Temporal Asymmetry of Lagrangian Coherent Structures

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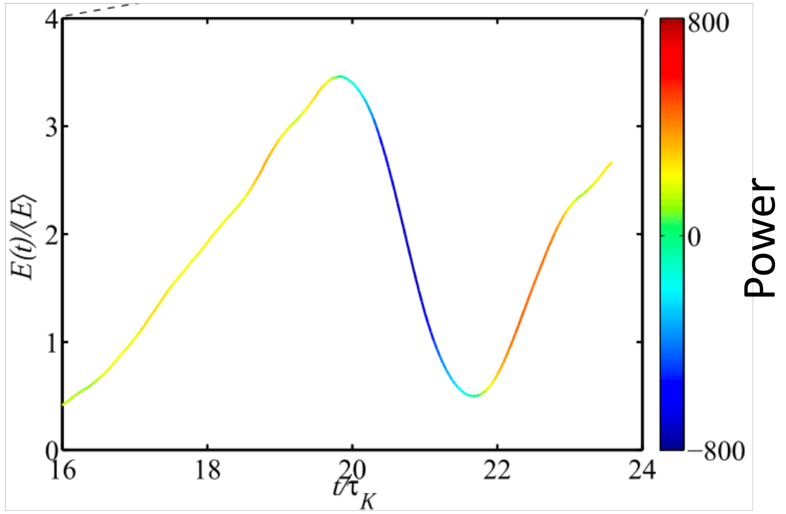
Time-reversibility in fluid dynamics

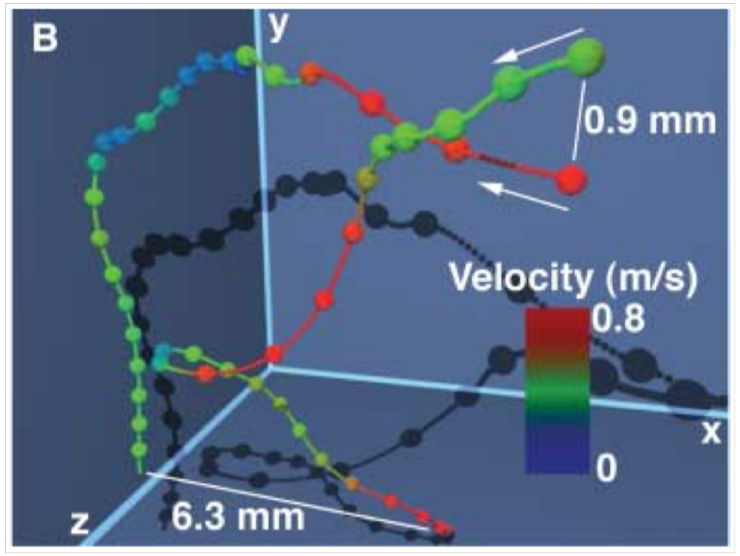
(G. I. Taylor 1960)

Time-asymmetry in fluid dynamics

- **Flight-crash events** (rapid particle deceleration)
  - Haitao et al, *PNAS* 2014



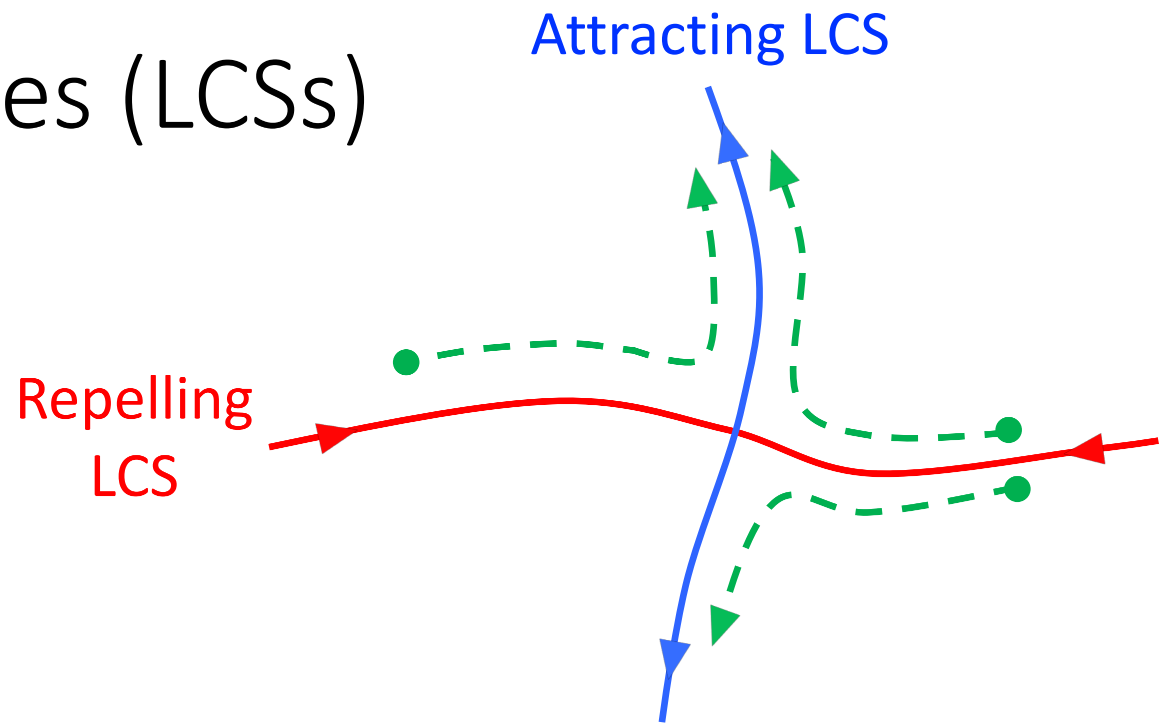
- **Pair dispersion** (in 2D, particles disperse faster in forward than backward time; opposite in 3D)
  - Faber & Vassilicos, *Phys. Fluids* 2009
  - Jucha et al, *PRL* 2014
  - Buaria et al, *Phys. Fluids* 2015
  - Bragg et al, *Phys. Fluids* 2016



Bourgoin et al, *Science* 2006

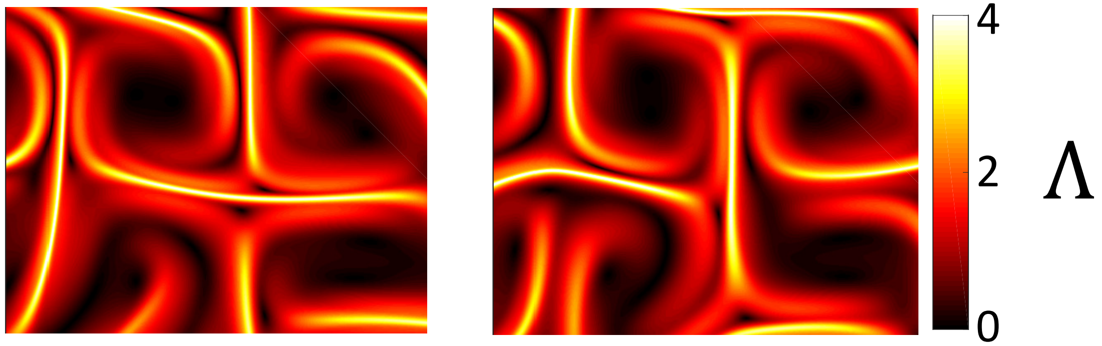
Lagrangian Coherent Structures (LCSs)

