Charging for Diversifiable Risk and Proud to Do It:
Multiline Insurance Pricing with a Distortion Risk Measure

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CHARGING FOR DIVERSIFIABLE RISK AND PROUD TO DO IT: MULTILINE INSURANCE PRICING WITH A DISTORTION RISK MEASURE PART 1

CAS WEBINAR SERIES

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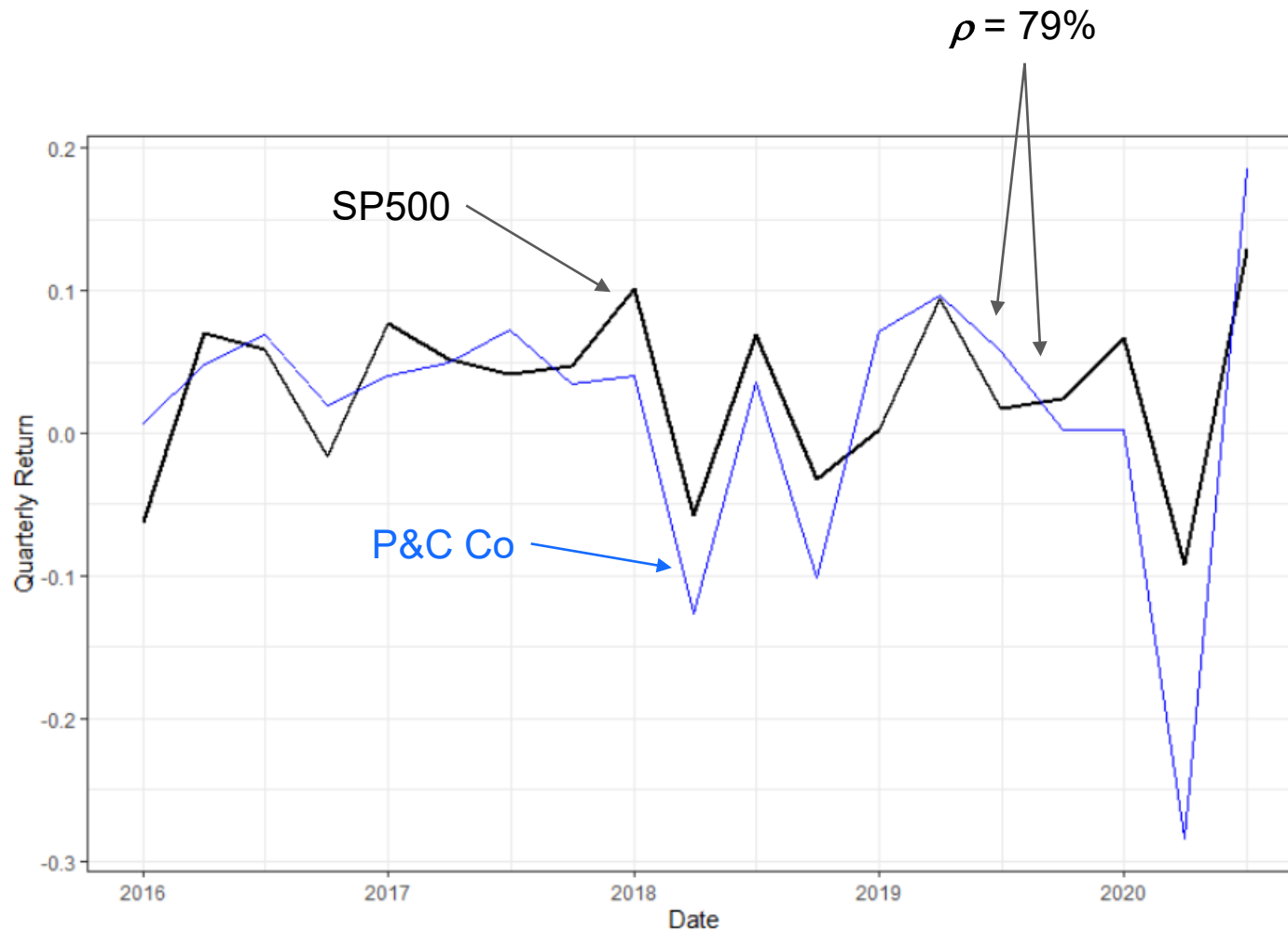New York

What's it about?

- Idiosyncratic (diversifiable) insurance risk is priced in the real world

- A short discussion of why this might be

- A long discussion of how to handle it "systematically"

Capital Asset Pricing Model
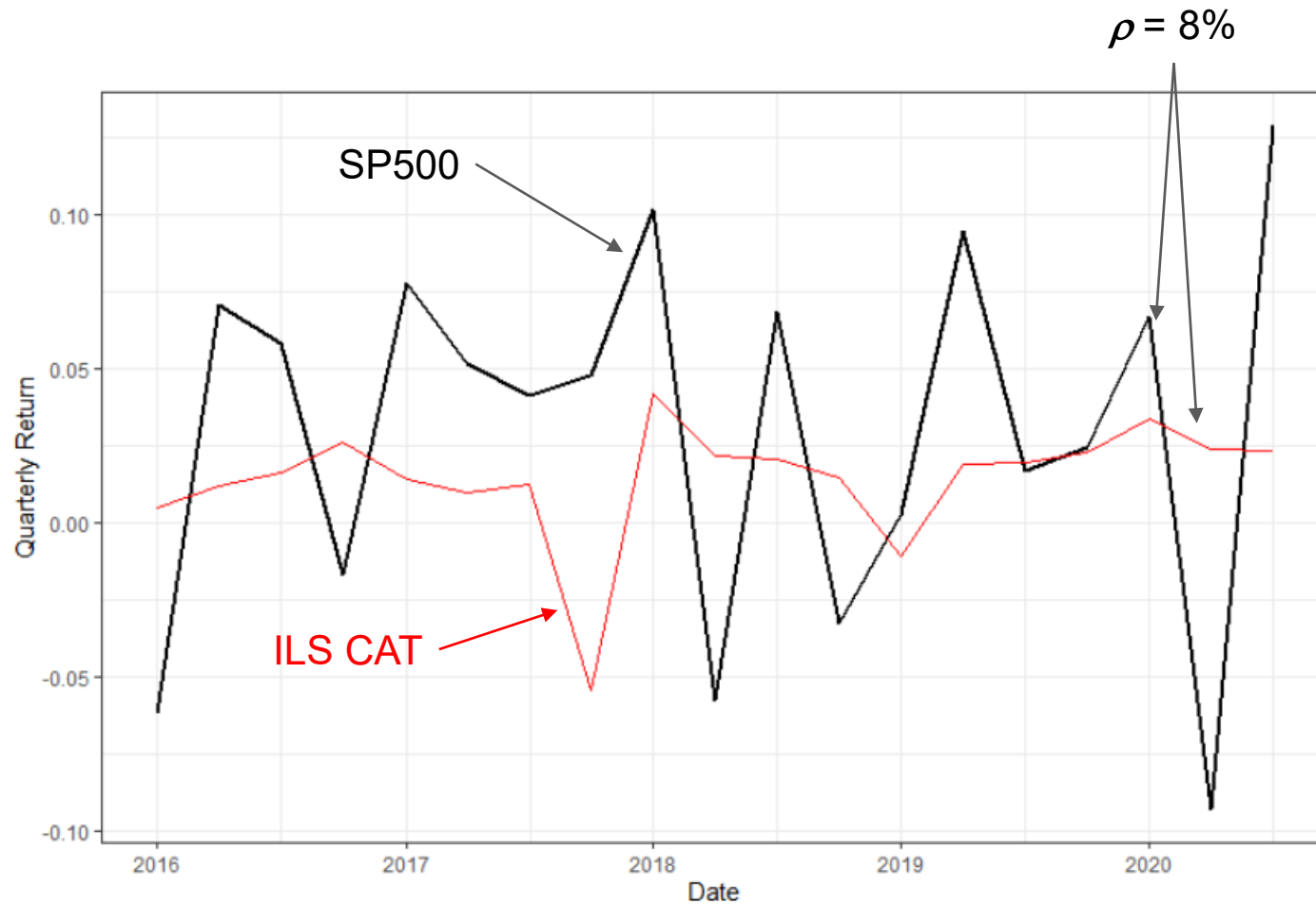
$$E[r] = r_f + \beta(r_m - r_f)$$

$$\beta = \frac{cov(r, r_m)}{\sigma^2(r_m)} \longleftarrow \textit{Systematic risk}$$

Systematic risk

- Systematic risk = correlated with financial markets
- Earns risk premium beyond expected value

Idiosyncratic risk

$\rho = 8\%$

SP500

ILS CAT

- Diversifiable, therefore should not earn a risk premium

But it does!

- Long term average returns
  - Specialty insurers: 7-9%
  - Reinsurers: 9%
  - ILS: 7.5%

- Total cost of capital breakdown:
  - Risk cost ~ 7% ← *Diversifiable, should be zero*
  - Frictional cost ~ 2%

Why?

- Violation of "perfect market" assumptions
    - Instantaneous, liquid trading
    - Long and short positions
    - Symmetric information
    - Complete market
   - No transaction costs

- Negative reasons

- Any positive reasons?

Winner's Curse

- K competitors have independent, unbiased estimates of loss cost
  - The one with the lowest estimate wins the business

- Probability that the winning estimate is too low: $1 - (1/2)^K \rightarrow 1$
  - "Common value auction"
  - Extensive theory
  - *Solution:* bid higher than your estimate

Ambiguity Aversion

- When data is thin or nonexistent, relying on expert judgment, etc.
  - Don't know the exact probabilities

- Ellsberg Paradox: Win $100 if…

Bet on Red ✓	vs	Bet on Black
1/3		??

Bet on Red or Yellow	vs	Bet on Black or Yellow ✓
??		2/3
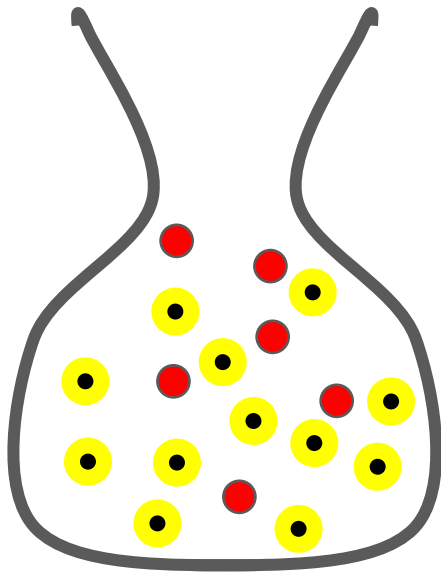
30 red
60 black OR yellow
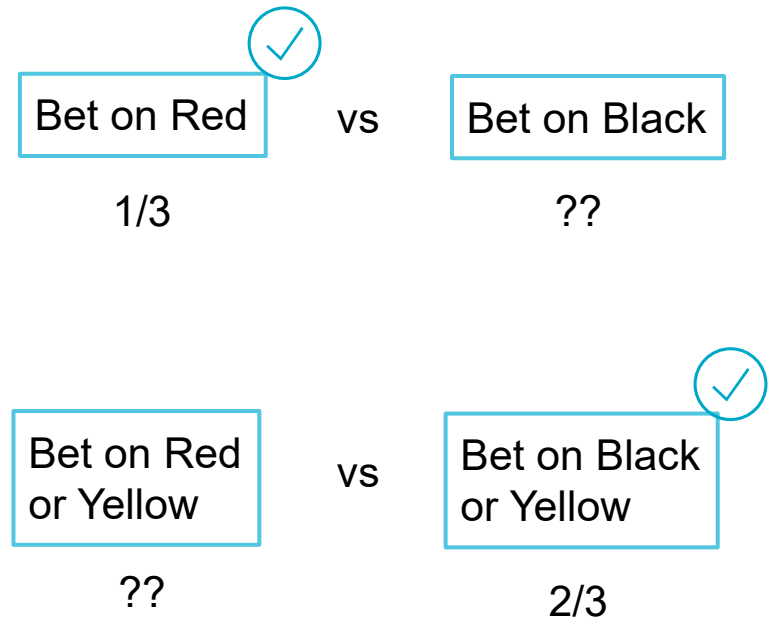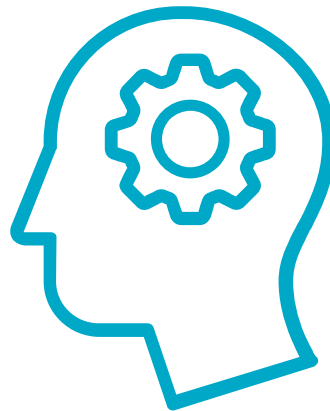
Inconsistent with utility theory
Solution: assume the worst
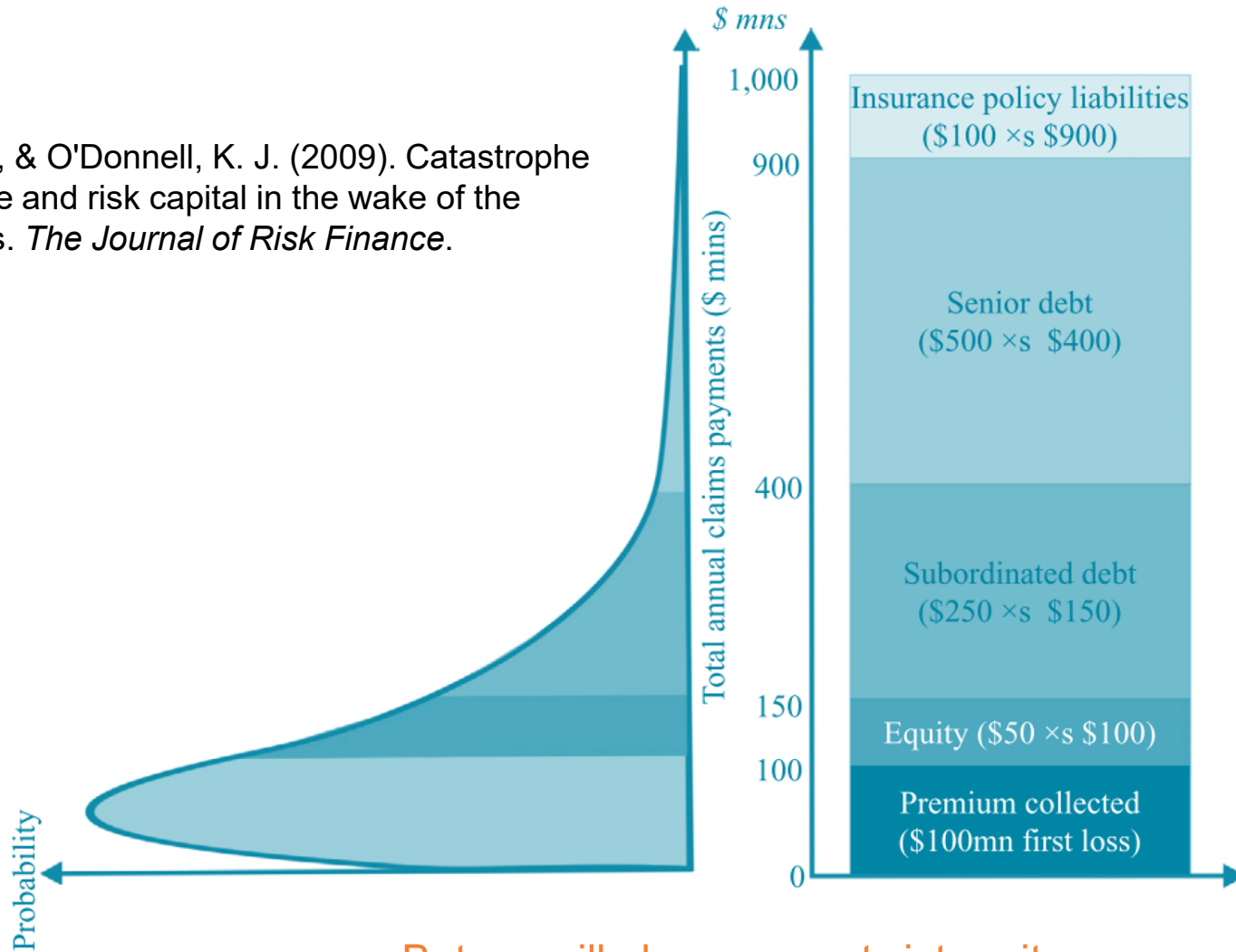
Bottom line

- P&C risk mostly idiosyncratic (diversifiable)
  - reasons to add a risk load.

- Do it right
  - Price whole portfolio
  - Allocate in a logical, consistent manner

A new (?) way of looking at risk capital

Culp, C. L., & O'Donnell, K. J. (2009). Catastrophe reinsurance and risk capital in the wake of the credit crisis. *The Journal of Risk Finance*.

