MS64: Emerging Models of Resilience

SIAM DS17 • May 22, 2017

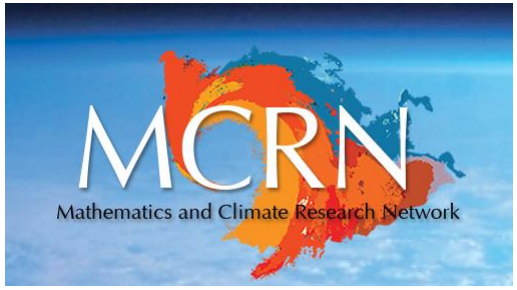
3:45-4:10 Introduction
Katherine Meyer, University of Minnesota

4:15-4:40 Break

4:45-5:10 A Flow-Kick Framework for Exploring Resilience
Alanna Hoyer-Leizel, Mount Holyoke College
Sarah Iams, Harvard University
Ian Klasky, Victoria Lee, and Stephen Ligtenberg, Bowdoin College
Katherine Meyer, University of Minnesota
Mary Lou Zeeman, Bowdoin College and Cornell University, USA

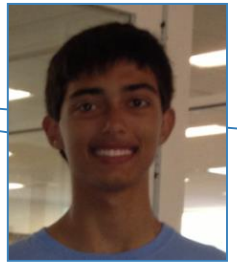
5:15-5:40 Emergence and Resilience of a New Alternative State in the Gulf of Maine
Steven Dudgeon, California State University, Northridge
Peter Petraitis, University of Pennsylvania

Mentors and Collaborators



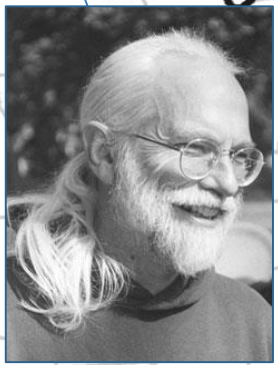
Mary Lou Zeeman

Stephen Lightenberg



Ian Klasky

Torey Lee



Richard McGehee,

Erika Bussman, Alison Shaw

Sarah Pierro

Alanna Hoyer-Leitzel

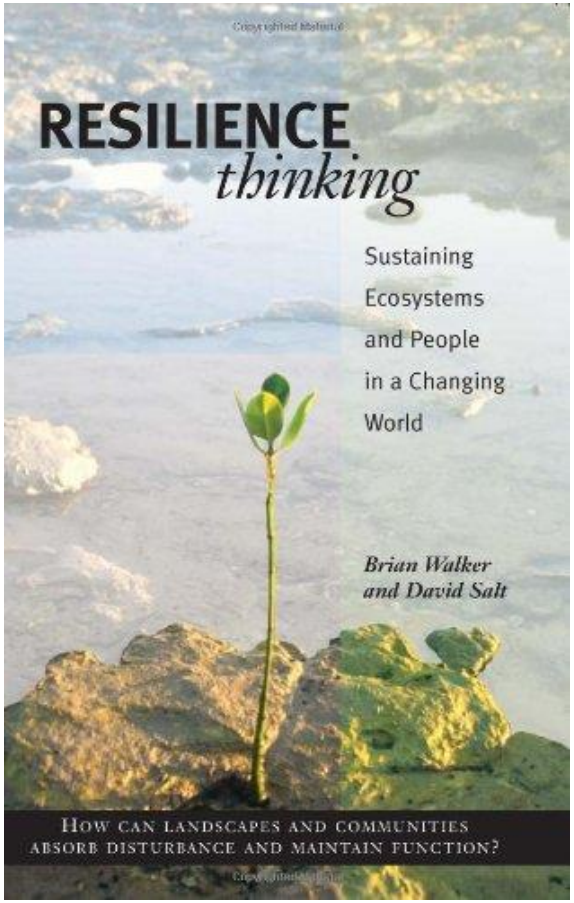
Sarah Iams

Emerging Models of Resilience
-Introduction-

1. Why resilience?

2. Resilience quantification

3. An example



Resilience:

*"[T]he **capacity** of [a] system to **absorb change** and disturbances and still **retain** its basic **structure and function**"*

