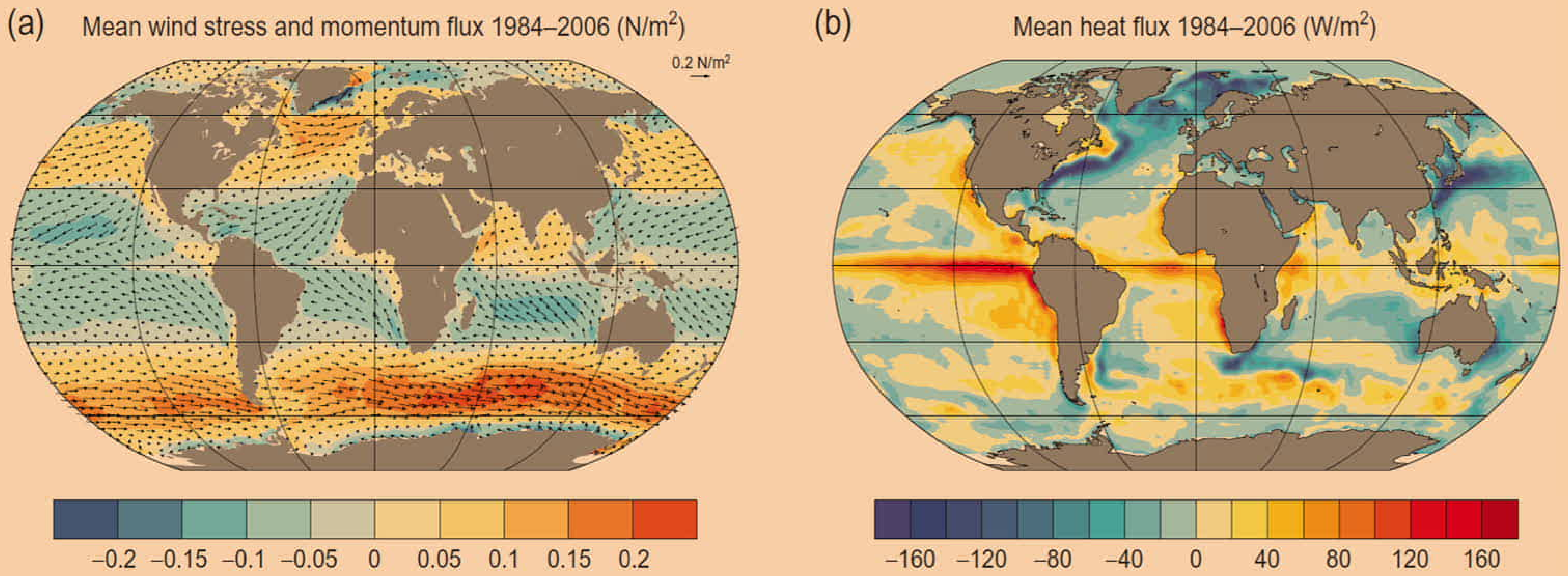
Alistair Adcroft

PRINCETON UNIVERSITY

NOAA GFDL

Ocean forcing

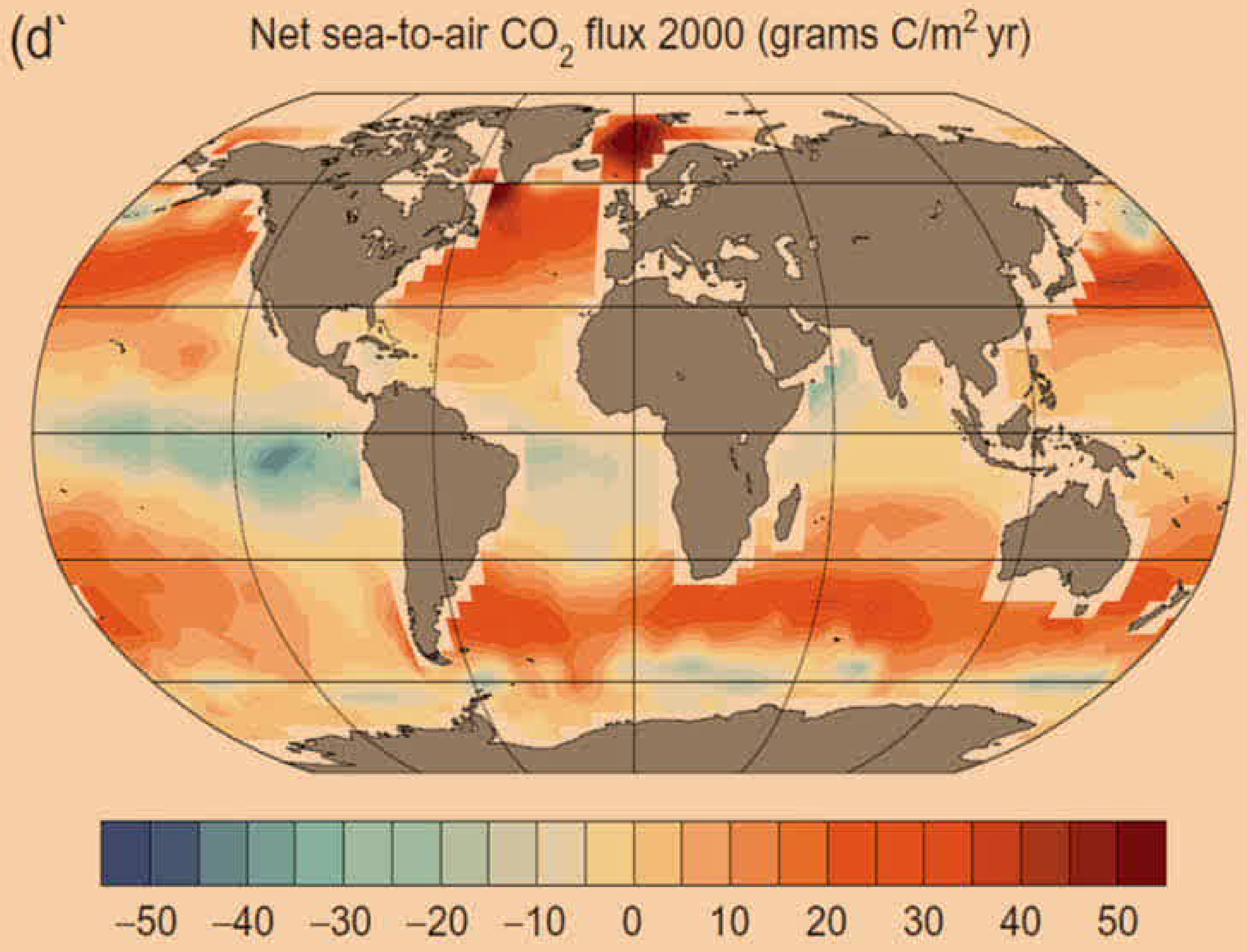
- Primarily forced from above by wind stress, and heat and mass fluxes



Large & Yeager, 2009

Role of the Ocean in the Carbon Cycle

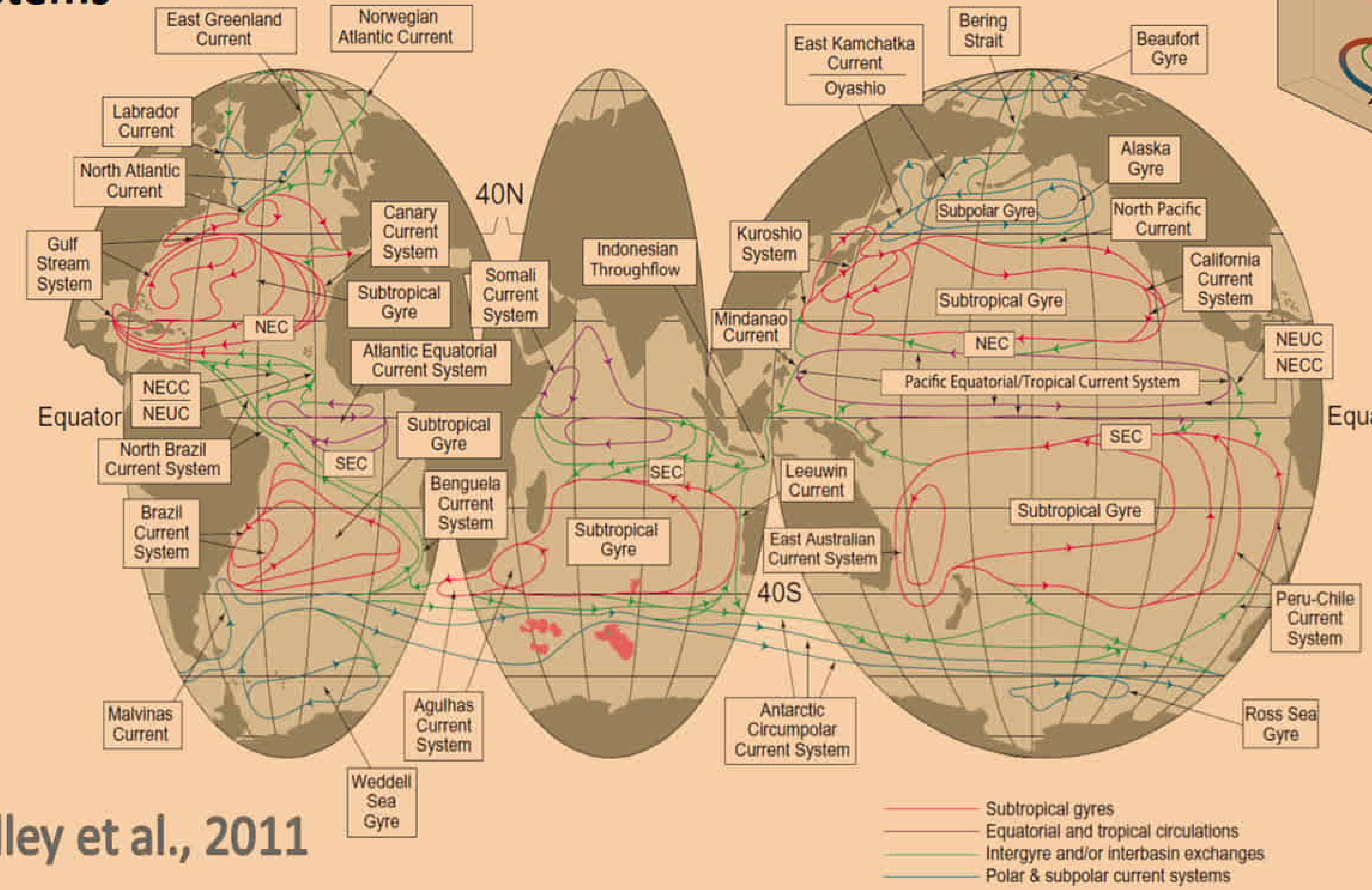
- The ocean has absorbed ~40% of anthropogenic carbon emissions (Ciais & Sabine, 2013, IPCC fifth assessment report)
- Leads to ocean acidification
- Much interest in understanding how the carbon cycle works and the role of the ocean