Figure 3
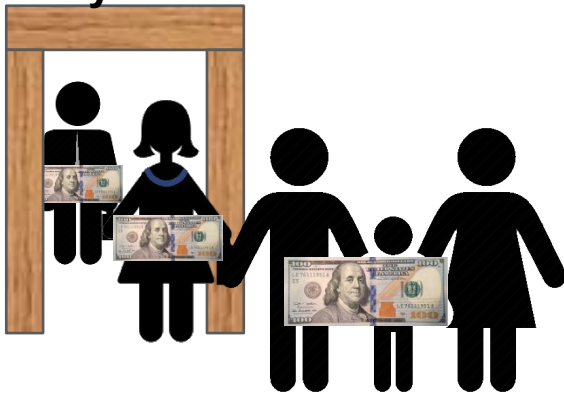

But we will show a new twist on it

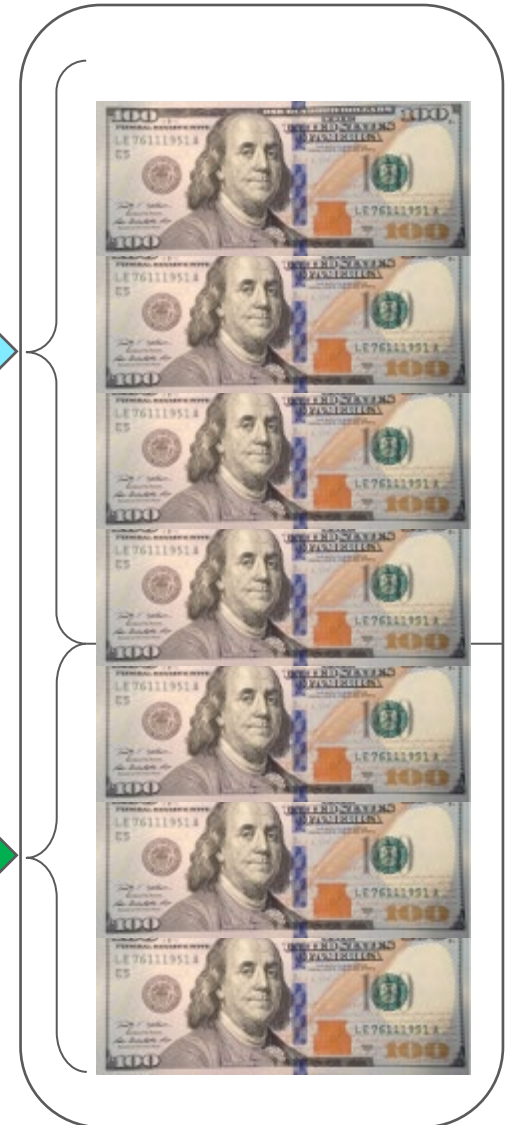# How we usually think about operations: (1) funding

Investors
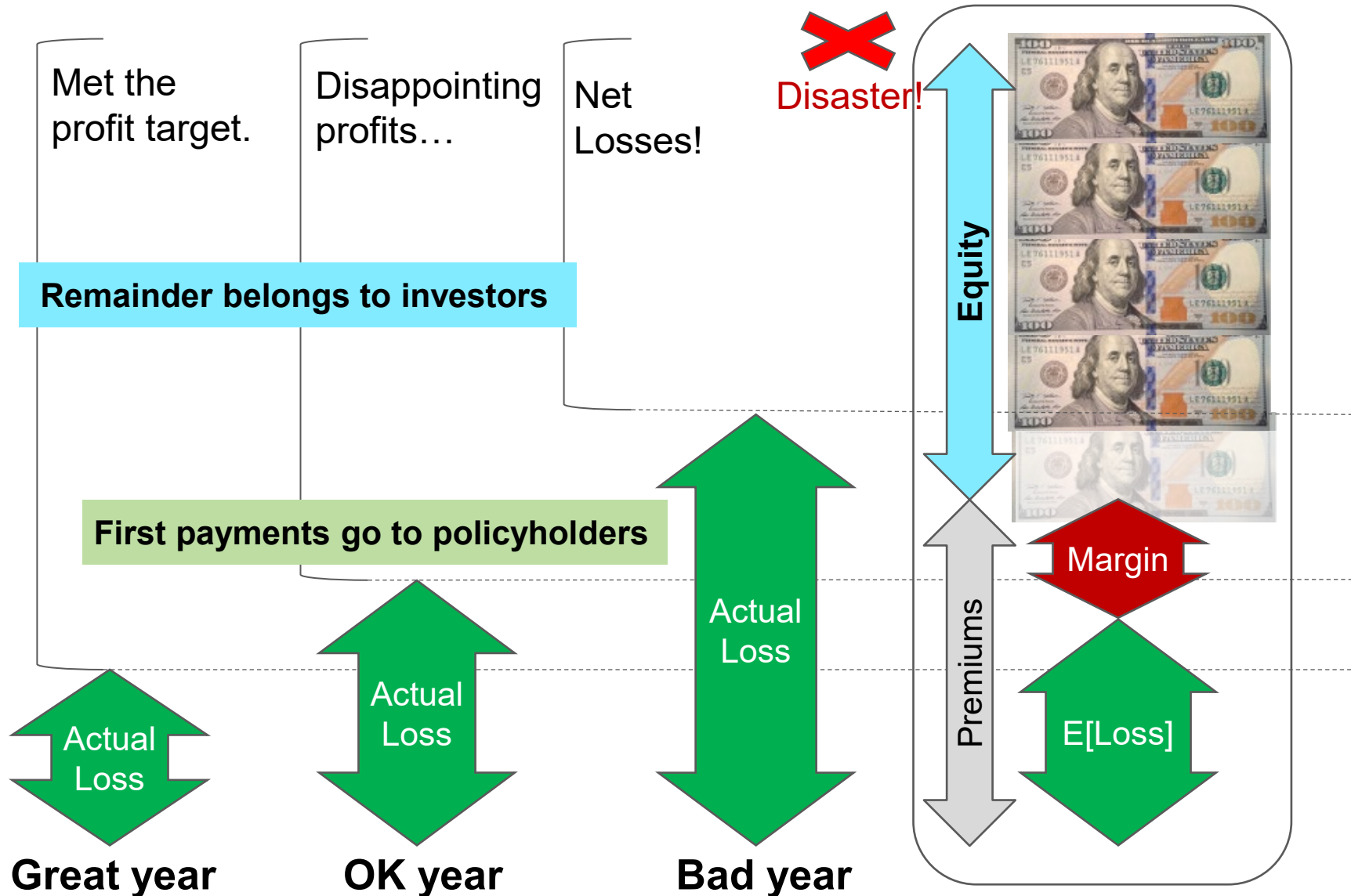

Equity

Buying residual value (providing the safety)

Policyholders

Premiums

Buying cover (at a safety level)

How we usually think about operations: (2) claim payment

Met the profit target.

Disappointing profits…

Net Losses!

Disaster!

Remainder belongs to investors

First payments go to policyholders

Actual Loss

Actual Loss

Actual Loss

Actual Loss

Actual Loss

Great year

OK year

Bad year

Equity

Premiums

Margin

E[Loss]

What if you had to fund each asset unit (layer) individually?

Investors

Investor is not going to supply *all* the funding

Policyholders

Policyholders will have to supply expected loss and risk margin.
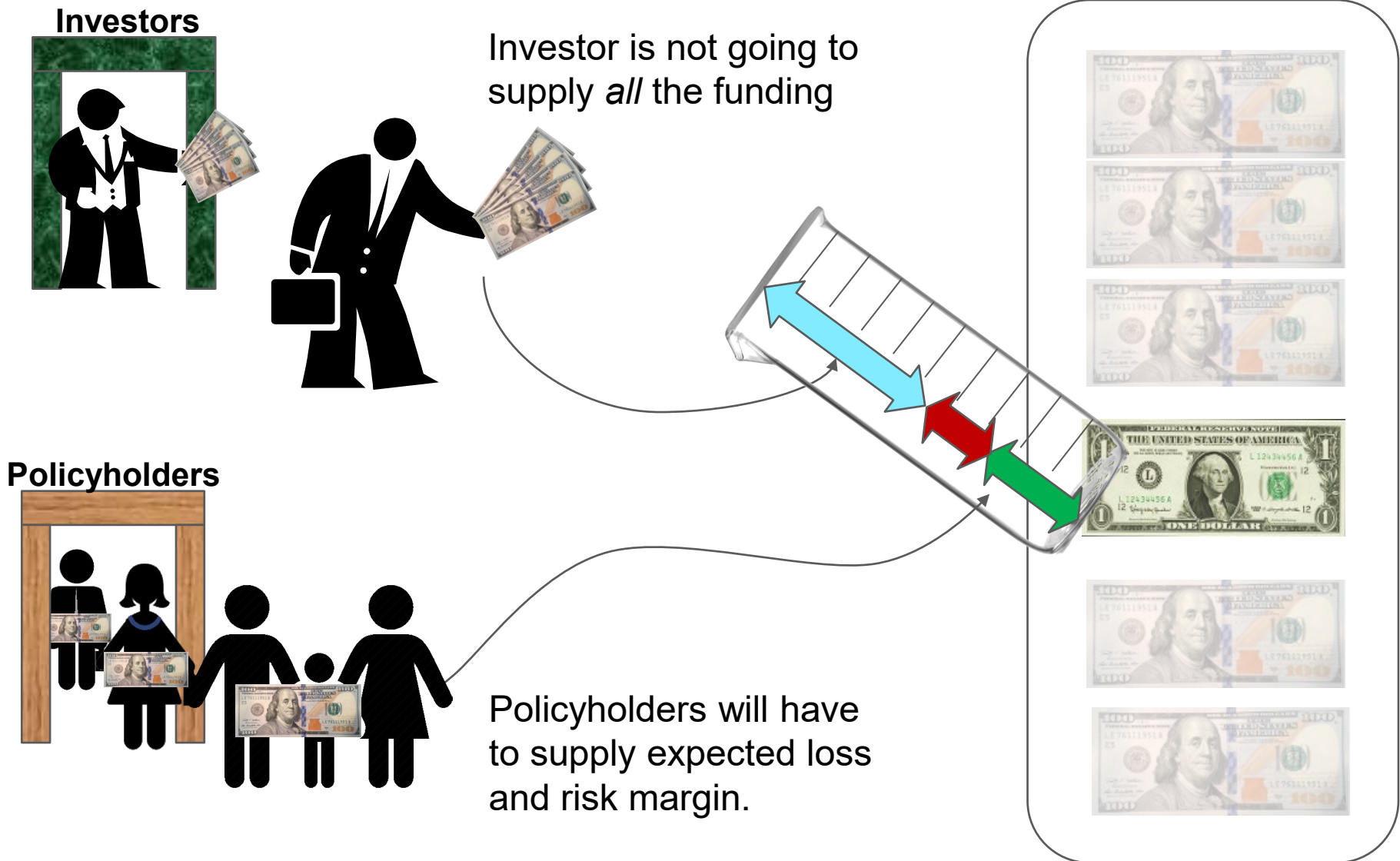
What do we know about a thin layer on the portfolio aggregate loss?

Probability of attachment $s = S(x) = 1 - F(x)$
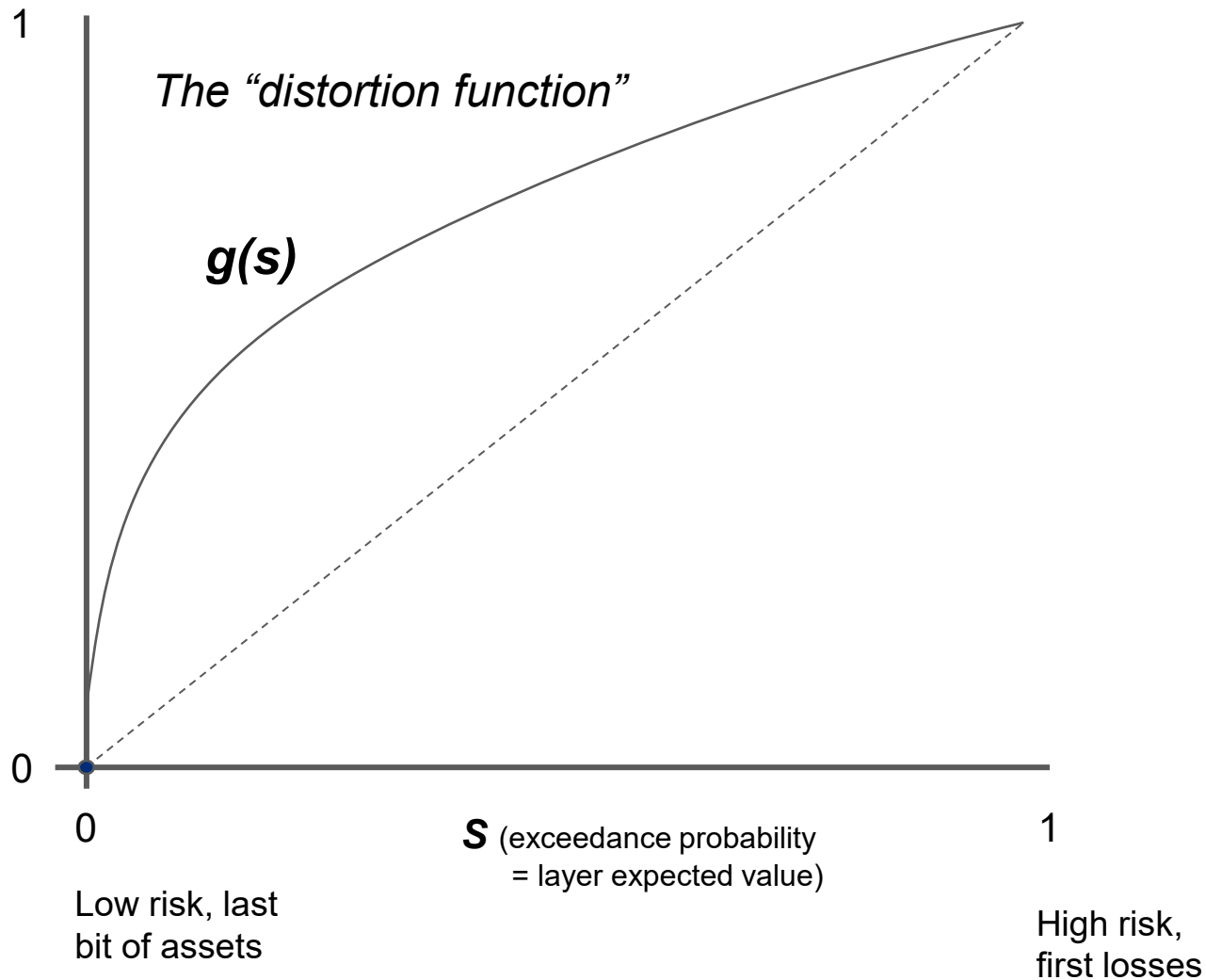
Probability of exhaustion $s - \varepsilon$

