A High-Order Low-Order Algorithm with Exponentially-Convergent Monte Carlo for Thermal Radiative Transfer

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Outline

1. Introduction
2. Basic Equations
3. Implicit Monte Carlo
4. Holo Method
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Overview

- Modeling thermal radiation transport in the High-Energy Density Physics regime.
- Temperatures on order of $1M^{\circ}$ Kelvin or more.
- Radiation emitted proportional to T^4 and can be scattered and absorbed.
- Significant energy and momentum may be exchanged with material.
- Radiative transfer simulations important in modeling:
    - Material under extreme conditions
    - Inertial confinement fusion
    - Supernovae
    - Other types of astrophysical phenomena.

The thermal radiative transfer equations

- The 1D grey equations:

$$\frac{1}{c}\frac{\partial I}{\partial t} + \mu\frac{\partial I}{\partial x} + \sigma_t I(x,\mu) = \frac{1}{4\pi}\sigma_s\phi + \frac{1}{4\pi}\sigma_a acT^4\,, \tag{1}$$

$$C_v\frac{\partial T}{\partial t} = \sigma_a\phi - \sigma_a acT^4\,, \tag{2}$$

$$\phi(x) = 2\pi\int_{-1}^{+1} I(x,\mu)\mathrm{d}\mu\,. \tag{3}$$

- Fundamental unknowns radiation intensity $I(x,\mu)$ and material temperature T.
- Absorption cross section (σ_a) strong function of T.
- Equations nonlinear and may be tightly coupled.

The Implicit Monte Carlo method

- Equations often solved with Monte Carlo (MC) via the Implicit Monte Carlo (IMC) method.
- Temperature unknown discretized in time and space.
- Material energy equation linearized over time step and eliminated from transport equation.
  - Results in discrete emission and effective scattering terms.
  - Linear transport equation solved with standard MC algorithm; then temperature updated.
- Drawbacks
  - Effective scattering cross section can be very large.
  - Nonlinearities not converged.
  - Reconstruction of linear source shape in cell required.

An alternative High-Order Low-Order approach

- HOLO concept independent of Monte Carlo:
  - Discretize both equations in time using backward Euler.
  - Define fixed spatial grid; assume T and ϕ have discrete LD spatial dependence within each cell.
  - Take two angular moments and two spatial moments of transport equation and two spatial moments of temperature equation.
  - Moment equations closed with infomation from transport solution.
  - Moment solutions provide $\tilde{\phi}$ and $\tilde{T}$ for scattering and emission sources in transport equation.
  - Left side of transport equation exactly inverted with ECMC to get closures.

An alternative High-Order Low-Order approach

- HOLO iteration
  - Assume diffusive moment closures.
  - Nonlinearly solve LO moment system for LD ϕ and T.
  - Compute scattering and emission sources for transport equation
  - Invert left side of transport equation via ECMC to obtain I.
  - Compute new closures.
  - Return to Step 2 until convergence.

An alternative High-Order Low-Order approach

- Advantages:
  - ECMC reduces statistical errors to negligible levels, so nonlinear convergence is achieved.
  - ECMC solves pure absorber problem - nothing but ray tracing - compatible with exascale.
  - LD temperature representation makes linear reconstuction unnecessary.
  - Rapid convergence for diffusive problems.
  - No ray effects.

ECMC

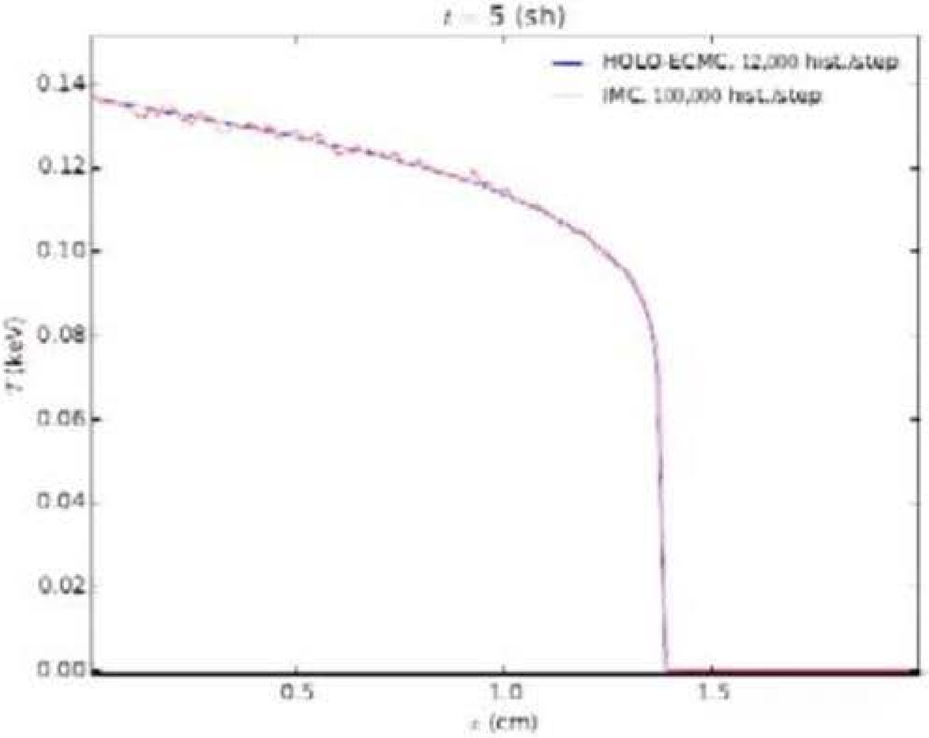
- Express transport equation as $LI = Q$.
- First batch is standard MC.
- Project MC solution (exact) onto space-angle LD FEM space to obtain $\tilde{I}^{(0)}$.
- Compute residual for projection: $R^{(0)} = Q - L\tilde{I}^{(0)}$.
- Perform standard MC batch for error: $L\epsilon^{(0)} = R^{(0)}$.
- Project error onto space-angle LD FEM space to obtain $\tilde{\epsilon}^{(0)}$
- Add error to solution estimate to obtain new estimate: $\tilde{I}^{(1)} = \tilde{I}^{(0)} + \tilde{\epsilon}^{(0)}$.
- Repeat process until convergence.