Takahashi, et al., 2009

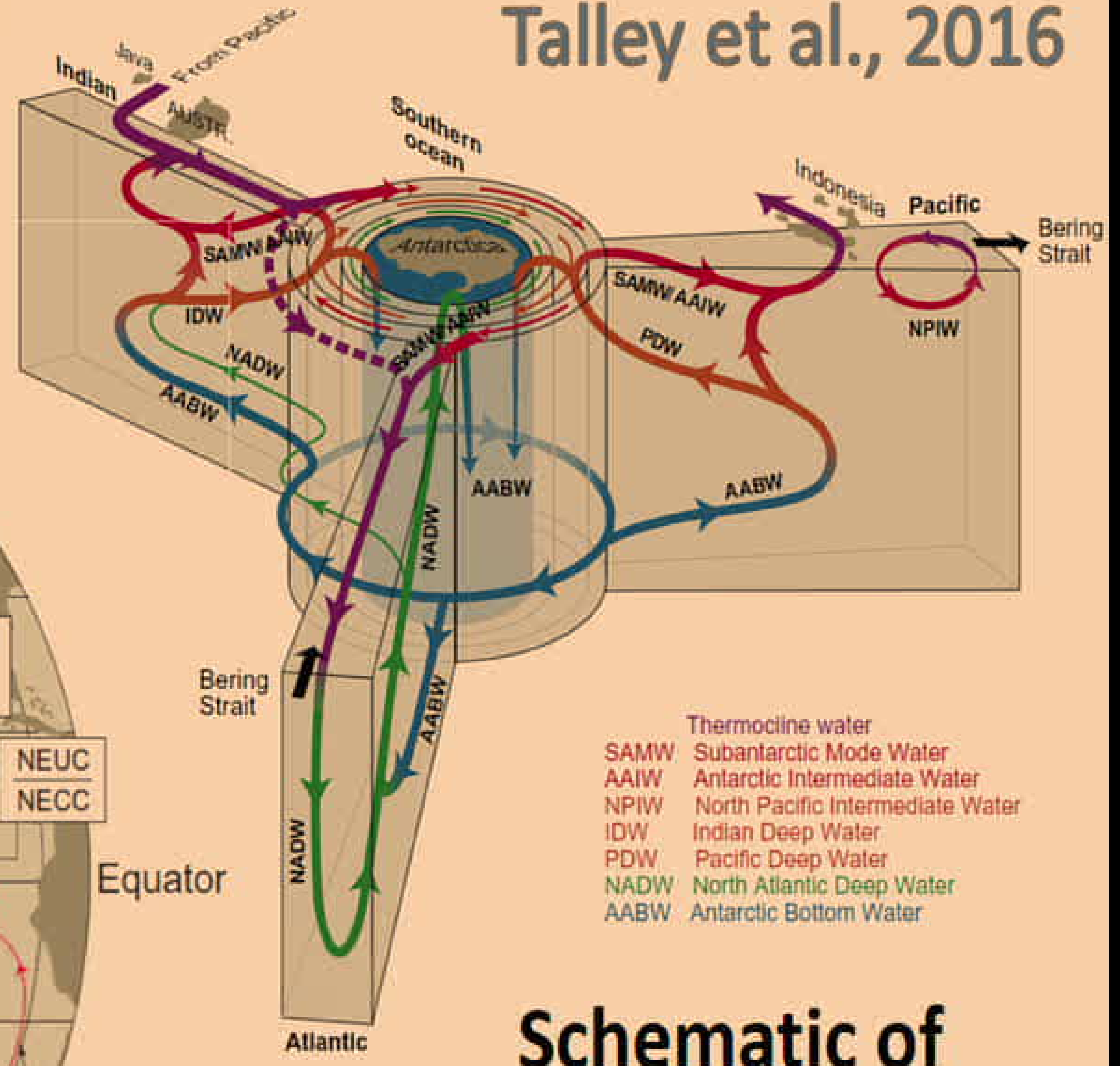
General circulation of the ocean

Schematic of surface current systems



Talley et al., 2011

Talley et al., 2016

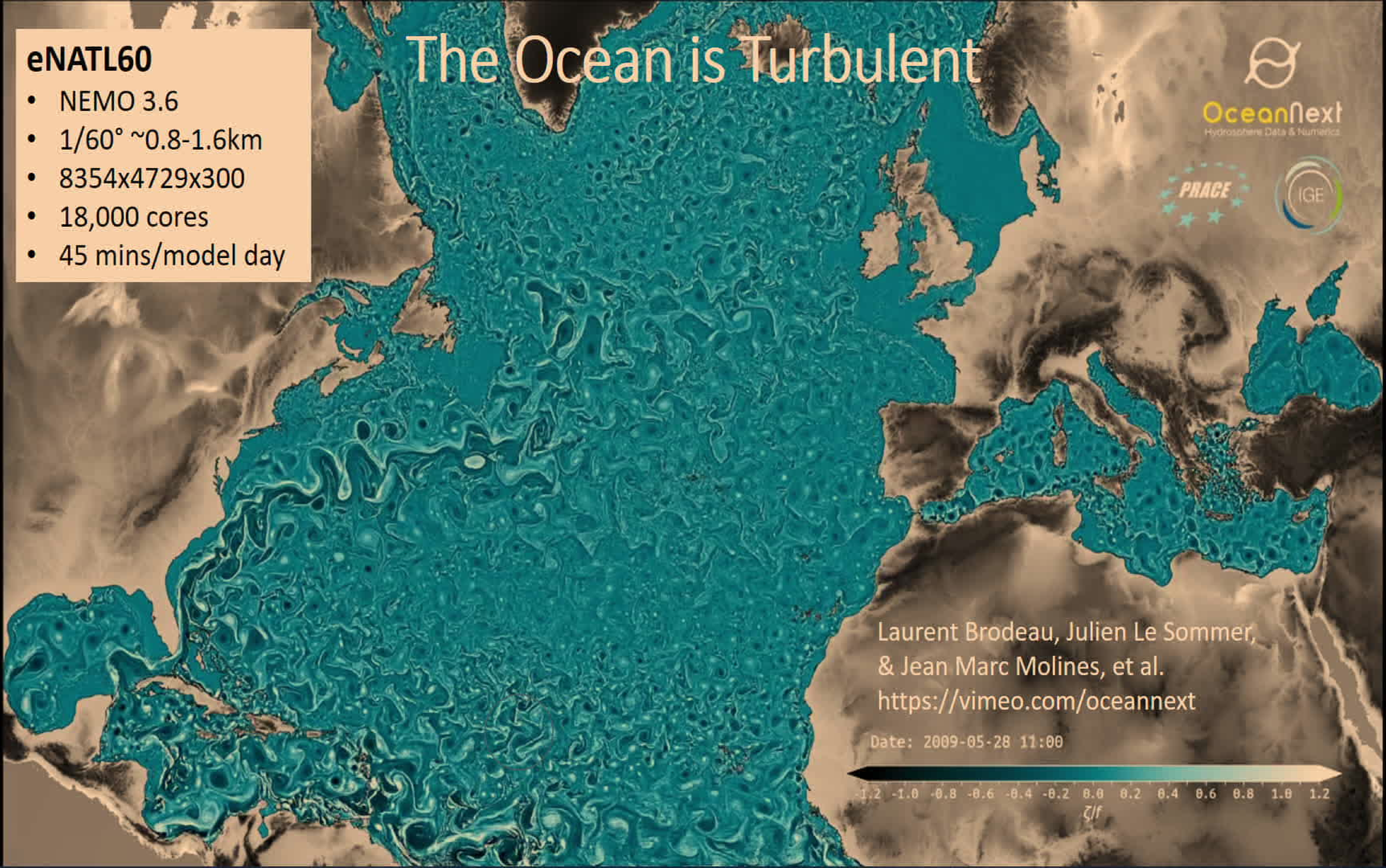


Schematic of deep circulation

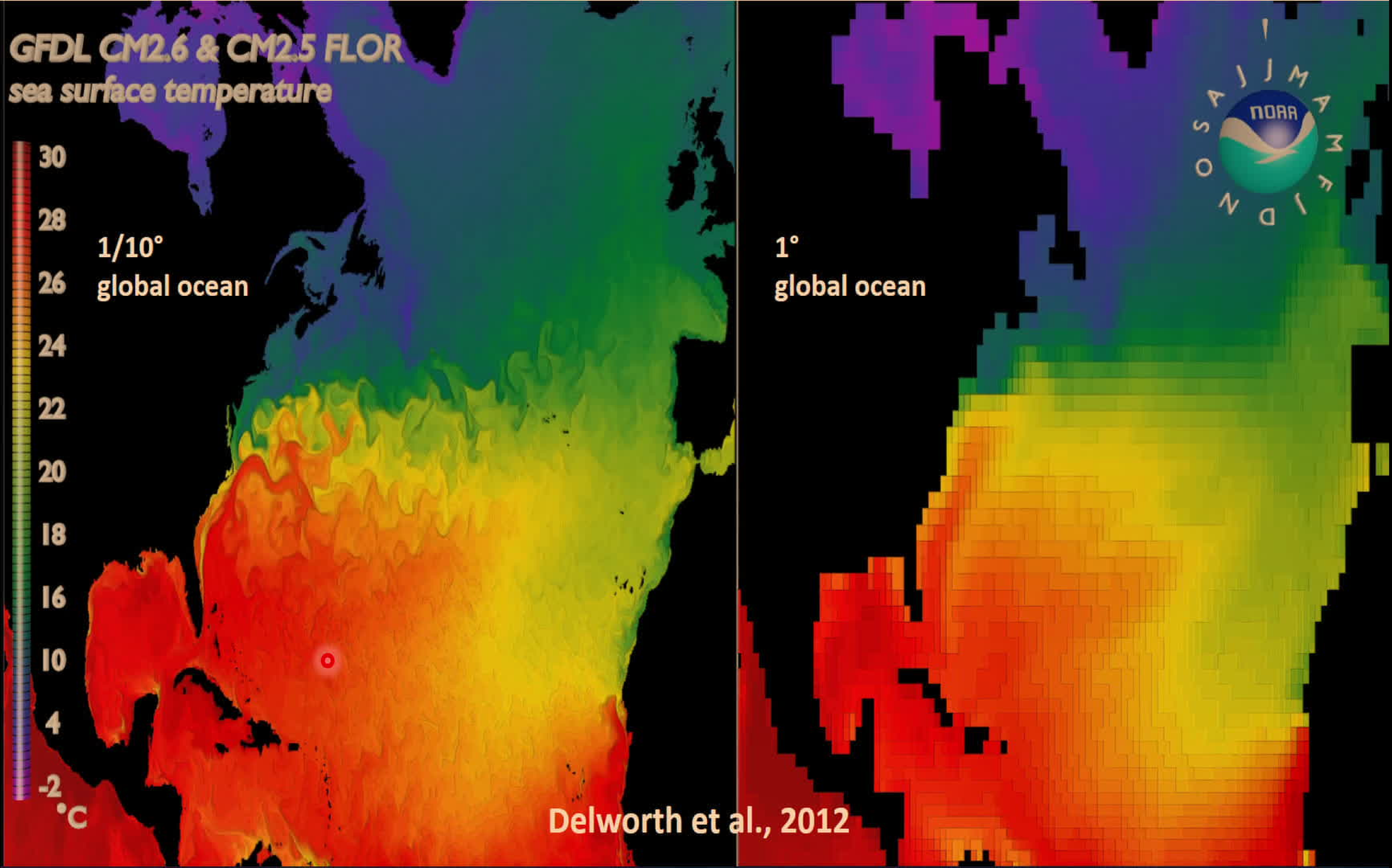
The Ocean is Turbulent

eNATL60

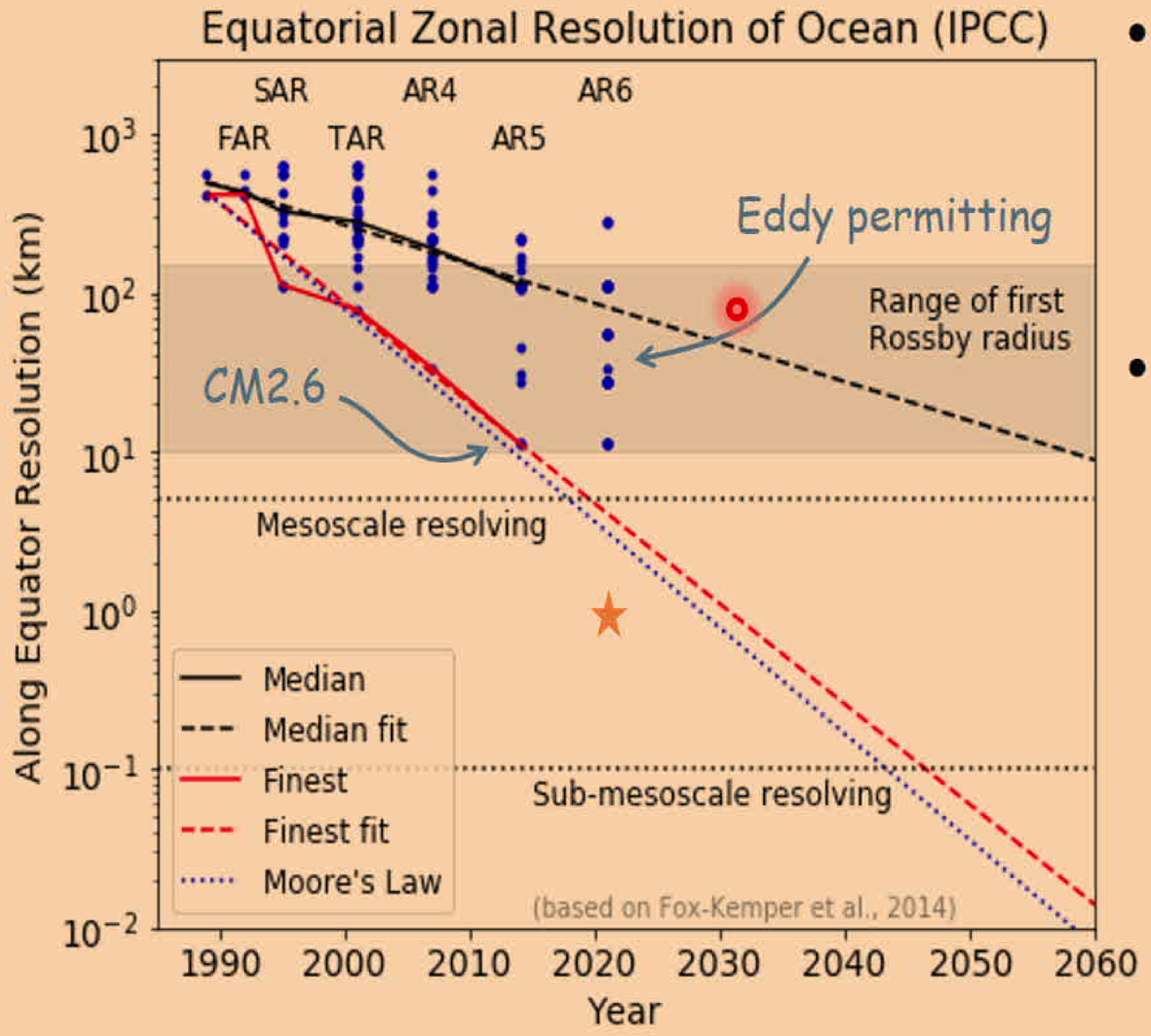
- NEMO 3.6
- 1/60° ~0.8-1.6km
- 8354x4729x300
- 18,000 cores
- 45 mins/model day



Laurent Brodeau, Julien Le Sommer,
& Jean Marc Molines, et al.
https://vimeo.com/oceannext


GFDL CM2.6 & CM2.5 FLOR
sea surface temperature
30
28
26
24
22
20
18
16
10
4
-2
°C
1/10°
global ocean
1°
global ocean
Delworth et al., 2012

Climate models are expensive



