On the Dynamics of Influence and Appraisal Networks

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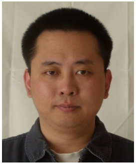
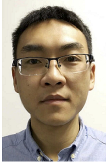
Acknowledgments

Ana MirTabatabaei
Apple



Peng Jia
Discover Financial

Wenjun Mei
UCSB



Xiaoming Duan
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Noah E. Friekin
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Kyle Lewis
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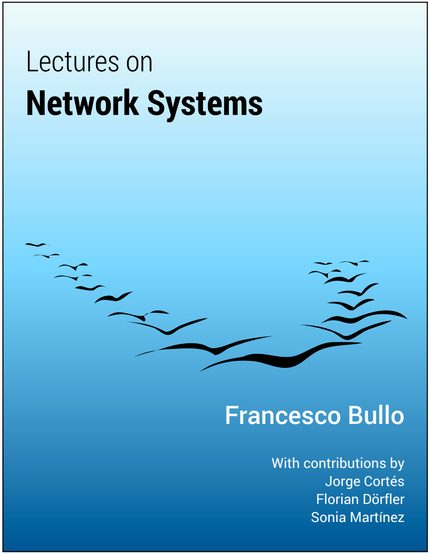
New text "Lectures on Network Systems"



Lectures on Network Systems, ver .95
For students: free PDF for download
For instructors: slides and answer keys
http://motion.me.ucsb.edu/book-lns/

Linear Systems:

1. motivating examples from social, sensor and compartmental networks
2. matrix and graph theory, with an emphasis on Perron–Frobenius theory and algebraic graph theory
3. averaging algorithms in discrete and continuous time, described by static and time-varying matrices
4. positive and compartmental systems, described by Metzler matrices

Nonlinear Systems:

5. formation control problems for robotic networks
6. coupled oscillators, with an emphasis on the Kuramoto model and models of power networks
7. virus propagation models, including lumped and network models as well as stochastic and deterministic models
8. population dynamic models in multi-species systems

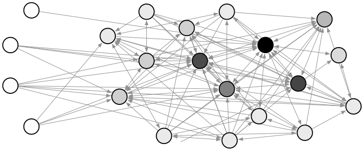
Dynamics and learning in social systems

Dynamic phenomena on dynamic social networks

1. opinion formation, information propagation, collective learning, task decomposition/allocation/execution
2. interpersonal network structures, e.g., influences & appraisals

Questions on collective intelligence, rationality & performance:

- wisdom of crowds, group think, and democracy versus autocracy
- collective learning or lack thereof
- discovery/propagation/abandonment of truth



Dynamics and learning in social systems
Mathematical sociology + systems/controls

opinion dynamics over influence networks

- seminal works: French '56, Harary '59, DeGroot '74, Friedkin '90
- recently: bounded confidence, learning, social power
- key object: row stochastic matrix

dynamics of appraisal networks and structural balance

- seminal works: Heider '46, Cartwright '56, Davis/Leinhardt '72
- recently: dynamic balance, empirical studies
- key object: signed matrix

Not considered today:

- other dynamic phenomena (epidemics)
- static network science (clustering)
- game theory and strategic behavior (network formation)

Selected literature on math sociology and systems/control

- F. Harary, R. Z. Norman, and D. Cartwright. *Structural Models: An Introduction to the Theory of Directed Graphs.* Wiley, 1965 (Research Center for Group Dynamics, Institute for Social Research, University of Michigan)
- M. O. Jackson. *Social and Economic Networks.*¡ Princeton Univ Press, 2010
- D. Easley and J. Kleinberg. *Networks, Crowds, and Markets: Reasoning About a Highly Connected World.* Cambridge University Press, 2010
- N. E. Friedkin and E. C. Johnsen. *Social Influence Network Theory: A Sociological Examination of Small Group Dynamics.* ¡ Cambridge University Press, 2011
- A. V. Proskurnikov and R. Tempo. A tutorial on modeling and analysis of dynamic social networks. Part I. *Annual Reviews in Control*, 43:65–79, 2017

exploding literature on opinion dynamics in sociology, physics, social networks

Outline

1. Influence systems: the mathematics of social power

   P. Jia, A. MirTabatabaei, N. E. Friedkin, and F. Bullo. "Opinion Dynamics and The Evolution of Social Power in Influence Networks." *SIAM Review*, 57(3):367-397, 2015

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Social power along issue sequences

Deliberative groups in social organization

- government: juries, panels, committees
- corporations: board of directors
- universities: faculty meetings

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Natural social processes along sequences

- opinion dynamics for single issue?
- levels of openness and closure along sequence?
- influence accorded to others? emergence of leaders?

