Large Scale Fusion of Energy Resolved Compton Scatter and Attenuation-Based X-Ray Data for Materials Characterization

Hamideh Rezaee[1], Eric L. Miller[1], Brian Tracey[1] , Misha Kilmer[2]

[1]Department of Electrical and Computer Engineering, Tufts University
[2]Department of Mathematics, Tufts University

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Outline

- Introduction
- Background
  - Compton Scattering Tomography
  - Attenuation-Based Tomography
- Forward Model
- Inverse Problem
  - Density Reconstruction
  - Photoelectric Reconstruction
- Simulation Results
- Conclusion and Future Work

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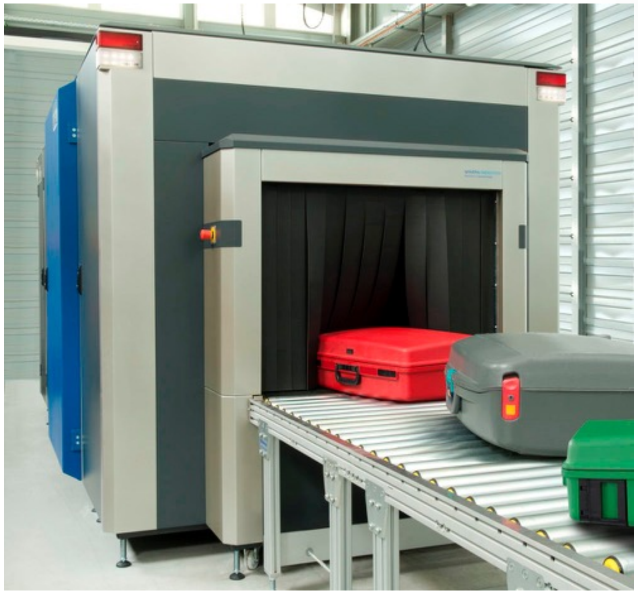
Introduction

- Challenges
  - Application: Luggage screening
  - Few fixed (non-rotating) sources
  - Fixed energy-resolved detectors
- Goals
  - Improving detection performance
  - Material characterization
  - Recovery of the mass density and photoelectric absorption coefficient
  - Artifact reduction
- Solution
  - Compton scatter = additional information
  - Fusing attenuation data and Compton scatter data
  - Energy resolved detectors



http://www.passengerterminaltoday.com/viewnews.php?NewsID=42596

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Compton Scatter

Change in energy

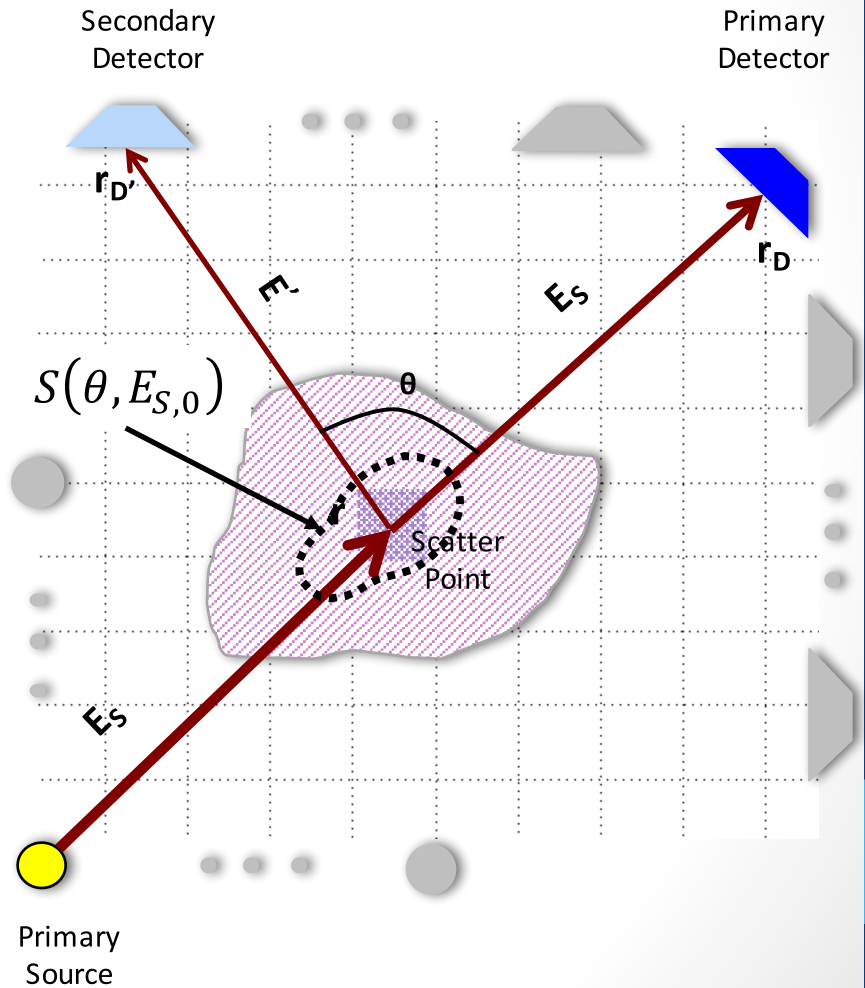
$$E' = \frac{E_S}{1+\gamma\left(1-\cos(\theta(\mathbf{r},\mathbf{r}_D,\mathbf{r}_{D'}))\right)}$$

Change in direction

$$\theta(\mathbf{r},\mathbf{r}_D,\mathbf{r}_{D'}) = \cos^{-1}\left(\frac{\mathbf{r}-\mathbf{r}_D}{|\mathbf{r}-\mathbf{r}_D|}\cdot\frac{\mathbf{r}-\mathbf{r}'_D}{|\mathbf{r}-\mathbf{r}'_D|}\right)$$

Intensity of scattering

$$S(\mathbf{r},\theta,E_S) = \frac{N_A}{2}\frac{\mathrm{d}\sigma_{KN}(E_S,\theta)}{\mathrm{d}\Omega}$$



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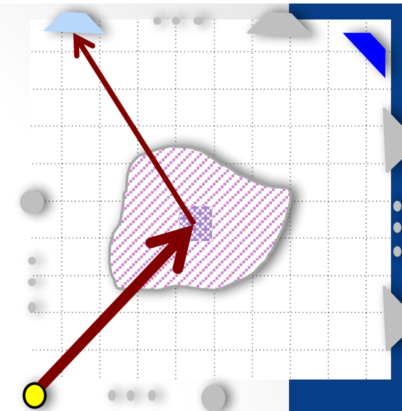
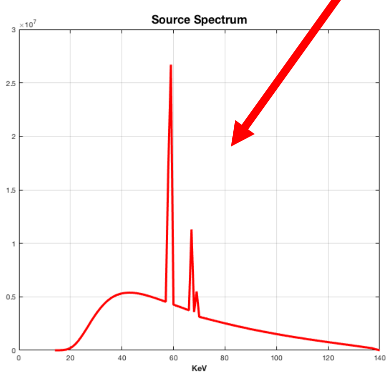
Compton Scatter



- Compton Scatter- Continuous form

$$g_S(\mathbf{r}_{D'}, E') = \int \mathrm{I}(E_S) \left[\int \Omega_{D'} f(\mathbf{r}_{D'}, \mathbf{r}, E') S(r, \theta, E_S) f(\mathbf{r}, \mathbf{r}_S, E_S) \delta_{\mathbf{r}_D, \mathbf{r}_S}(\mathbf{r}) \rho(\mathbf{r}) \mathrm{d}\mathbf{r} \right] \mathrm{d}E_S$$



$$f(\mathbf{r}_2, \mathbf{r}_1, E) = \exp\left(- \int \mu(\mathbf{r}', E) \delta_{\mathbf{r}_2, \mathbf{r}_1}(\mathbf{r}') \mathrm{d}\mathbf{r}' \right)$$

$$\mu(\mathbf{r}, E_S) = N_A \frac{Z(\mathbf{r})}{A(\mathbf{r})} f_{KN}(E_S) \rho(\mathbf{r}) + f_p(E_S) p(\mathbf{r})$$