- "Cutting edge" resolution was keeping up with an 18 month doubling of compute capability
  - Not the median – much slower progress!
- Resolution of some **climate** ocean models only recently crossing the "eddy permitting" threshold
  - Atmosphere is more expensive at the same resolution
  - Models are becoming more "complex" over time
    - More components to Earth System
  - Climate modeling needs ensembles

Role of horizontal resolution in GFDL coupled models

- Delworth at el., 2012, coupled model series (CM2.1, CM2.5, CM2.6):
  - 200km, 100km, 50 km atmosphere
  - 1°, ¼° and 0.1° ocean
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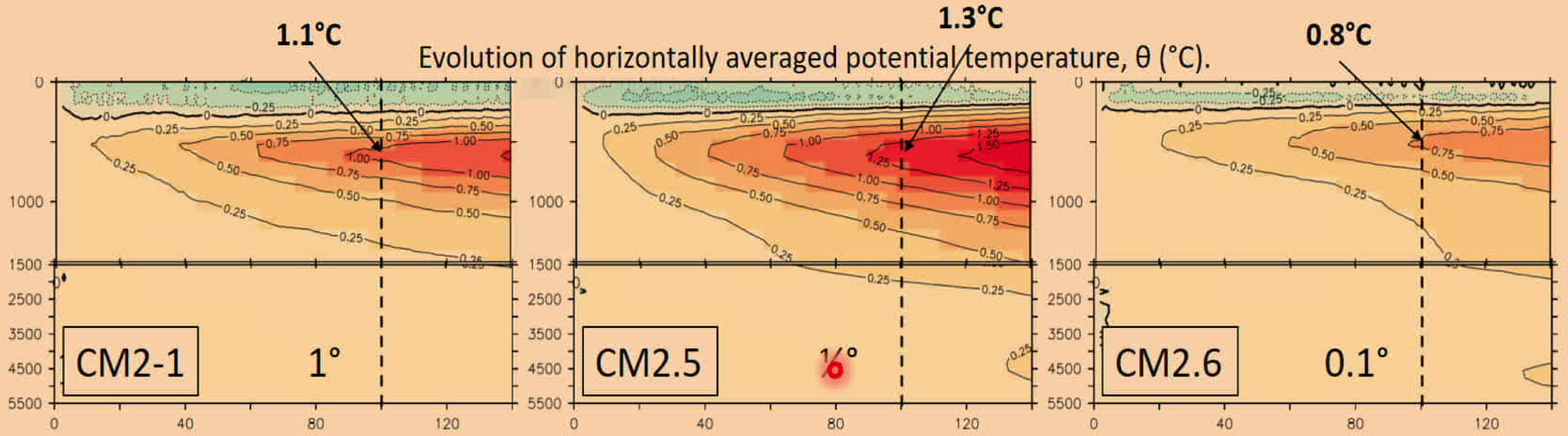
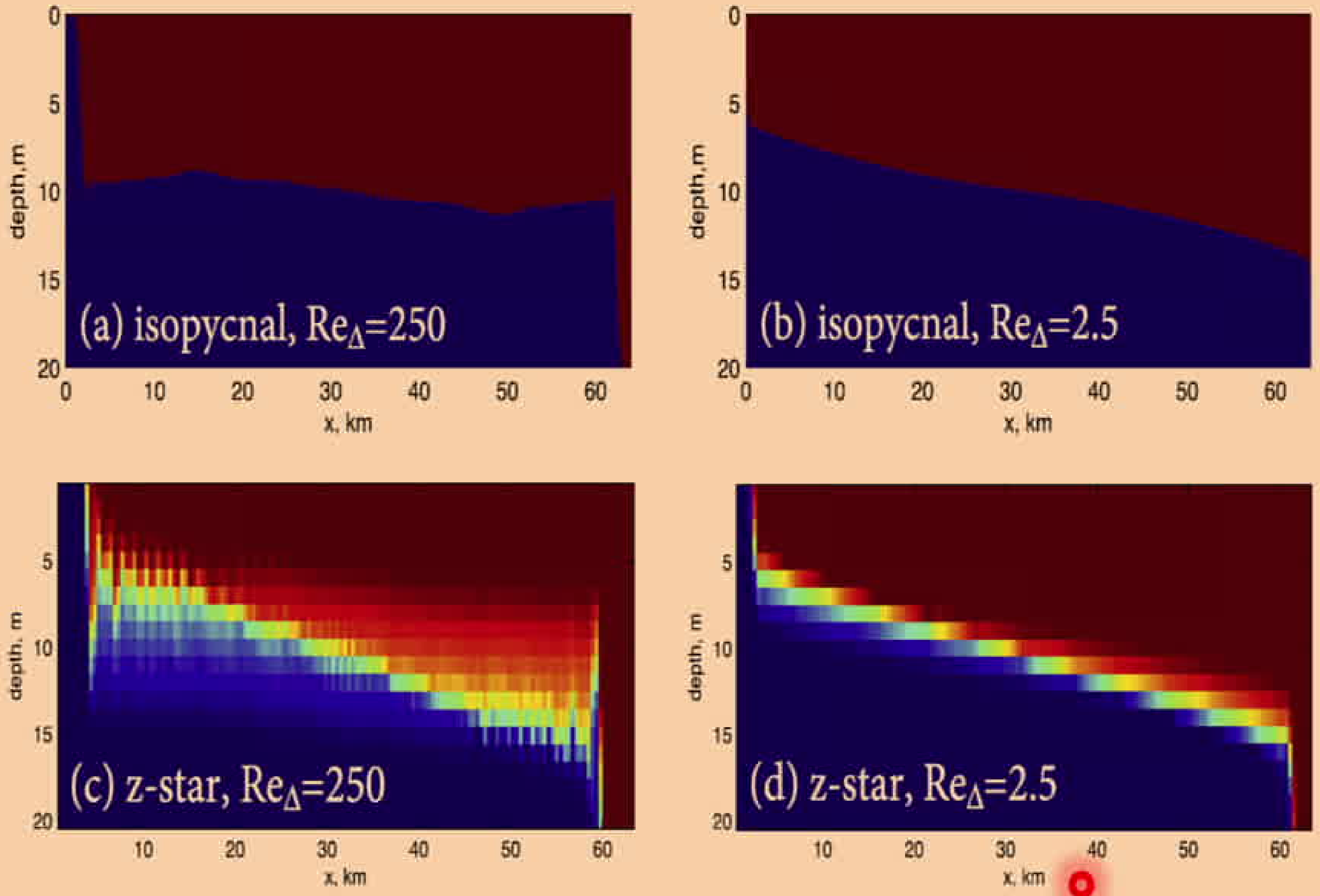