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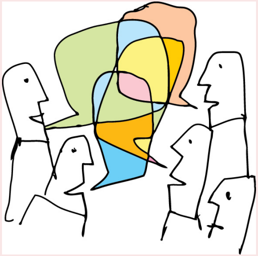
Groupthink = "deterioration of mental efficiency . . . from in-group pressures," by I. Janis, 1972

Wisdom of crowds = "group aggregation of information results in better decisions than individual's" by J. Surowiecki, 2005

Selected literature on social power & reflected appraisal

- J. R. P. French. A formal theory of social power. *Psychological Review*, 63(3):181–194, 1956
- M. H. DeGroot. Reaching a consensus. *Journal of the American Statistical Association*, 69(345):118–121, 1974

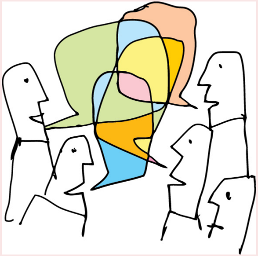
- C. H. Cooley. *Human Nature and the Social Order*. Charles Scribner Sons, New York, 1902
- V. Gecas and M. L. Schwalbe. Beyond the looking-glass self: Social structure and efficacy-based self-esteem. *Social Psychology Quarterly*, 46(2):77–88, 1983
- N. E. Friedkin. A formal theory of reflected appraisals in the evolution of power. *Administrative Science Quarterly*, 56(4):501–529, 2011



DeGroot averaging model for opinion dynamics

$$y(k+1) = Ay(k)$$

Opinion dynamics and social power along issue sequences



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Consensus under mild assumptions:

$$\lim_{k\to\infty} y(k) = (v_{\text{left}}(A)\cdot y(0))\mathbb{1}_n$$

where $v_{\text{left}}(A)$ is **social power**

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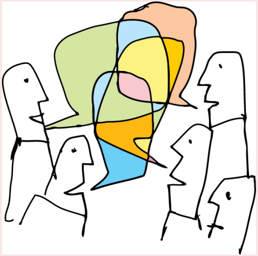

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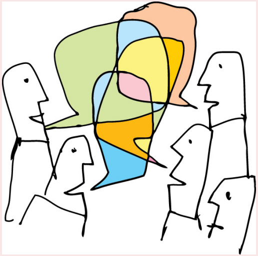

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- $A_{ii} =: x_i$ are **self-weights / self-appraisal**
- let W_{ij} be **relative interpersonal accorded weights**
  define $A_{ij} =: (1 - x_i)W_{ij}$ so that

$$A(x) = \text{diag}(x) + \text{diag}(\mathbb{1}_n - x)W$$

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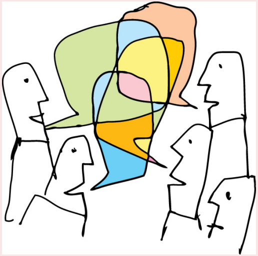

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- $v_{\text{left}}(W) = (w_1, \dots, w_n) =$ dominant eigenvector for W

Opinion dynamics and social power along issue sequences

Reflected appraisal phenomenon (Cooley 1902 and Friedkin 2011)

along issues $s = 1, 2, \ldots,$ *individual dampens/elevates self-weight according to prior influence centrality*

self-weights ⇦ relative control on prior issues = social power

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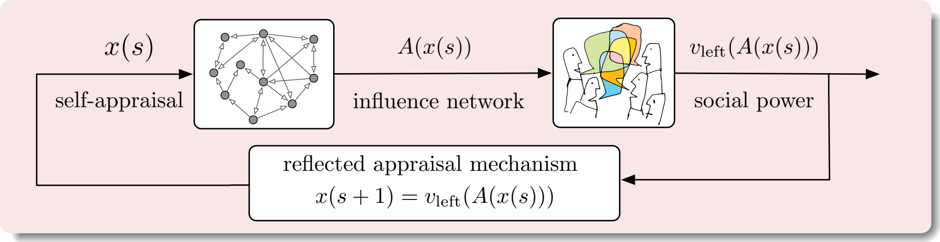
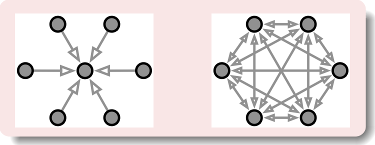


Dynamics of the influence network



Existence and stability of equilibria?
Role of network structure and parameters?
Emergence of *autocracy* and *democracy*?

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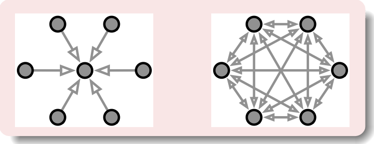


Existence and stability of equilibria?
Role of network structure and parameters?
Emergence of *autocracy* and *democracy*?

