Applications of Erosion to Debris and Mudflows

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Outline

1. The Montecito Debris and Mudflows
2. Erosion
3. Numerical Method
4. DEM (Digital Elevations Models)
5. Simulations

Outline

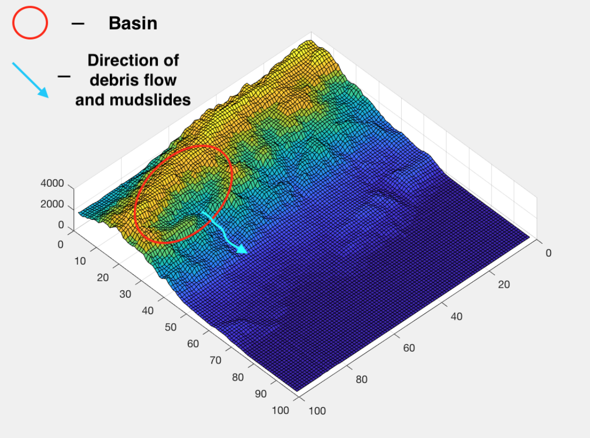
Debris Flow

On January 9, 2018, a month after Santa Barbara and Ventura counties were scorched by the Thomas Fire, heavy rains in Montecito, California produced mudslides which engulfed certain areas of the town, and left other areas unscathed.

The model used in (Staley et al., 2017) reduces the factors, that cause a mudslide/debris flow event to occur, to an essential four:

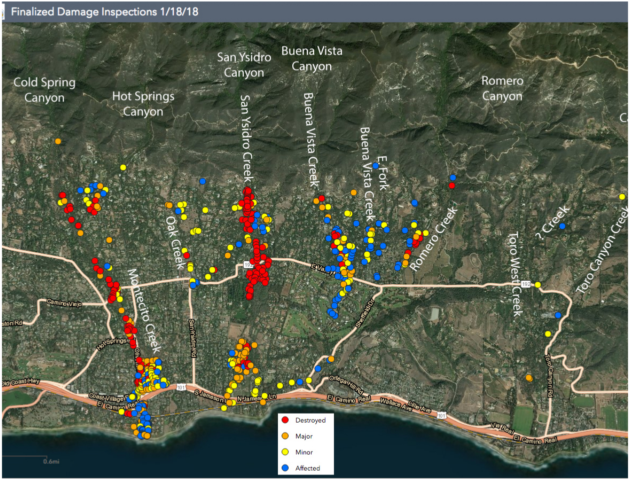
- terrain steepness
- the intensity of the wildfire
- surface properties or sediment availability or erodibility
- the intensity of rainfall

The digital elevation model (DEM)



Aerial view of a basis and canyon on the Montecito/Santa Barbara mountains

The damaged areas



Areas worst hit

Improved Predictions

- The magnitude and destructive power of what happened in Montecito and Santa Barbara county on January 9th took most by surprise.
- Furthermore, the amount and duration of wildfires have been increasing gradually (Westerling et al. 2006), meaning areas with little to no history of wildfires or debris/mudflow could soon find themselves in a situation similar to the one in Montecito (Cannon and DeGraff, 2009).
- If further improved, this model holds the potential to one day be the basis of a preemptive simulation, a prediction of, how powerful a debris- or mudflow would be and in which areas it would hit the hardest.

Outline

The BBS Equations

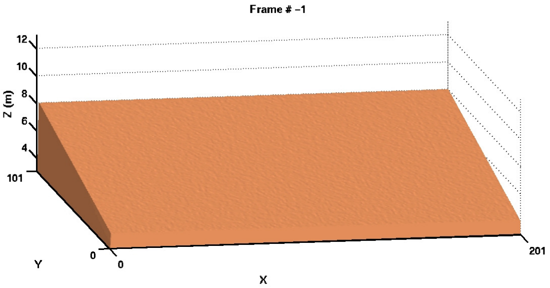
- Let $H = z + h$ be the *height of the free water surface*, where z is the height of the land surface and h is the water depth.

$$\eta^2 \frac{\partial h}{\partial t} = \nabla \cdot \left[h^{3/2} \, |\nabla H|^{1/2} \mathbf{u} \right] + R, \tag{1}$$

$$\frac{\partial H}{\partial t} = \nabla \cdot \left[h^{10/3} |\nabla H|^3 \mathbf{u} \right] - \delta h^{3/2} |\nabla H|. \tag{2}$$