Spurious diabatic mixing

Ilicak et al., 2012
Peterson et al., 2014

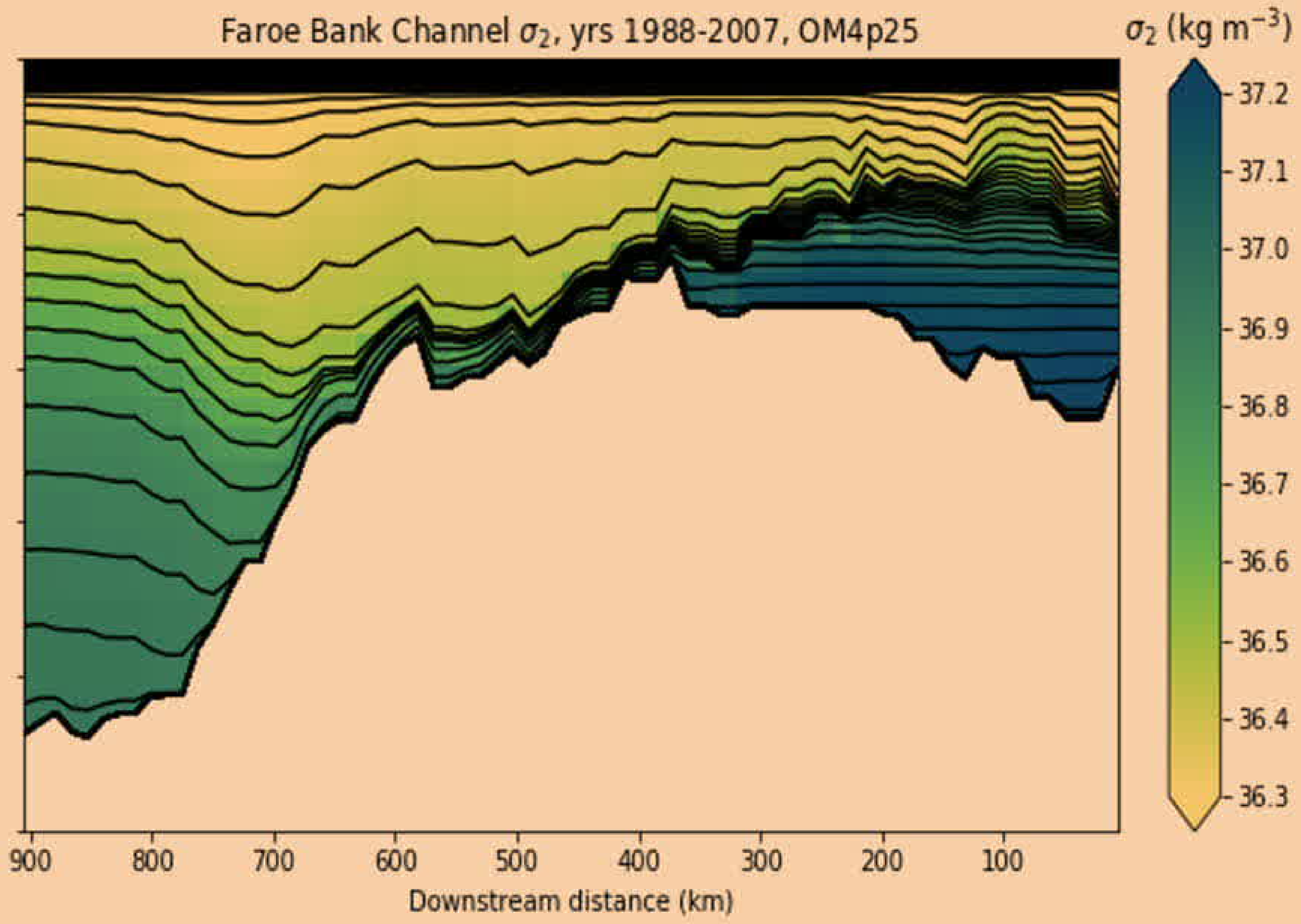


Lock exchange test
MPAS-O

- Grid Reynolds number controls noise and variance at grid scale
- We were(are) in habit of using highest possible Re_Δ that is still stable
- Limiters turn on more often at high Re_Δ
  - Limiters are useful (they avoid unphysical solutions)
- Note the Isopycnal model has no spurious mixing
  - by definition - immiscible layers

Hybrid vertical coordinate

- Isopycnal models near adiabatic in interior
  - But lose resolution in unstratified regions (near surface)
  - also have problems with the non-linear equation of state
- Bleck, 2002, introduced a hybrid coordinate
  - Z (height) in the well mixed regions
  - Isopycnal in the near-adiabatic interior
- Bleck, 2002, also introduced A.L.E. to ocean modeling (HYCOM)



(Boussinesq) Primitive Equations in z-coordinates

- Large scale ocean dynamics are very nearly incompressible

$$\rho = \rho(\theta, S, g\rho_o z)$$

$$g\rho + \partial_z p = 0$$

Rotating frame

$$\partial_t \boldsymbol{u} + (f\hat{z} + \nabla_z \times \boldsymbol{u}) \times \boldsymbol{u} + w\partial_z \boldsymbol{u} + \frac{1}{\rho_o}\nabla_z p = \nabla \cdot \boldsymbol{\sigma}$$

$$\nabla_z \cdot \boldsymbol{u} + \partial_z w = 0$$

$$\partial_t \theta + \nabla_z \cdot \theta \boldsymbol{u} + \partial_z \theta w = \nabla \cdot \boldsymbol{F}_q$$

$$\partial_t S + \nabla_z \cdot S\boldsymbol{u} + \partial_z S w = \nabla \cdot \boldsymbol{F}_s$$

$$\partial_t C_l + \nabla_z \cdot C_l \boldsymbol{v} + \partial_z C_l w = \nabla \cdot \boldsymbol{F}_{C_l} + \sum_k N_{lk}(C_l, C_k)$$

$$L_d \sim \frac{NH}{f}$$

Parameterizations of unresolved physical processes

Parameterizations of bio-geochemical interactions

- Barotropic gravity waves, internal gravity waves, Rossby waves, …

$\sqrt{gH} \sim 200$ m/s $\quad NH \sim 2$ m/s

Primitive equations in Lagrangian coordinates

- Transform to a general vertical coordinate, *r*
- Integrate vertically over layers between *r* values
- Use Lagrangian method in vertical

$$\rho_k = \rho(\theta_k, S_k, g\rho_o z)$$

$$\rho_k \delta_k \Phi + \delta_k p = 0$$

$$\partial_t \boldsymbol{u}_k + (f\hat{z} + \nabla_r \times \boldsymbol{u}_k) \times \boldsymbol{u}_k + \frac{\rho}{\rho_o} \nabla_r \Phi_k + \frac{1}{\rho_o} \nabla_r p_k = \nabla \cdot \boldsymbol{\sigma}$$

$$\partial_t h_k + \nabla_r \cdot h_k \boldsymbol{u}_k = 0$$

$$\partial_t h_k \theta + \nabla_r \cdot \theta h_k \boldsymbol{v}_k = \nabla_r \cdot h_k \boldsymbol{F}_q + \delta_k F_q^v$$

$$\partial_t h_k S + \nabla_r \cdot S h_k \boldsymbol{v}_k = \nabla_r \cdot h_k \boldsymbol{F}_s + \delta_k F_s^v$$

$$\partial_t h_k C + \nabla_r \cdot C h_k \boldsymbol{v}_k = \nabla_r \cdot h_k \boldsymbol{F}_C + \delta_k F_C^v + N$$

- Vertically remap occasionally
- No tangling (because Lagrangian method in only vertical direction)
- Works trivially for vanished layers

Two general coordinate algorithms

Eulerian

$$\partial_z p = -g\rho(z, S^n, \theta^n)$$

$$v_h^{n+1} = v_h^n + \Delta t\left(-\frac{1}{\rho_o}\nabla_z p + \cdots\right)$$

$$\delta_k w = -\nabla_z \cdot h v_h^{n+1}$$

$$h\theta^{n+1} = h\theta^n - \Delta t\begin{bmatrix}\nabla_z \cdot \left(h v_h^{n+1}\theta^n\right) + \\ \delta_k(w\theta^n) + \cdots\end{bmatrix}$$

$$\boxed{\frac{\Delta t w}{\Delta z} < 1}$$

A.L.E.