Expected Loss on line $\sim s$

Hypothesis: s is all we *need* to know to price the layer

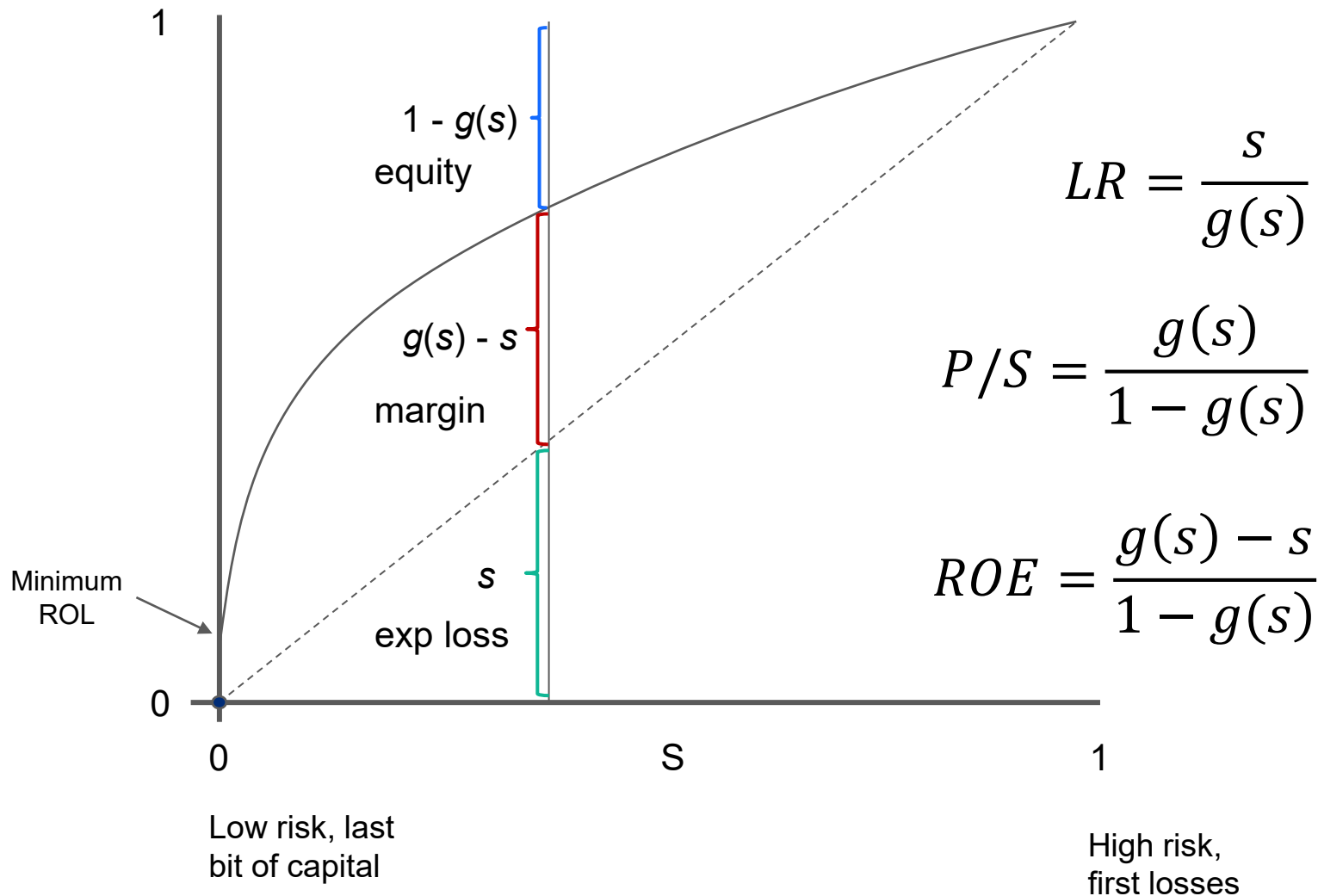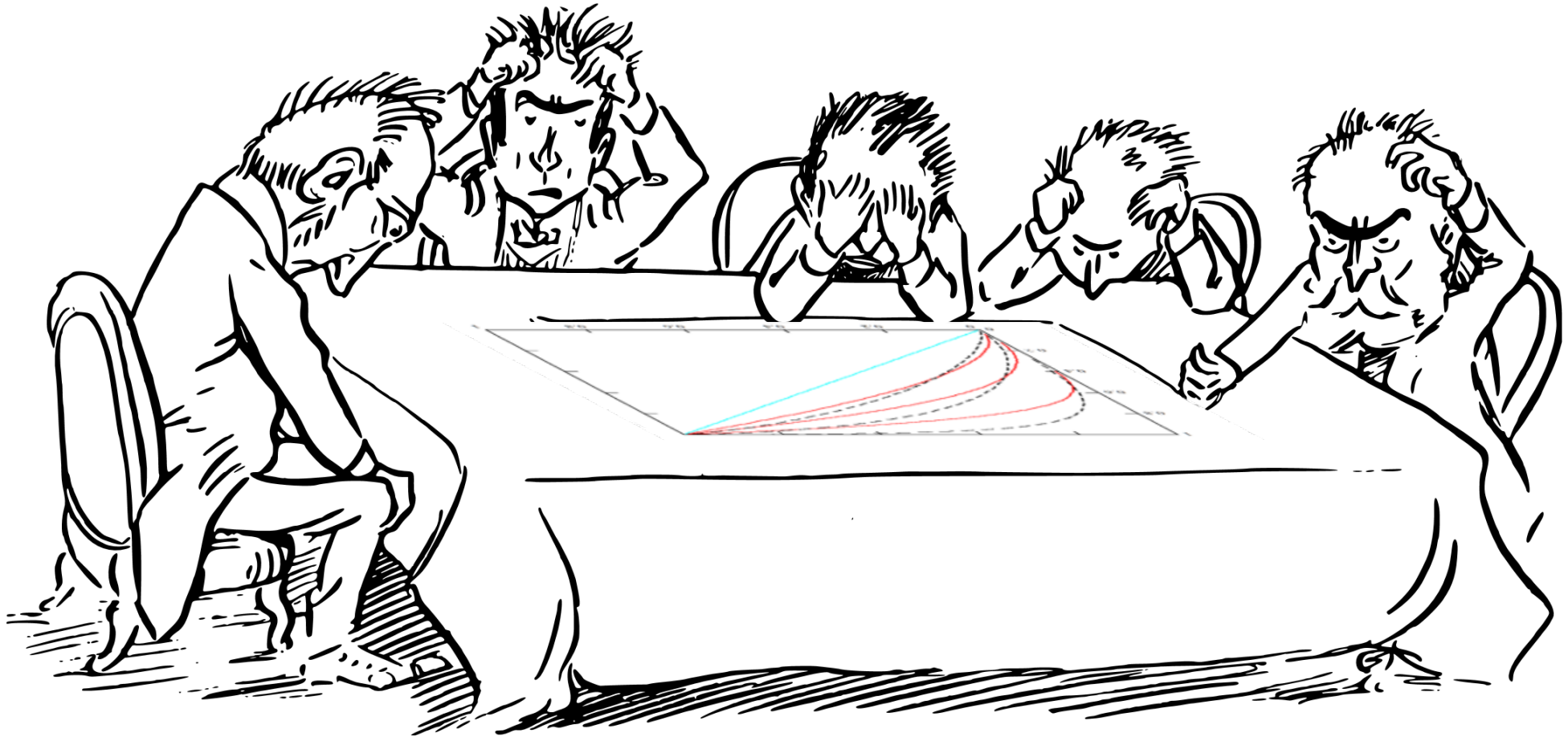Total Loss $= x$

Assumption: there is a functional relationship between layer LOL and layer ROL

The "distortion function"

g(s)

1

0

0

S (exceedance probability = layer expected value)

1

Low risk, last bit of assets

High risk, first losses

$$LR = \frac{s}{g(s)}$$

$$P/S = \frac{g(s)}{1 - g(s)}$$

$$ROE = \frac{g(s) - s}{1 - g(s)}$$

1 - g(s)
equity

g(s) - s
margin

s
exp loss

Minimum ROL

0

S

1

Low risk, last
bit of capital

High risk,
first losses

Where does that *g*(*s*) function come from?

https://go.guycarp.com/cas2018

How it looks back in the scenario-loss domain

Low Pr{Loss},
last bit of assets

a

$g{\circ}S(x)$

$S(x)$

x
(Loss)

Cumulative
probability F(x)

High Pr{Loss},
first losses

0   1

0

Scenarios
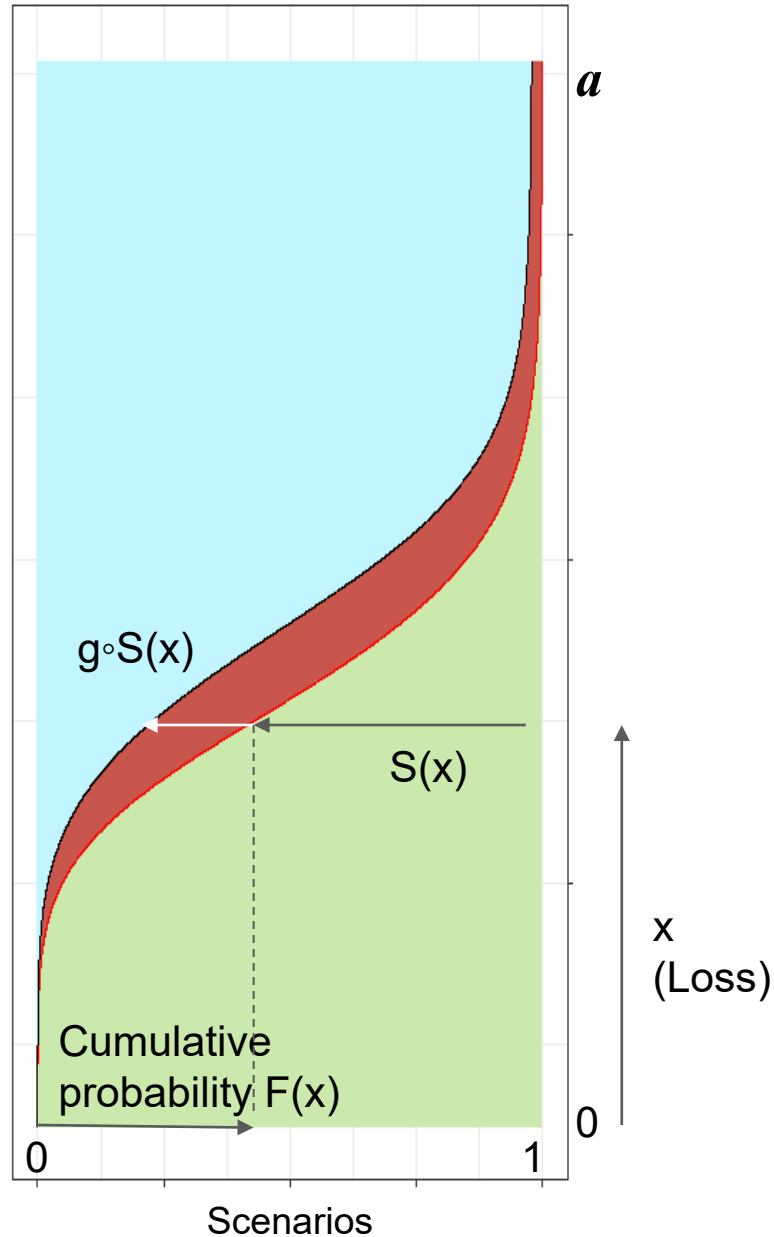
The new perspective on where premium and equity sit
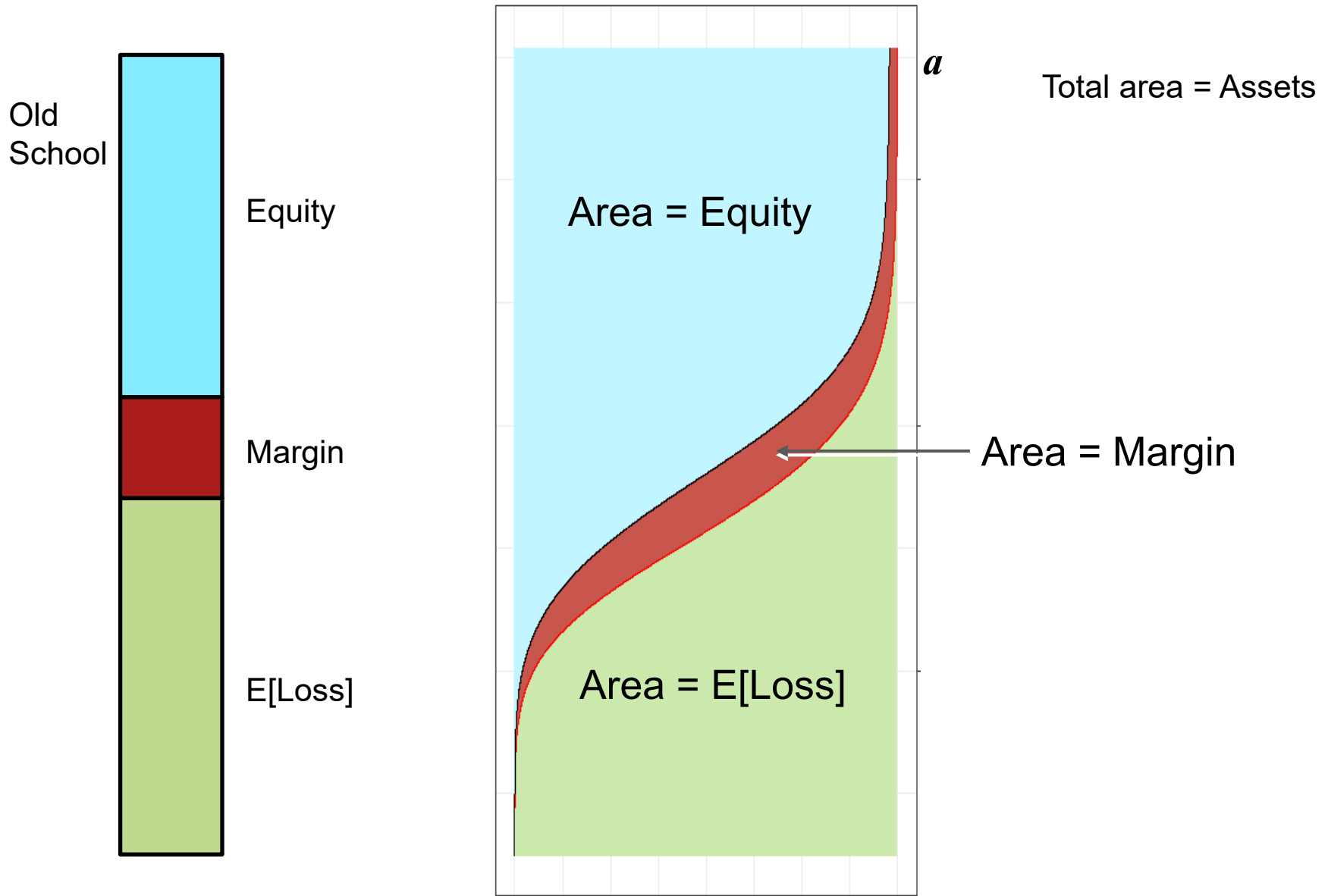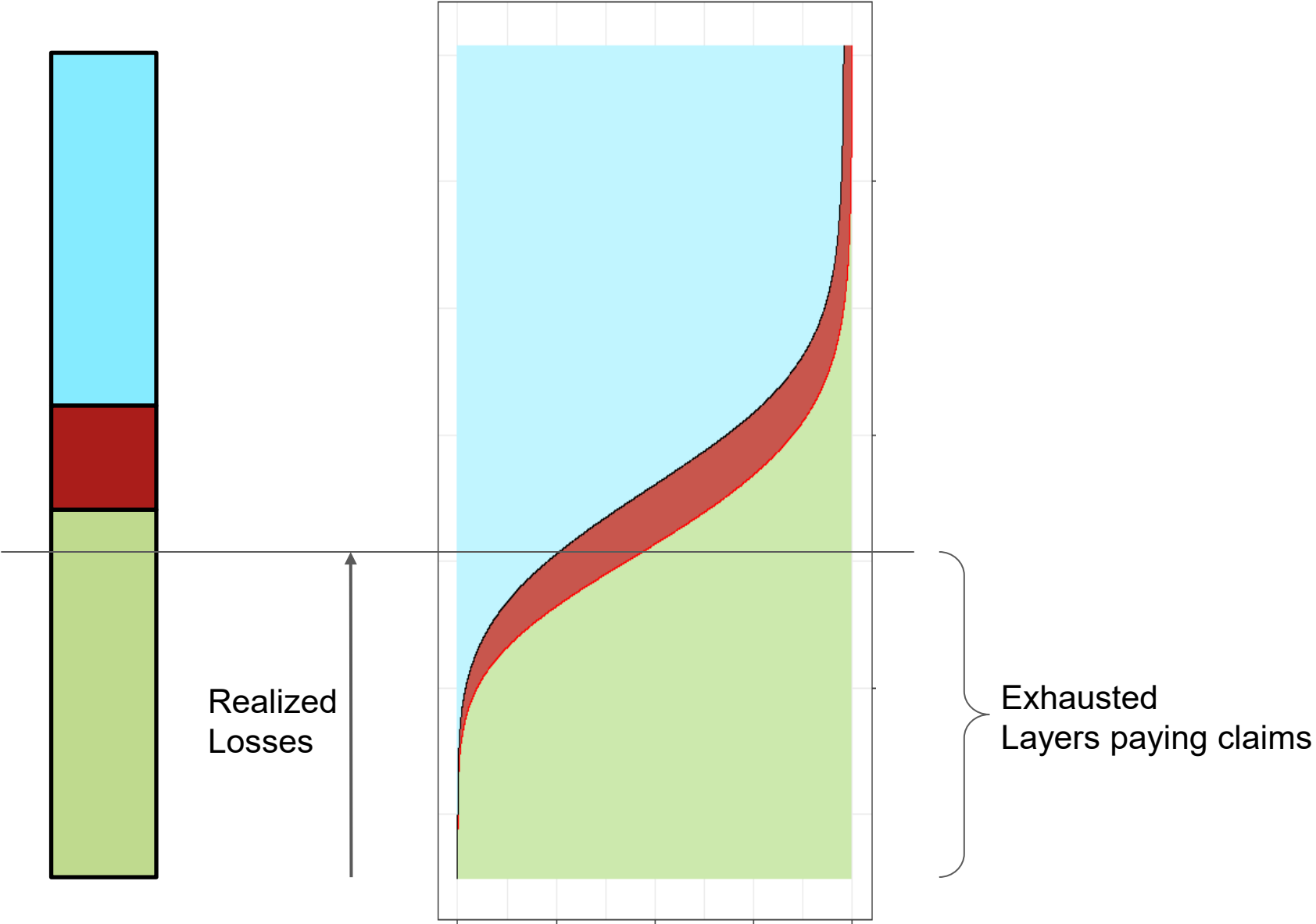
Old School

Equity

Margin

E[Loss]