ECMC

- Works as long as error estimated uniformly and well represented by FEM space.
- Error reduction geometric with number of batches.
- Eventually convergence stagnates.
- FEM refinement required to maintain convergence.
- ECMC does not make a difficult problem easy - it efficiently reduces statistical error to negligible levels if standard MC is uniformly effective.

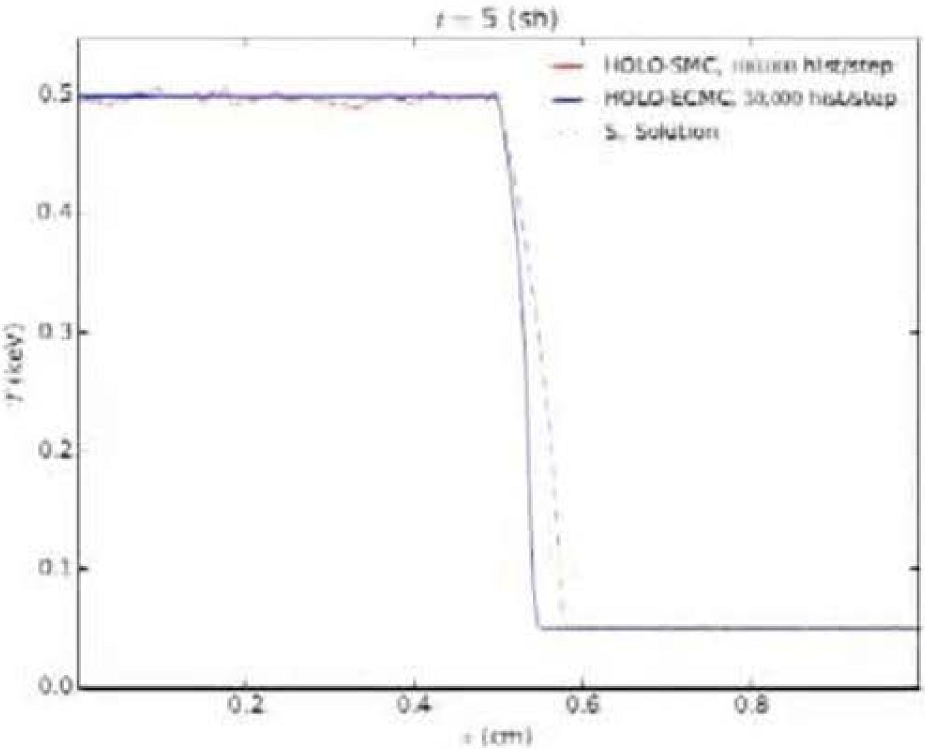
Marshak Wave Test Problem

- From equilibrium, a radiation source is applied at the left boundary, $\sigma \propto T^{-3}$.
- Plot of transient solution for $T_r = \sqrt[4]{\phi/ac}$ after 5 shakes, 200 x cells



Comparison of statistical noise for standard and ECMC HO solvers

- Two material problem
- One HO solve, with a *fixed number of histories* per time step, for two different HO solvers: a comparison of ECMC with 3 batches and standard MC (SMC), as well as S_2 solution



Current & Future Development

- Currently developing strategies for dealing with unresolvable solutions.
- Can accurately reproduce IMC results with HOLO method
  - Pure absorber histories are more efficient than standard MC simulations
  - Nonlinearities iteratively converged.
  - Linear shape within a cell mitigates teleportation error
  - Very efficient for diffusive problems
- Future work: Implement in 2 spatial dimensions to demonstrate elimination of ray effects in both time and space.