- Compton Scatter- Discrete form

$$\mathbf{g}_S = \mathbf{K}_S(\boldsymbol{\rho}, \mathbf{p}) \boldsymbol{\rho}$$

scattered data

discretized scattering system matrix

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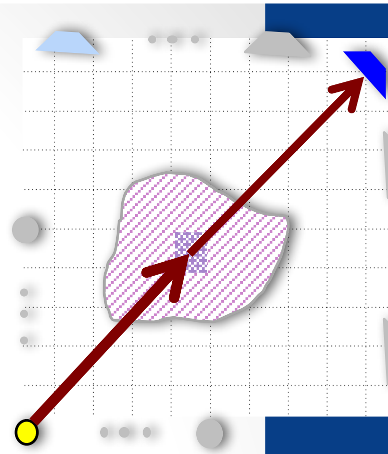
Attenuation



- Attenuation- Continuous form

$$g(\mathbf{r}_S, \mathbf{r}_D) = \int I(E_S) \left[\exp\left(- \int \mu(\mathbf{r}', E_S) \delta_{\mathbf{r}_D, \mathbf{r}_S}(\mathbf{r}') \mathrm{d}\mathbf{r}' \right) \right] \mathrm{d}E_S$$

- Attenuation- Discrete form

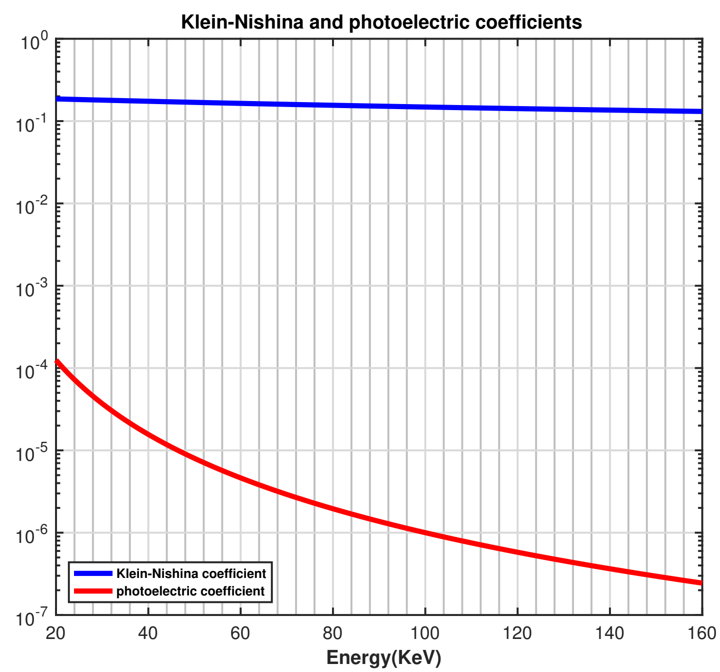
$$g(i,m) = \int_{E_m - \frac{\Delta E}{2}}^{E_m + \frac{\Delta E}{2}} I(E_S) \left[\exp\left(-[\mathbf{A}]_i \boldsymbol{\mu}(E_S) \right) \right] \mathrm{d}E_S$$

$$g(i,m) \approx \left[\exp\left(-[\mathbf{A}]_i \boldsymbol{\mu}(E_m) \right) \right] \int_{E_m - \frac{\Delta E}{2}}^{E_m + \frac{\Delta E}{2}} I(E_S) \, \mathrm{d}E_S$$

$$g_A(i,m) = -\log\left(\frac{\mathbf{g}(i,m)}{\tilde{I}_m} \right) = [\mathbf{A}]_i \boldsymbol{\mu}(E_m)$$

$$\mathbf{g}_A = \mathbf{K}_{A,\rho} \boldsymbol{\rho} + \mathbf{K}_{A,p} \mathbf{p}$$

attenuation data | discretized attenuation system matrix



Detector resolution: 5KeV

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Forward Model

- **Attenuation model**

$$\mathbf{g}_A = \mathbf{K}_{A,\rho}\boldsymbol{\rho} + \mathbf{K}_{A,p}\mathbf{p} + \mathbf{w}_A$$

- **Compton scatter model**

$$\mathbf{g}_S = \mathbf{K}_S(\boldsymbol{\rho}, \mathbf{p})\boldsymbol{\rho} + \mathbf{w}_S$$

- Poisson statistics are the "right" way of modeling this problem.
- For simplicity, here we focus on the additive white Gaussian noise case

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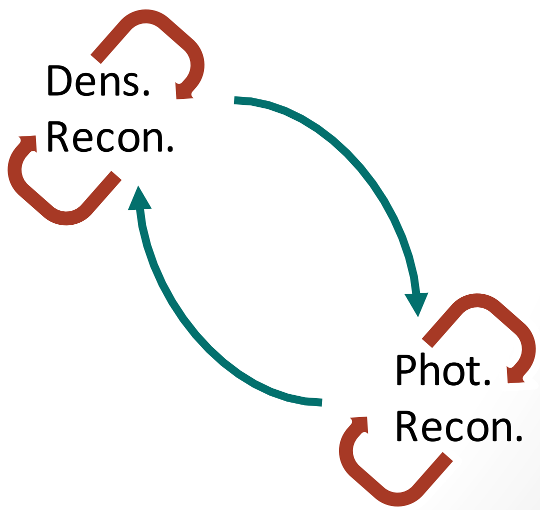
Inverse Problem

$$(\hat{\boldsymbol{\rho}}, \hat{\mathbf{p}}) = \underset{\boldsymbol{\rho},\mathbf{p}}{\operatorname{argmin}}\, \underbrace{w_1\|\mathbf{g}_S - \mathbf{K}_S(\boldsymbol{\rho},\mathbf{p})\boldsymbol{\rho}\|_2^2 + w_2\|\mathbf{g}_A - \mathbf{K}_{A,\rho}\boldsymbol{\rho} - \mathbf{K}_{A,p}\mathbf{p}\|_2^2}_{\text{data mismatch}} + \underbrace{R_\rho(\boldsymbol{\rho}) + R_p(\mathbf{p}|\mathbf{I}^{ref})}_{\text{regularization}}$$

- Cyclic coordinate decent method
  - Iterative density reconstruction
  - Iterative photoelectric reconstruction
- Regularization
  - Gradient-based and edge-preserving regularizations for density
  - Non-local means (NLM) for photoelectric
- Initialization
  - Multi-scaling approach



Regularization

- Gradient-based regularization $R_\rho(\boldsymbol{\rho}) = \lambda_\rho \|\mathbf{L}\boldsymbol{\rho}\|_2^2$
  - Penalize all high differences even edges
- Edge-preserving regularization [1]

$$R_{\rho,l}(\boldsymbol{\rho}) = \lambda_{\rho,l} \|\mathbf{D}^{(l)}\mathbf{L}\boldsymbol{\rho}\|_2^2$$

  - Diagonal elements determine whether a pixel belongs to the edge map
  - Closer to one: enforce smoothness
  - Closer to zero: should be preserved

$\mathbf{M} = \mathbf{D}^{(l)}\mathbf{L}$ **OR** $\mathbf{M} = \mathbf{L}$

Inputs:

- $D^{(0)} = I$
- **L gradient matrix**
- **Estimate of** ρ **for** $\mathbf{k} = 0,1, \ldots$

1: **for iterations** $\mathbf{k} = 1, \ldots$
2: **Set** $v = D^{(k-1)} L \rho_{\mathbf{k}-1}$
3: **Normalize** v **by setting** $v \leftarrow v / \|v\|_\infty$
4: **Map** d **to** $[0,1]$ **by defining** $d := 1 - v.^p$
5: **Define** $D := diag(D)$
6: **Update** $D^{(\mathbf{k})} \leftarrow D D^{(\mathbf{k}-1)}$
7: **end**

[1] Oguz Semerici, "Image Formation Methods for Dual Energy and Multi-Energy Computed Tomography," PhD Thesis, Dept. of ECE Tufts University, October 2012.