- Analogs of classical invariant manifolds for unsteady flows
- FTLE: $\Lambda(\boldsymbol{x}, t, T) = \frac{1}{T}\log(S)$, where $S = \sqrt{\lambda_{\max}}$ uses max eigenvalue of Cauchy-Green strain tensor
- ±T gives forward/backward FTLEs



Generalization of saddles defined over interval [t, t+T]

Forward FTLEs Backward FTLEs



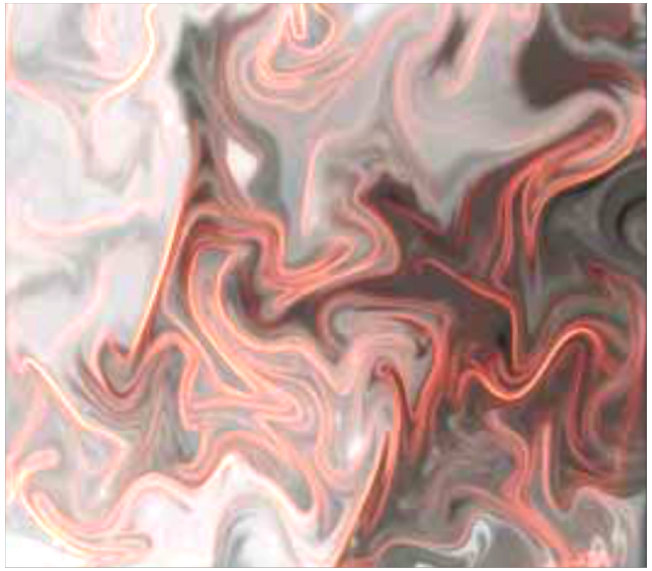
Forward FTLE ridges are most repelling material lines

Backward FTLE ridges are most attracting material lines

Lagrangian coherent structures (LCSs)

- Dominant barriers to transport in unsteady flows
  - Table-top experiments
  - Atmospheric and oceanic flows
  - Biological flows (e.g. blood)
- Review articles:
  - Peacock & Haller, *Phys. Today* 2013
  - Haller, *Ann. Rev. Fluid Mech.* 2015
  - Allshouse & Peacock, *Chaos* 2015
- Little research regarding time-asymmetry of LCS