For strongly connected W and non-trivial initial conditions

1. **unique fixed point** $x^* = x^*(w_1, \dots, w_n)$
2. **convergence = forgets initial condition**

$$\lim_{s\to\infty} x(s) = \lim_{s\to\infty} v_{\text{left}}(A(x(s))) = x^*$$

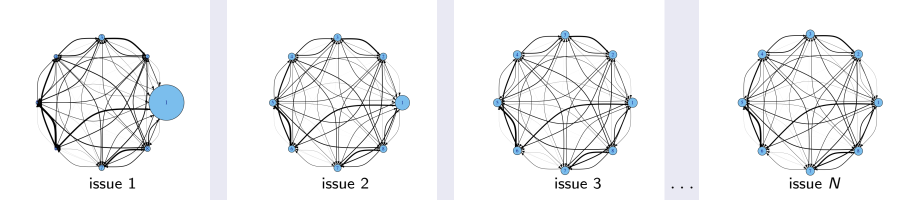
3. **accumulation of social power and self-appraisal**
   - fixed point x^* has same ordering of $(w_1, \dots, w_n)$
   - x^* is an extreme version of $(w_1, \dots, w_n)$

Emergence of democracy

If W is doubly-stochastic:

1. the non-trivial fixed point is $\frac{\mathbb{1}_n}{n}$
2. $\lim_{s \to \infty} x(s) = \lim_{s \to \infty} v_{\text{left}}(A(x(s))) = \frac{\mathbb{1}_n}{n}$

- Uniform social power
- No power accumulation = evolution to democracy

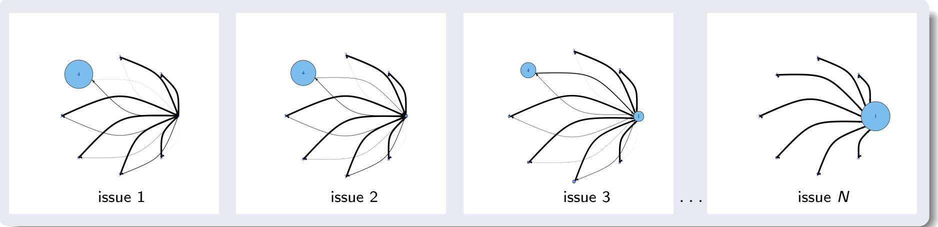


Emergence of autocracy

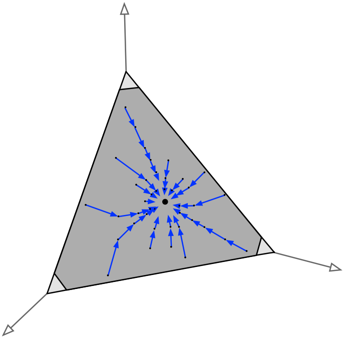
If W has star topology with center j:

1. there are no non-trivial fixed points
2. $\lim_{s \to \infty} x(s) = \lim_{s \to \infty} v_{\text{left}}(A(x(s))) = \mathbb{e}_j$

- Autocrat appears in center node of star topology
- Extreme power accumulation = evolution to autocracy



Analysis methods



1. existence of x^* via
   Brower fixed point theorem
2. **monotonicity**:
   $i_{\max}$ and $i_{\min}$ are forward-invariant

$$i_{\max} = \operatorname{argmax}_j \frac{x_j(0)}{x_j^*}$$

$$\Longrightarrow \qquad i_{\max} = \operatorname{argmax}_j \frac{x_j(s)}{x_j^*}, \quad \text{for all subsequent } s$$

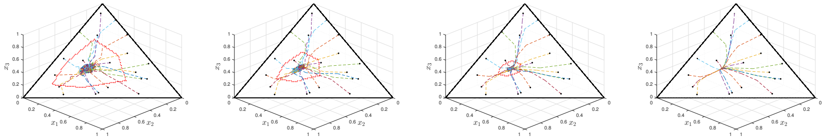
3. convergence via variation on classic **"max-min" Lyapunov function**:

$$V(x) = \max_j \left(\ln \frac{x_j}{x_j^*} \right) - \min_j \left(\ln \frac{x_j}{x_j^*} \right) \qquad \text{strictly decreasing for } x \neq x^*$$

Stochastic models with cumulative memory

Other extensions: modified models, reducible W, periodic W ...