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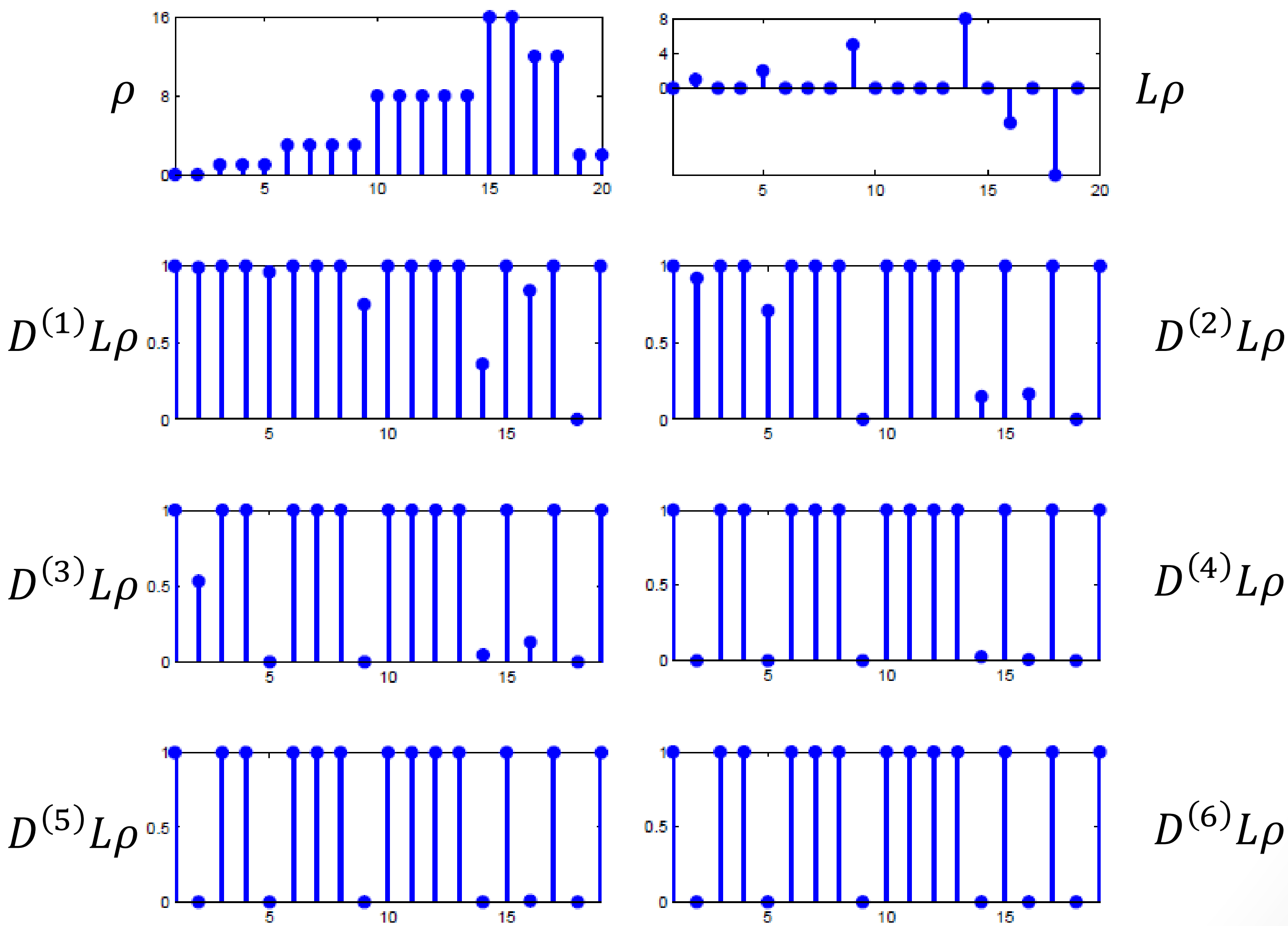
Edge-Enhancing Regularization



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Density Reconstruction

$$\hat{\boldsymbol{\rho}}_{n+1} = \underset{\boldsymbol{\rho}}{\operatorname{argmin}}\, w_1 \|\mathbf{g}_S - \mathbf{K}_S(\boldsymbol{\rho}, \hat{\mathbf{p}}_n)\boldsymbol{\rho}\|_2^2 + w_2 \|\mathbf{g}_A - \mathbf{K}_{A,\rho}\boldsymbol{\rho} - \mathbf{K}_{A,p}\hat{\mathbf{p}}_n\|_2^2 + R_\rho(\boldsymbol{\rho})$$

$$\hat{\boldsymbol{\rho}}_{n+1} = \underset{\boldsymbol{\rho}}{\operatorname{argmin}} \left\| \begin{bmatrix} \sqrt{w_1}\,\mathbf{g}_S \\ \sqrt{w_2}\,(\mathbf{g}_A - \mathbf{K}_{A,p}\hat{\mathbf{p}}_n) \\ 0 \end{bmatrix} - \begin{bmatrix} \sqrt{w_1}\,\mathbf{K}_S(\boldsymbol{\rho}, \hat{\mathbf{p}}_n) \\ \sqrt{w_2}\,\mathbf{K}_{A,\rho} \\ \sqrt{\lambda}\,\mathbf{M}(\boldsymbol{\rho}) \end{bmatrix} \boldsymbol{\rho} \right\|_2^2$$

$$\hat{\boldsymbol{\rho}}_{n+1} \equiv \underset{\boldsymbol{\rho}}{\operatorname{argmin}} \left\| \tilde{\mathbf{g}} - \tilde{\mathbf{K}}(\boldsymbol{\rho})\boldsymbol{\rho} \right\|_2^2$$