Area = Equity

Area = Margin

Area = E[Loss]

a

Total area = Assets

After operations financial reporting

Realized Losses

Exhausted Layers paying claims

After operations financial reporting

Lost equity

Realized Losses

Premium to pay losses

Exhausted Layers paying claims

After operations financial reporting

Returned equity

Lost equity

Realized Losses

Return of residual value

Profit more than makes up for lost equity

Premium to pay losses

Exhausted Layers paying claims
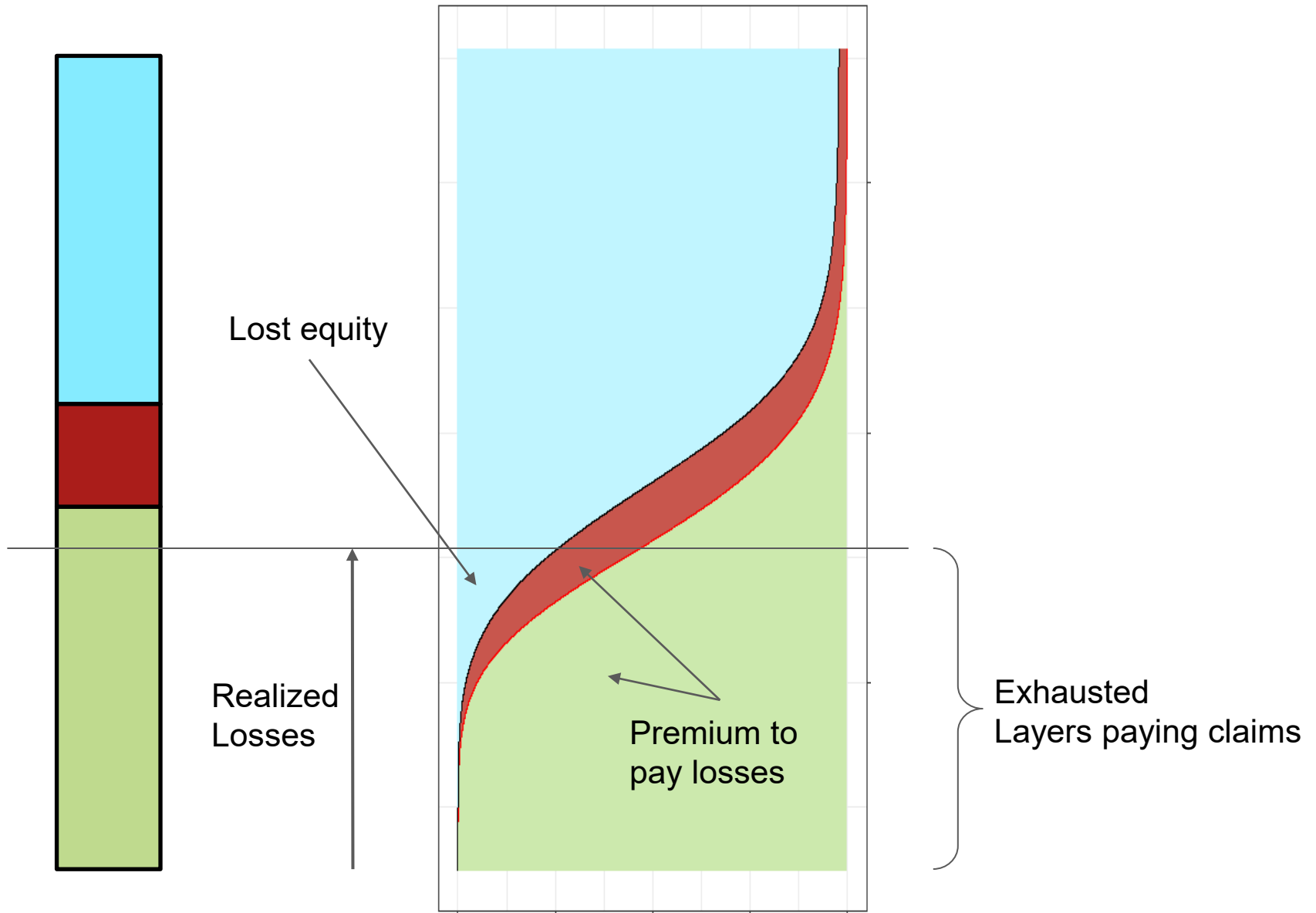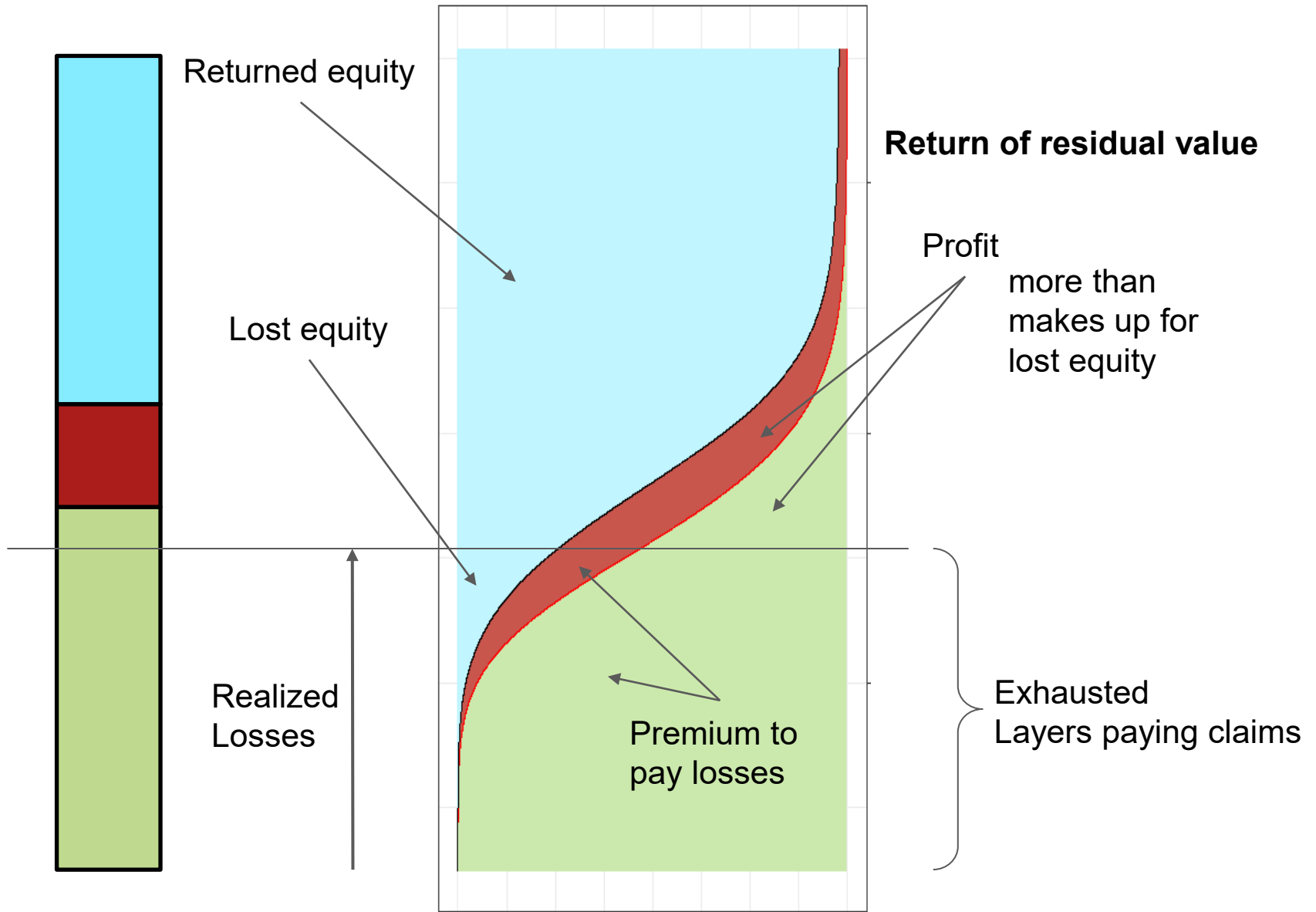
Visualizing the expectation – conventional view

$$E[L \wedge a] = \int_0^a x \, dF(x) + aS(a)$$

a

S(a)

X

X (Loss)

0

dF(x)
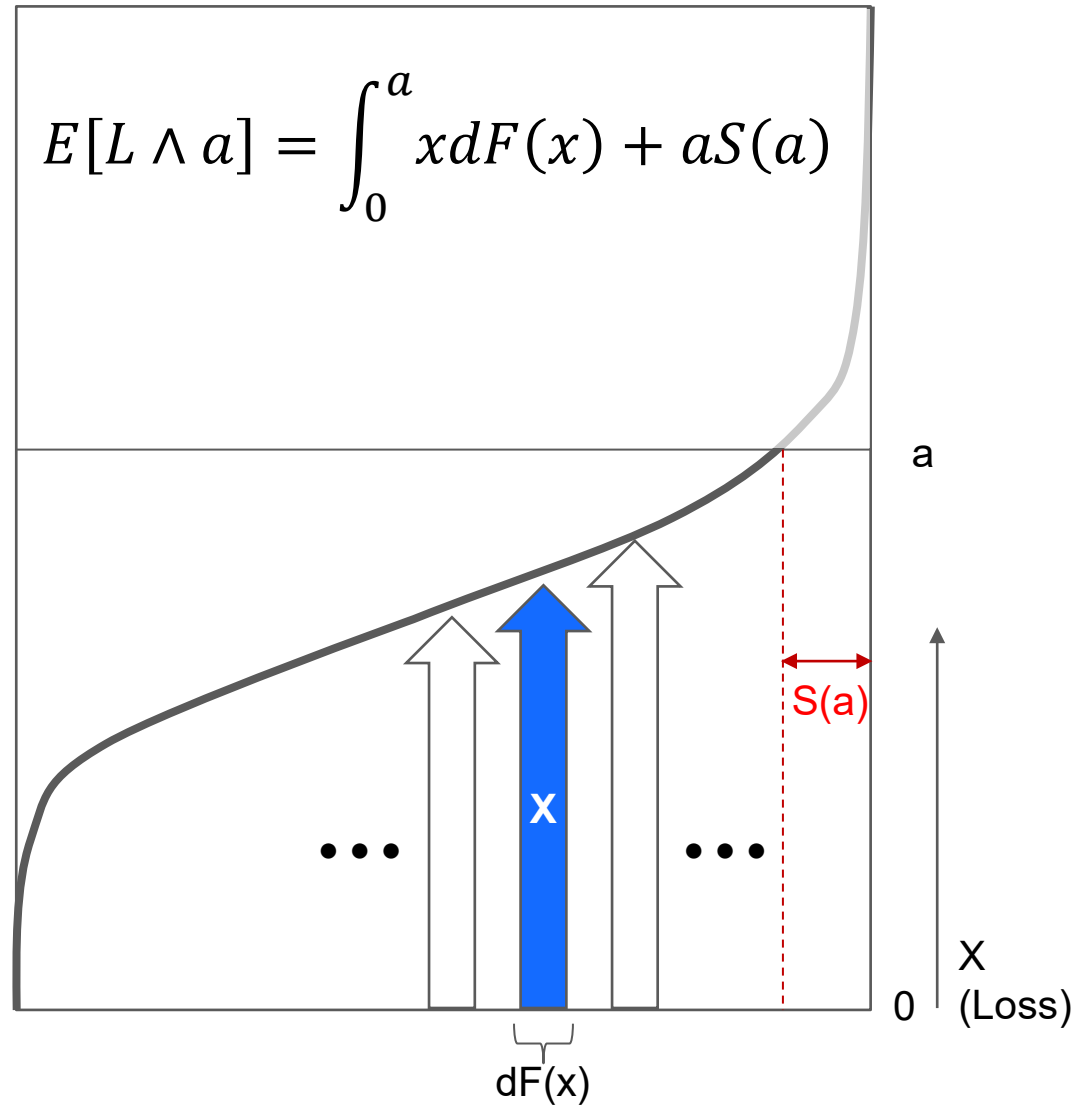
Visualizing the expectation – layer view

$$E[L \wedge a] = \int_0^a S(x)dx$$

a

S(X)

dx

X
(Loss)

0
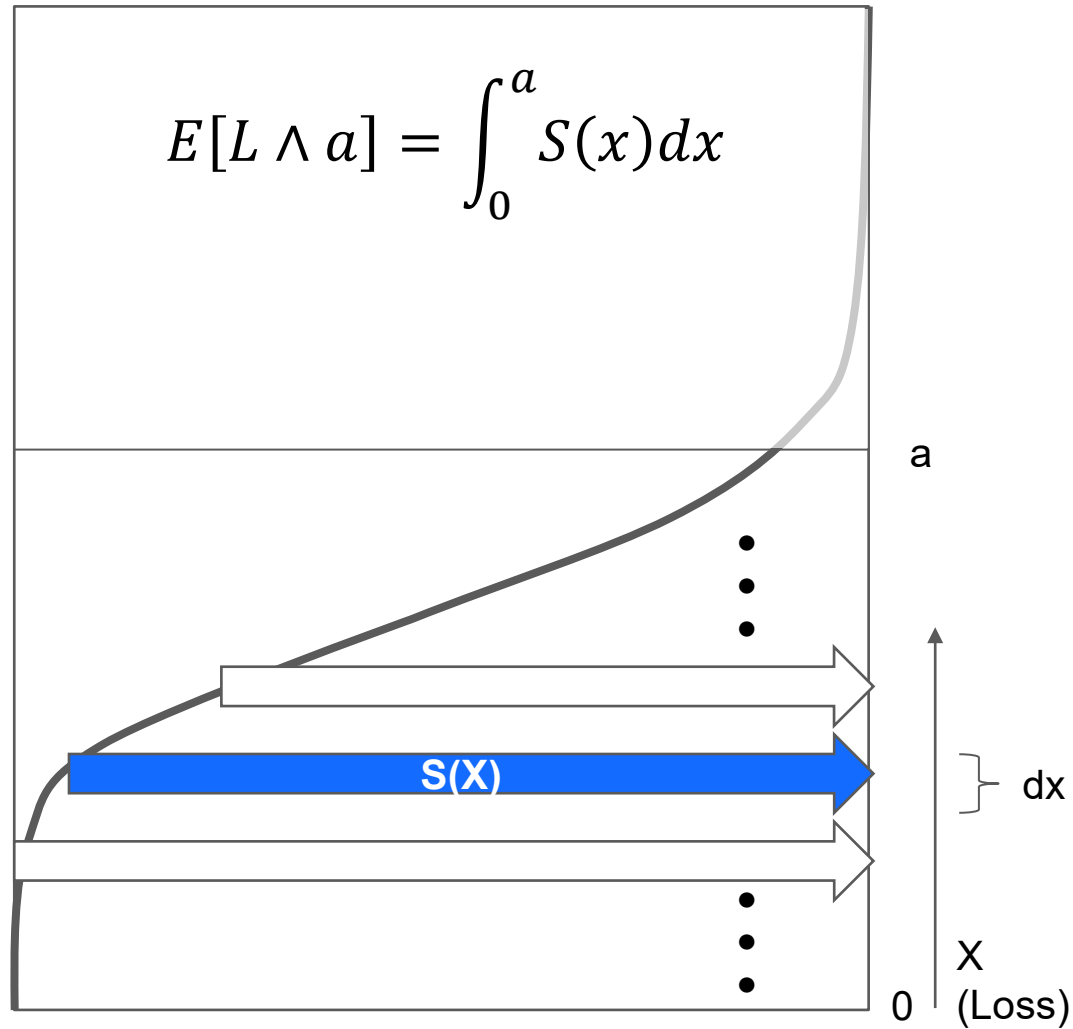
Probability distortion implies pricing

Expected loss (LEV)

$$E[L \wedge a] = \int_0^a S(x)dx = \int_0^a xdF(x) + aS(a)$$

Probability distortion implies pricing

Expected loss (LEV)

$$E[L \wedge a] = \int_0^a S(x)dx = \int_0^a xdF(x) + aS(a)$$

distorted probability   transformed cdf

Required premium
Distorted expected loss

$$E_g[L \wedge a] = \int_0^a g\big(S(x)\big)dx = \int_0^a xdG(x) + ag\big(S(a)\big)$$
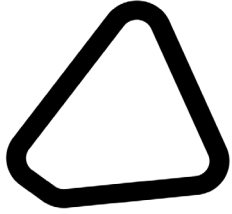
Pause

- What you've seen so far:
  - Pricing curves do not violate finance theory
  - Thinking about layers of assets
  - Each consists of premium + equity
  - Margin is cost of capital
  - Expected loss *s* determines layer funding
  - Functional relationship *g(s)*

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Charging for Diversifiable Risk and Proud to Do It: Multiline Insurance Pricing With a Distortion Risk Measure, Part 2

Stephen J. Mildenhall

CAS Webinar Series, September 24, 2020

Section 1: Risk Margins by Line without Allocation

Loss payments: who gets what in default?

- Sold insurance promises

$$X = X_1 + \cdots + X_n$$

- **Equal priority** payment to line i with assets a

$$X_i(a) = \begin{cases} X_i & X \le a \\ a\,(X_i/X) & X > a \end{cases}$$

$$= X_i \frac{X \wedge a}{X}$$

$$= \frac{X_i}{X} X \wedge a$$

- $\dfrac{X \wedge a}{X} =$ fixed payment pro rata factor applied to loss from all lines

- $\dfrac{X_i}{X} =$ variable share of available assets for line i

- $X \wedge a$ amount of assets **available** to pay claims

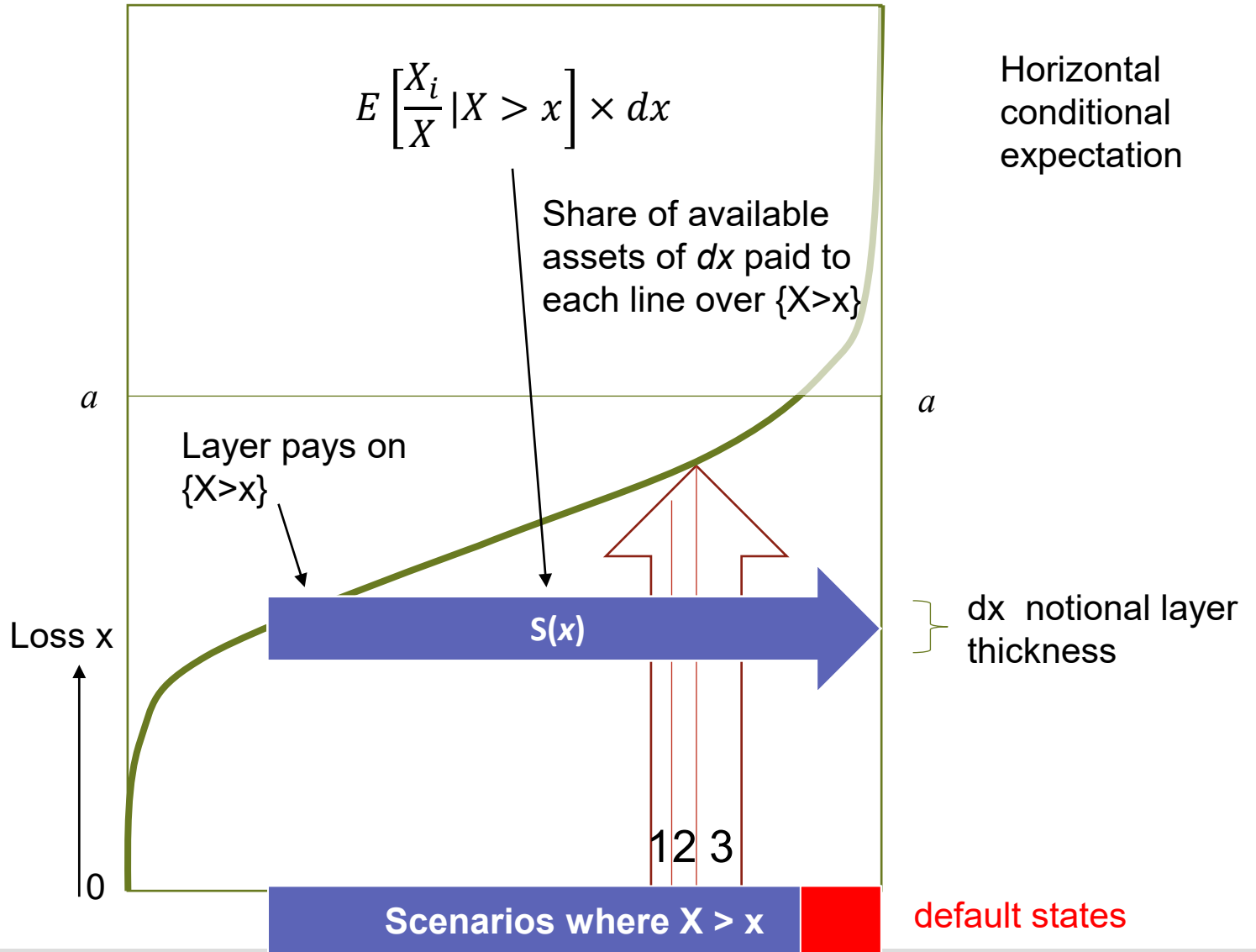- $X_i(a)$ sum to $X \wedge a$, limited losses