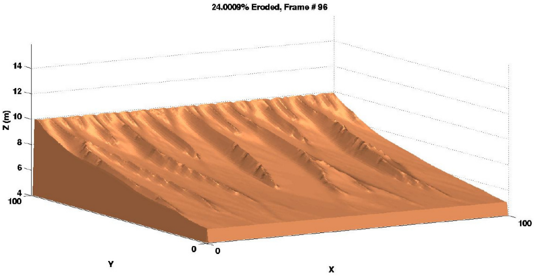
- $\mathbf{u} = \frac{\nabla H}{|\nabla H|}$ is the unit normal down the gradient of the water surface, R is the rainfall rate and η is small.
- The second term in Equation (2) models erosion and is inspired by Kramer and Marder 1992.

Initial Surface



The initial water surface, a flat ridge

A Typical Surface Simulated by David



A Pattern of Ridges and Valleys, at 60% eroded

Instabilities

Linearize the PDEs around the initial surface we get two instabilities:

- If the PDE (2) has no erosion term, the dispersion relation becomes

$$\omega = \frac{5}{3} d^{\frac{2}{3}} c^{\frac{1}{2}} [(2-d)k_1^2 + (\frac{1}{2} - 3d)k_2^2],$$

  where d is small. It shows that all the spatial frequencies are unstable and that the highest frequencies grow the fastest.

- If the erosion term is included we get an additional instability

$$\omega = \frac{3}{2}\delta - k_1^2.$$

  This instability gives rise to river channels.

Outline

Numerical Methods

- If the smallest frequencies grow the fastest, we have a real problem numerically.
- In nature there is a natural (lower) cutoff, when the scale of the grain size is reached.
- Nonlinearities also saturate the exponential growth of the instabilities.
- How does one capture this numerically? Cattan and Birnir (2017)

More Numerics

- Answer: Implicit methods work, explicit methods do not capture the small scales.
- Small viscosity is build into the Crank-Nickolson/Upwind scheme in a very controlled way. It is small and decreases with the discretization size.
- Both Predictor-Corrector and Crank-Nickolson/Upwind schemes capture the large scale features of the landscape.
- The number of ridges and the number of valleys are the same and the half-width of the valleys.

Scaling of the Variogram

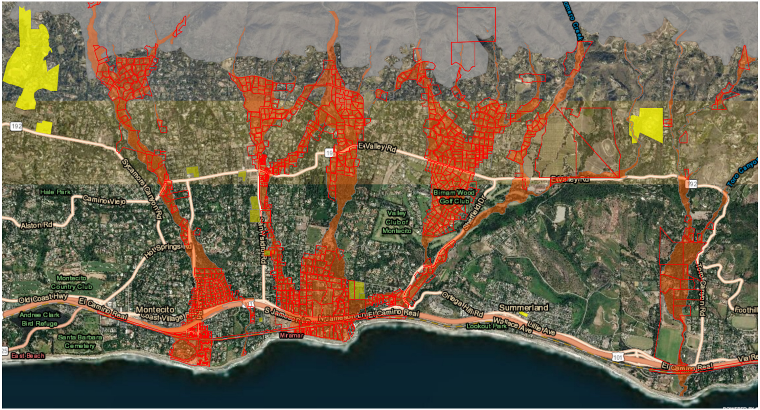
- The variogram

$$V_f(\mathbf{x}, t) = < |f(\mathbf{x} + \mathbf{y}, t) - f(\mathbf{y}, t)|^2 >^{\frac{1}{2}} \qquad (3)$$

  tion differences as a function of distances of separation (or lag) $|\ \mathbf{x}\ |$.
- This function, known as the *variogram, height-height correlation function*, *roughness function*, or *width function*, characterizes the roughness of the surface.
- The variogram is just the second structure function from turbulence.
- Crank-Nickolson/Upwind produces the scaling exponents $1/2$ for h and $3/4$ for H, see B., Smith and Merchant (2001). $3/4$ gives Hack's Law.