1. assume noisy interpersonal weights $W(s) = W_0 + N(s)$
   assume noisy perception of social power
   $x(s+1) = v_{\text{left}}(A(x(s))) + n(s)$
   Thm: practical stability of x^*



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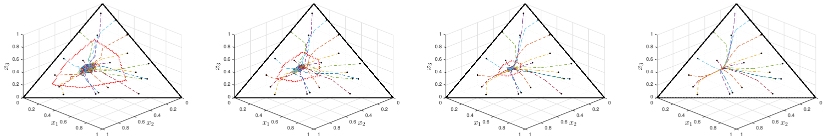

2. assume self-weight := cumulative average of prior social power

$$x(s+1) = (1-\alpha(s))x(s) + \alpha(s)\Big(v_{\text{left}}(Ax(s)) + n(s)\Big)$$

Thm: a.s. convergence to x^* (under technical conditions)

Recent extensions on social power evolution

- G. Chen, X. Duan, N. E. Friedkin, and F. Bullo. Stochastic models for social power dynamics over influence networks. *IEEE Trans. Autom. Control*, May 2017. Submitted

- Z. Xu, J. Liu, and T. Başar. On a modified DeGroot-Friedkin model of opinion dynamics. In *Proc ACC*, pages 1047–1052, Chicago, USA, July 2015

- X. Chen, J. Liu, M.-A. Belabbas, Z. Xu, and T. Başar. Distributed evaluation and convergence of self-appraisals in social networks. *IEEE Trans. Autom. Control*, 62(1):291–304, 2017

- M. Ye, J. Liu, B. D. O. Anderson, C. Yu, and T. Başar. On the analysis of the DeGroot-Friedkin model with dynamic relative interaction matrices. In *Proc IFAC World C*, Toulouse, France, July 2017

- P. Jia, N. E. Friedkin, and F. Bullo. Opinion dynamics and social power evolution over reducible influence networks. *SIAM J Ctrl Optm*, 55(2):1280–1301, 2017

- Z. Askarzadeh, R. Fu, A. Halder, Y. Chen, and T. T. Georgiou. Stability theory in ℓ_1 for nonlinear Markov chains and stochastic models for opinion dynamics. arXiv preprint arXiv:1706.03158, 2017

Summary (Social Influence)

New perspective on influence networks and social power

- dynamics and feedback in influence networks
- novel mechanism for power accumulation / emergence of autocracy

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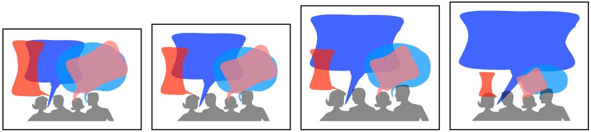
Open directions

- measurement models and empirical validation
- intervention strategies for optimal decision making:

No one speaks twice, until everyone speaks once



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Experiments on opinion formation and influence networks

domains: risk/reward choice dilemmas, analytical reliability, resource allocation

- **30 groups of 4 subjects** in a face-to-face discussion
- **sequence of 15 issues** in domain of **risk/reward choice dilemmas**:

  what is your minimum level of confidence (scored 0-100) required to accept a risky option with a high payoff rather than a less risky option with a low payoff

- **"please, reach consensus"** pressure

- On each issue, each subject recorded (privately/chronologically):
  1. **an initial opinion** prior to the-group discussion,
  2. **a final opinion** after the group-discussion (3-27 mins),
  3. **an allocation of "100 influence units"**
     ("these allocations represent your appraisal of the relative influence of each group member's opinion on yours").

Postulated mechanisms for single-issue opinion dynamics

Averaging (DeGroot model))

$$y(k+1) = Ay(k)$$
$$\lim_{k \to \infty} y(k) = (c^\top y(0))\mathbb{1}_n$$

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Averaging + attachment to initial opinion (prejudice, F-J model)

$$y(k+1) = Ay(k) + \Lambda y(0)$$
$$\lim_{k\to\infty} y(k) = V \cdot y(0), \qquad \text{for } V = (I_n - A)^{-1}\Lambda$$
$$c = V^\top \mathbb{1}_n / n$$

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level of closure:	a_{ii}	diagonal entries of influence matrix
social power:	c_i	entries of centrality vector

(1/3) Prediction of individual final opinions

balanced random-intercept multilevel longitudinal regression

	(a)	(b)	(c)
F-J prediction		0.897***	1.157***
		(0.018)	(0.032)
initial opinions			−0.282***
			(0.031)
log likelihood	-8579.835	-7329.003	-7241.097

Standard errors are in parentheses; * $p \leq 0.05$, ** $p \leq 0.01$, *** $p \leq 0.001$; maximum likelihood estimation with robust standard errors; $n = 1,800$.