Expected loss formulas

$$E[X \wedge a] = \int_0^a S(x)dx$$

$$E[X_i(a)] = ??$$

Visualizing expected loss by line and layer and total

$$E\left[\frac{X_i}{X}\,|X>x\right]\times dx$$

Horizontal conditional expectation

Share of available assets of *dx* paid to each line over {X>x}

a

a

Layer pays on {X>x}

S(*x*)

dx notional layer thickness

Loss x

0

12 3

Scenarios where X > x

default states

Visualizing expected loss by line and layer and total

$$E[X_i(a)] = \int_0^a E\left[\frac{X_i}{X}\,|X > x\right] S(x)\,dx$$

Vertical integration, including chance $S(x)$ layer pays

a

a

Layer pays on $\{X>x\}$

$S(x)$

dx notional layer thickness

Loss x

0

12 3
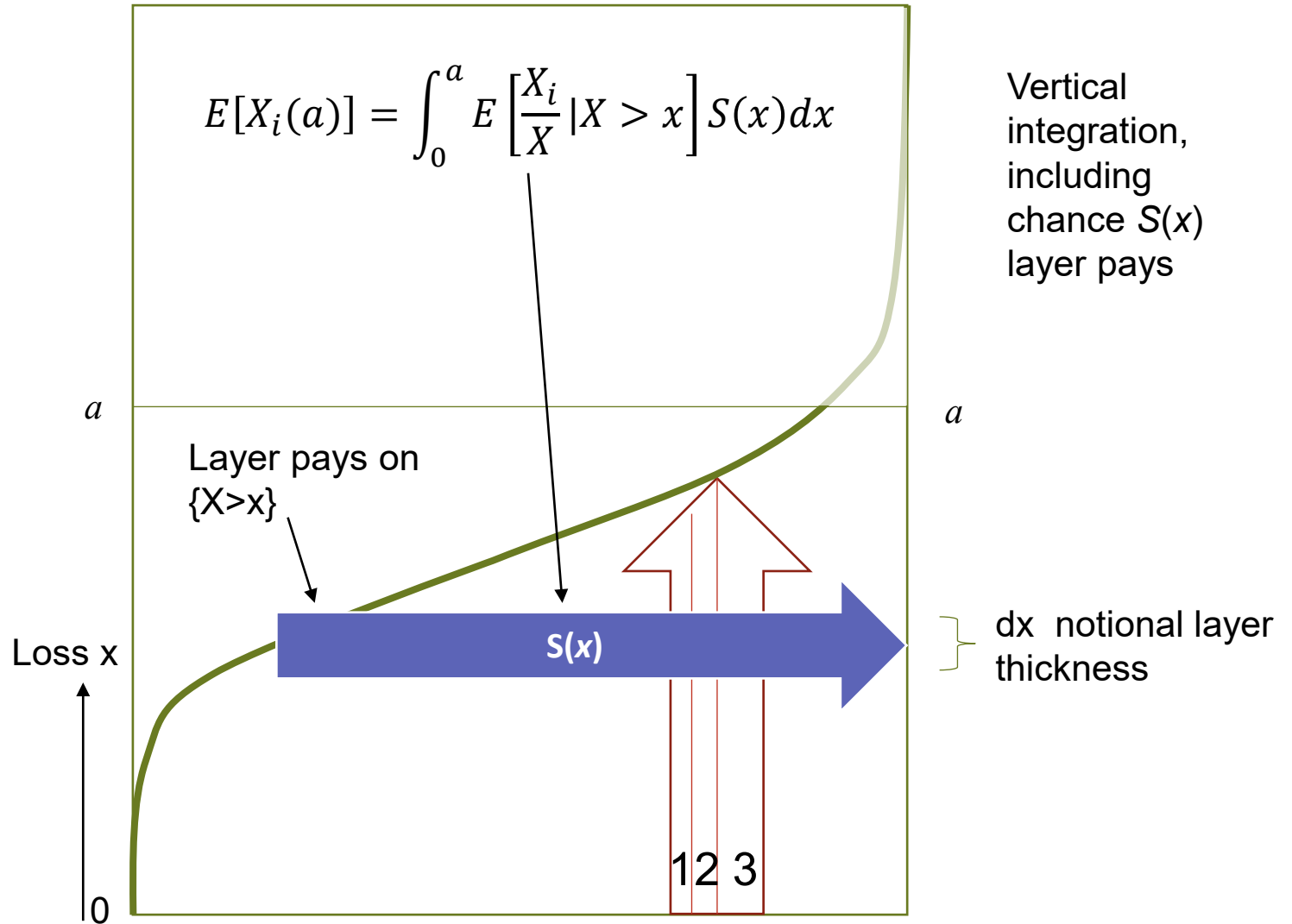
Expected loss and premium by line and layer and total

$$\bar{L}_i(a) = E[X_i(a)] = \int_0^a E\left[\frac{X_i}{X}|X > x\right] S(x)dx = \int_0^a \alpha_i(x)S(x)dx$$

$$\underbrace{\phantom{E\left[\frac{X_i}{X}|X > x\right]}}_{\alpha_i(x)}$$

$$\bar{P}_i(a) = E_g[X_i(a)] = \int_0^a E_g\left[\frac{X_i}{X}|X > x\right] g(S(x))dx = \int_0^a \beta_i(x)g(S(x))dx$$

$$\underbrace{\phantom{E_g\left[\frac{X_i}{X}|X > x\right]}}_{\beta_i(x)}$$

α_i, β_i functions add-up: $\sum \alpha_i(x) = E\left[\frac{X_1 + \cdots + X_n}{X}|X > x\right] = 1$

Expected loss and premium by line and layer and total

Loss cost density $\qquad L_i(x) = \alpha_i(x)S(x)$

Premium density $\qquad P_i(x) = \beta_i(x)g\big(S(x)\big)$

$\Rightarrow$ Margin density $\qquad M_i(x) = P_i(x) - L_i(x)$

$$= \beta_i(x)g\big(S(x)\big) - \alpha_i(x)S(x)$$

- Integrate density to get total
- Everything you need to price!
- All quantities add-up
- Not an arbitrary allocation...no choices

Assumptions

- Price with g
- Equal priority in default

Independence of X_i **not** required

Three subtle points

- E_g is not additive, the risk adjustment depends on X and ρ

- Allocation of an allocation: is risk adjustment based on X or $X \wedge a$?
  - It can matter...it doesn't for SRMs
  - Comonotonic additive

- Non-uniqueness: is risk adjustment (conditional measure) unique?
  - No...but it doesn't matter for SRMs
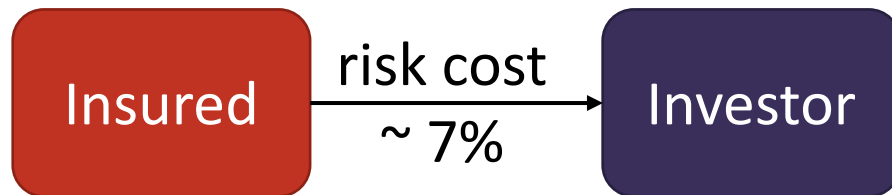 - Law invariant and comonotonic additive

Section 2: Frictional Costs and Allocation

Frictional costs of capital

Sidecar, ILS, Alternative Cat Capital: risk cost from SRM

Insured → **risk cost ~ 7%** → Investor

Equity Capital: risk cost + frictional costs

Insured → **total cost ~ 10%** → Insurer → **risk cost ~ 7% / frictions ~ 3%** → Investor

- Frictional costs of capital: investors don't like committing permanent equity capital
  - Don't trust management: principal/agent problems
  - Double taxation
  - Regulatory restrictions
- Frictional costs = tax on capital → must allocate capital
- Insurers exist because they lower ambiguity costs (no cat models)

Law invariant assumption

A **law invariant** risk measure is function of the distribution of outcomes but does not distinguish by cause of loss

Therefore return can't vary by line within a layer

For a given **layer**, all LOBs must have the same ROE

$$r(x) = \frac{M(x)}{Q(x)} = r_i(x) = \frac{M_i(x)}{Q_i(x)}$$

Spectral risk measures are law invariant

Implied layer capital allocation by line

$$\frac{M(x)}{Q(x)} = \frac{M_i(x)}{Q_i(x)} \implies Q_i(x) = \frac{M_i(x)}{M(x)} Q(x)$$

$$Q_i(x) = \underbrace{\frac{\beta_i(x)g\big(S(x)\big) - \alpha_i(x)S(x)}{g\big(S(x)\big) - S(x)}}_{\text{Capital allocation}} \underbrace{\{1 - g\big(S(x)\big)\}}_{\text{Capital in layer}}$$

Capital allocation   Capital in layer

...unique layer capital allocation!

Risk margin and capital allocation can be negative!

- **Risk margin** and allocation can be **negative** if $\beta_i(x)$ sufficiently less than $\alpha_i(x)$

$$\frac{\beta_i(x)g\big(S(x)\big) - \alpha_i(x)S(x)}{g\big(S(x)\big) - S(x)}$$

- When is $\beta_i(x) < \alpha_i(x)$? For relatively thin tailed lines!

- **Risk margin** across all lines $g(S(x)) - S(x)$ is always **positive**

- Allocated risk margin always positive for independent lines if capital standard sufficiently strong

Risk cost of capital varies by amount of assets

- Total risk margin is a function of total assets

$$\overline{M}(a) = \int_0^{\boxed{a}} g(S(x)) - S(x)\,dx$$

- Total capital also varies with assets

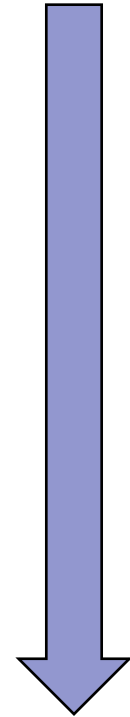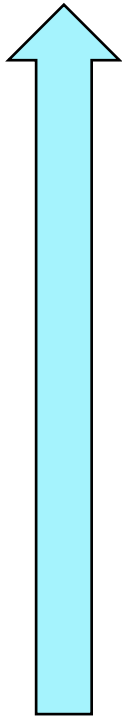$$\overline{Q}(a) = \int_0^a 1 - g(S(x))\,dx$$

- Hence risk cost of capital (M / Q) varies with assets

- Total cost of capital adds frictional costs proportional to allocated capital
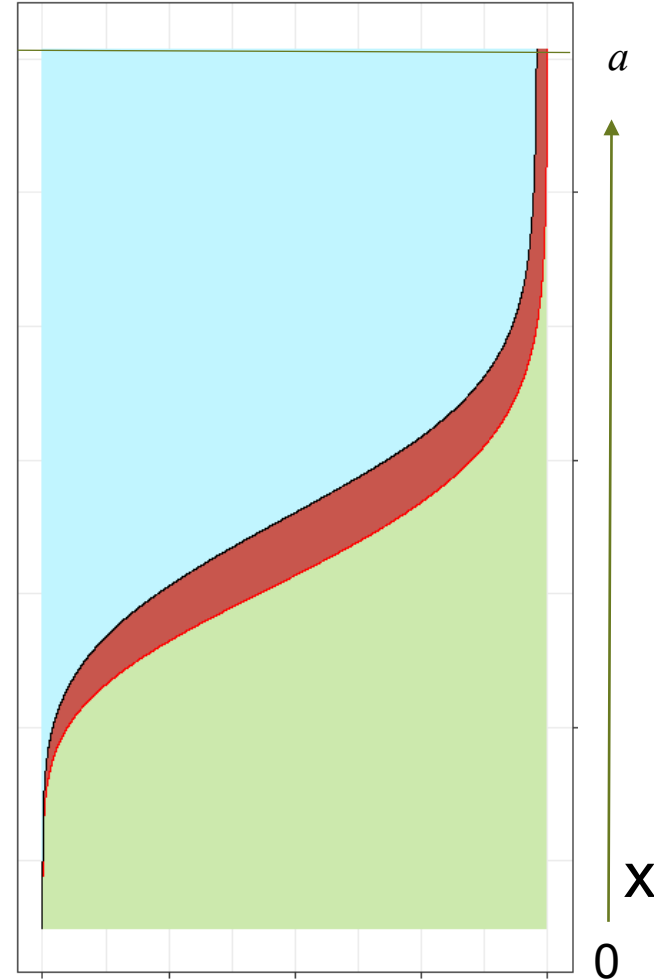
Equity and risk margin vary by layer in complex manner

More equity
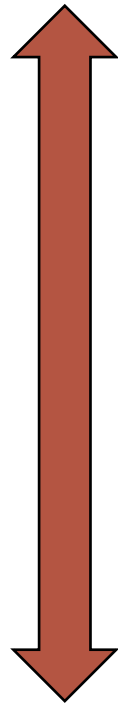
Higher risk return

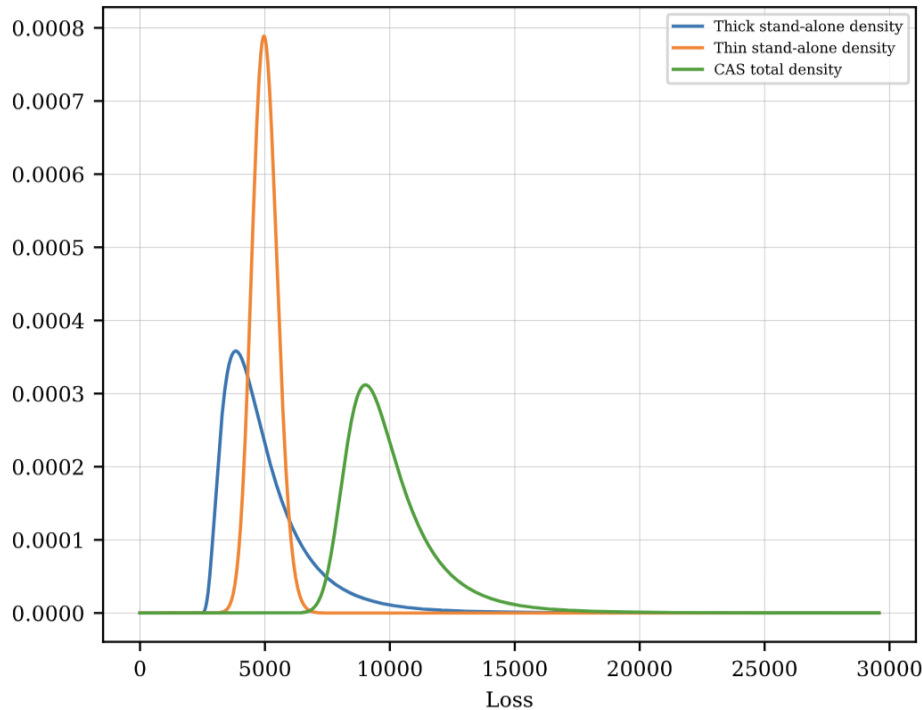Variable cost?