Hirt et al. , 1974

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$$v_h^{n+1} = v_h^n + \Delta t\left(-\frac{1}{\rho_o}\nabla_z p + \cdots\right)$$

$$\delta_k(w^* + w_g) = -\nabla_r \cdot h^n v_h^{n+1}$$

$$h^{n+1} = h^n + \Delta t \delta_k(w_g)$$

$$h^{n+1}\theta^{n+1} = h^n\theta^n - \Delta t\begin{bmatrix}\nabla_r \cdot \left(h^n v_h^{n+1}\theta^n\right) \\ +\delta_k(w^*\theta^n) + \cdots\end{bmatrix}$$

$$\boxed{\frac{\Delta t w^*}{\Delta z} < 1} \qquad w^* = w - w_g$$

Leclair & Madec, 2011, use this form

Lagrangian + remap

$$\partial_z p = -g\rho(z, S^n, \theta^n)$$

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$$h^{n+1} = h^n - \Delta t \nabla_r \cdot h^n v_h^{n+1}$$

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Remap

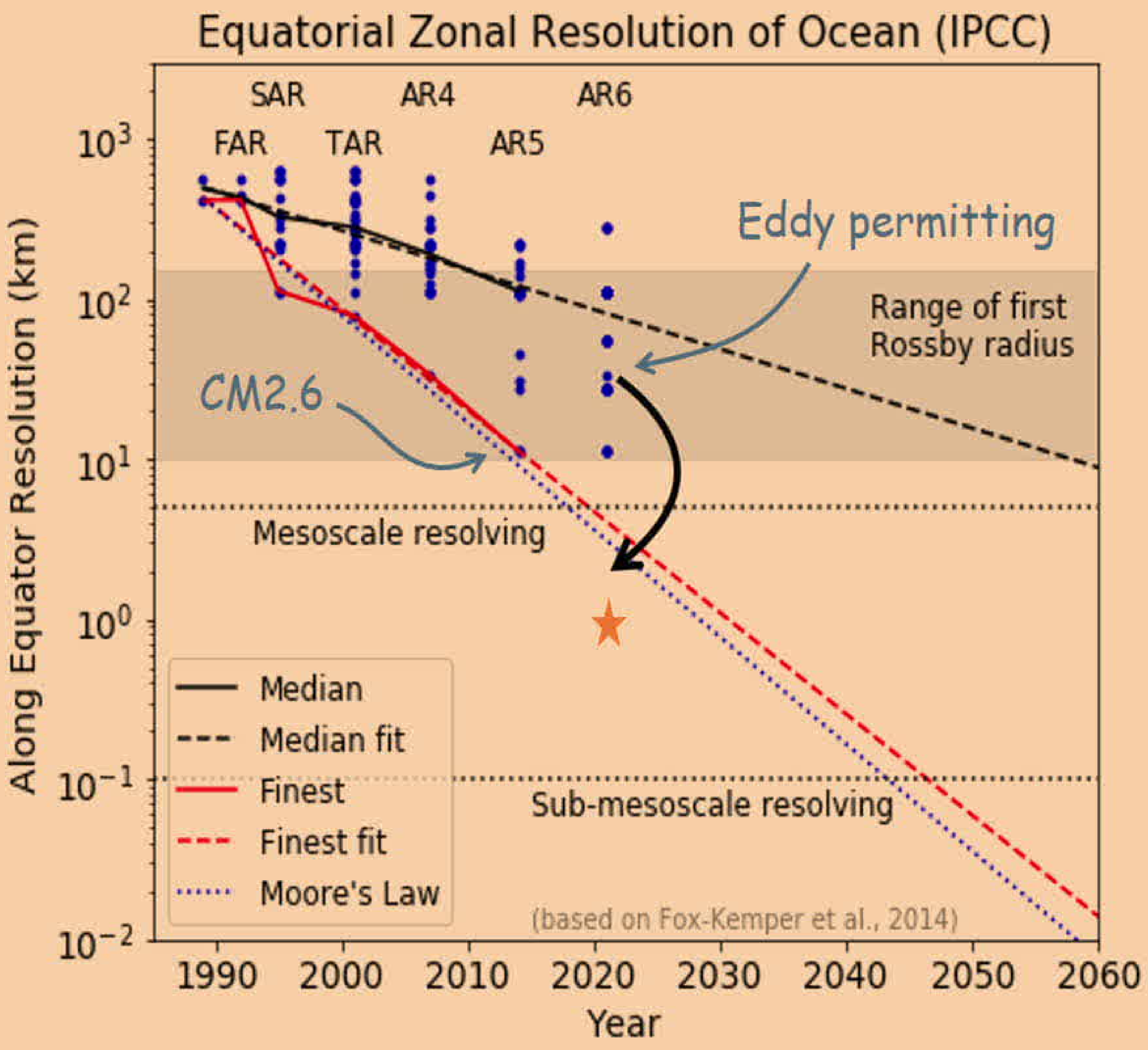
$$h^{n+1} \leftarrow \delta_k Z\left(z^\dagger\right)$$

$$\theta^{n+1} = \theta^\dagger\left(Z(z^\dagger)\right)$$

Bleck, 2002

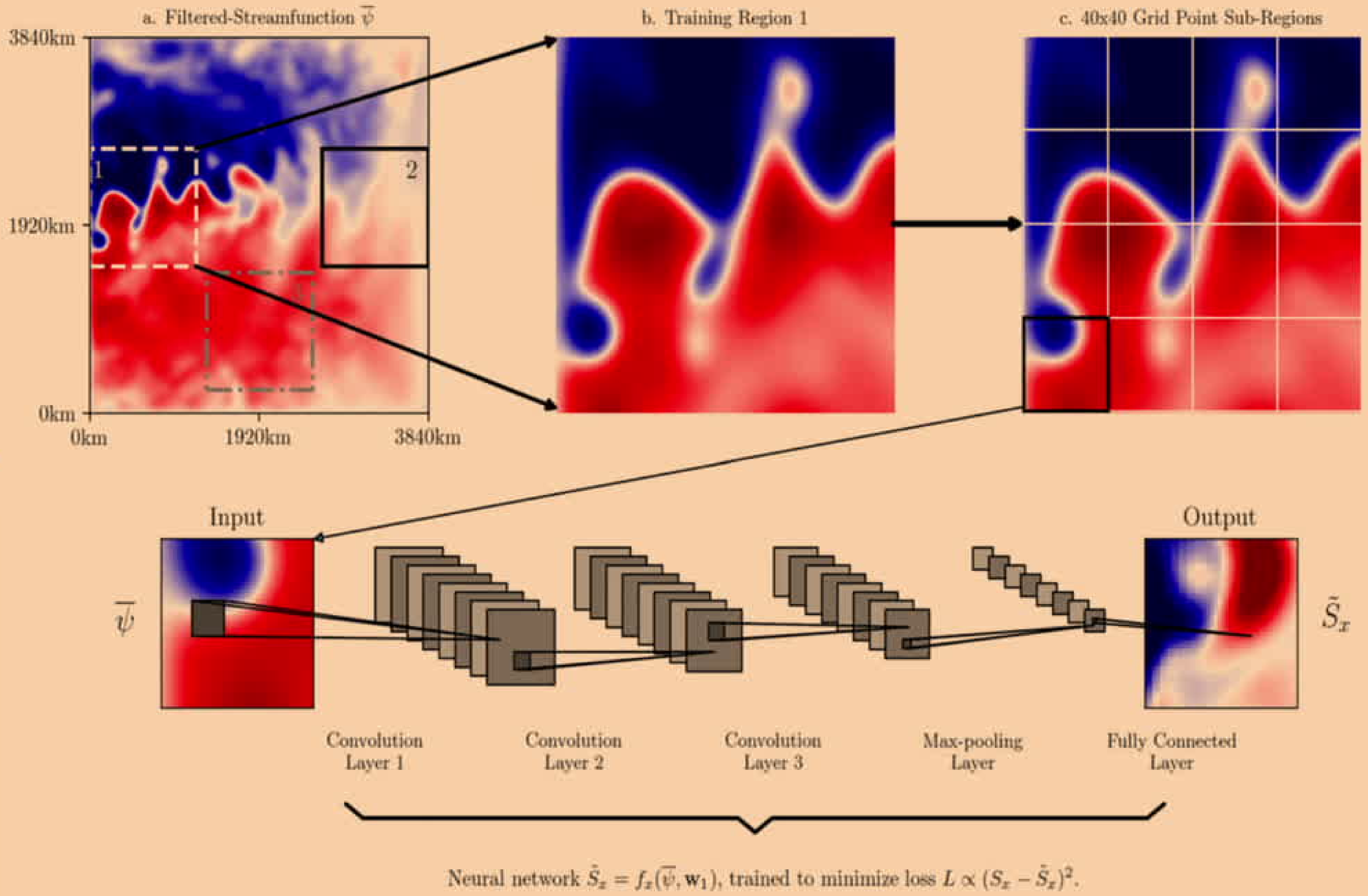
Can we jump off the curve?

- To jump from 25km to 2km horizontal resolution:
  - 140 times the horizontal cells
- Would need to run at same rate to avoid other components waiting for ocean
  - Use a lot more cores
  - Avoid reducing time step by 12?



Machine learned parameterizations

- Can we replace the expensive operations (e.g. parameterizations) with cheaper learned models?



Bolton & Zanna, 2018

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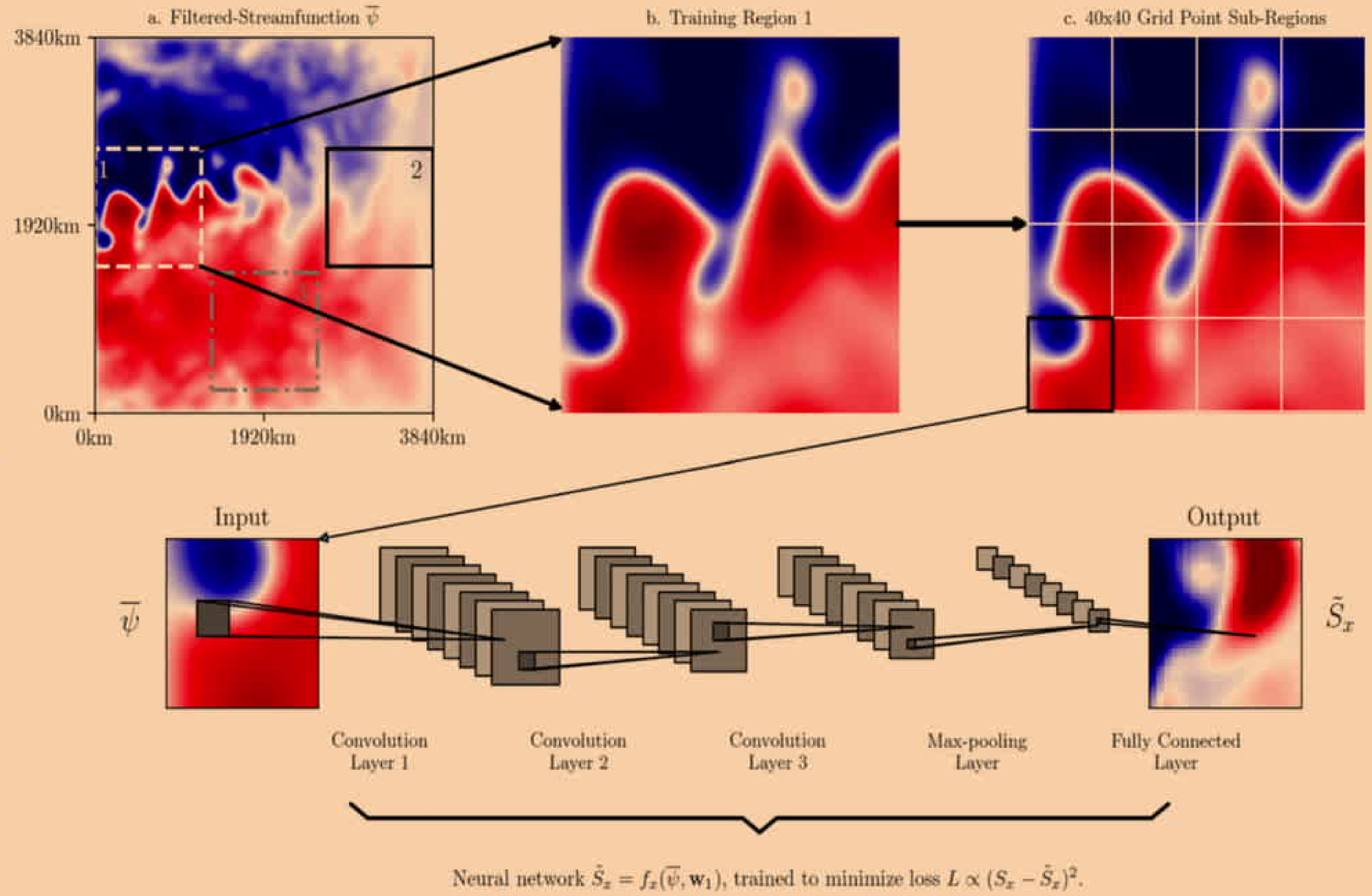