FJ averaging model is predictive for risk/reward choice dilemmas

(2/3) Prediction of individual level of closure

balanced random-intercept multilevel longitudinal regression

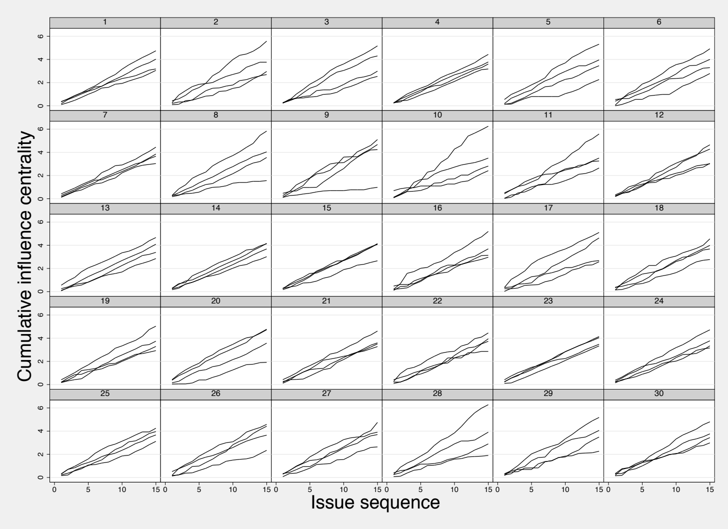
individual's "closure to influence" as predicted by:

- individual's prior centrality $c_i(s)$
- individual's time-averaged centrality $\bar{c}_i(s) = \frac{1}{s}\sum_{t=1}^{s} c_i(t)$

	(a)	(b)	(c)
$c_i(s)$		0.336^{***}	
$\bar{c}_i(s)$			0.404^{**}
s		0.002	-0.018^{***}
$s \times c_i(s)$		0.171	
$s \times \bar{c}_i(s)$			0.095^{***}
log likelihood	-367.331	-327.051	-293.656

prior and cumulative prior centrality predicts individual closure

(3/3) Prediction of cumulative influence centrality



individuals accumulate influence centralities at different rates,
and their time-average centrality stabilizes to constant values

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Appraisal systems and collective learning

Teams and tasks

- individuals with skills
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- related through networks of interpersonal appraisals and influence

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Natural social processes along sequences

- how is task decomposed, assigned and executed?
- how do individuals learn about each other?
- how does group performance evolve?

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models/conditions for learning correct appraisals and achieving optimal assignments

model/conditions for failure to learn and correctly assign

A group dynamic process: the development of a Transactive Memory System

- **TMS** studied in Applied Psychology & Organization Science
- **members' collective understanding of which members possess what skills and knowledge, based on sequence of transactions:**

A group dynamic process: the development of a Transactive Memory System

- **TMS** studied in Applied Psychology & Organization Science
- **members' collective understanding of which members possess what skills and knowledge, based on sequence of transactions:**
  1. as members observe the task performances of other members
  2. their understanding of "who knows what" tends to become more accurate and more similar
  3. leading to greater coordination and integration of members' knowledge
  4. tasks assigned to members most likely to possess the appropriate skills.
- empirical research (different team types and settings) shows positive relationship between TMS development and **team performance**

Selected literature on learning in appraisal systems

- D. M. Wegner. Transactive memory: A contemporary analysis of the group mind. In B. Mullen and G. R. Goethals, editors, *Theories of Group Behavior*, pages 185–208. Springer Verlag, 1987

- K. Lewis. Measuring transactive memory systems in the field: Scale development and validation. *Journal of Applied Psychology*, 88(4):587–604, 2003

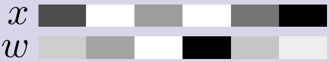
- J. R. Austin. Transactive memory in organizational groups: the effects of content, consensus, specialization, and accuracy on group performance. *Journal of Applied Psychology*, 88(5):866, 2003

- A. Nedić and A. Ozdaglar. Distributed subgradient methods for multi-agent optimization. *IEEE Trans. Autom. Control*, 54(1):48–61, 2009

- A. Jadbabaie, A. Sandroni, and A. Tahbaz-Salehi. Non-Bayesian social learning. *Games and Economic Behavior*, 76(1):210–225, 2012

Tasks, skills and assignments

- team: n individuals with skills $x > \mathbb{0}_n$, $x_1 + \cdots + x_n = 1$
- decomposable tasks, assignment percentages $w > \mathbb{0}_n$, $w_1 + \cdots + w_n = 1$



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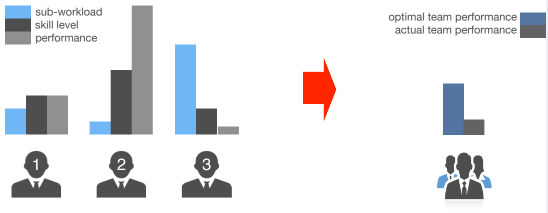
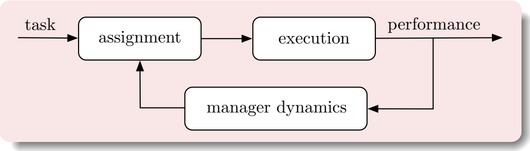
- individual performance: $p_i(w) = x_i / w_i$
- team performance: maximized at $w^* = x$