a

x

0

Section 3: Thick- and Thin-Tailed Example

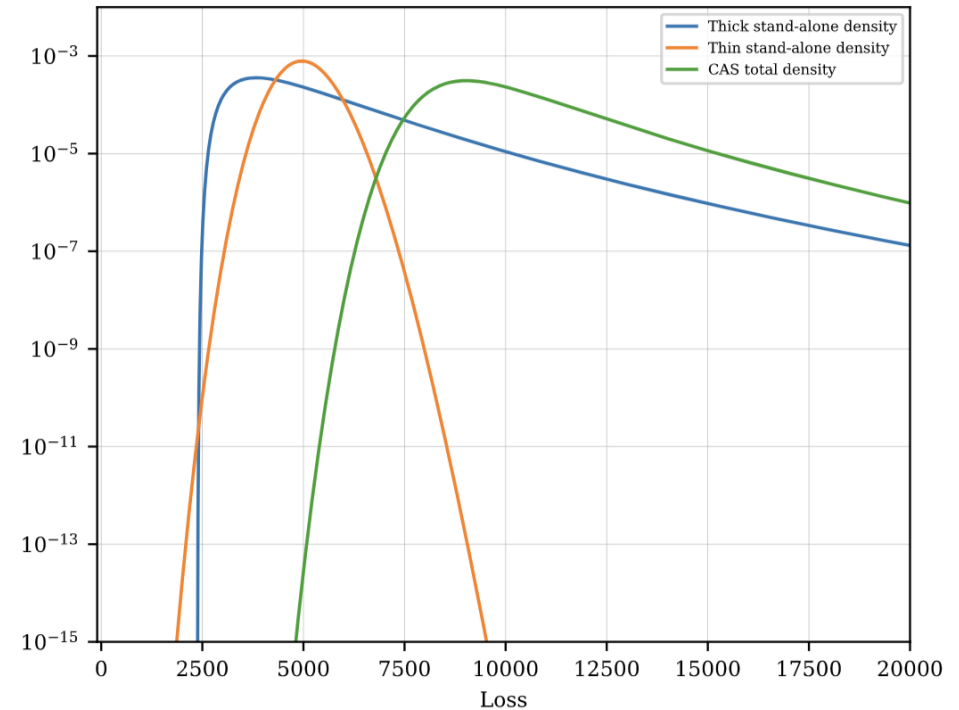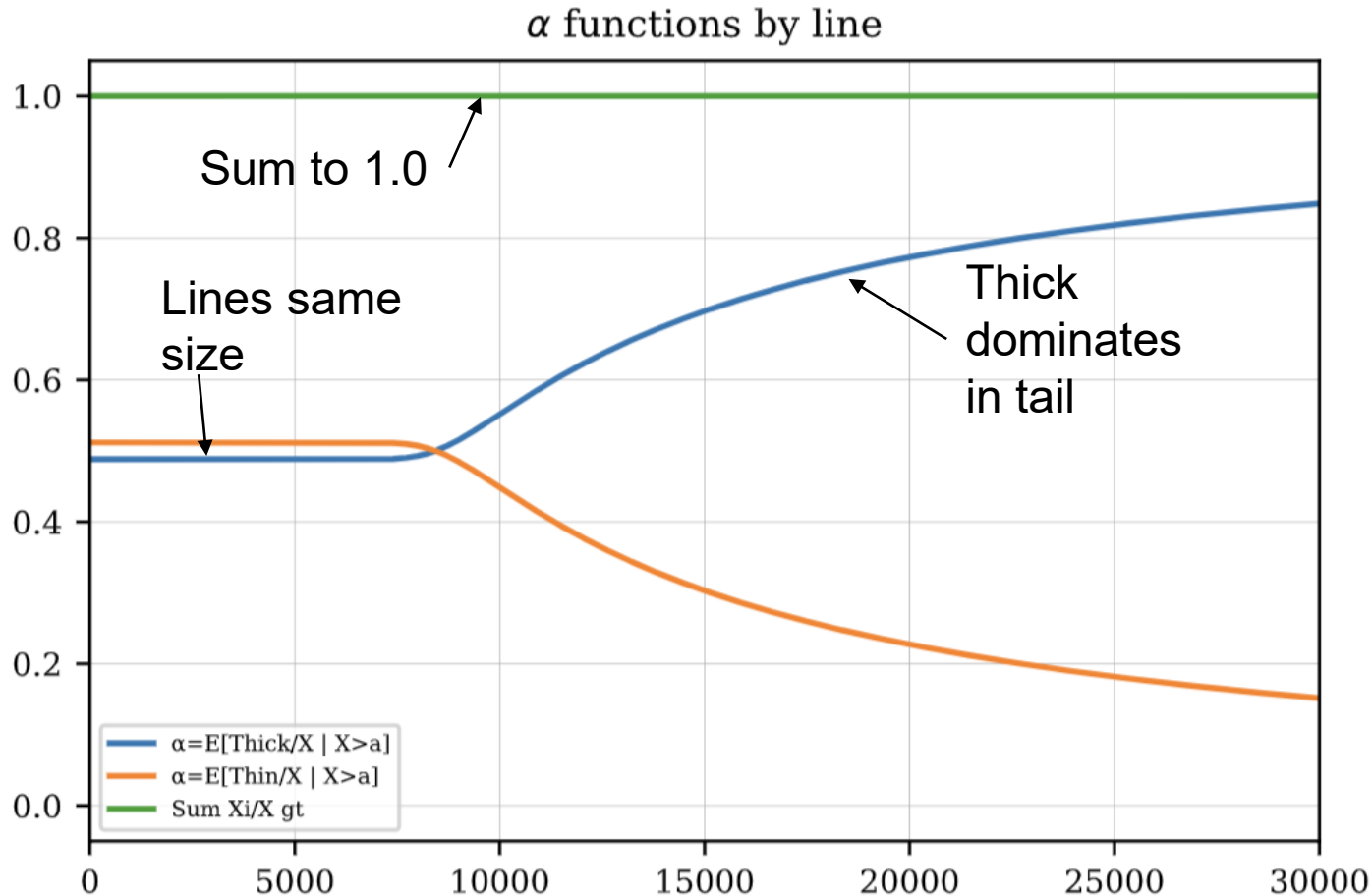# Example: Thick and Thin two-line model

- Lines independent, convenience only
- Lines same size, each has EL = 5000
- Line CVs are 36% and 10%, overall CV = 18.9%
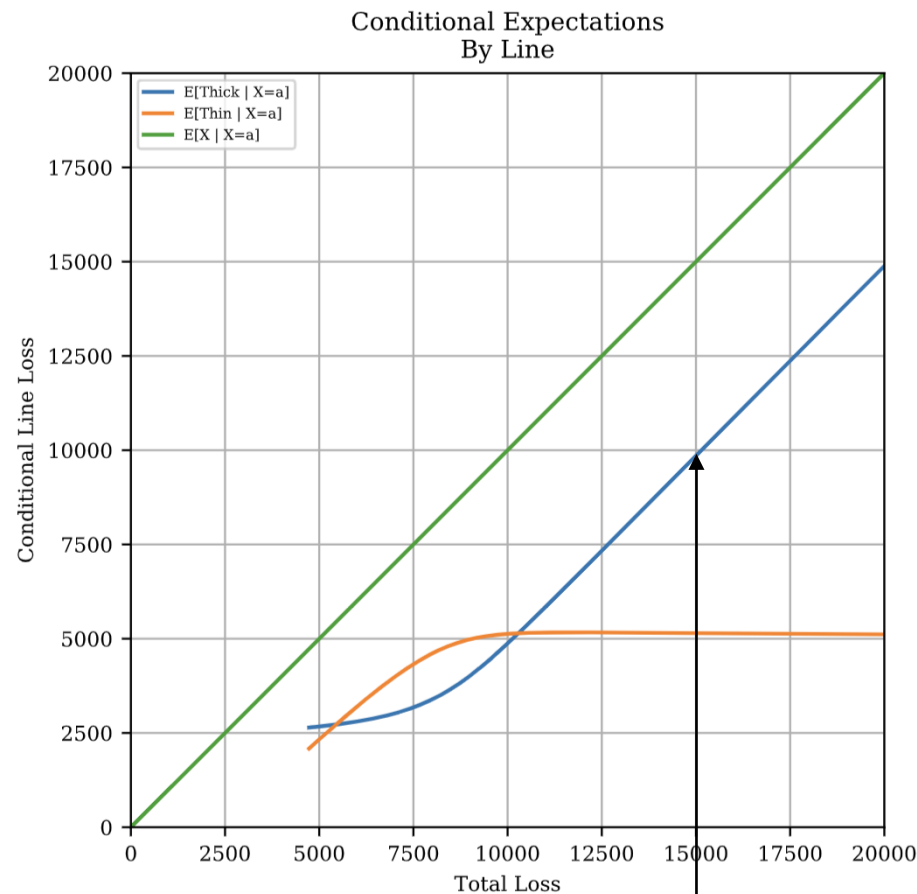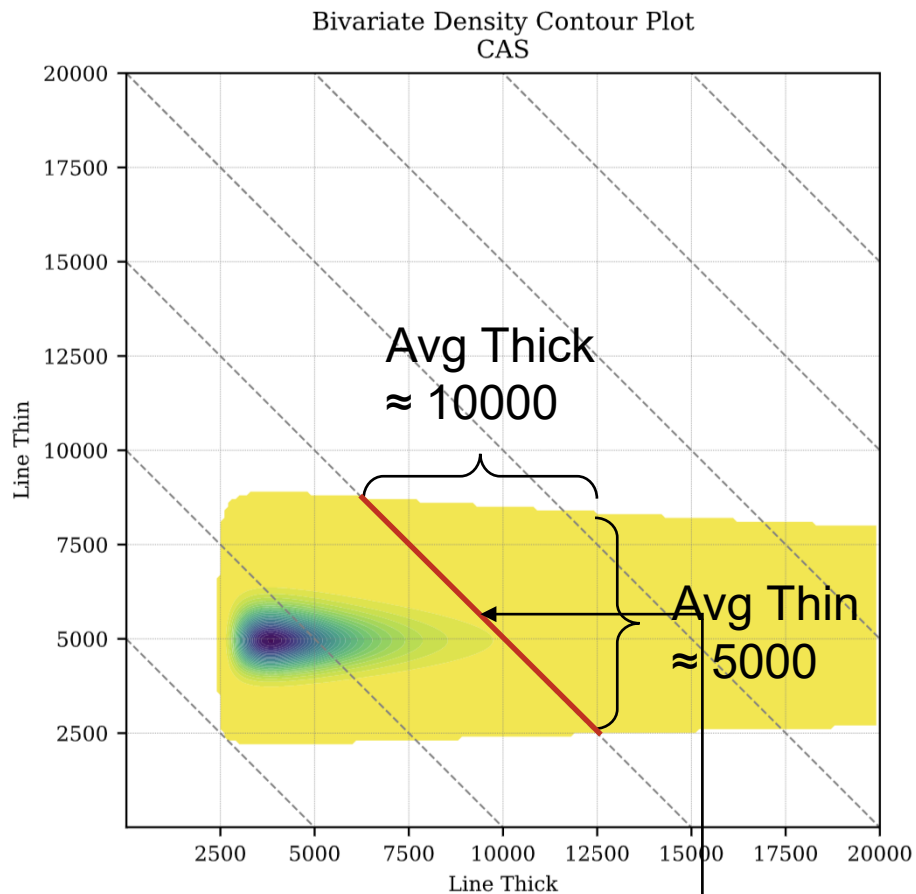- Pricing: Wang distortion to 10% ROE at 20,000 assets, LR = 91.7%

alpha function: calculates expected loss by line

- $\alpha_i(x) = E[X_i / X \mid X > x]$ as a function of x

α functions by line

$$\alpha_i(x)S(x) = \int_x^\infty \frac{E[X_i \,|\, X = t]}{t} f_X(t)dt$$

$E[X_i \,|\, X=x]$: building block function for alpha and beta

Bivariate Density Contour Plot
CAS

Avg Thick ≈ 10000

Avg Thin ≈ 5000

Conditional Expectations By Line

E[Thick | X=a]
E[Thin | X=a]
E[X | X=a]

Conditional on X = 15000
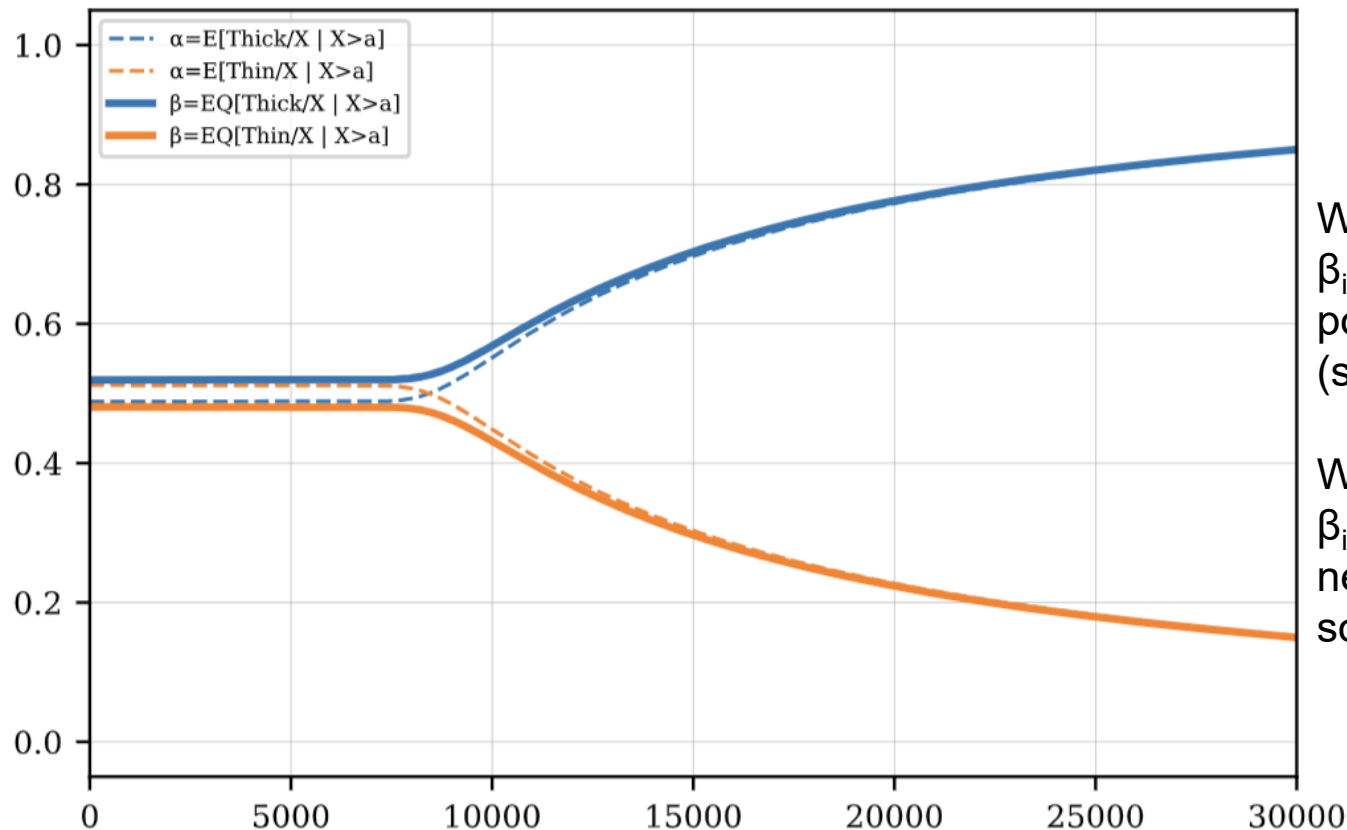
beta function: calculates premium by line

- $\beta_i(x) = E_g[X_i / X \mid X > x]$ (solid line) calculates premium

- Risk adjusted version of α, putting more weight on right tail

β functions by line

Legend:
- - - α=E[Thick/X | X>a]
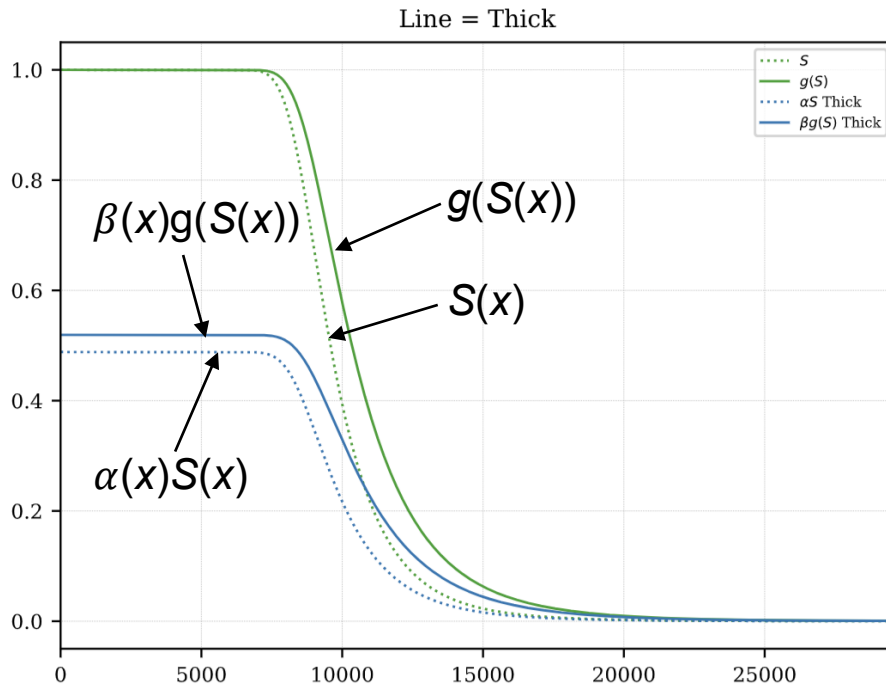- - - α=E[Thin/X | X>a]
— β=EQ[Thick/X | X>a]
— β=EQ[Thin/X | X>a]

When $\alpha_i(x)$ **inc**reases $\beta_i(x)$ is **above** $\alpha_i(x)$, positive margins = **Thick** (solid above dashed)

When $\alpha_i(x)$ **dec**reases $\beta_i(x)$ is **below** $\alpha_i(x)$, negative margins for some layers = **Thin**

Margins by asset layer, by line and tail behavior

Line = Thick

Line = Thin

β(x)g(S(x))

g(S(x))

S(x)

α(x)S(x)

Negative margins

Thick...α$_i$(x) **inc**reases...β$_i$(x) **above** α$_i$(x)