Density Reconstruction

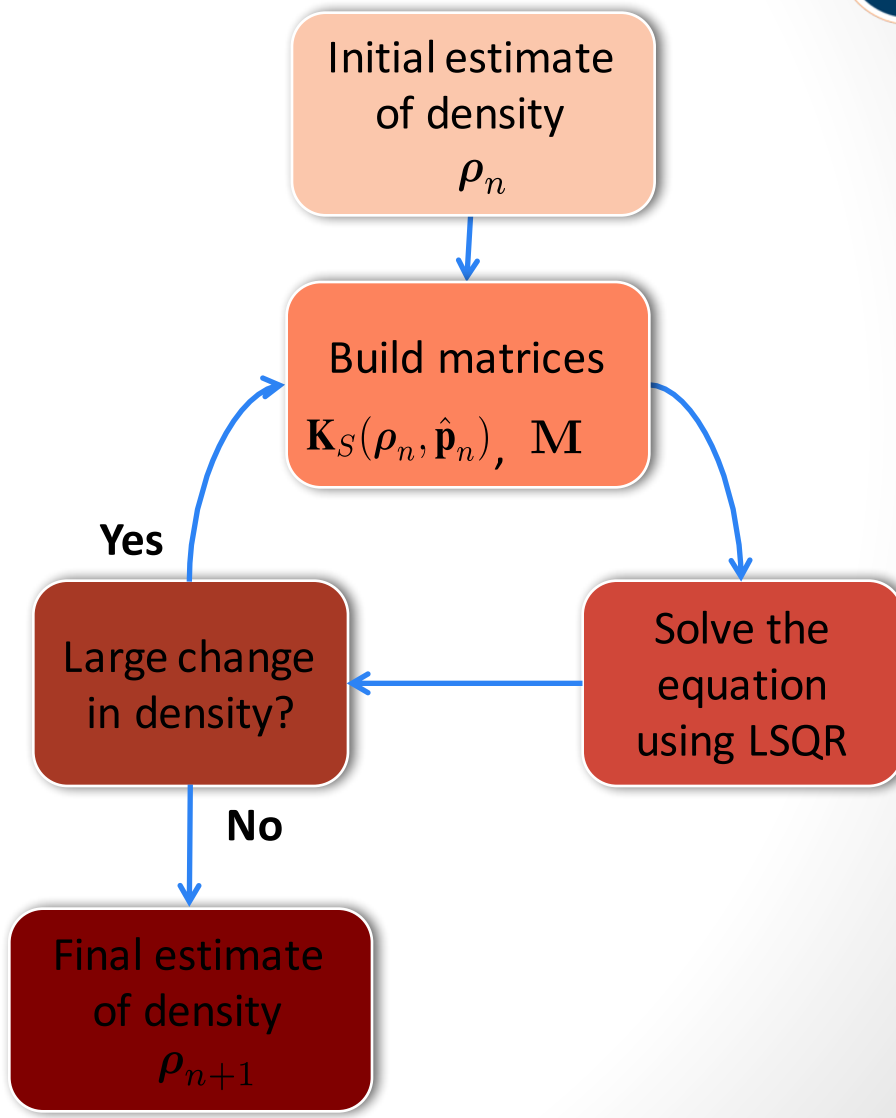
$$\hat{\boldsymbol{\rho}}_{n+1} \equiv \underset{\boldsymbol{\rho}}{\operatorname{argmin}} \left\| \tilde{\mathbf{g}} - \tilde{\mathbf{K}}(\boldsymbol{\rho})\boldsymbol{\rho} \right\|_2^2$$

- Solution
  - Iterative fixed-point approach
  - Linear least squares
  - LSQR method

- Starting with current estimation of density and photoelectric

- If edge-preserving regularization then update:

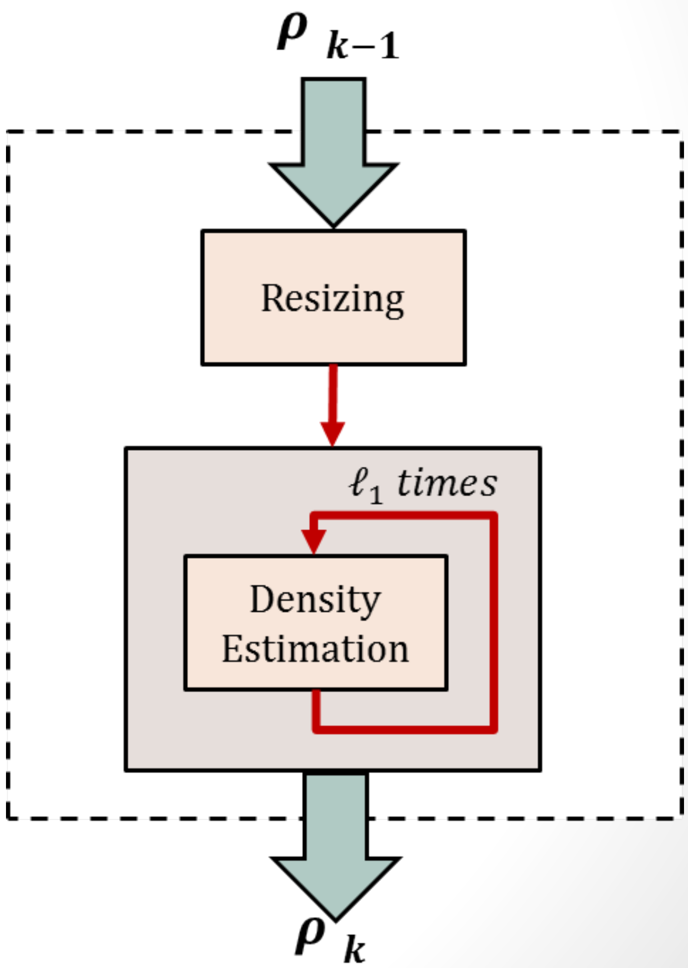
$$\mathbf{M} = \mathbf{D}^{(l)}\mathbf{L}$$



Density Reconstruction

$$\hat{\boldsymbol{\rho}}_{n+1} \equiv \underset{\boldsymbol{\rho}}{\operatorname{argmin}} \left\|\tilde{\mathbf{g}} - \tilde{\mathbf{K}}(\boldsymbol{\rho})\boldsymbol{\rho}\right\|_2^2$$

- Initial guess
  - Constant background image
- Assuming several scales with different resolutions for discretized model and density
- Starting with the coarse scale representation and constant density image
- Using the final estimation of previous scale for the next scale



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Photoelectric Reconstruction

$$\hat{\mathbf{p}}_{n+1} = \underset{\mathbf{p}}{\operatorname{argmin}}\, w_1 \left\|\mathbf{g}_S - \mathbf{K}_S(\hat{\boldsymbol{\rho}}_n, \mathbf{p})\hat{\boldsymbol{\rho}}_n\right\|_2^2 + w_2\|\mathbf{g}_A - \mathbf{K}_{A,\rho}\hat{\boldsymbol{\rho}}_n - \mathbf{K}_{A,p}\mathbf{p}\|_2^2 + R_p(\mathbf{p}|\mathbf{I}^{ref})$$

- Solution
  - Non-linear least squares
  - Levenberg-Marquardt method
- Regularization
  - Non-local means (NLM) [2] $R_p(\mathbf{p}|\mathbf{I}^{ref}) = R_{NLM}(\mathbf{p}|\hat{\boldsymbol{\rho}}_{n=1}) = \lambda_p\|(\mathbf{I} - \mathbf{W})\mathbf{p}\|_2^2$
- Initial guess
  - Constant background image

[2] Brian H. Tracey and Eric L. Miller, "Stabilizing dual-energy X-ray computed tomography reconstructions using patch-based regularization," Inverse Problems, 31(10), 05004, September 2015

NLM Regularization

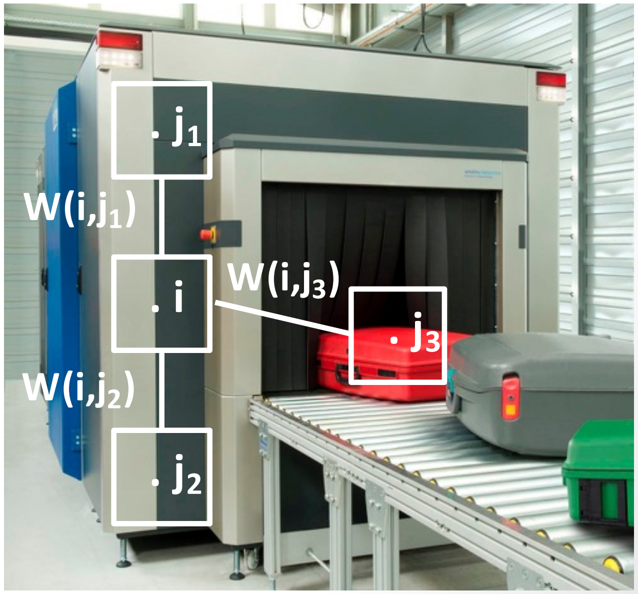
$$R_p(\mathbf{p}|\mathbf{I}^{ref}) = R_{NLM}(\mathbf{p}|\hat{\boldsymbol{\rho}}_{n=1}) = \lambda_p \|(\mathbf{I} - \mathbf{W})\mathbf{p}\|_2^2$$