2D flow example:
LCSs (red) guide the dye distribution

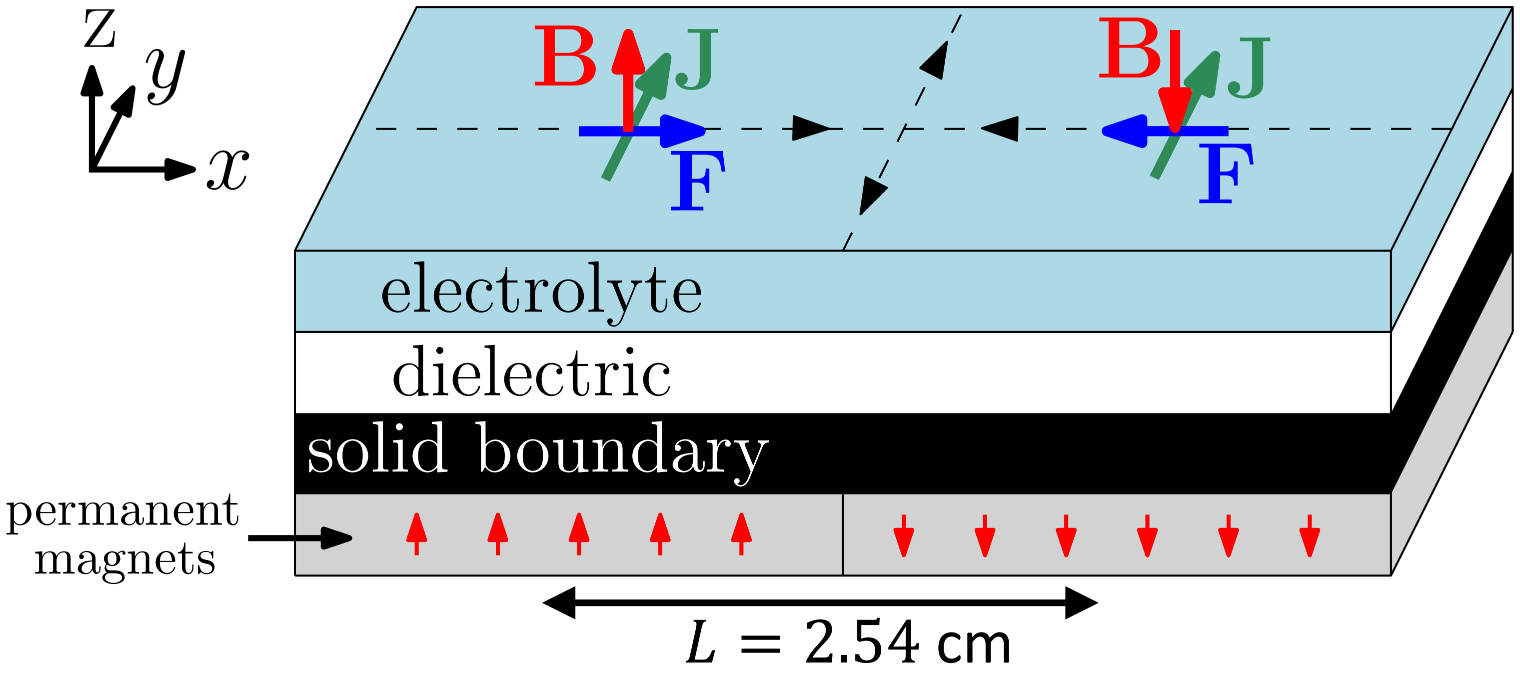


Voth, Haller, & Gollub. *PRL* 2002

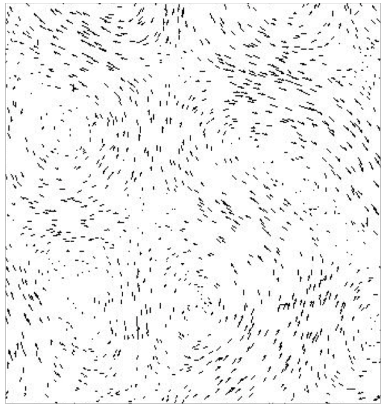
Experimental setup

Lorentz Force

$$\mathbf{F} = \mathbf{J} \times \mathbf{B}$$



Velocity measurements via particle tracking velocimetry

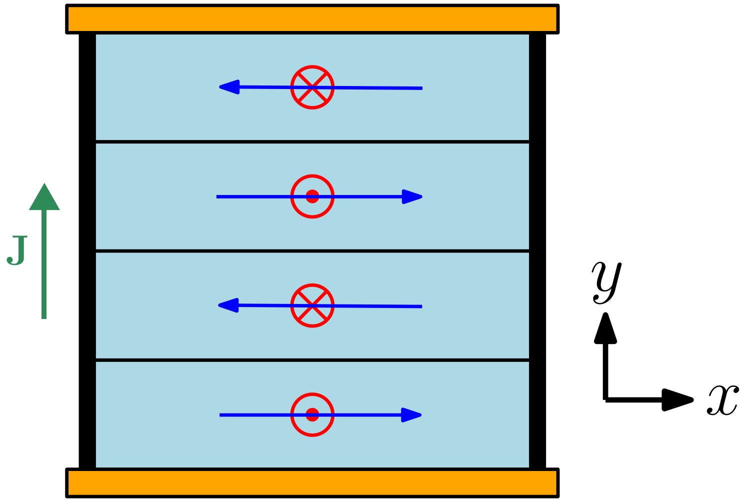


$$\mathrm{Re} = \frac{V_{rms}\, L}{\nu}$$

$$T_0 = \frac{L}{V_{\mathrm{rms}}}$$

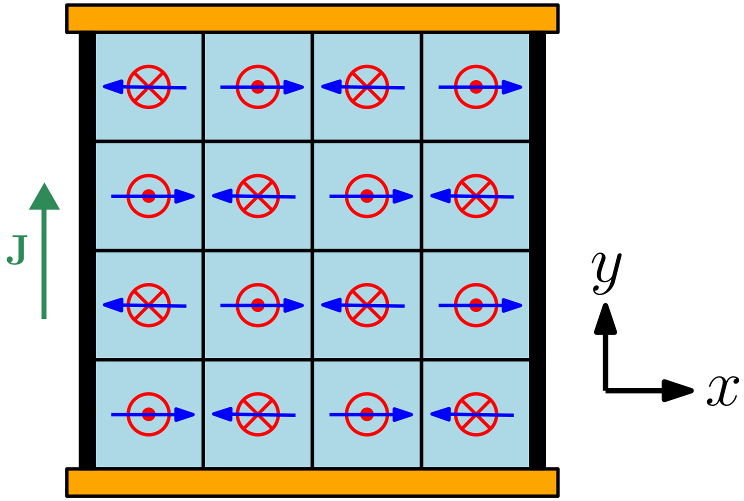
Kolmogorov flow

$$\mathrm{F} \approx A \sin(\pi y/L)\, \hat{x}$$



Checkerboard flow

$$\mathrm{F} \approx A \sin(\pi x/L) \sin(\pi y/L)\, \hat{x}$$

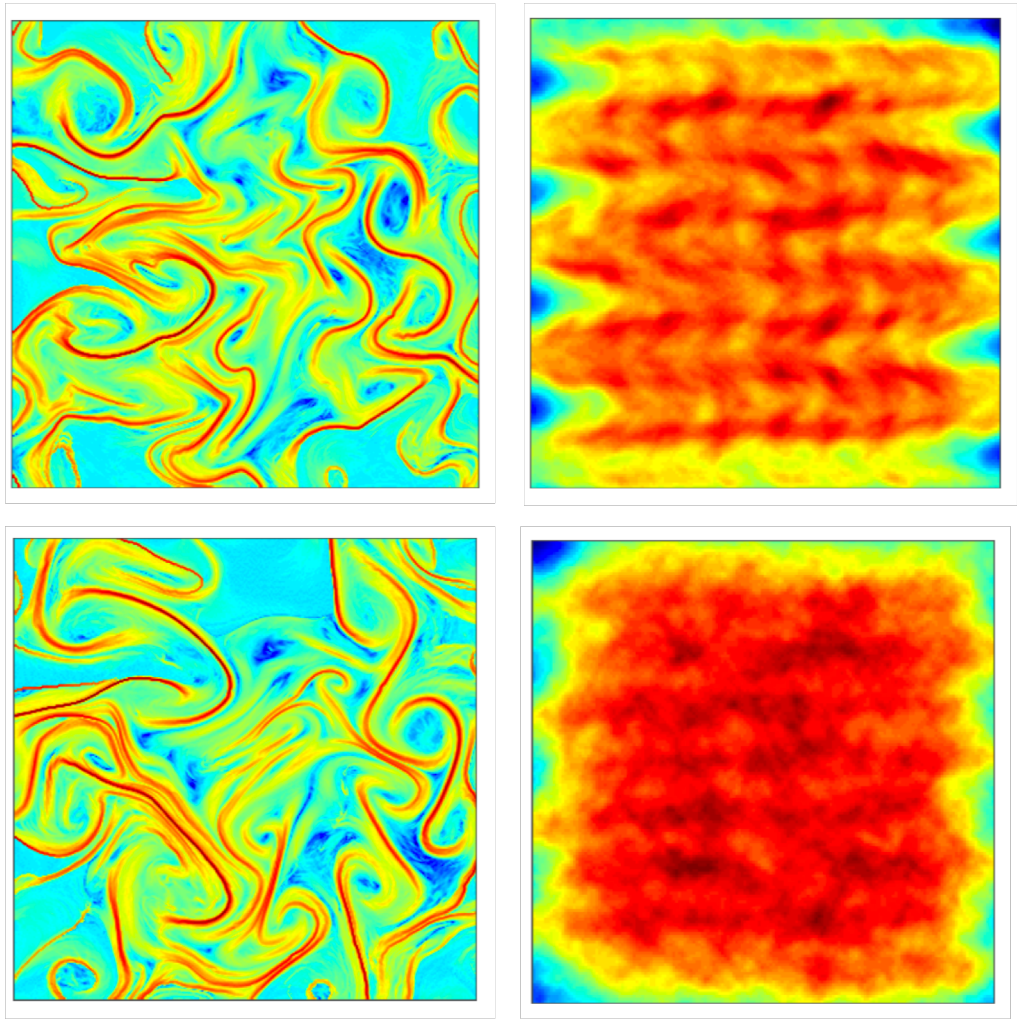


LCS Asymmetry (Ouellette et al. *Phys. Fluids* 2016)

Single FTLE field | Ensemble average

Forward FTLEs (Ridges are repelling LCSs)

Backward FTLEs (Ridges are attracting LCSs)



- Kolmogorov flow (stripes)
- Long time series at Re=235
  - 74k frames
  - 700 T_0
  - $\sim$20 min
- Clear asymmetry in the ensemble averages

Open questions about LCS asymmetry

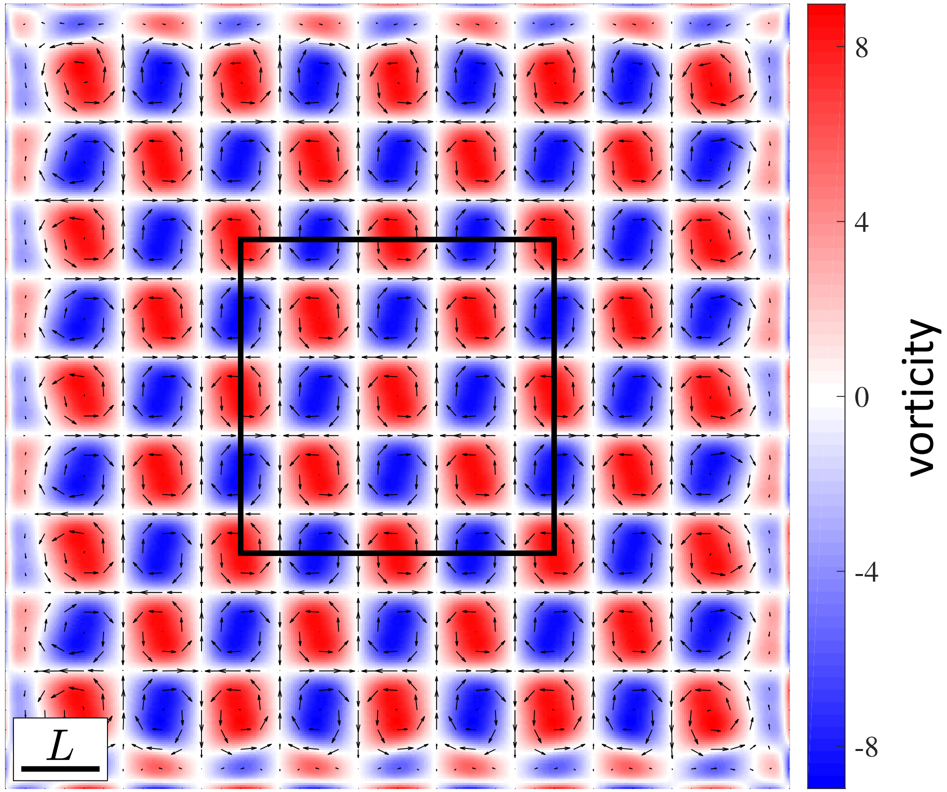
1. Present in different geometries?
2. How does it change with Re?
3. Is there a connection to other known turbulent phenomena?