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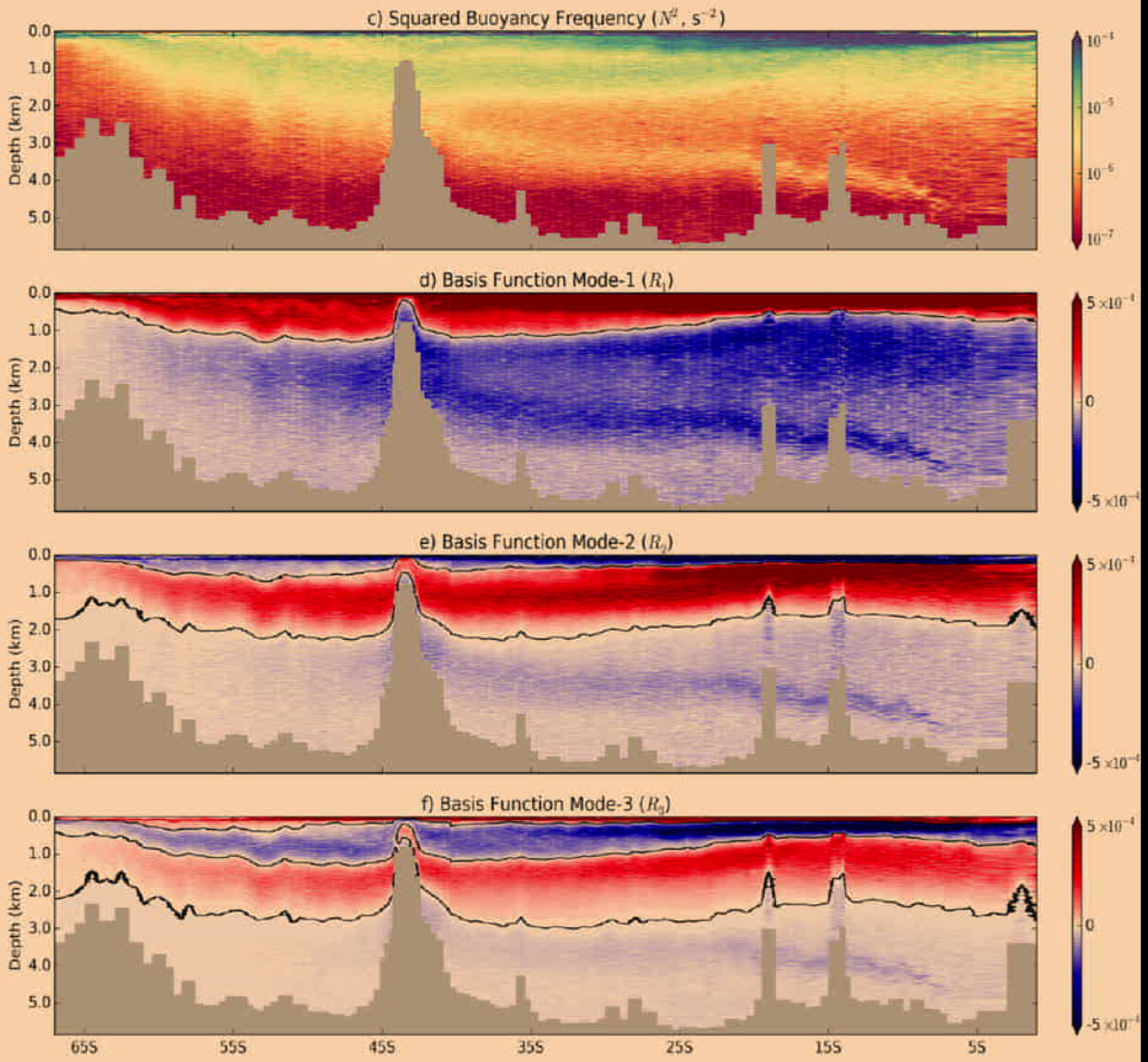
Vertical structure

- *Stewart et al.*, 2017 provide rule for vertical resolution required to globally represent baroclinic modes
  - 50 levels for first mode, 25 for each higher mode
  - assumes fixed non-distorted grid
- Can a vertical coordinate be designed to optimally represent these modes with fewer DOF?



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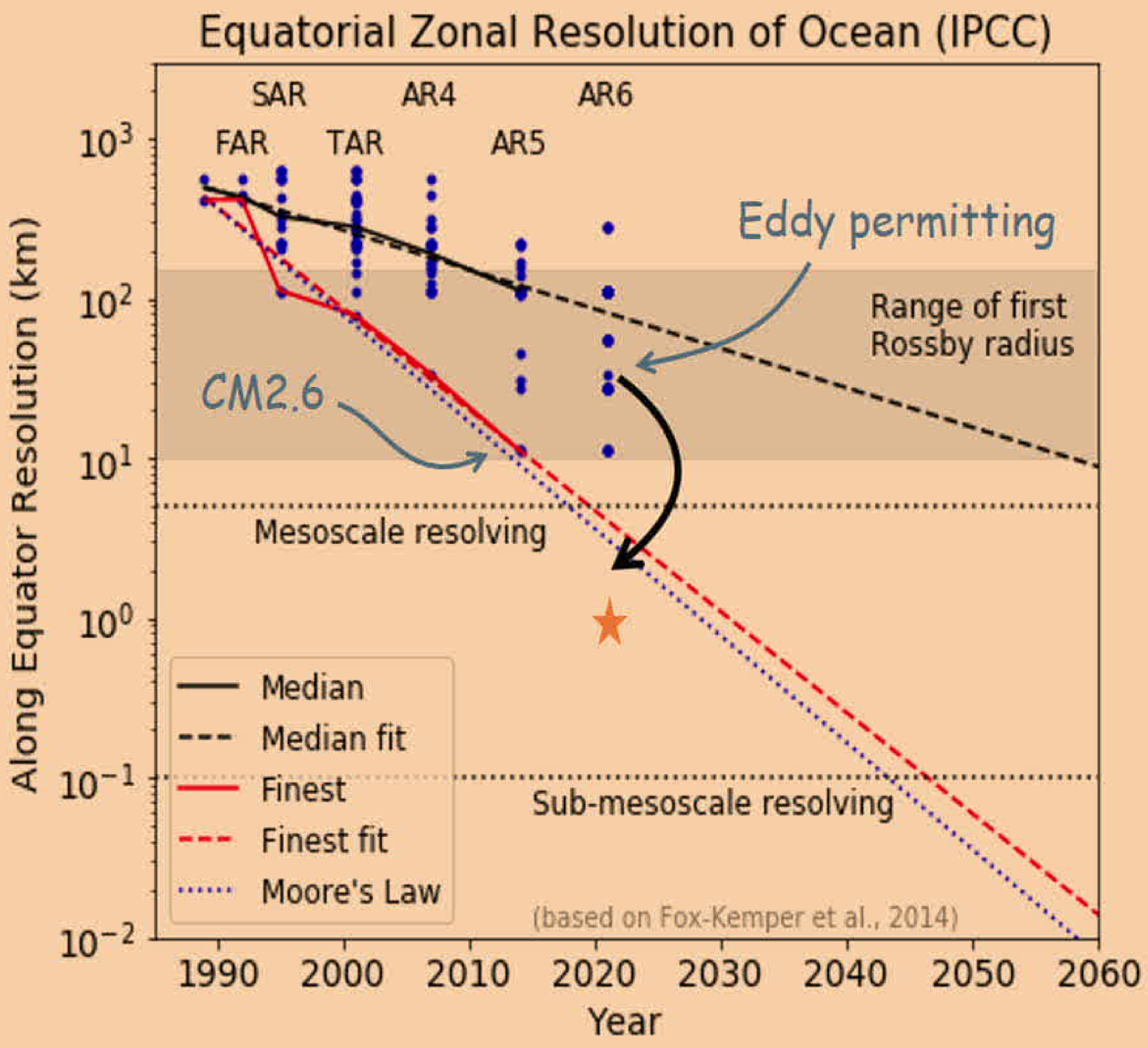

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Sub-cycling to minimize compute

$$\delta_k p = -\rho(z, S^n, \theta^n)\delta_k \Phi$$

$$v_h^{m+1} = v_h^m + \frac{1}{M}\frac{\Delta t}{\rho_o}(-\nabla_r p - \rho \nabla_r \Phi + \cdots)$$

$$h^{m+1} = h^m - \frac{1}{M}\Delta t \nabla_r \cdot (h^m v_h^{m+1})$$

Internal gravity waves $\frac{\Delta t c_{ig}}{\Delta x} < 1$

$$U^{l+1} = U^l + \frac{1}{L}\Delta t(-\nabla \eta^l + \cdots)$$

$$\eta^{l+1} = \eta^m - \frac{1}{L}\Delta t \nabla_r \cdot (HU^{l+1})$$

Barotropic gravity waves $\frac{\Delta t\sqrt{gH}}{L\Delta x} < 1$

$\times L$

$\times M$

$\times N$

$$h^* C^* = h^n C^n - M\Delta t\left[\nabla \cdot \left(\sum_{m=1}^{M} h^m v_h^{m+1} C^n\right)\right]$$