- Calculates weighting matrix using density estimation as reference image

- Reduce noise artifacts

$$W(i,j) = \frac{1}{Z(i)} \exp\left(-\frac{\sum_{\delta \in \Delta} \left(\boldsymbol{\rho}_{i+\delta}^{ref} - \boldsymbol{\rho}_{j+\delta}^{ref} \right)^2}{h^2} \right)$$

$$Z(i) = \sum_j W(i,j)$$



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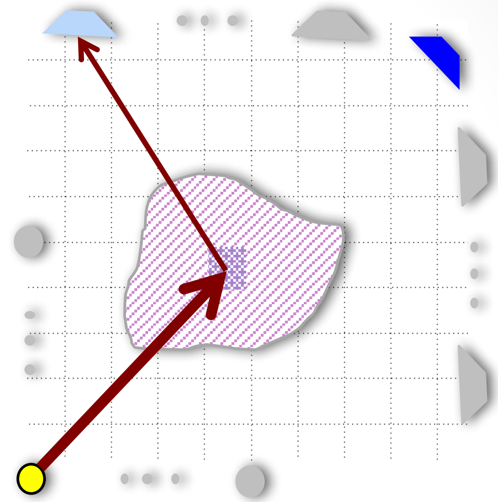
Results

System setup

- Grid size of $20cm \times 20cm$
- 3 Sources and 41 detectors
- Five representations of the grid
  $10 \times 10, 20 \times 20, \ldots, 50 \times 50$
- SNR ratio of 50 dB
- Initial guess
  - Density: Constant image
    $$\rho_0 = .1\ \text{g/cm}^3$$
  - Photoelectric:
    $$p_0 = 0$$



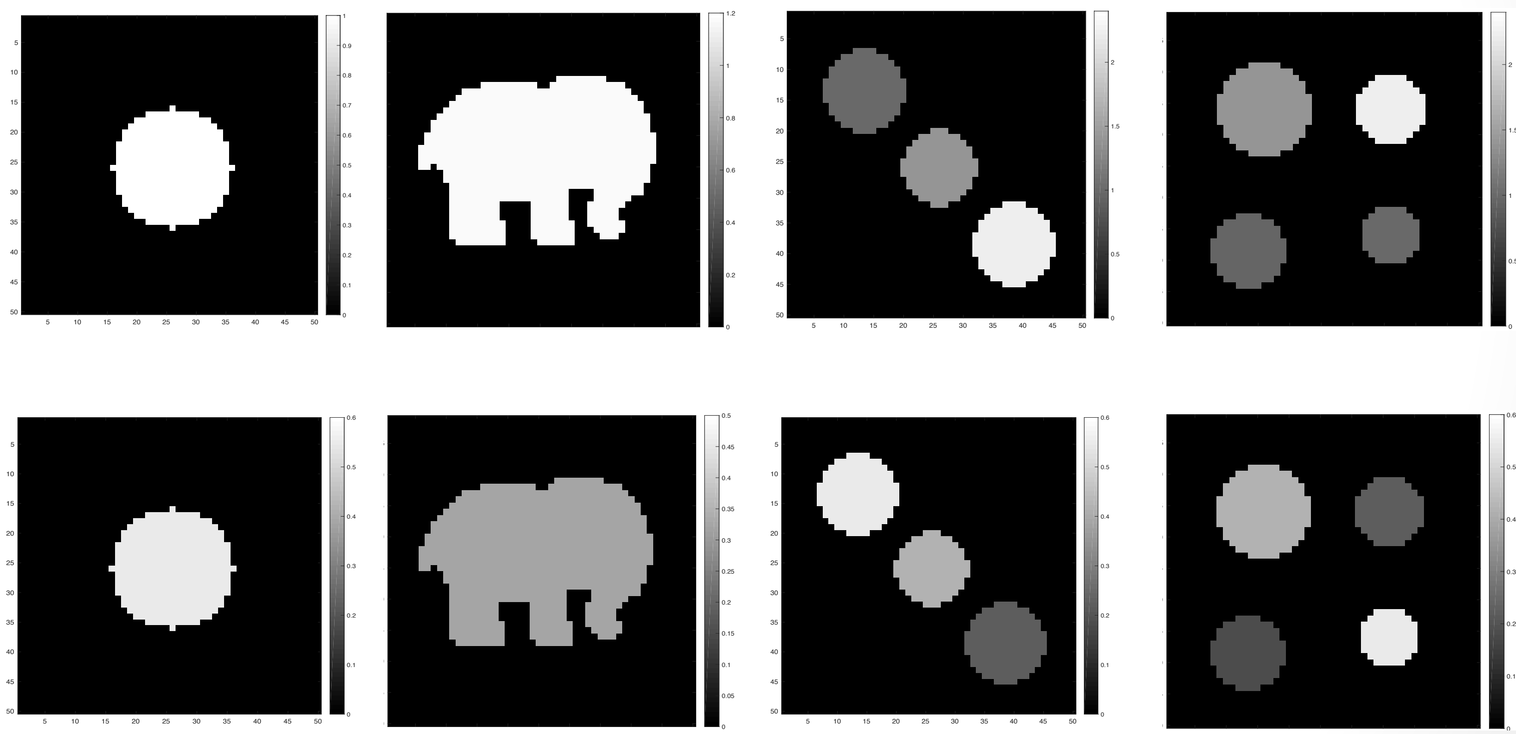
- Materials:
  - HDPE
  - Water
  - Graphite
  - Glass
- Size of the data
  $$1.05760e + 5$$
- Size of the system matrix
  $$(1.05760e + 5) \times 2500$$

Results

Phantom #1 | Phantom #2 | Phantom #3 | Phantom #4

Density

Photoelectric



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Results

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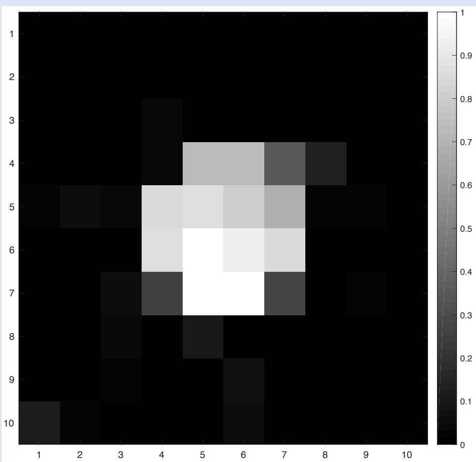
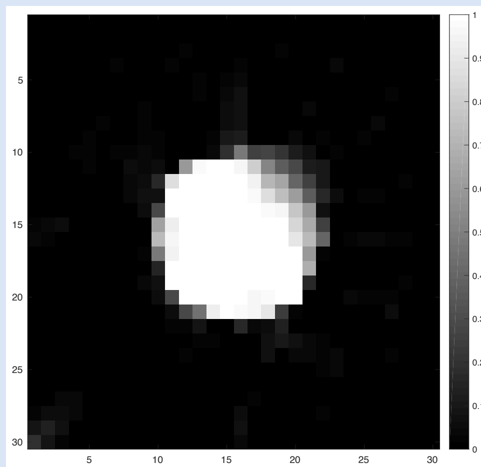
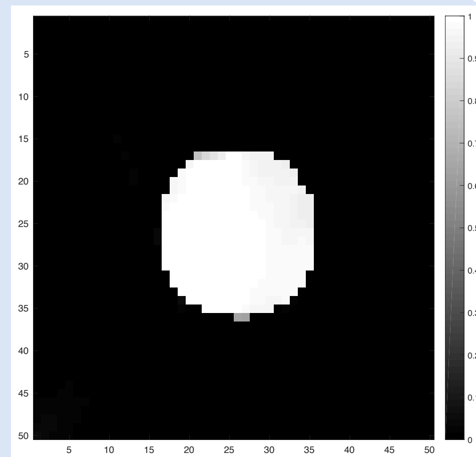
Density Estimation: Iterative Edge-Enhancing Regularization