β$_i$(x)g(S(x)) **above** α$_i$(x)S(x) since g(S)>S

Positive margins at all layers of capital
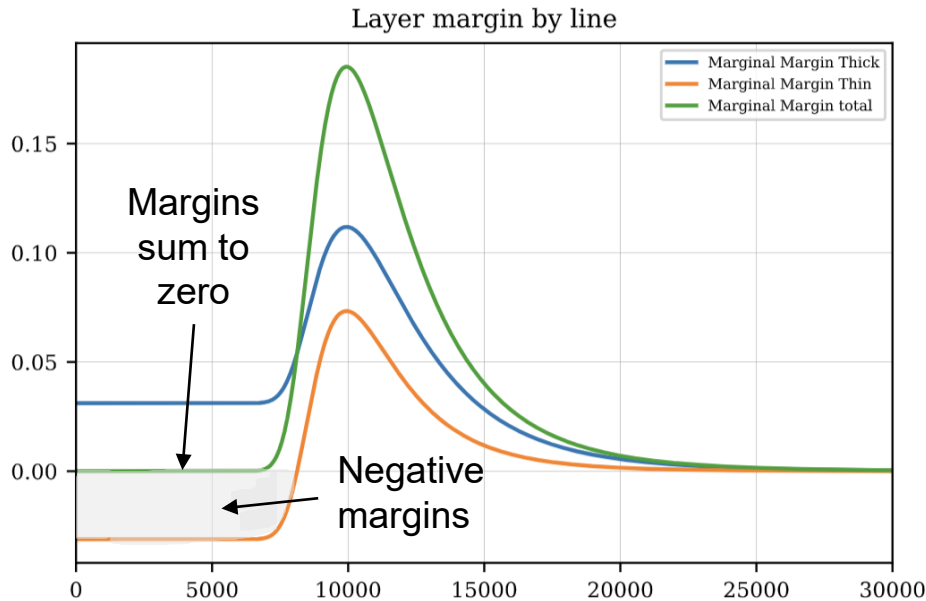
Thin...α$_i$(x) **dec**reases...β$_i$(x) **below** α$_i$(x)
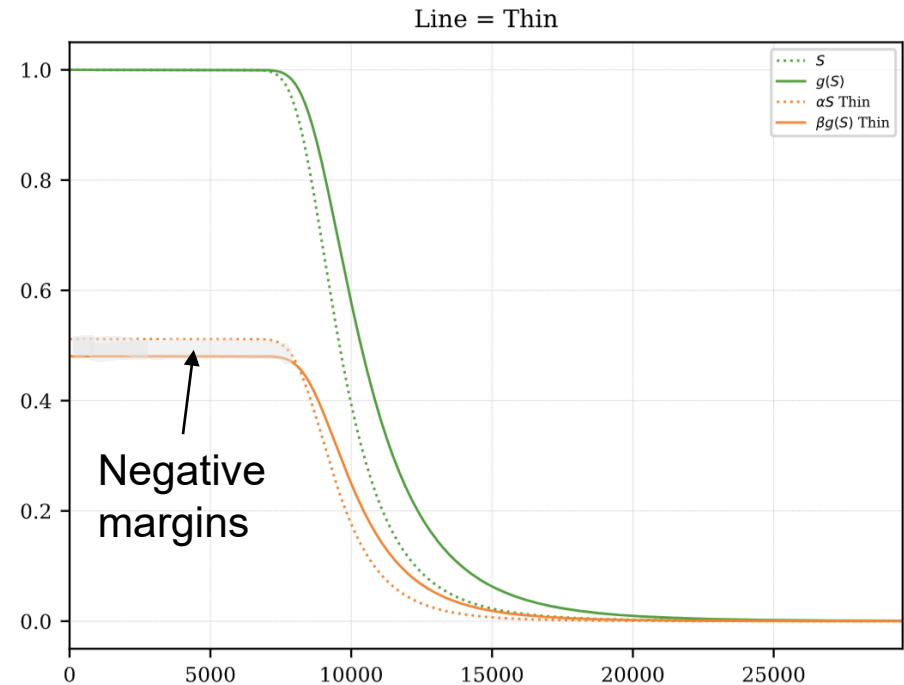
β$_i$(x)g(S(x)) may be **below** α$_i$(x)S(x)

Possible negative margins for low layers, g(1) = 1, and lower overall margin

Margin by asset layer, by line

Layer Margin Density = Solid - Dashed
$\beta_i(x)g(S(x)) - \alpha_i(x)S(x)$

Layer margin by line

Legend:
- Marginal Margin Thick
- Marginal Margin Thin
- Marginal Margin total

Margins sum to zero

Negative margins

Cumulative Margin
Integral of margin density

Total margin by line

Legend:
- Total Margin Thick
- Total Margin Thin
- Total Margin total

910
844
66

- Thick gains by pooling with thin in low layers

- Thick pays a positive margin to compensate thin for worse coverage

- Both lines benefit from better cover at high layers

- Both lines pay positive margin for incremental capital
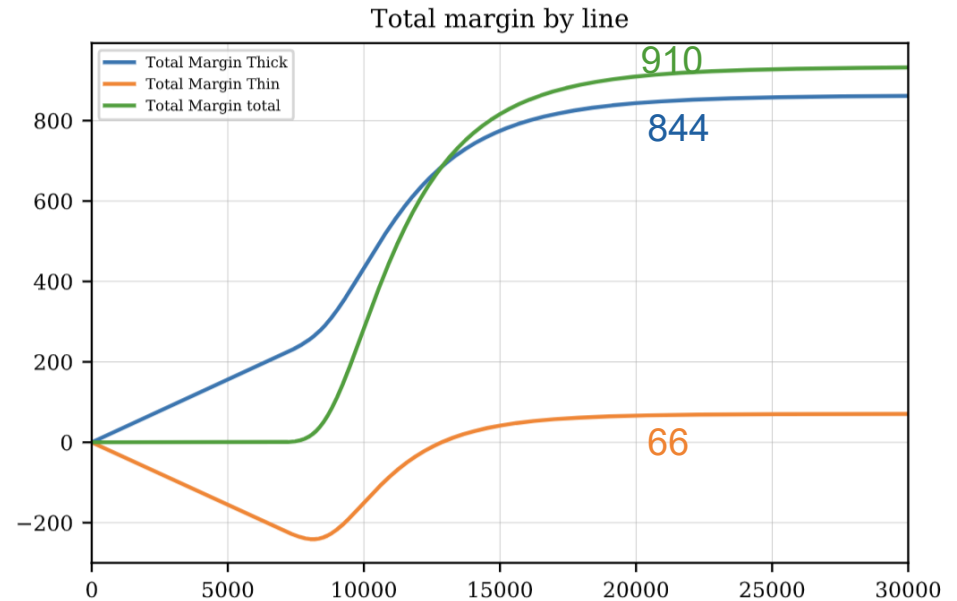
- Above 13K both lines pay positive margin but thin line cost reduced by coverage impacts of pooling with Thick

- Thin 2.5% cost of capital and thick 13.1%; overall cost calibrated to 10.0% (see appendix for details)

Pricing summary: **not** the tail wagging the dog

- Thick line double whammy
  - Higher capital need
  - Consumes more high relative cost tail capital
- Pooling helps Thick, hurts Thin
- **Margin driven by body, not default**

- Total margins (shaded, right)
  - Combined   **910**   (right)
  - Thick within total   844   (prev. sld.)
  - Thin within total   66   (prev. sld.)
  - Thick stand-alone   **872**   (right)
  - Thin stand-alone   **239**   (right)
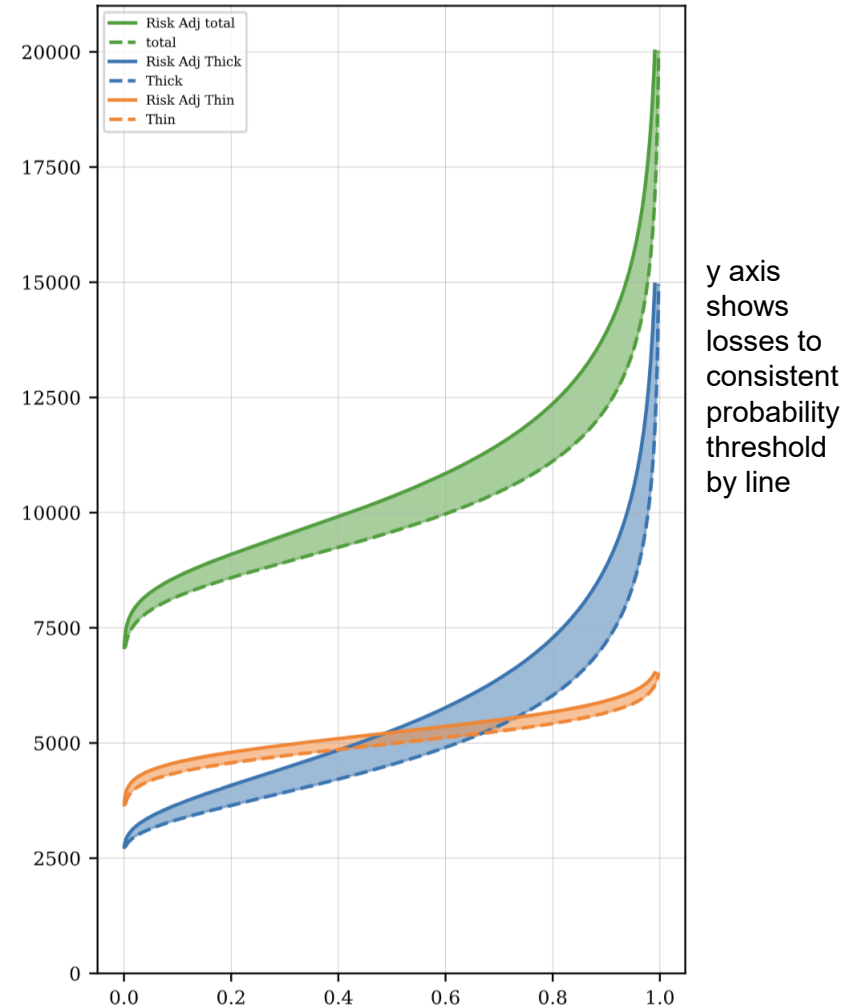
y axis shows losses to consistent probability threshold by line

Pricing summary: **not** the tail wagging the dog

- Thick line double whammy
  - Higher capital need
  - Consumes more high relative cost tail capital

- Pooling helps Thick, hurts Thin

- **Margin driven by body, not default**

- Adding Thin line hardly changes shape or area of Thick line margin!

- Thick blue, translated up by 5000, expected loss for Thin, is almost the same total, green

- Adding thin ≈ adding constant loss

y axis shows losses to consistent probability threshold by line
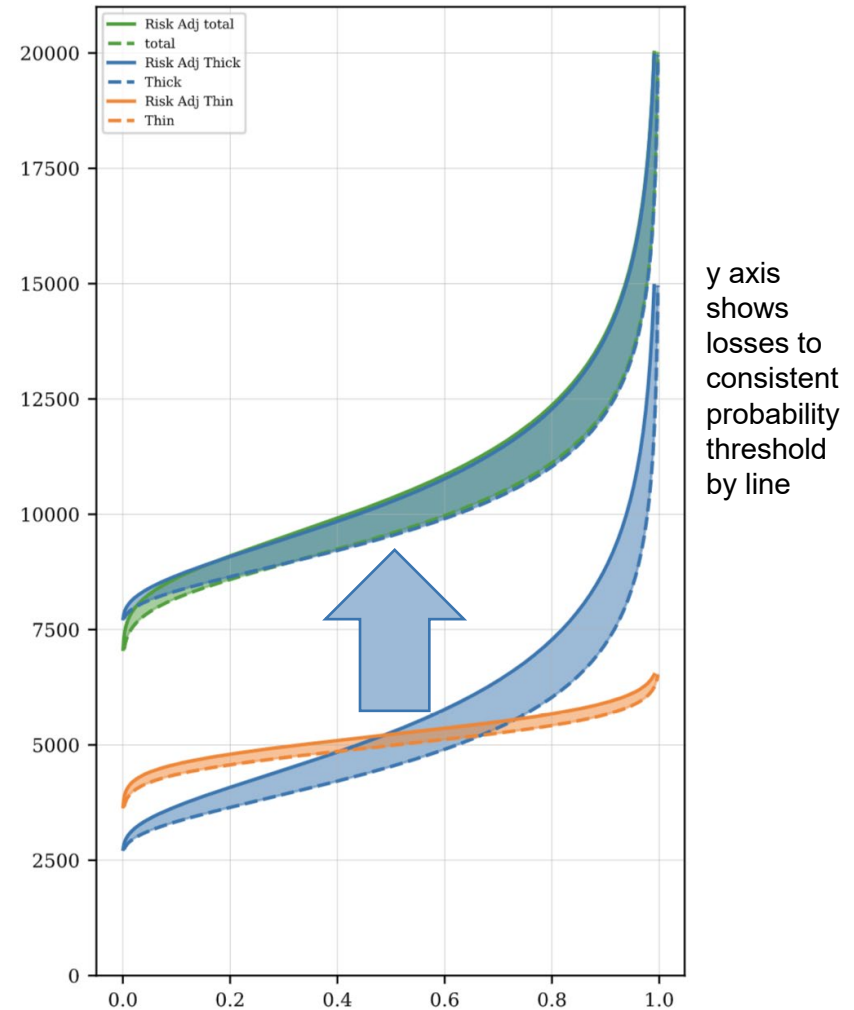
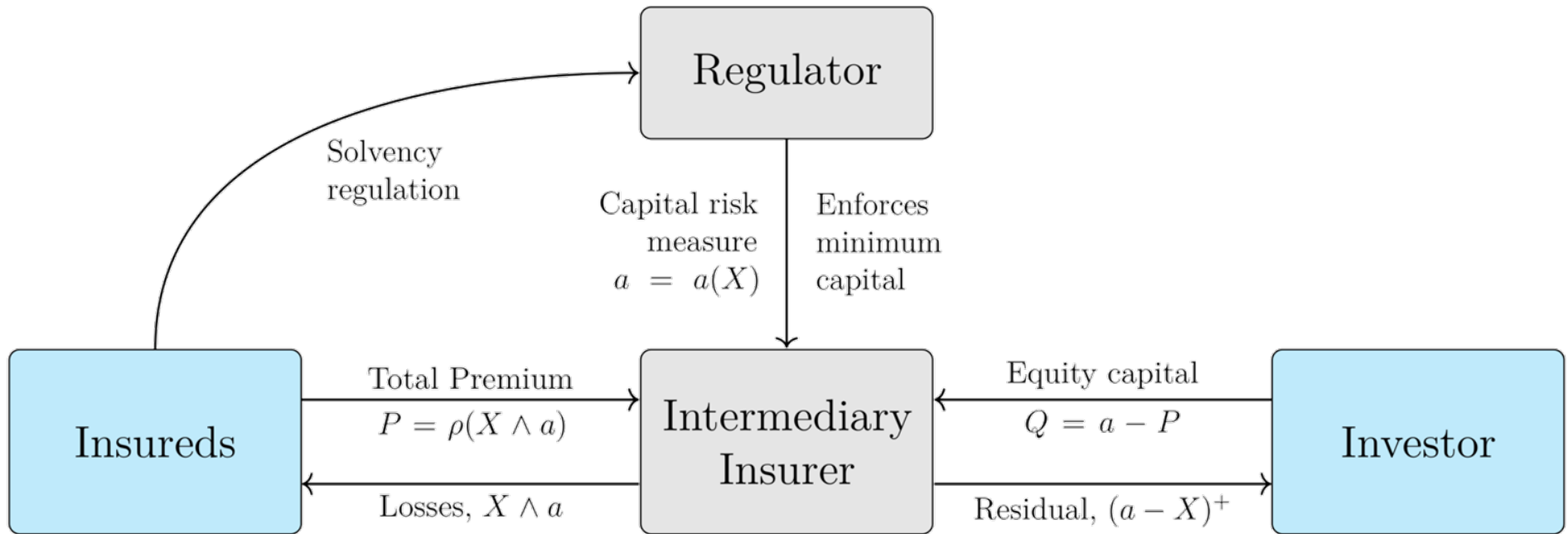## Where to find thin-tailed business?

Reserves!

Section 4: Market Structure Implications

Why is Florida homeowners written in monoline companies?