Tracers $\frac{M\Delta t u_h}{\Delta x} < 1$

$$h^{n+1} \leftarrow \delta_k Z(h^*)\,;\, C^{n+1} = C^*(Z(h^*))\,;\, \ldots$$

Vertical remap

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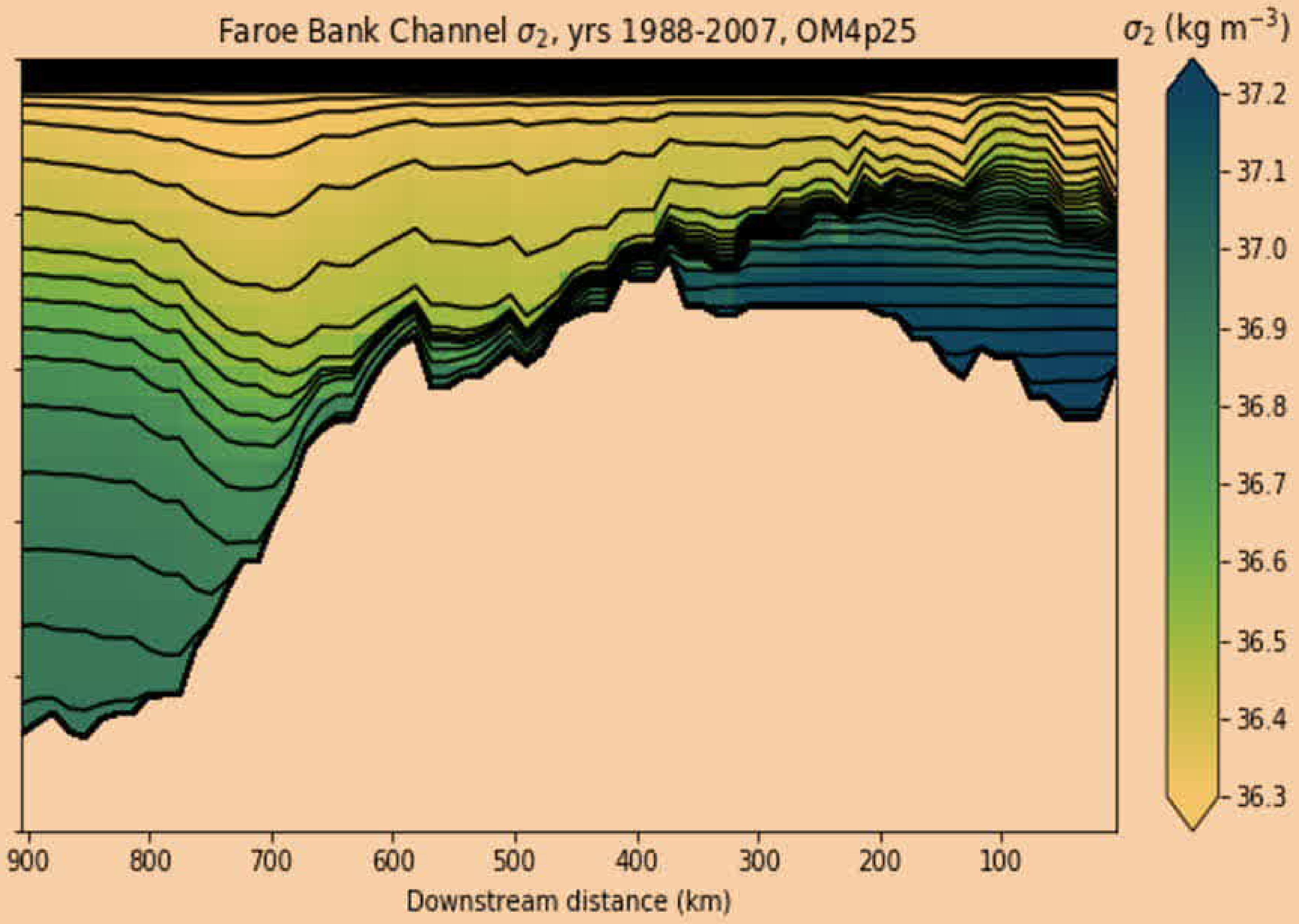

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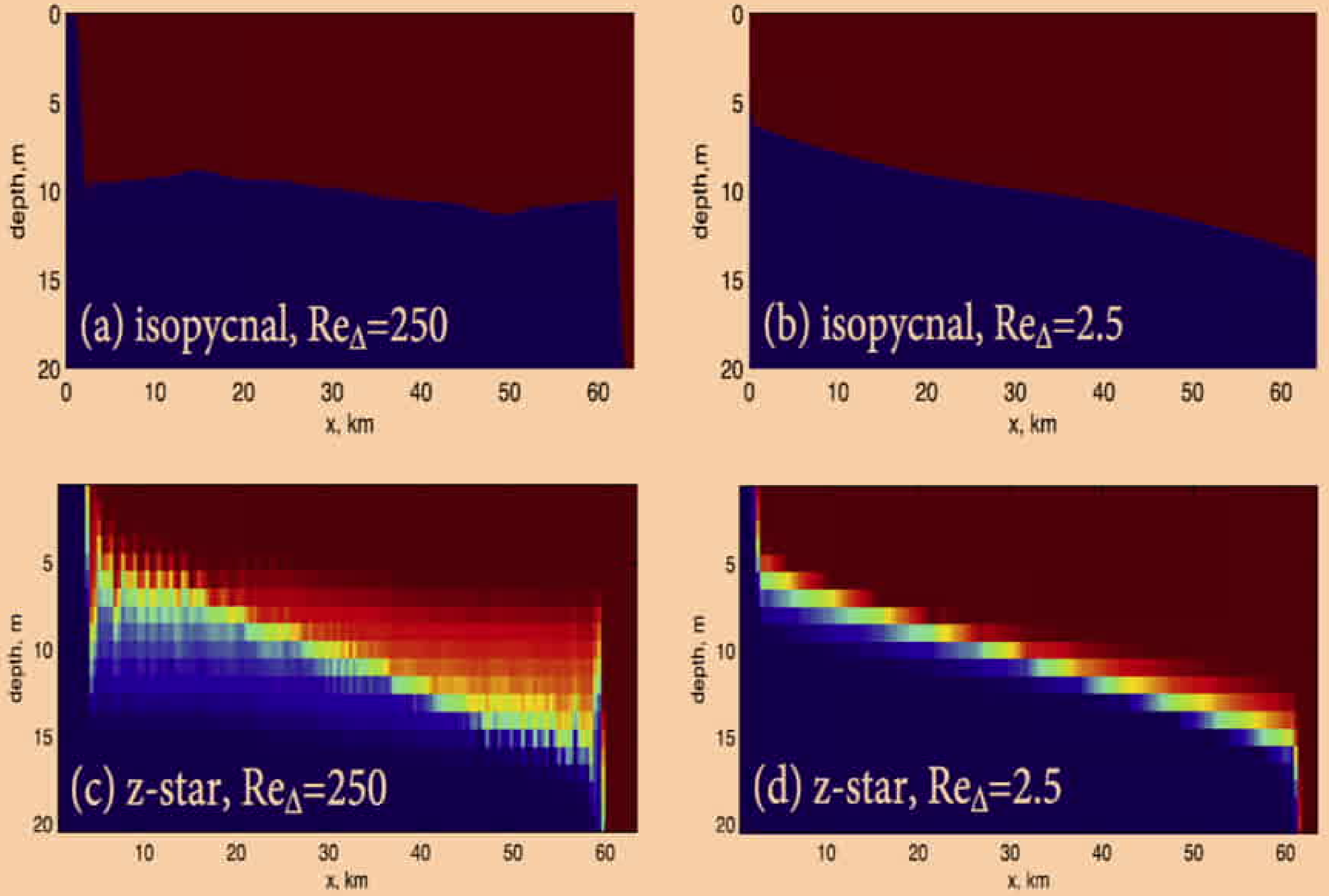

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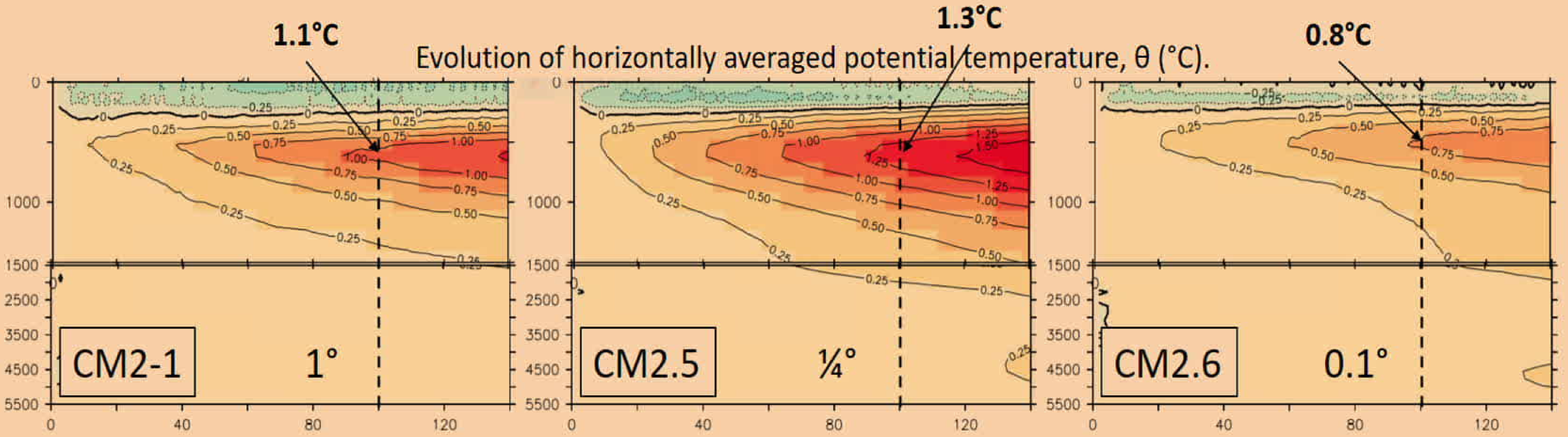
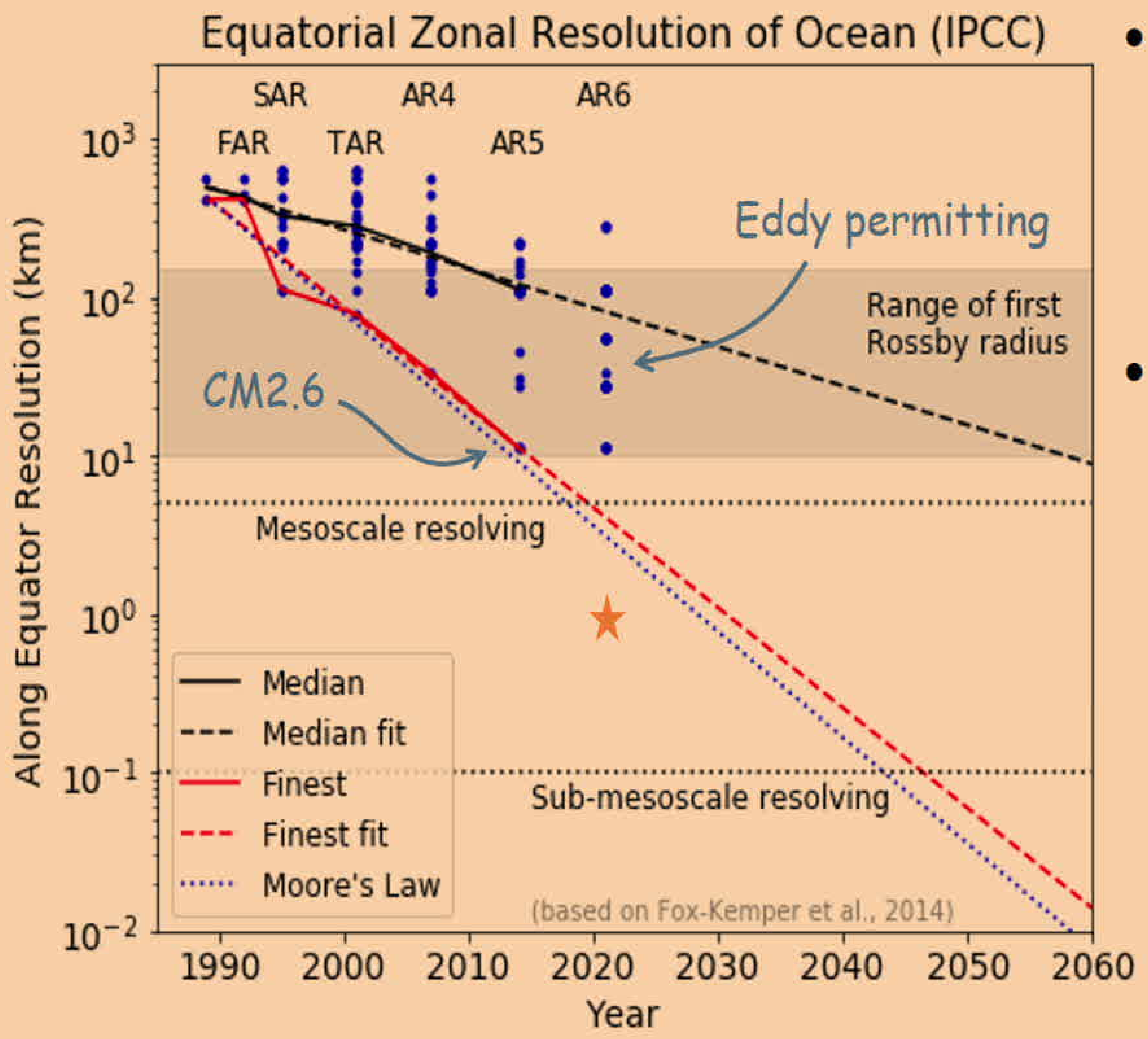