Scale 1
10×10



Scale 3
30×30

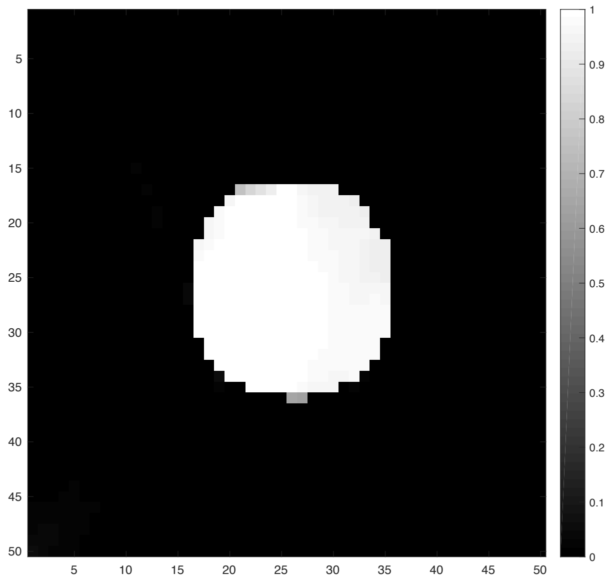


Scale 5
50×50

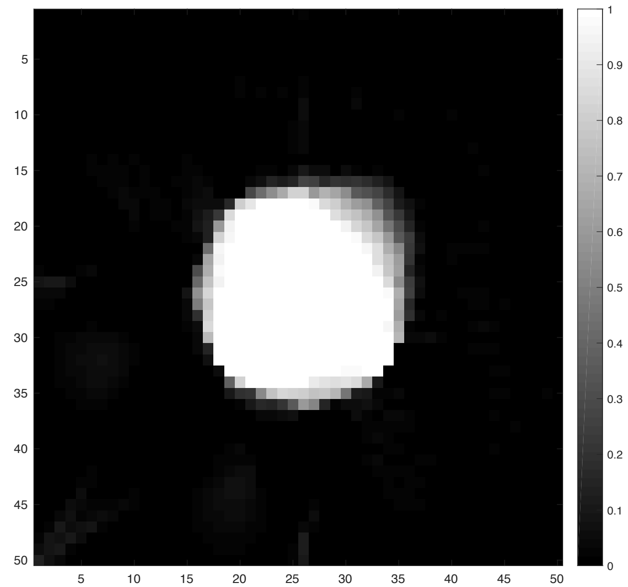
Results

Density Estimation

Edge-Enhancing Regularization



Gradient-based Regularization



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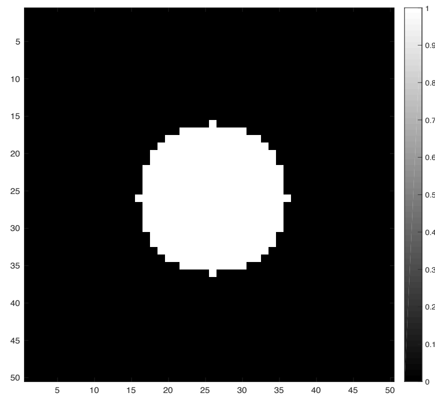
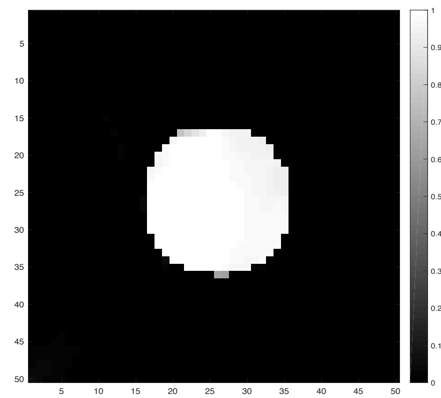
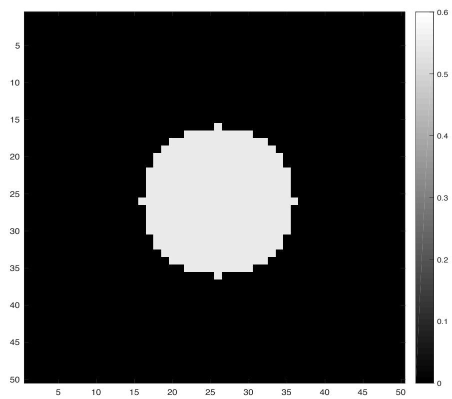
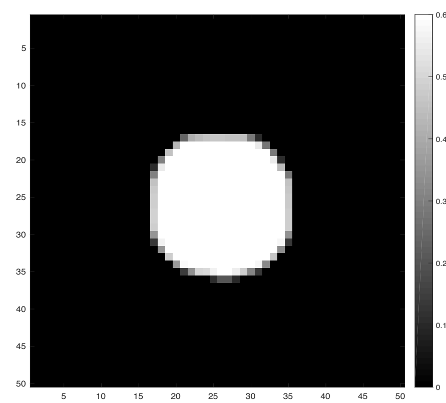
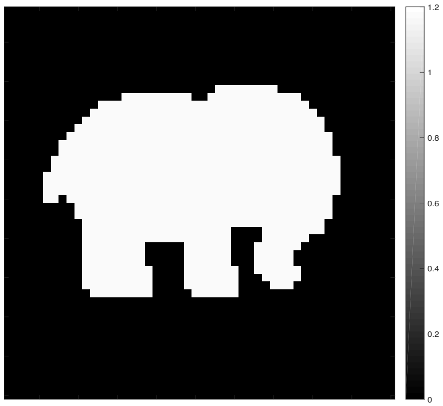
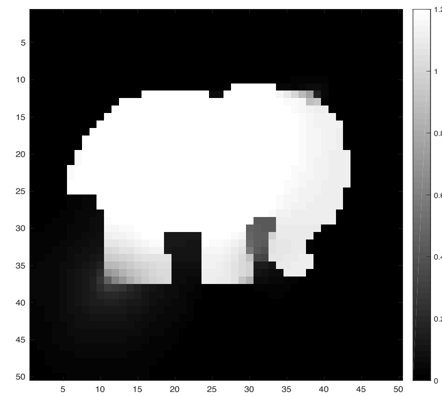
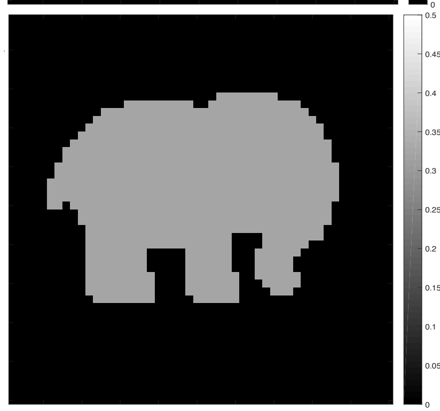
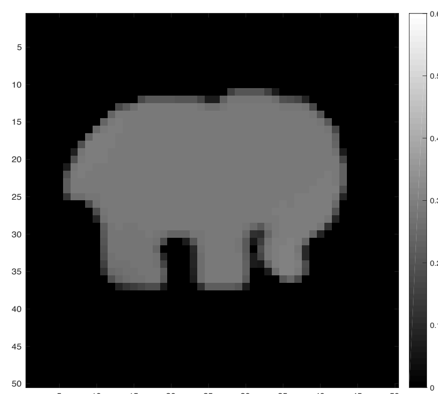
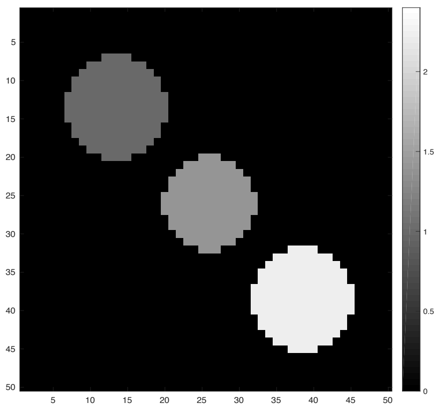
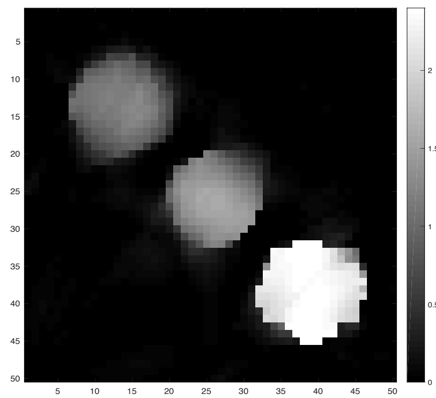
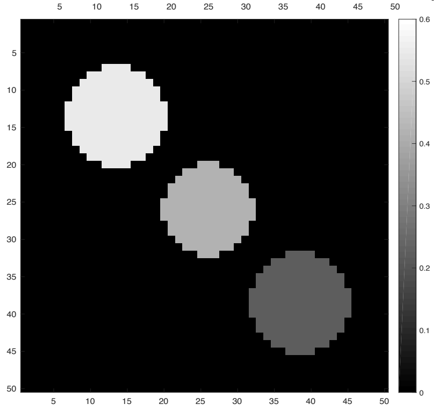
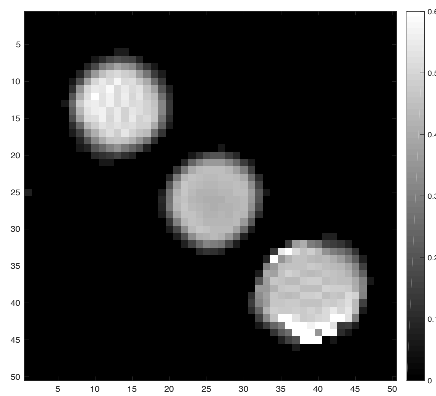
Results