Answer questions by studying the **checkerboard flow**

Use **experiments** and realistic **numerical simulations**

Realistic 2D numerical simulation

- Accurately captures details of experiment:
  - domain size
  - lateral boundary conditions (no-slip)
  - effect of velocity profile along z
  - electromagnetic forcing
- Full details:
  - Tithof et al, *J. Fluid Mech.* 2017.
  - Suri et al, *Phys. Fluids* 2014.
- Calculate LCSs in central 4x4 region for experiment and simulation

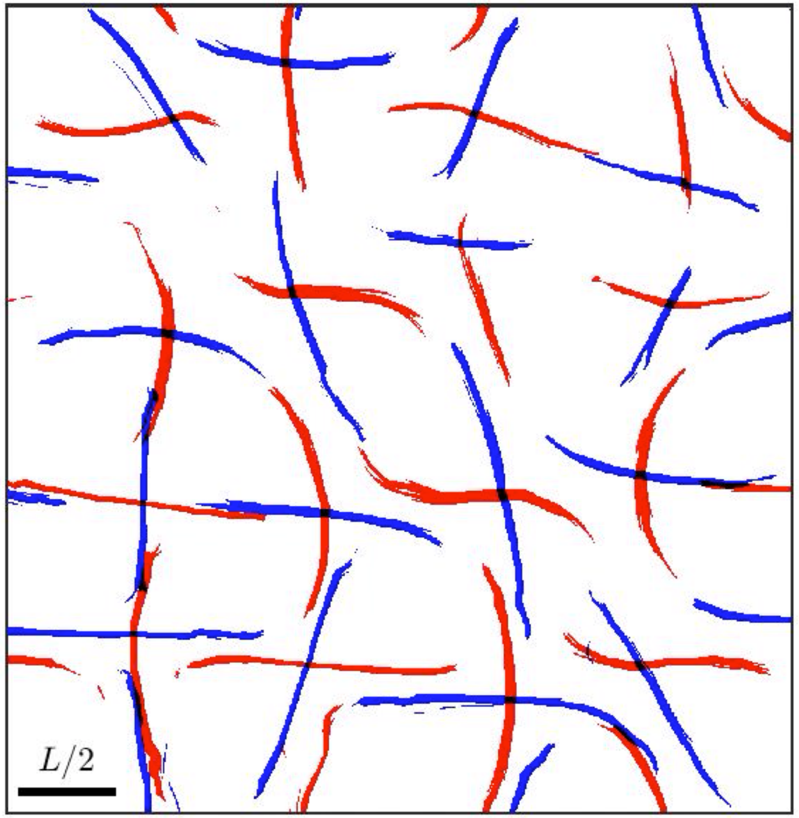


LCSs for $\mathrm{Re} \approx 130$ (Chaotic flow)

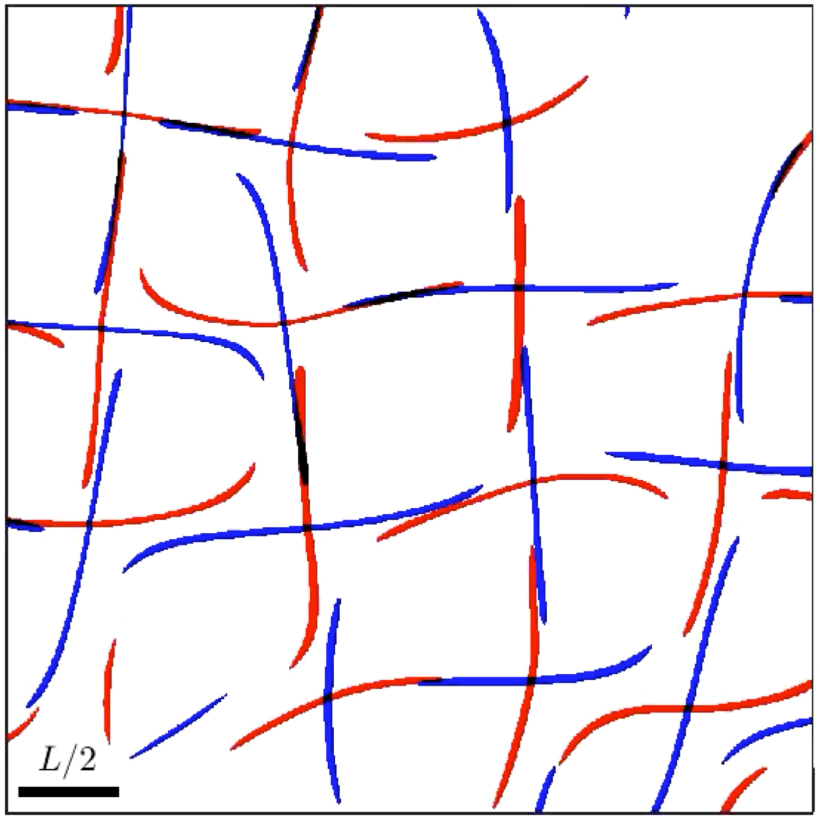
Repelling LCSs
(Forward FTLE ridges)

Attracting LCSs
(Backward FTLE ridges)

Experiment



Simulation



Ensembles computed for [t, t+T_0] as t varies

Pointwise LCS Probabilities for $\mathrm{Re} \approx 130$ (Chaotic flow)