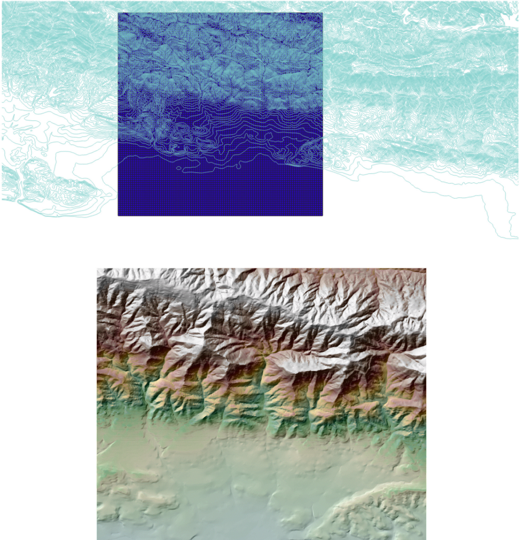
Outline

Debris Flows in Montecito, January 2017

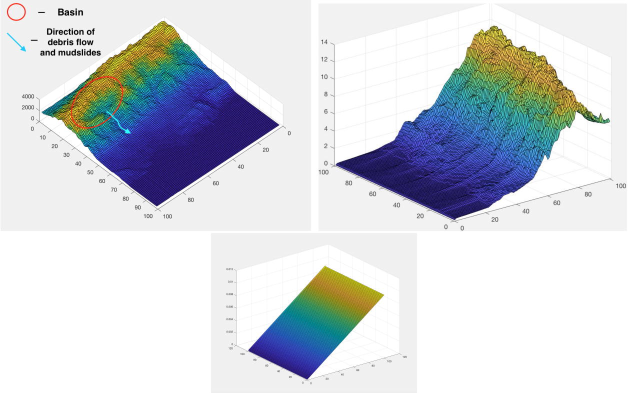


The risk area in Montecito

The Digital Elevation Model (DEM)



The DEM of the watershed



(a) Aerial view of a basis and canyon on the Montecito/Santa Barbara mountains. The initial surface (b) and rainfall (c) used for the simulation

The Results: A Debris Flow and a Mudflow

- The model replicated two different types of flows: a debris flow and a mud flow.
- The simulations produce a debris flow on the scale of hours and a mudflow that last for two to three days.
- The debris flow run all the way to the ocean, but much of the mudflow stops below the foothills.
- The contours of the flow follow the risk area on the map above.

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Debris Flow

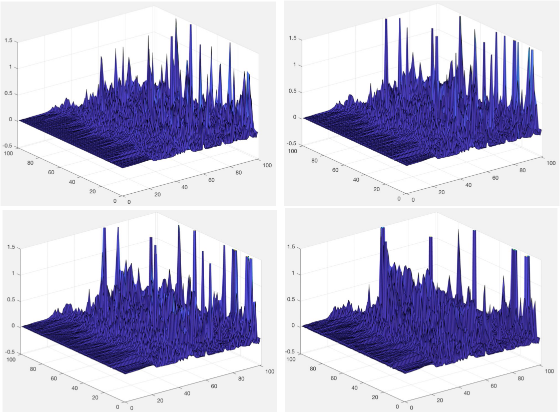

Fig. 6: Stop-frames of the debris flow ordered from top left to bottom right

Mud Flow

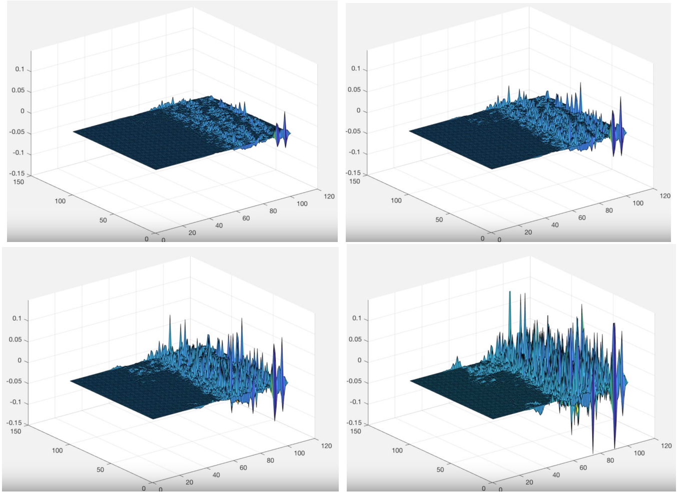

Fig. 8: Stop-frames of the mud flow ordered from top left to bottom right

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