	Phantom #1	Reconstruction	MSE
Density			0.0022
Photoelectric			0.0018

Results

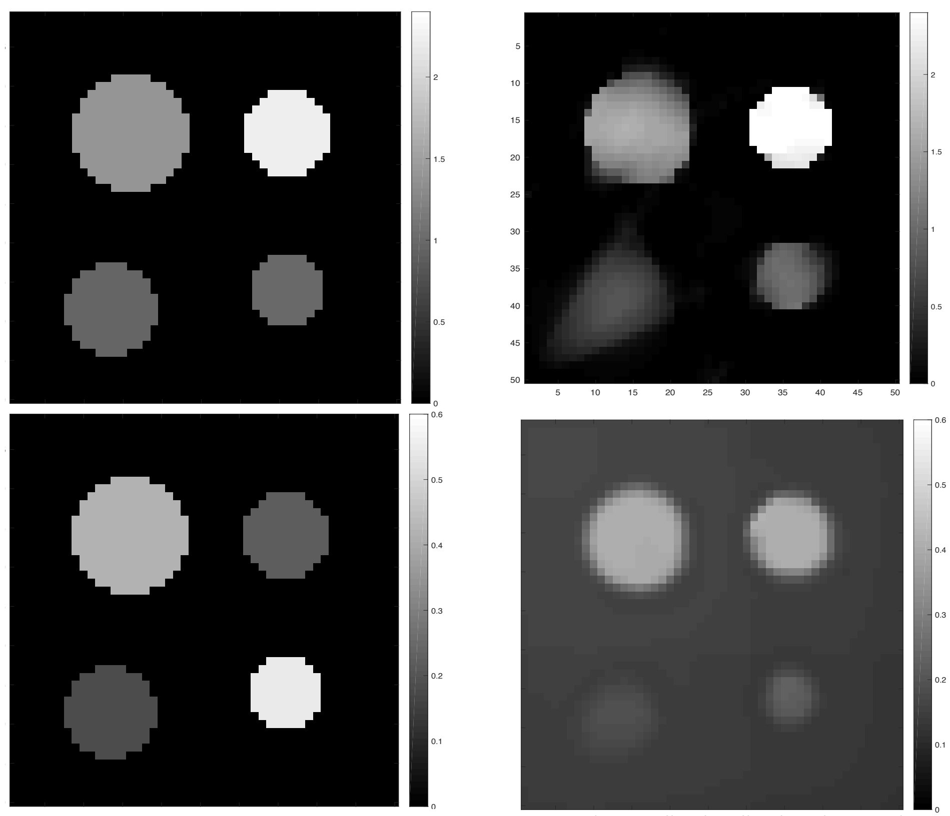
	Phantom #2	Reconstruction	MSE
Density			0.0084
Photoelectric			0.0017

Results

	Phantom #3	Reconstruction	MSE
Density			0.0416
Photoelectric			0.0105

Results

	Phantom #4	Reconstruction	MSE
Density			0.0430
Photoelectric			0.0242



Conclusion

- **Observation**

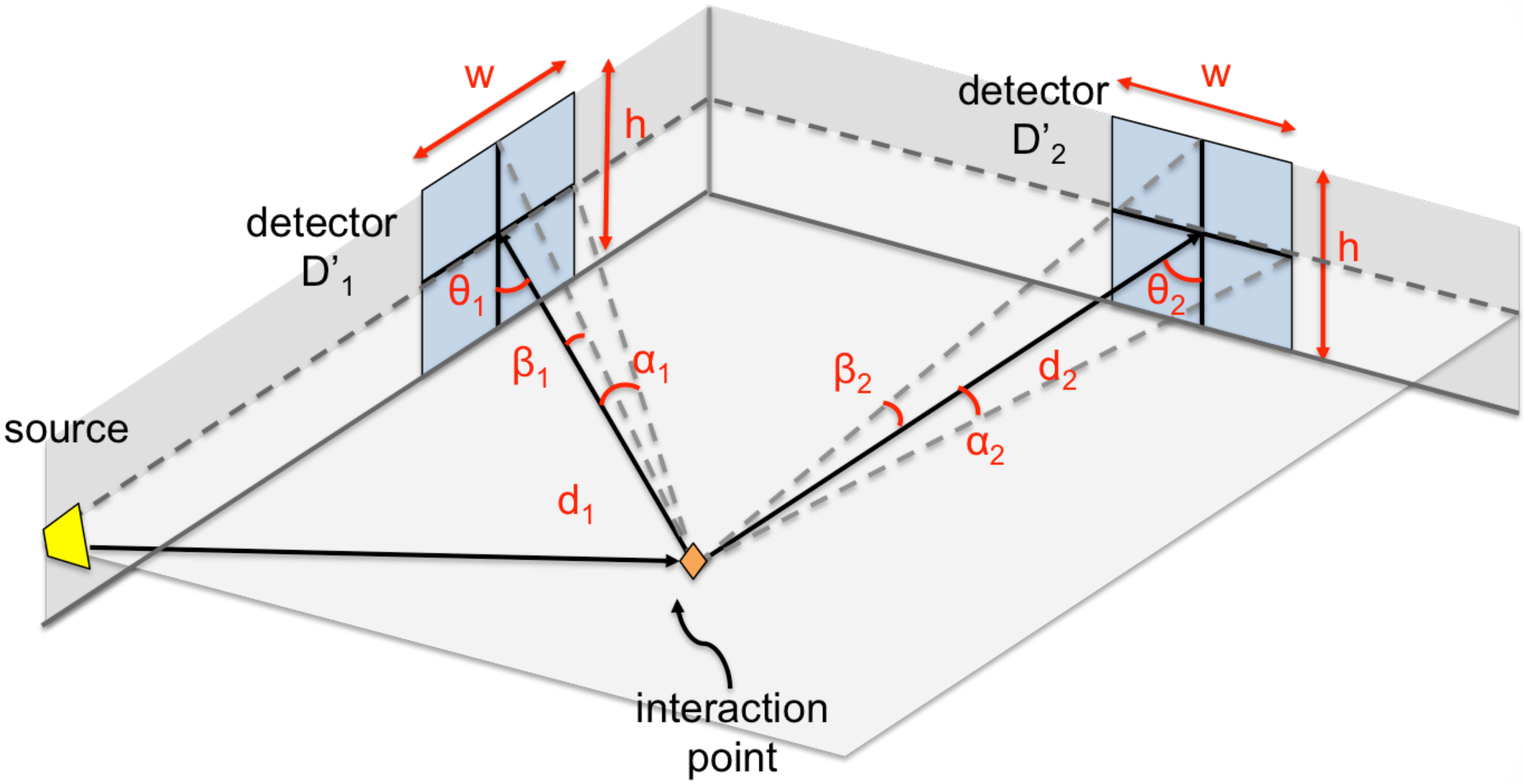
  - Compton Scattering model is successful in recovering density and photoelectric
  - Joint attenuation and Compton Scatter inversion increase the quality of imaging
  - Reconstruction doesn't require any prior information of the object in multi-scale approach
  - Edge-preserving results obtains more of the structure of the object
  - Stabilized reconstruction of photoelectric using "structural" regularization like NLM regularization