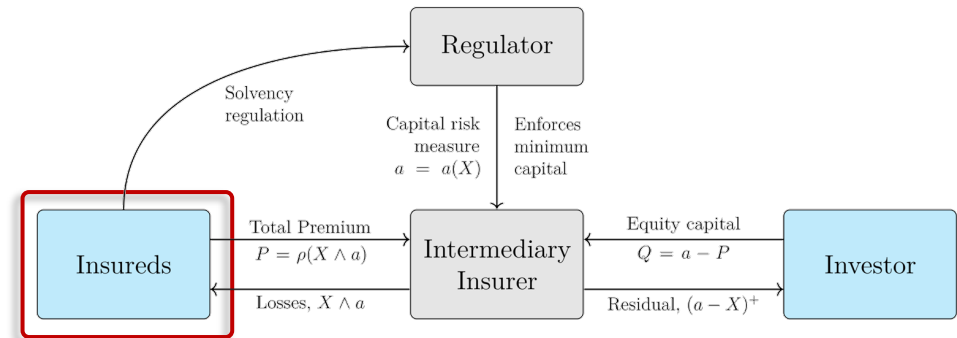# Four actors and their interactions

- One-period model, no expenses, no investment income, no taxes; risk transfer and not risk pooling = **no frictional costs**

Insured loss distributions

- **Two classes (lines) of insured**
  - X_0 Low-risk class: high frequency, low severity; **Illinois auto**
 - X_1 High-risk class: catastrophe exposed; **Florida home**

- Risk is a characteristic of class and not the individual insured

- Homogeneous loss model: distribution scales, no shape change
  - Results for a sub-pool of a class are proportional to the results for whole class, model loss ratio

Insured buying behavior

- **Face mandatory / quasi-mandatory insurance requirement**
  - 60% of premium (Aon Benfield, 2015)
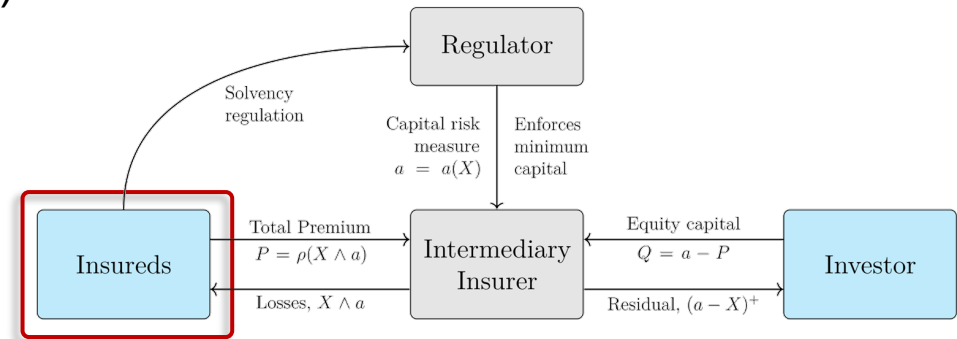
- Mandate is for third-party protection
  - Insureds do not care about insurer solvency, provided policy satisfies mandatory requirement

- Insureds are pure price buyers

Regulator

- **Solvency regulation necessary to ensure effectiveness of mandatory insurance**
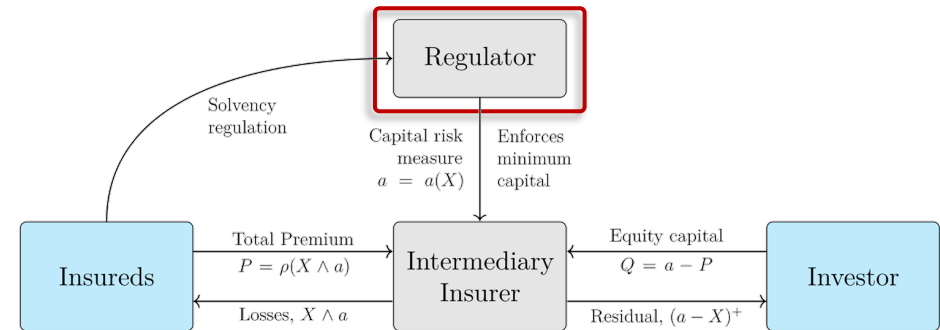
Incorporeal: regulator is a formula

- Regulatory capital standard risk functional a = a(X) = a(total risk)
  - **Value at Risk** (VaR) or tail value at risk

- No other regulation beyond capital standard

Intermediary insurer or pool

- **"Smart contract" incorporeal insurer or risk pool like a cat bond**
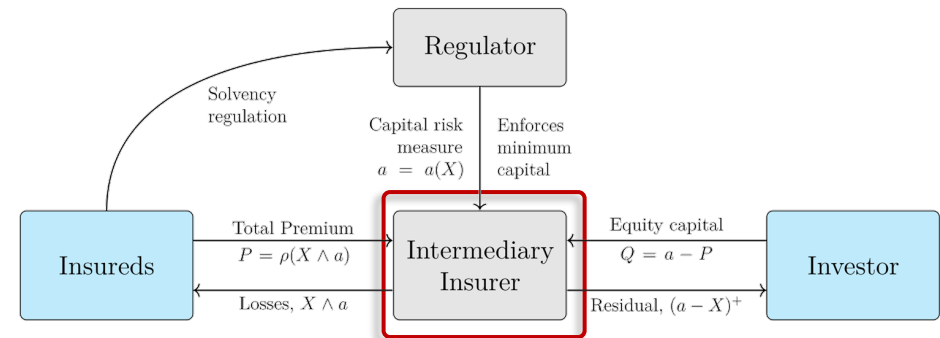
Incorporeal: insurer is a formula

- No frictional cost for investor to hold assets in insurer
  - No transaction costs, no taxes
  - No management: no principle-agent problems
  - Minimal regulation, no trapped capital

Investor: ultimate risk bearer

- **Ambiguity averse but not necessarily risk averse**
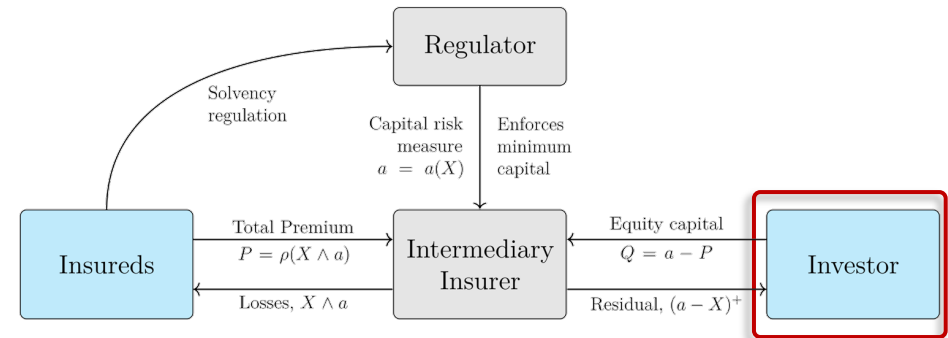
- Investors price using a **distortion risk measure** ρ, which prices any distribution X as ρ(X)
  - Use a spectral risk measure for ρ

How will risks pool?

- Monoline pools on the same class can merge by homogeneity

- There are only three possible market structures
  - Full pooling: one insurer
  - Two monoline insurers
  - One multiline pool insurer and one monoline insurer

- **Market defined by proportion t of risk class 1 in the pool, $0 \leq t \leq 1$,** and

$t = 0, 1$	two monoline pools
$t = 0.5$	full pooling
$0 < t < 0.5$	class 0 fully pooled, class 1 split between pool and monoline
$0.5 < t < 1$	class 1 fully pooled, class 0 split between pool and monoline

Example

Rate by Line and Total
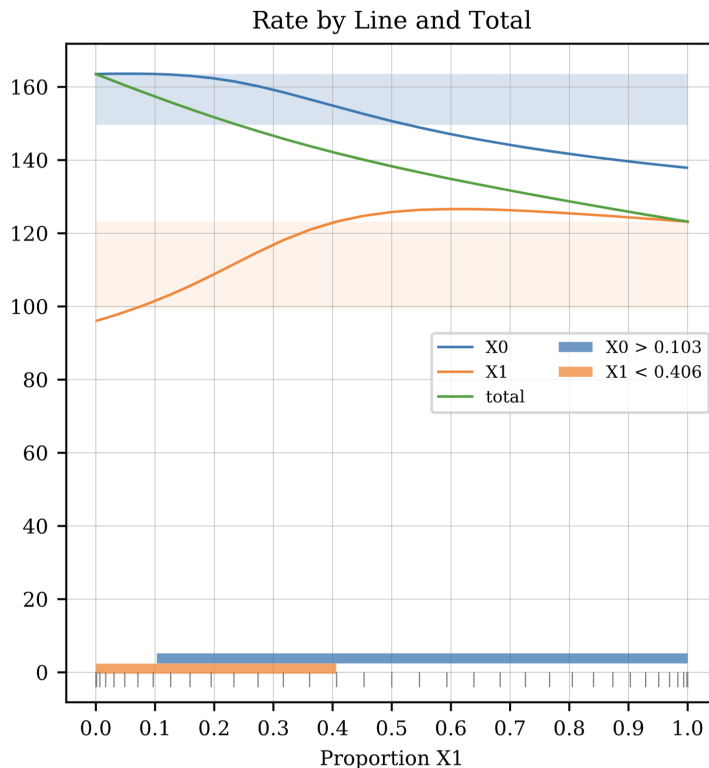
Assumptions
- Losses gamma distribution with CV 0.1 and 0.25
- Proportional hazard rho with 0.3 parameter
- Capital standard: 95% value at risk

- t, the proportion of X_1, on x-axis
- Lines show **rate** for each line
  - **Blue X_0 low, orange X_1 high risk**
  - Green: blended pool rate
- Expected unlimited loss, before insurer default
  - $X_0 = 150$
  - $X_1 = 100$
- Shaded bands at top show range from monoline loss cost and premium for each line
- Expensive pricing, weak capital standard

Example: partial pooling equilibrium solution

Rate by Line and Total
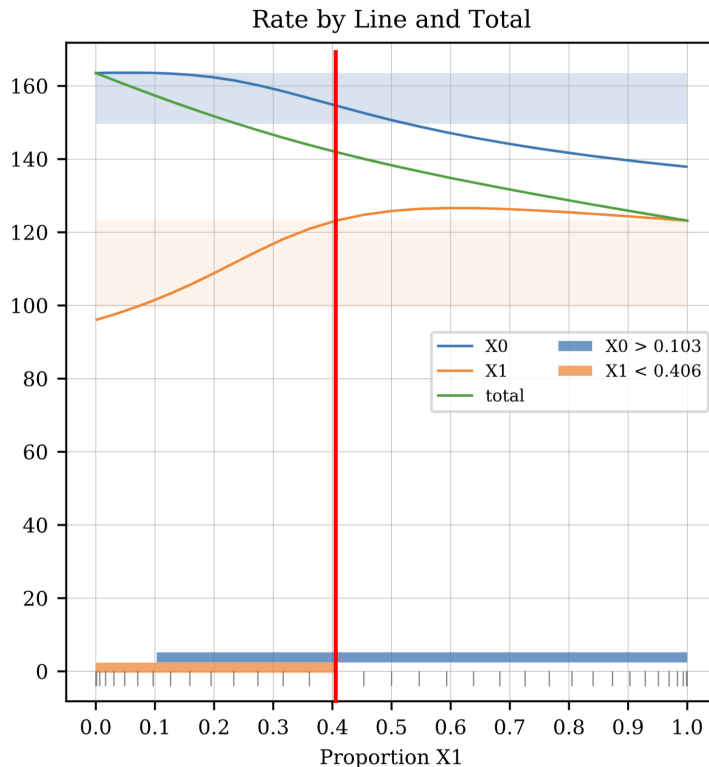
Hence Florida home-owners not fully pooled

- Equilibrium solution
  - **X_0 and 2/3rds of X_1 are pooled; remaining 1/3rd of X_1 written monoline, $t = 0.4$**

- Why?
  - $t > 0.4$: X_1 rate greater than monoline...X_1 will not pool
  - $t < 0.4$: X_1 insureds in pool get below monoline rate, with remainder monoline
  - Remainder will offer to pool with X_0 at slightly higher rate until equilibrium reached at $t = 0.4$
  - **X_1 pays monoline rate and X_0 captures all diversification benefit**

Example: why orange rate line bows up

Rate by Line and Total
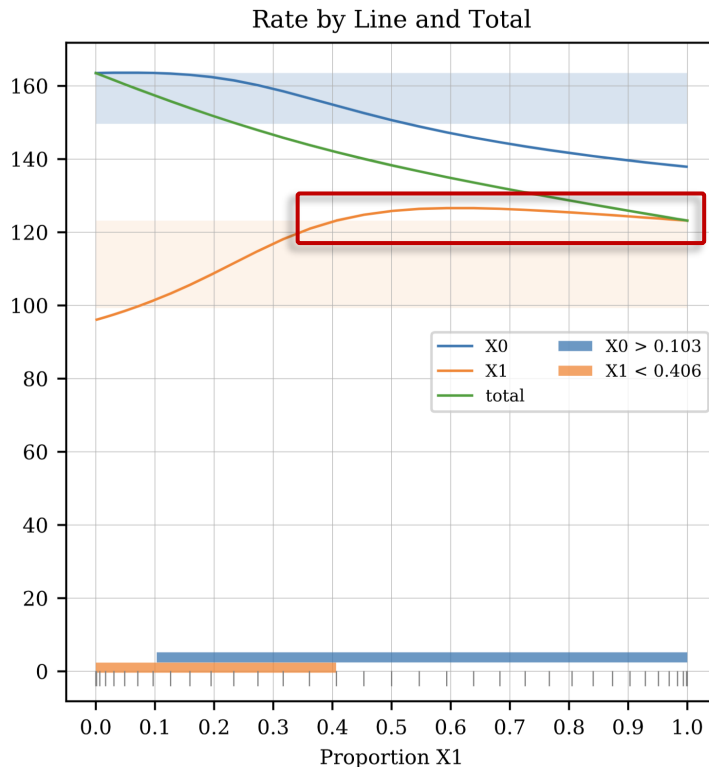
- Adding small amount of X_0 to X_1 advantages X_1
  - Small amount of X_0 like adding a constant liability (slide 25)
  - X_1 thicker tailed...more likely to "cause" insolvency
  - ...by equal priority it picks up a greater share of assets in default
- Bowing up does not occur with unlimited capital unless capital standard super-additive = green line bows up
- Two monoline pools with super-additive capital standard

Example: Full Pooling Outcome

Rate by Line and Total
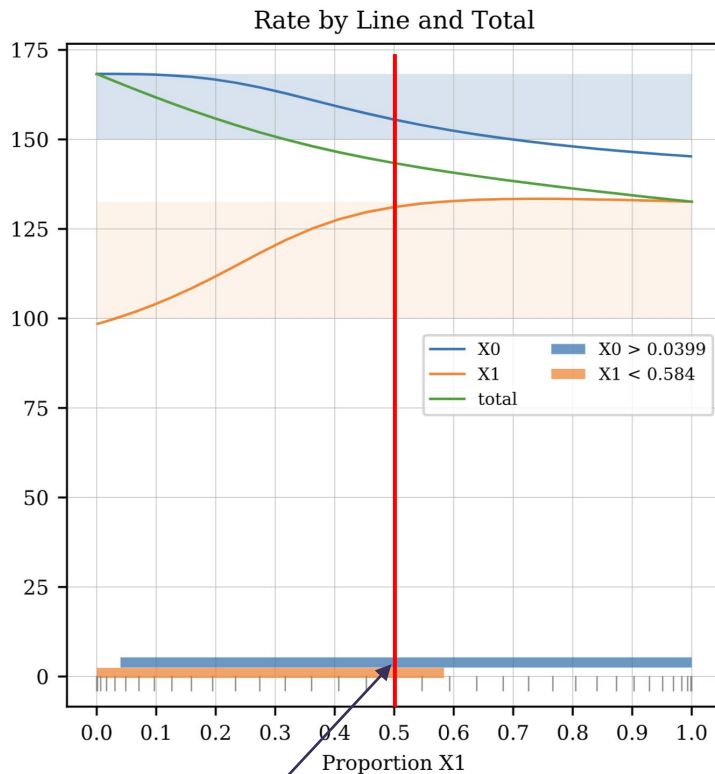
Feasible region overlap
includes 50/50 pool

- **When *t* = 0.5 is feasible for both lines it is an equilibrium solution**

- Why?
  - For *t* ≠ 0.5 some insureds are forced into monoline rate
  - Monoline insureds offer to pool at more advantageous rate
  - Original pool unravels

- At *t* = 0.5 all insureds pay lower multiline rate, and no rational action can cause pool to unravel

- Difference: capital standard 99.5% Value at Risk

Section 5: Conclusions

Conclusions

- Premium combines **fair value to customers** of contractual cash flows and **marginal frictional cost to insurer**

- Risk cost of capital varies by layer, line, and amount of capital in a complex manner, but can be determined without allocation

- Capital allocation needed to incorporate frictional cost of capital

- Capital standards can lead to incomplete pooling, e.g., Florida HO

- Additional resources
  - Introductory videos: http://go.guycarp.com/cas2018
  - Paper with details: https://arxiv.org/abs/2008.12427
  - Forthcoming book *Pricing Insurance Risk* (Wiley) due Summer 2021
  - aggregate Software: https://aggregate.readthedocs.io/

Appendix

Audit statistics and pricing summary

	Thick	Thin	total
Mean	5000	5000	10000
CV	0.364418	0.101493	0.189144
Skew	2.40723	0.158277	2.1551
EmpSkew	2.4055	0.158277	2.15259
P99.0	11645	6240	16712
P99.5	13212	6384	18274
P99.99	24537	7067	29578
MeanErr	-4.73138e-07	-1.22125e-15	-4.88351e-07
CVErr	-2.41911e-05	2.28706e-14	-3.92099e-05

line stat	Thick	Thin	total
EPD	0.001107	0.00027622	0.00069138
Loss	4994.5	4998.6	9993.1
Loss Ratio	0.85552	0.98691	0.91656
Margin	843.44	66.28	909.72
Premium	5837.9	5064.9	10903
P/S Ratio	0.90642	1.9066	1.1985
Equity	6440.6	2656.6	9097.2
ROE	0.13096	0.02495	0.1

- Example produced using aggregate Python package https://github.com/mynl/aggregate
- pip install aggregate
- Aggregate portfolio specification:

- Pricing results calibrated to 10% return at 20000 assets, p=0.997, using a Wang transform
- $P + Q$ = 10903 + 9097 = 20000
- $(P - L) / Q$ = (10903 – 9993) / 9097 = 0.1

```
port CAS
        agg Thick 5000 loss 100 x 0 sev lognorm 10 cv 20 mixed sig 0.35 0.6
        agg Thin  5000 loss 100 x 0 sev lognorm 10 cv 20 poisson
```

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Graphic note: County size scaled to AAL estimates for hurricane, earthquake and severe weather using Gastner & Newman algorithm