Detour: manager dynamics

Model description

- observes indiv. performance
- adjusts sub-task assignment

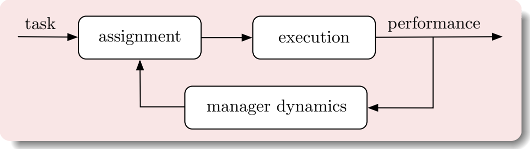


$$\frac{dw_i}{dt} = w_i\Big(p_i(w) - \sum_{k=1}^{n} w_k p_k(w)\Big)$$

Detour: manager dynamics

Model description

- observes indiv. performance
- adjusts sub-task assignment

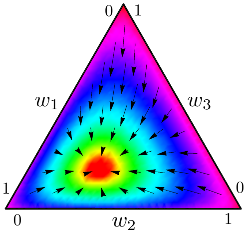


$$\frac{dw_i}{dt} = w_i \left(p_i(w) - \sum_{k=1}^{n} w_k p_k(w) \right)$$

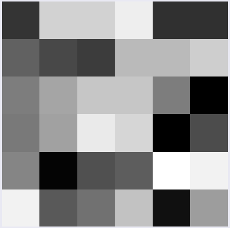
Theorem (Learning/optimality in manager)

$$\lim_{t\to\infty} w(t) = w^* = x$$

- *manager learns individuals' skills*
- *assignments asymp optimal*

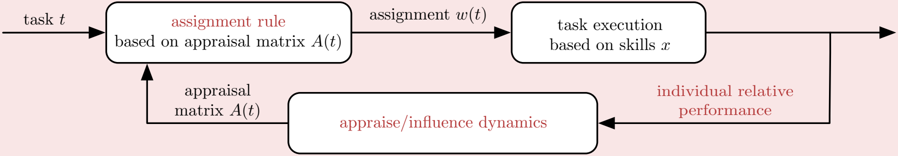


Assign/appraise/influence dynamics

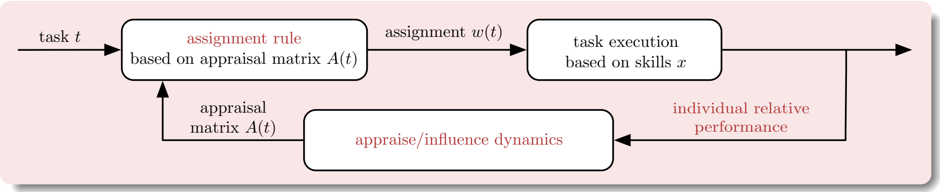


Network of interpersonal appraisals

- a_{ij} = **individual i's evaluation of x_j**
- $A = (a_{ij})_{n \times n}$ is row-stochastic
- weighted digraph



Assign/appraise/influence dynamics: Model assumptions

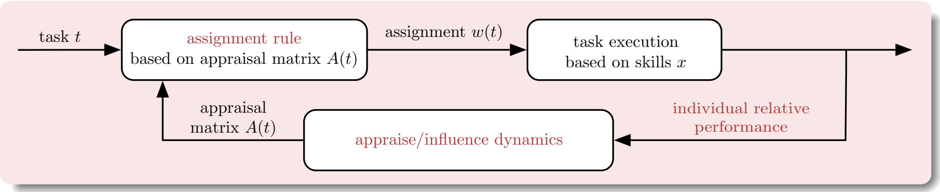


1. assignment rules:

appraisal average: $w(t) = \frac{1}{n}\mathbb{1}_n^\top A(t)$

appraisal centrality: $w(t) = v_{\text{left}}\big(A(t)\big)$ (eigenvector centrality score)

Assign/appraise/influence dynamics: Model assumptions

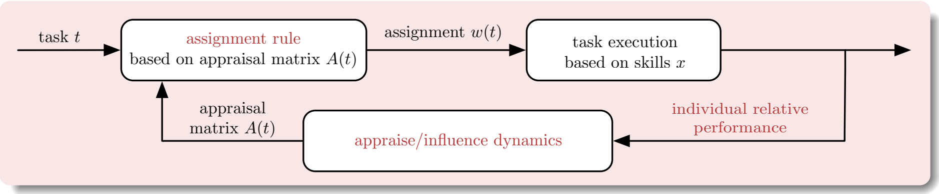


2. relative performance:
individual i observes a feedback signal

$$\phi_i = (\text{performance by } i) - (\text{average performance of observed subgroup})$$
$$= p_i - \sum_k m_{ik} p_k,$$

where $\{m_{ij}\}$ row-stochastic encodes an observation graph

Assign/appraise/influence dynamics: Model assumptions



3. appraise dynamics:
individual i updates appraisals via feedback signal:

- if $\phi_i > 0$, then $a_{ii}(t) \nearrow$ and $a_{ij}(t) \searrow$
- "simplest dynamics" to mantain $A(t)$ primitive and row-stochastic

4. influence dynamics:
individuals engage in consensus opinion formation

- continuous-time DeGroot (Laplacian flow)
- influence matrix = appraisal $A(t)$

Assign/appraise/influence dynamics: Equations

appraise dynamics: "simplest dynamics"

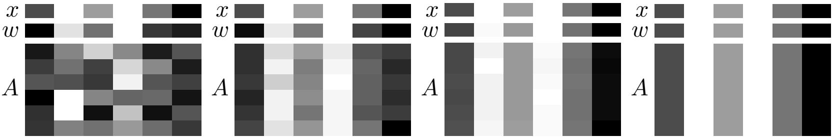
$$\begin{cases} \dot{a}_{ii} & = a_{ii}(1 - a_{ii})\phi_i \\ \dot{a}_{ij} & = a_{ii} a_{ij} \phi_i \end{cases}$$

influence dynamics: continuous-time DeGroot

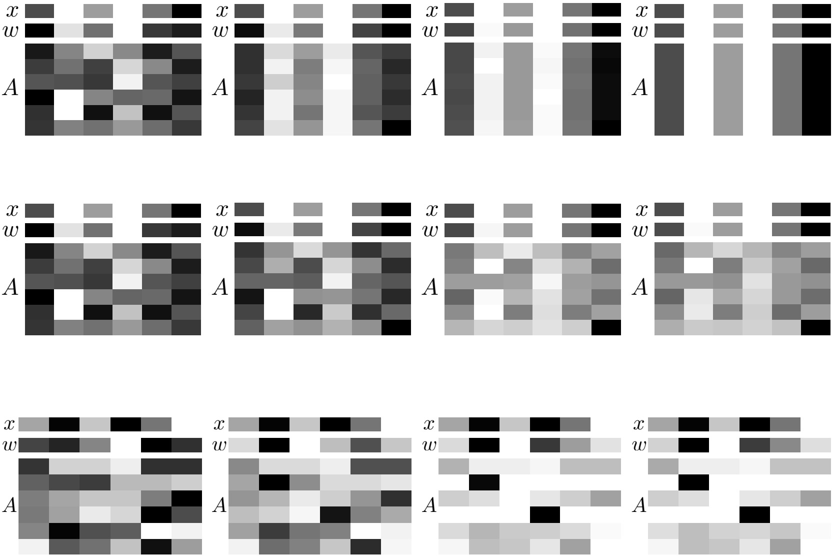
$$\dot{A}(t) = -\big(I_n - A(t)\big)A(t)$$

$$\begin{aligned} \dot{A} &= \lambda_1 F_{\text{appraise}}(A, \phi) + \lambda_2 F_{\text{influence}}(A) \\ &= \lambda_1 \operatorname{diag}(\phi(t)) \operatorname{diag}(A(t))(I_n - A(t)) - \lambda_2 \big(I_n - A(t)\big) A(t) \\ &= \dots \end{aligned}$$

What could happen?



What could happen?



Asymptotic learning and/or optimality in nominal settings

standing assumptions:

- $A(0)$ irreducible with positive diagonal
- appraisal centrality

Theorem (assign/appraise/influence dynamics)

If observation graph has globally reachable node, then

1. **collective learning**: $\lim_{t\to\infty} A(t) = \mathbb{1}_n x^\top$
2. **optimal assignment**: $\lim_{t\to\infty} w(t) = v_{\text{left}}(A^*) = w^*$

Theorem (assign/appraise (no influence))

If observation graph is strongly connected, then

1. *incorrect learning*: $\lim_{t\to\infty} A(t) = A^*$
2. **optimal assignment**: $\lim_{t\to\infty} w(t) = v_{\text{left}}(A^*) = w^*$