Experiment

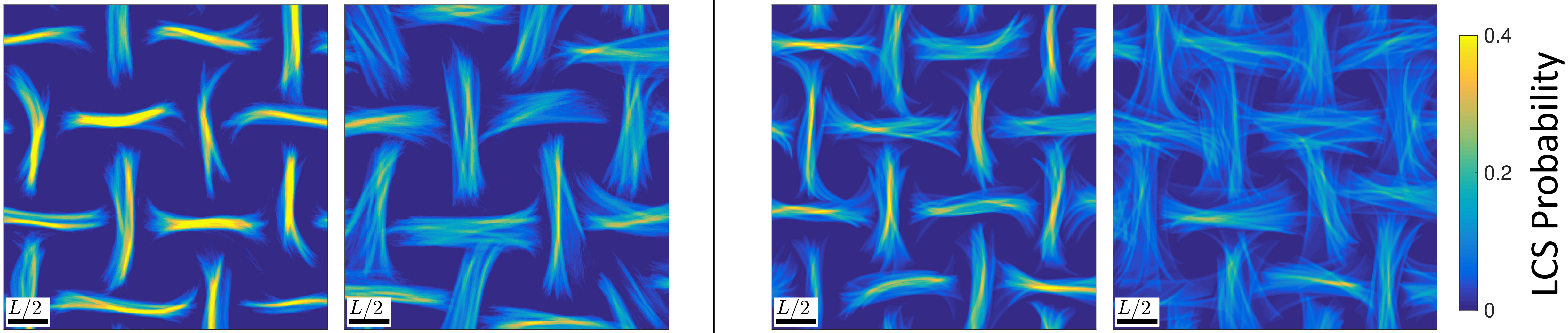
Repelling LCSs

Attracting LCSs

Simulation

Repelling LCSs

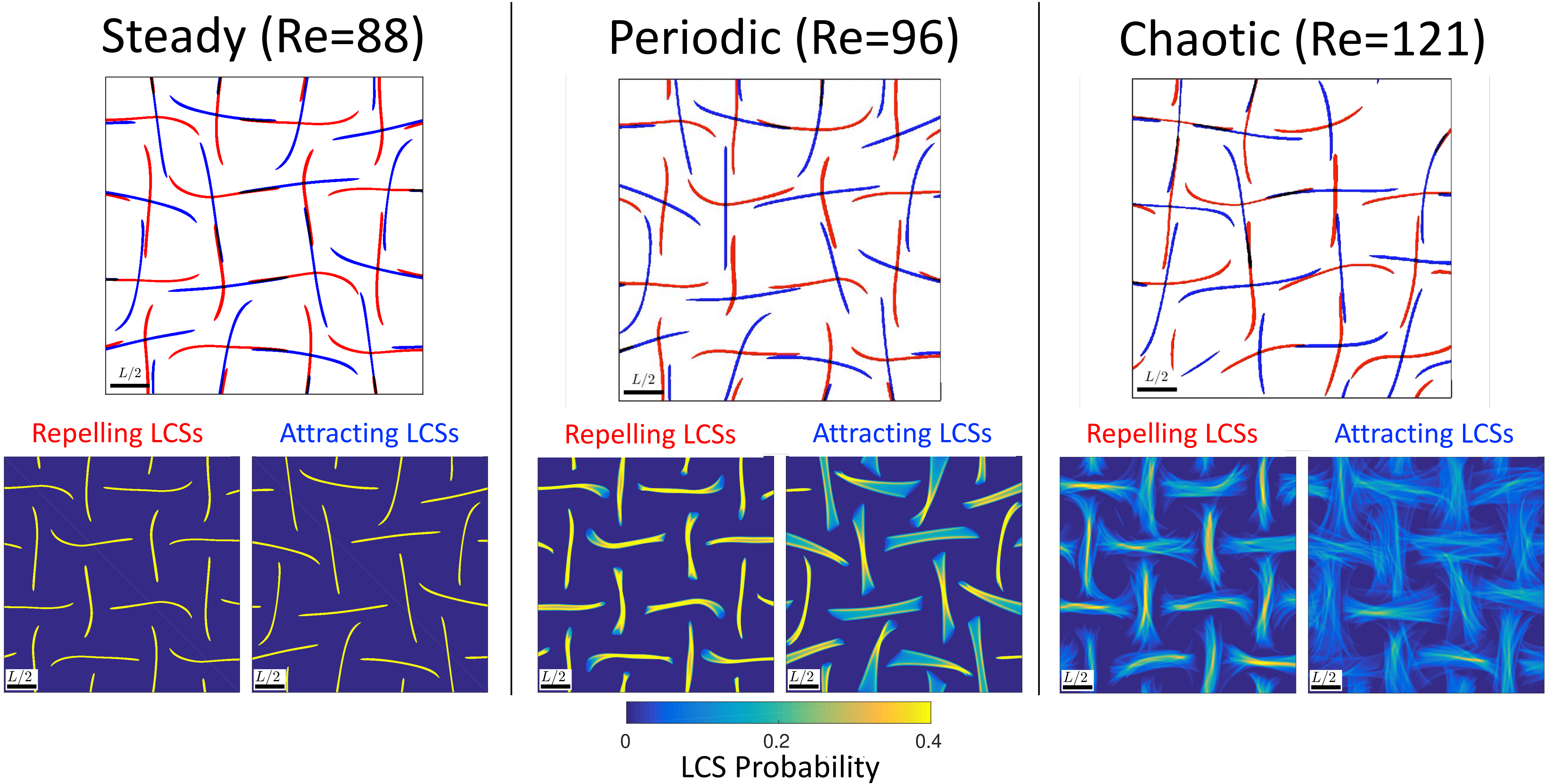
Attracting LCSs



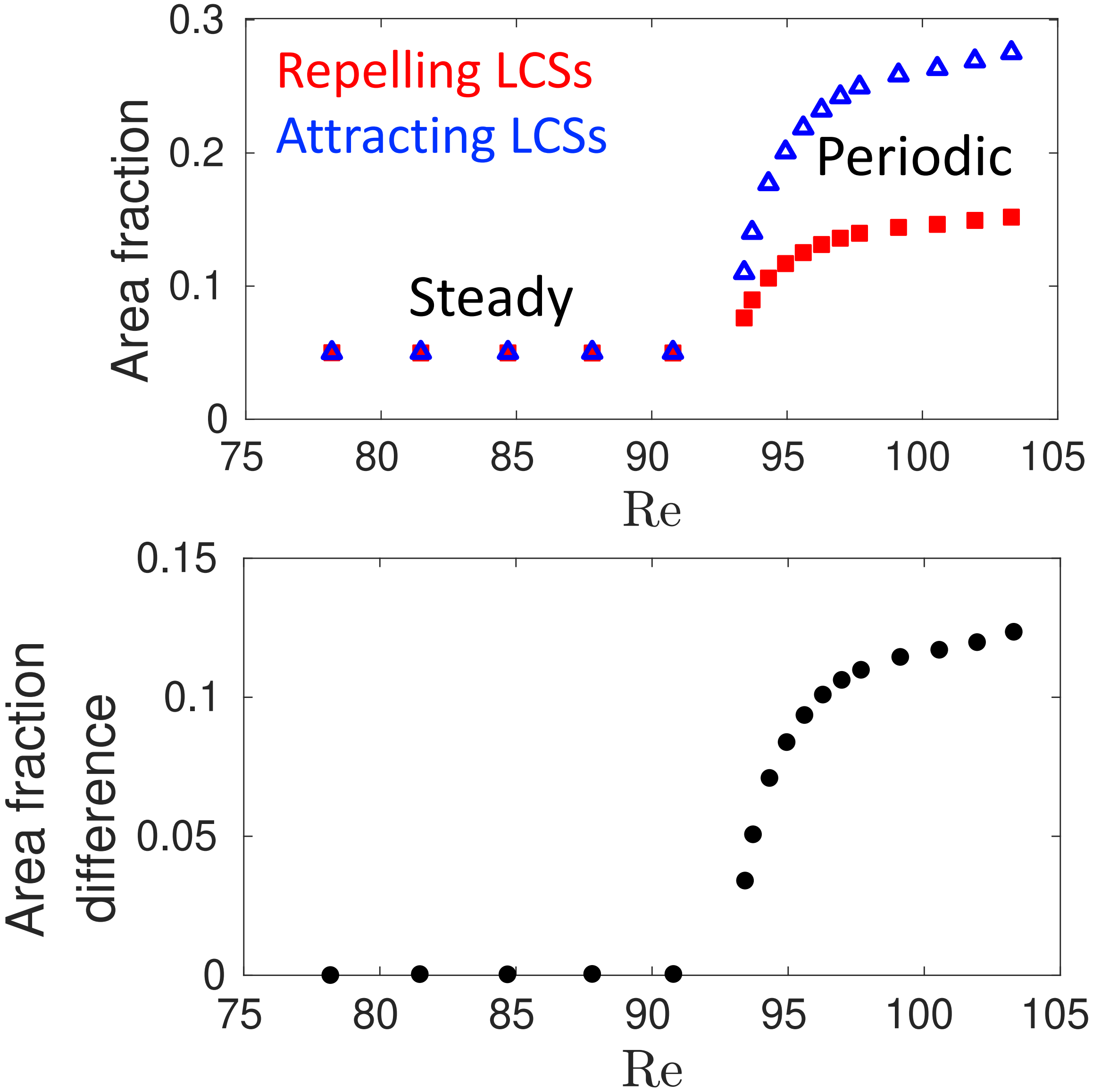
Answer to Q1: Asymmetry is also present in the checkerboard configuration