- **Need to do**

  - Explore performance on more complex phantoms and real data
  - Modify noise model
  - Improve photoelectric reconstruction

Backup Slides

Solid Angle Calculation



$$\Omega_{D'} = 4 \arcsin\left(\sin\left(\alpha\right) \times \sin\left(\beta\right)\right)$$

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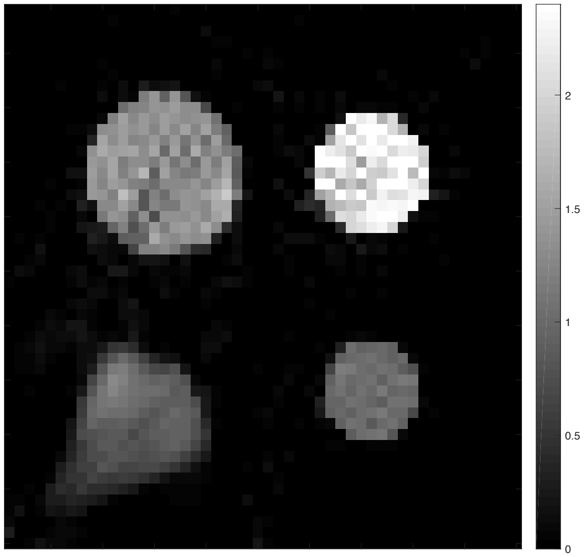
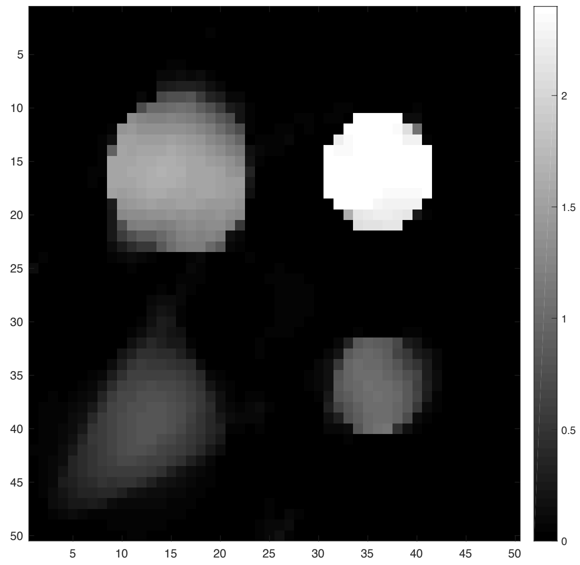
Results

Density Estimation

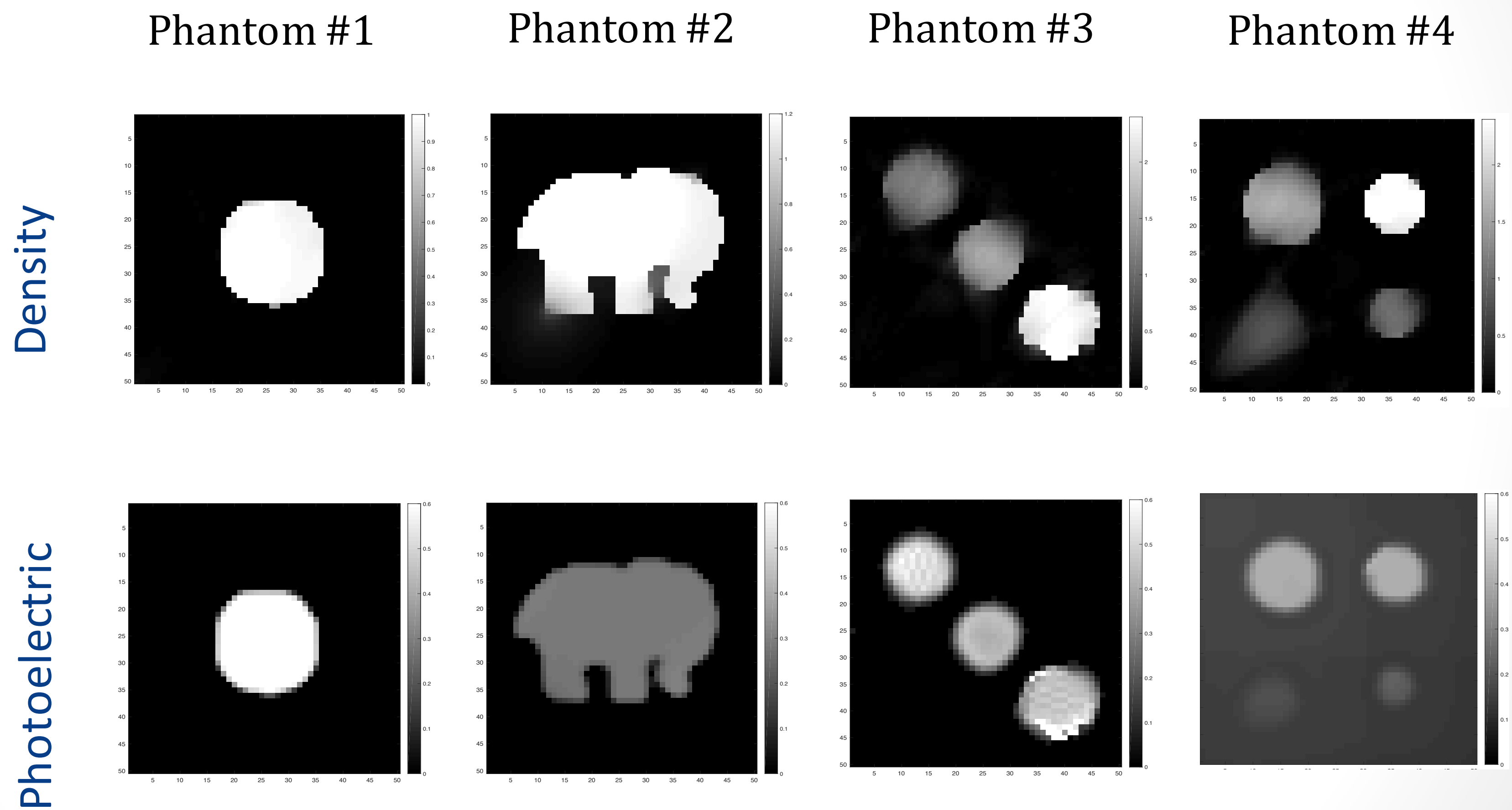
Scatter Data



Attenuation & Scatter Data



Results



Phantom #1
Phantom #2
Phantom #3
Phantom #4
Density
Photoelectric