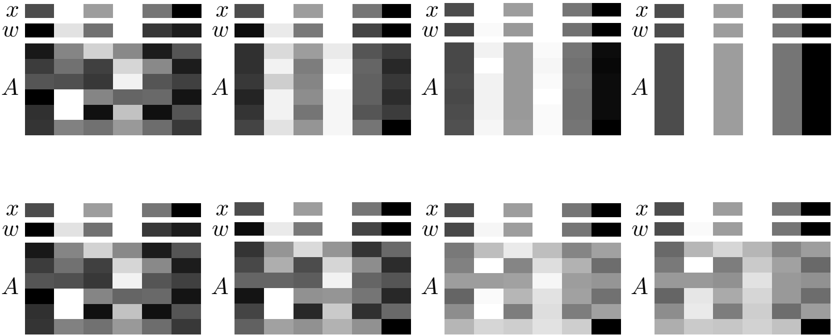
Remarkably, assignment dynamics is again replicator

$$\dot{w}_i = w_i\Big(a_i\phi_i(w) - \sum_{k=1}^{n} w_k a_k \phi_k(w)\Big)$$

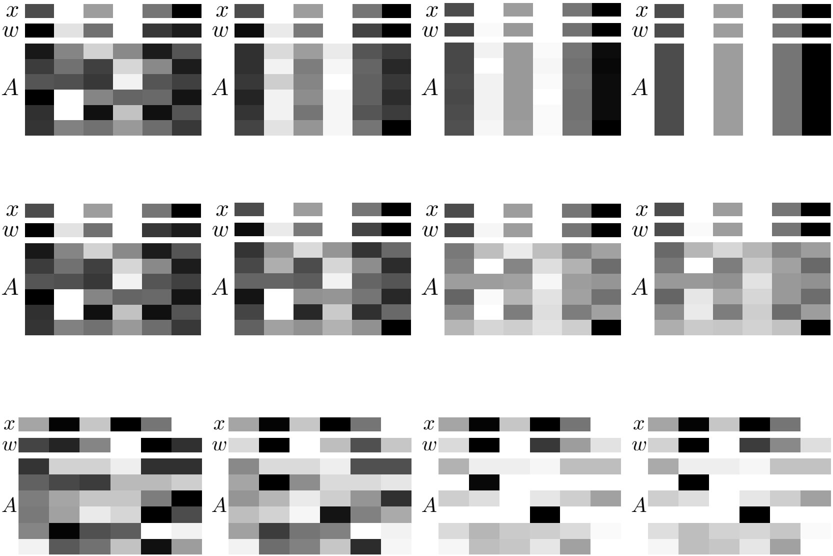
recall manager dynamics:

$$\dot{w}_i = w_i\Big(\phi_i(w) - \sum_{k=1}^{n} w_k \phi_k(w)\Big)$$

Assign/appraise/influence versus assign/appraise



Assign/appraise/influence versus assign/appraise



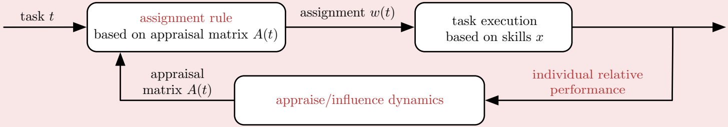
Causes of failure to learn/optimize

Incorrect learning and suboptimal assignment if:

1. assignment rule: appraisal average (and no influence dynamics)
2. appraise dynamics: weaker assumptions on observation graph
3. influence dynamics: prejudice model (F-J + model)

Lessons learned: Minimum conditions for collective learning

1. individual performance proportional to skill/workload, & appraisals are updated upon observation of relative performance
2. objectives: asympt optimal assignment and/or collective learning
3. 3 key activities: assign/appraise/influence



Lessons learned:
Minimum conditions for collective learning

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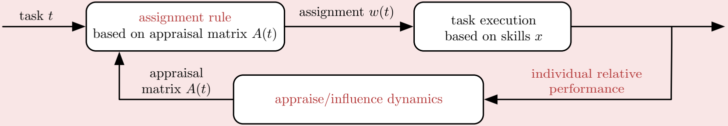


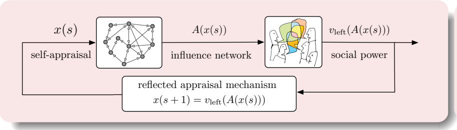
Lessons learned

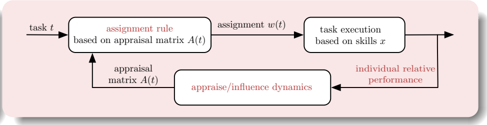
1. observation graph: better connectivity properties $\implies$ better learning
2. assign: appraisal centrality $>$ appraisal average
3. influence / consensus formation helps
   unless prejudice (no learning nor optimality)

Summary

Contributions

- dynamics and feedback in sociology and organization science
- domains: risk/reward choice dilemmas, decomposable tasks
- a new perspective on social power, self-appraisal, influence networks
- a new explanation of team learning and rationality





Next steps

1. extend the math to explain more behaviors
2. validate models with controlled experiments / massive online data