Q2: How does this asymmetry change with Re? Use simulations

Series of bifurcations



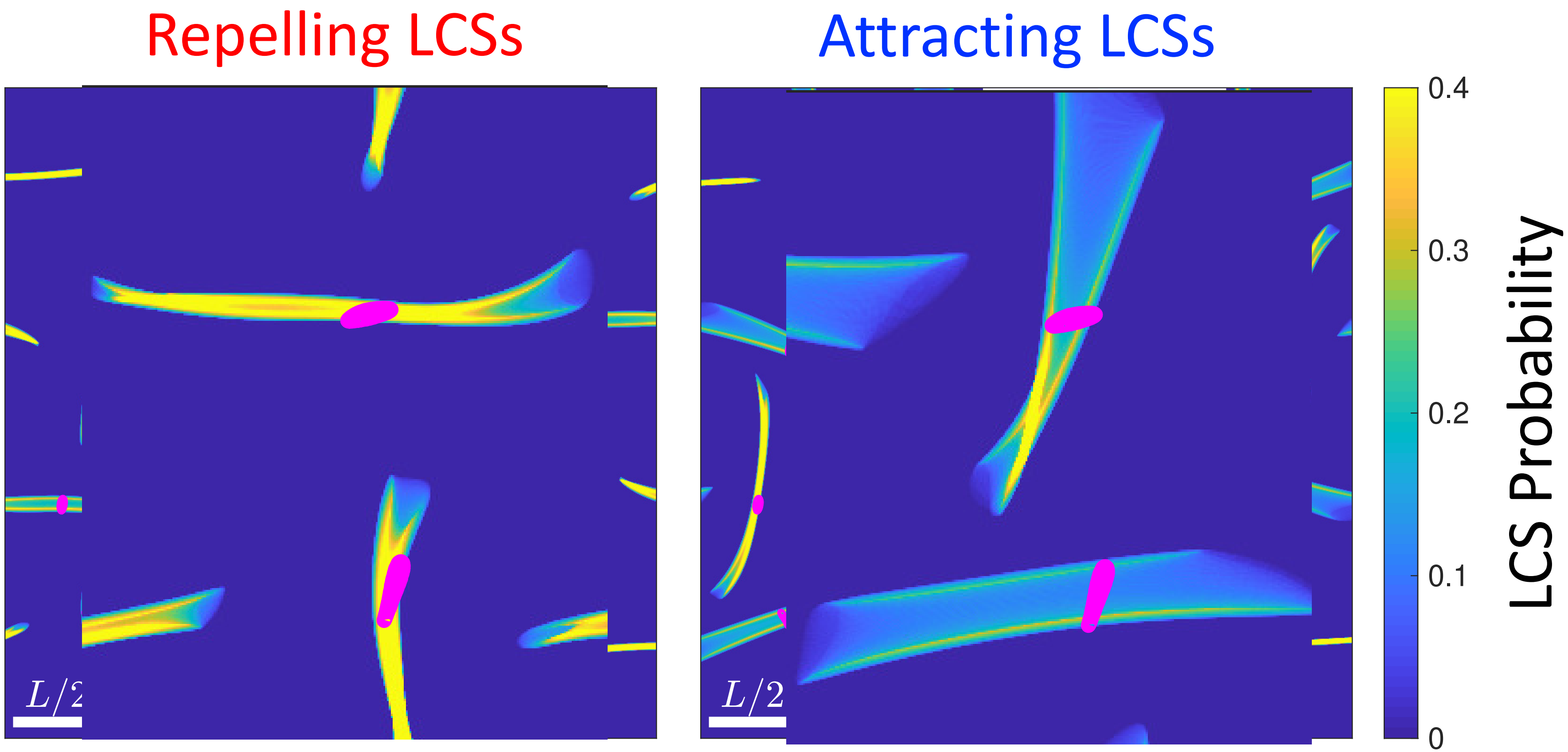
Area Fractions for Nonzero Probability of Finding an LCS

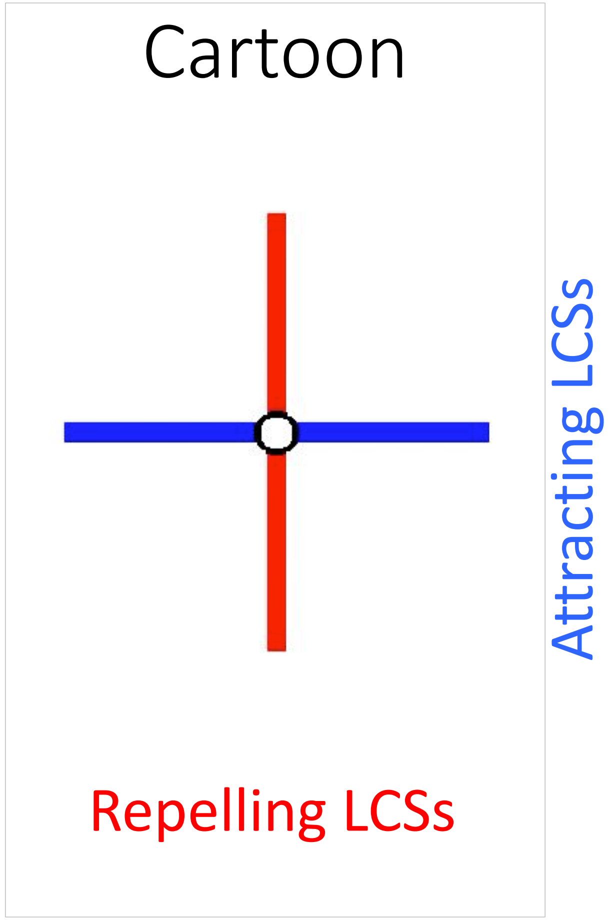


Q2. How does this asymmetry change with Re?

It grows with Re. The periodic orbit plays an important role.

What is the mechanism of the asymmetry?