Climate models are expensive



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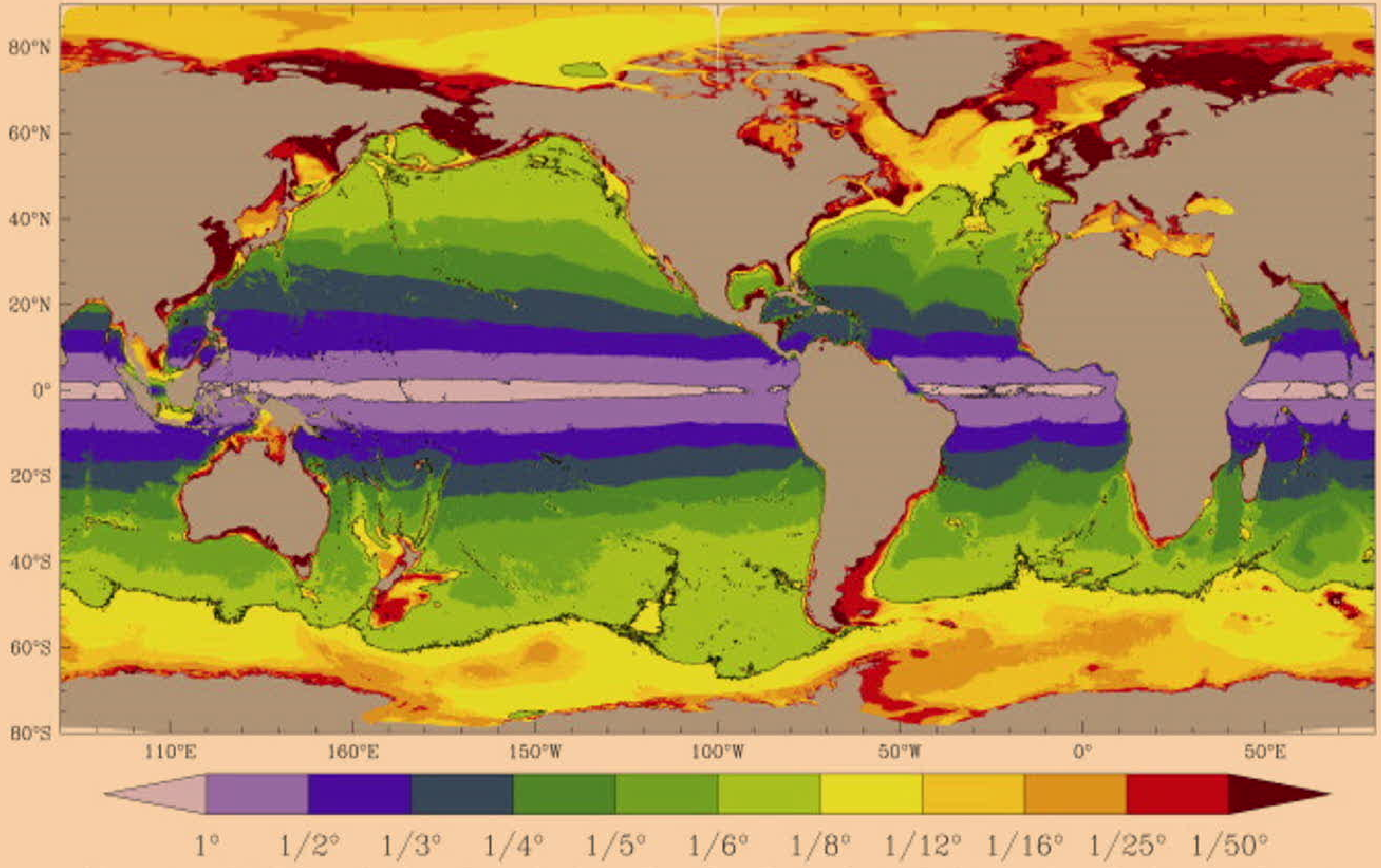
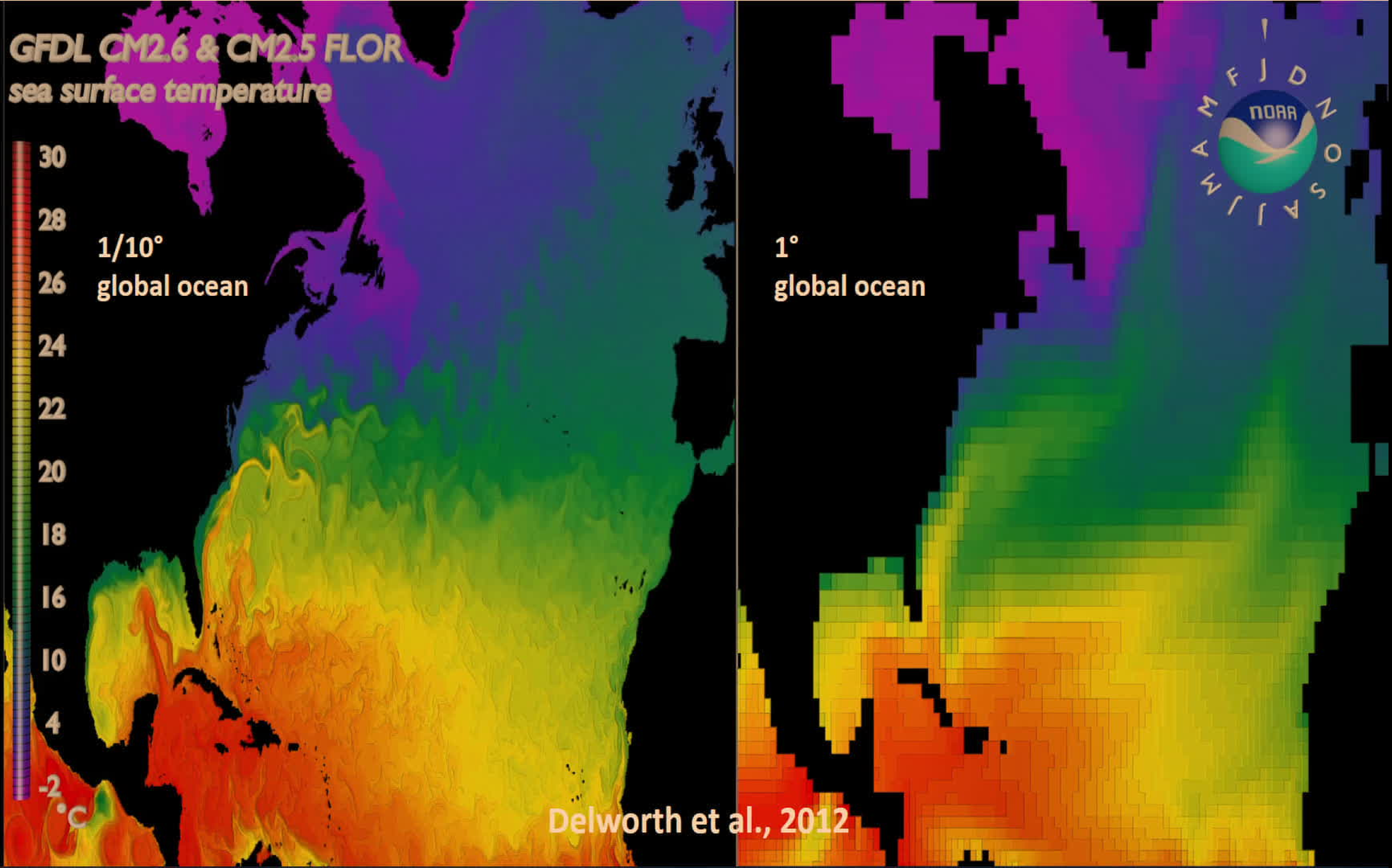
What resolution do we need?

Hallberg, 2013



Mercator Grid Resolution Required to Resolve Baroclinic Deformation Radius with 2 Δx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