Efficient integration of fractional beam equation with space-time noise

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joint work with Zhaopeng Hao

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Motivation of fractional modeling

$$ {}_0\partial_t^{\alpha} u(x) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{\frac{d^n}{d\xi^n} u(\xi)}{(t-\xi)^{\alpha-n+1}} d\xi, \quad t > 0, \qquad (1) $$

with $n - 1 < \alpha < n$ where n is a natural number.
damping

- $\alpha = 0$ the term represents a restoring force
- $\alpha = 1$ it represents a classical viscous damper

• Effect of fractional order in a similar but linear model (Lorenzo et al 2014) with $\alpha \in [0, 1]$ on power spectral density in frequency domain.

◇ Providing different damping effects in models is big in application of fractional calculus.

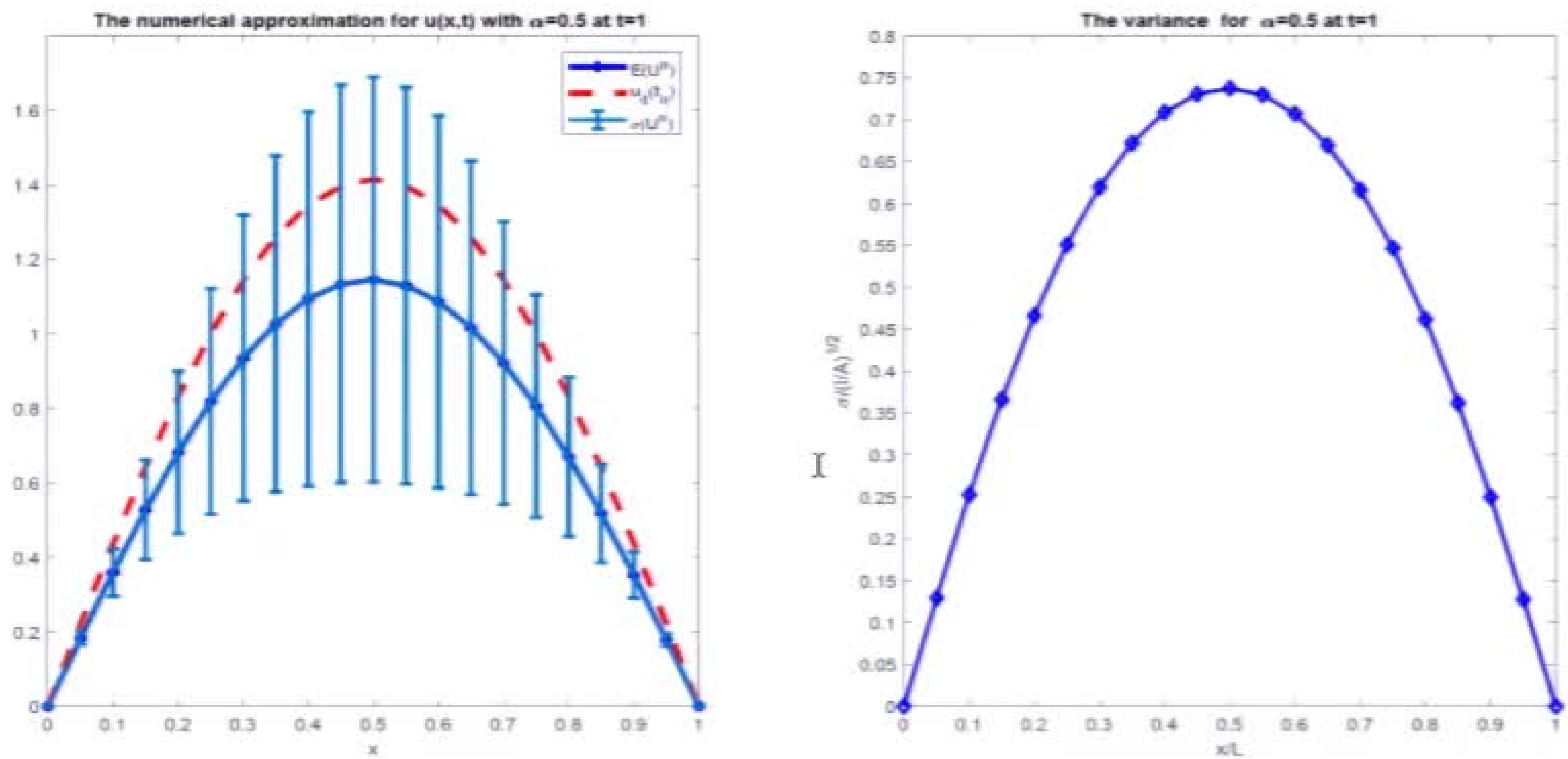

Figure: Numerical approximation for $u(x,t)$, $\alpha = 0.5$

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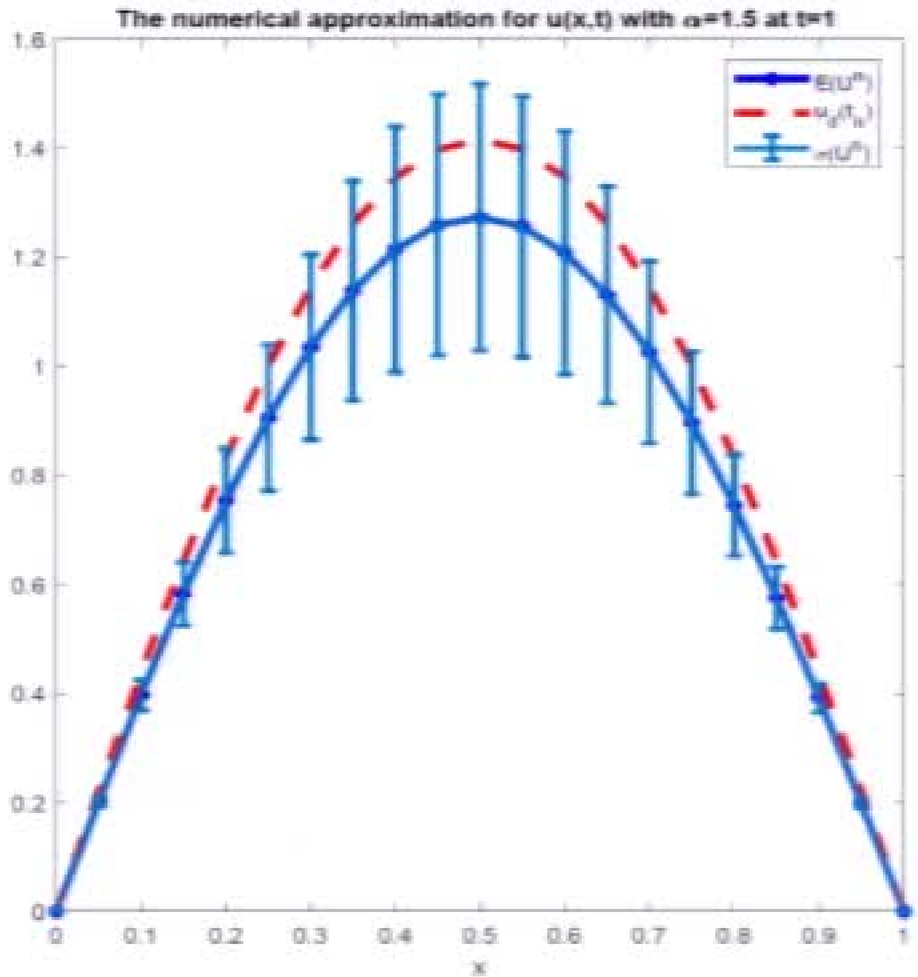

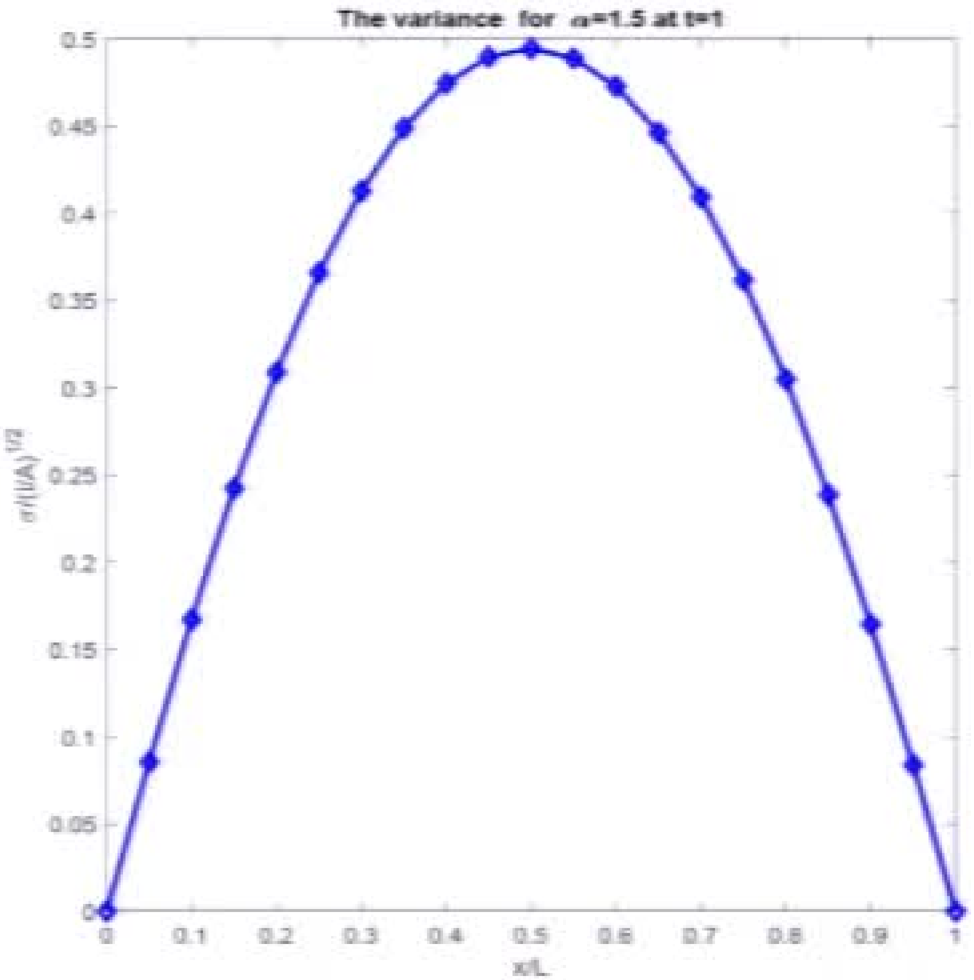

Figure: Numerical approximation for $u(x, t)$, $\alpha = 1.5$

larger $\alpha \implies$ larger mean but smaller variance

Conclusion and ongoing work

- Higher-order scheme in time: Is $O(\tau^{3/2})$ possible?
- Error estimates for numerical methods
    - Analysis: regularity of solutions
- Develop efficient method for high dimensional cases, fully nonlinear problems

Thanks for your attention!