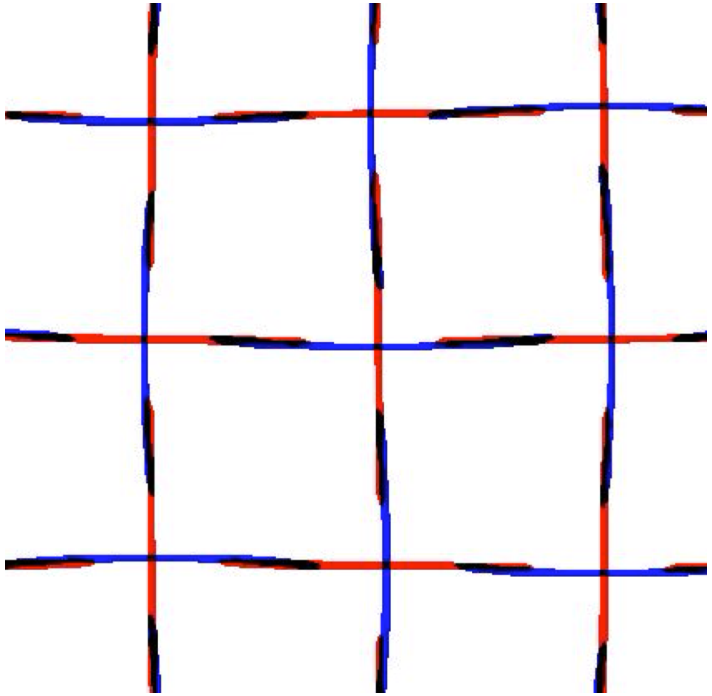




Hyperbolic points move preferentially along the repelling LCSs

A toy model

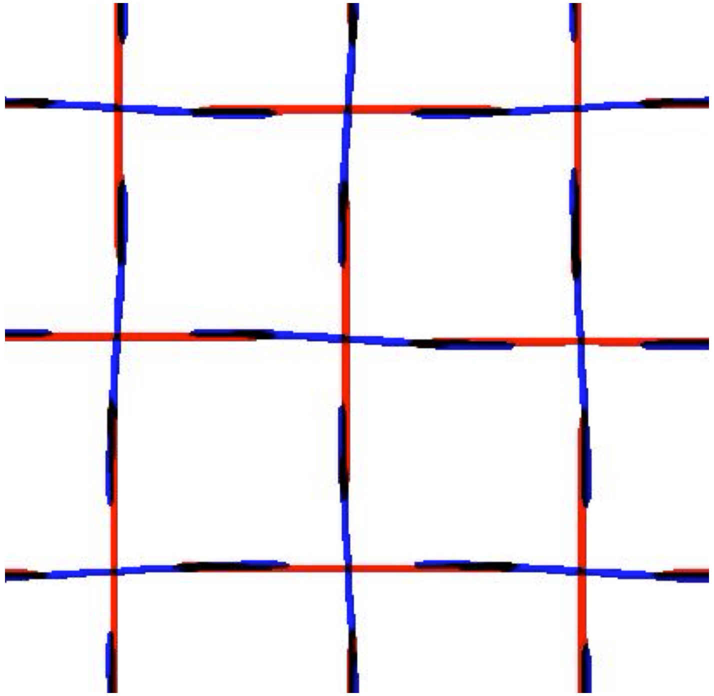
Same trend (hyperbolic points move along repelling LCSs)



$$\psi = \sin(2\pi x)\sin(2\pi y) + \sin(\omega t)\,[\,\sin(2\pi y)\sin(\pi x + \pi y) + \sin(2\pi x)\cos(\pi x - \pi y)\,]$$

$$\vec{u} = \frac{\partial \psi}{dy}\hat{x} - \frac{\partial \psi}{dx}\hat{y}$$

Opposite trend (hyperbolic points move along attracting LCSs)



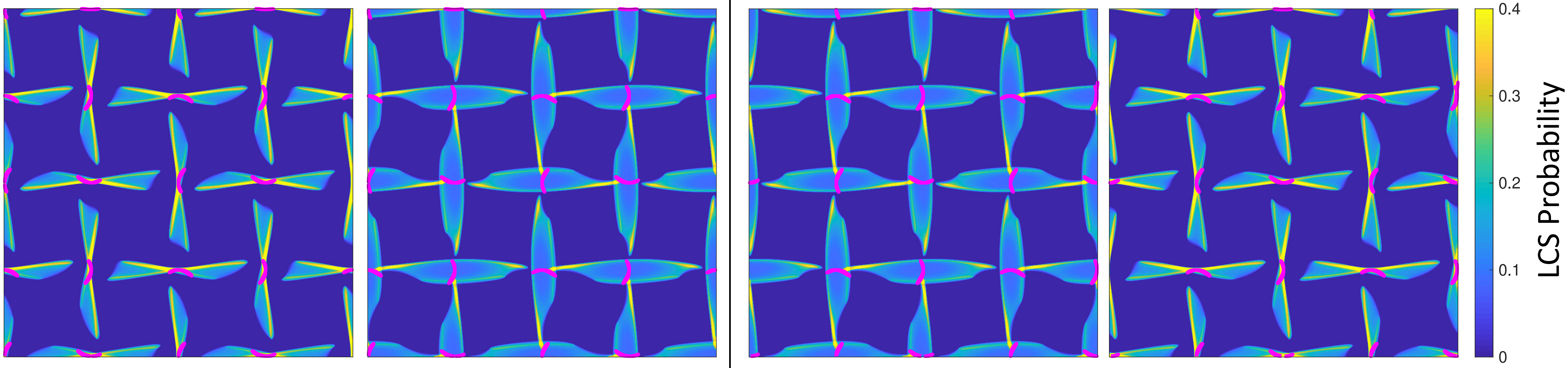
$$\psi = \sin(2\pi x)\sin(2\pi y) + \sin(\omega t)\,[\,\sin(2\pi y)\cos(\pi x + \pi y) + \sin(2\pi x)\sin(\pi x - \pi y)\,]$$

$$\vec{u} = \frac{\partial \psi}{dy}\hat{x} - \frac{\partial \psi}{dx}\hat{y}$$

Toy model reproduces same and opposite area fractions

Same trend (hyperbolic points move along repelling LCSs)

Opposite trend (hyperbolic points move along attracting LCSs)



Area Fraction: 0.23

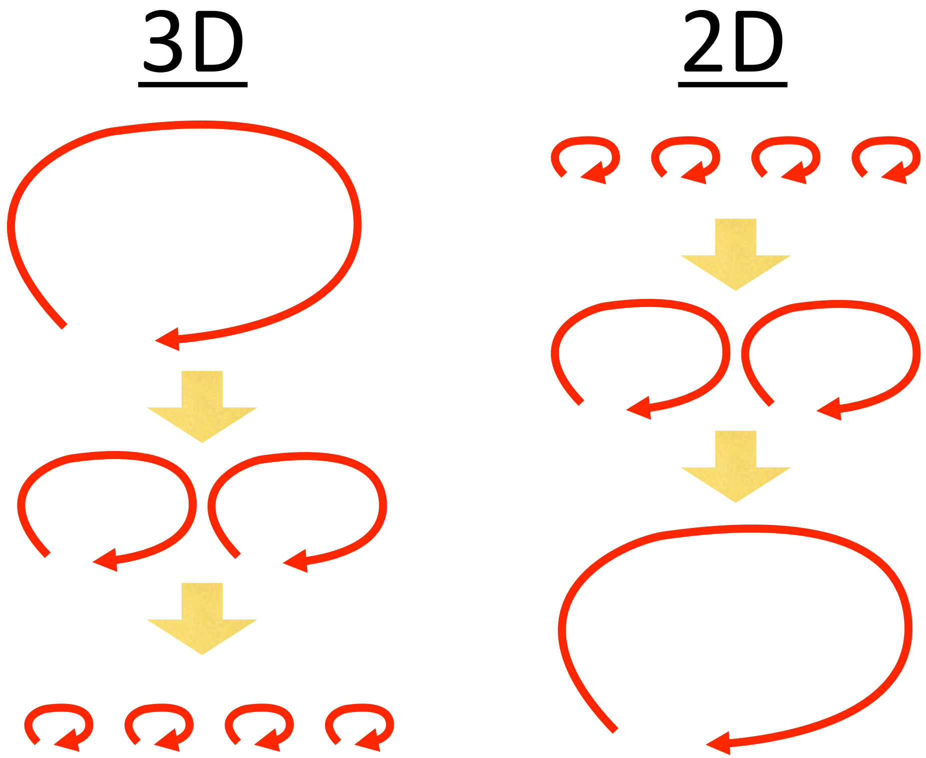
Area Fraction: 0.33

Area Fraction: 0.33

Area Fraction: 0.23

Energy flux across length scales

Nonlinearity of Navier-Stokes couples different length scales



3D: energy cascade (net flow to smaller length scales)

2D: inverse energy cascade (net flow to larger length scales)

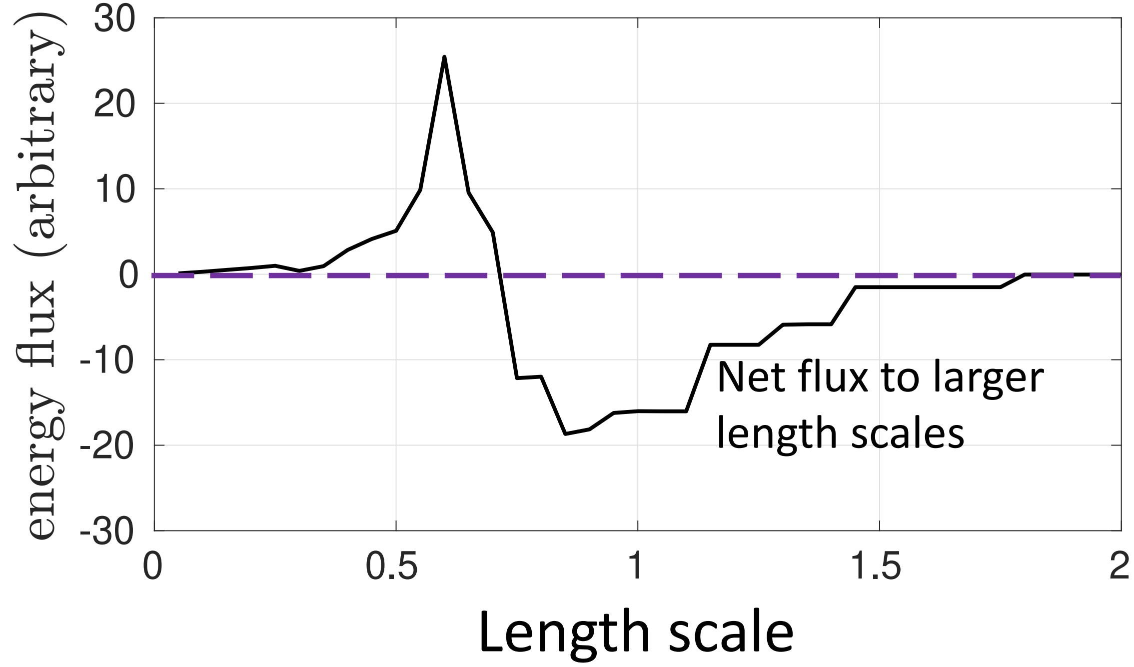
Compute energy transfer using **filter-space techniques**

- Eyink, *J. Stat. Phys.* 1995
- Rivera et al, *PRL* 2003
- (many more)

Test our toy model using this technique

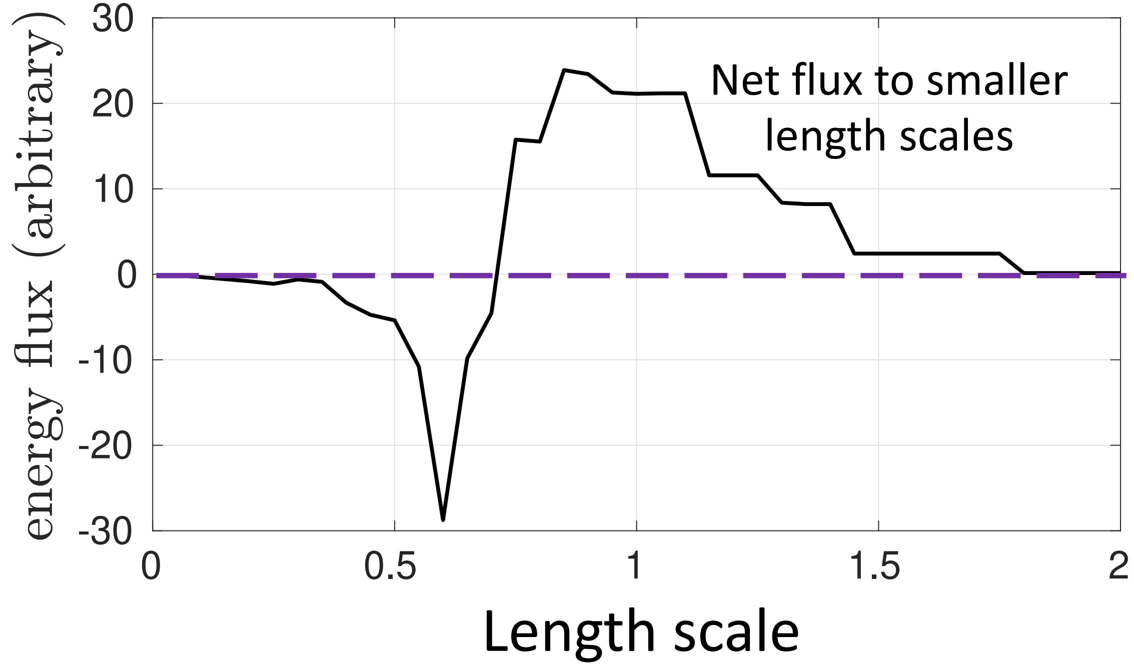
Toy model energy flux

Same trend (hyperbolic points move along repelling LCSs)



Agrees with expectations for a 2D flow!

Opposite trend (hyperbolic points move along attracting LCSs)



Disagrees with expectations for a 2D flow!

Conclusions

- LCSs exhibit a temporal asymmetry
  - Attracting LCSs vs repelling LCSs
  - Present in 2 different geometries
  - Implications for forecasting mixing
- Asymmetry grows with Re
- Hyperbolic points move preferentially along repelling LCSs
- Toy model: suggests link to inverse energy cascade of 2D turbulence
- Future work: explore LCS asymmetry in 3D