- Brian Walker and David Salt

Example

Oligotrophic Lake

Eutrophic Lake

Carpenter et al., *Ecological Applications,* 1999

http://resac.gis.umn.edu/water/regional_water_clarity/content/project_summary.htm

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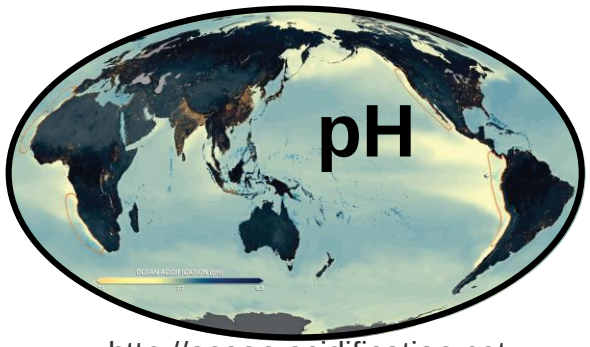
Resilience *of* What *to* What?

- attractor
- basin of attraction
- set in state space

- disturbances…

Carpenter et al., *Ecosystems,* 2001

Disturbances



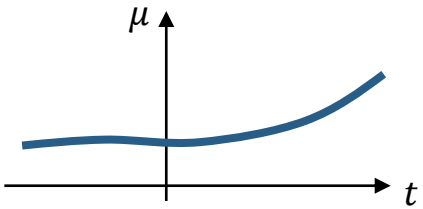
http://ocean-acidification.net

http://www.alaskajournal.com

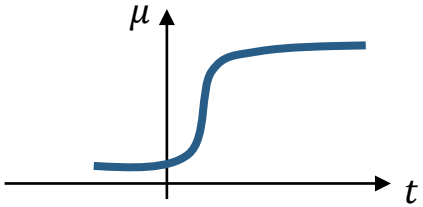
http://news.psu.edu/

continuous ⟷ **discrete**

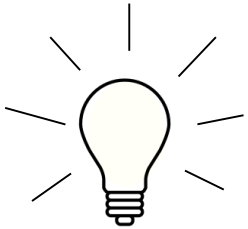
$\mu(t)$ gradual



... or rapid

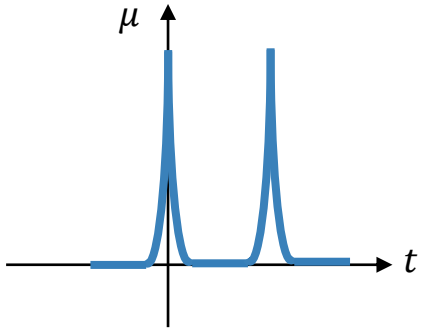


... or periodic ...

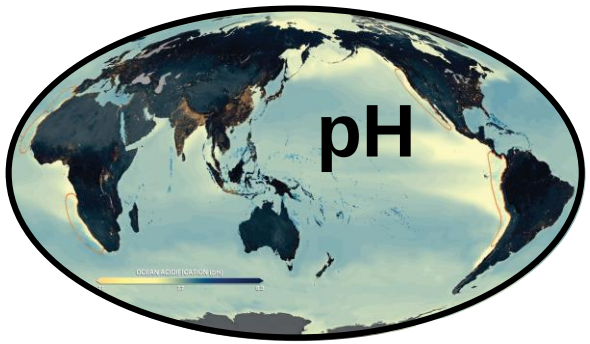


$x' = f(x, \mu(t))$

$\mu(t) \sim$ Dirac δ



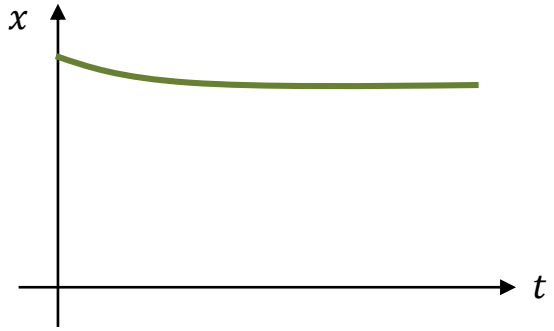
Disturbances

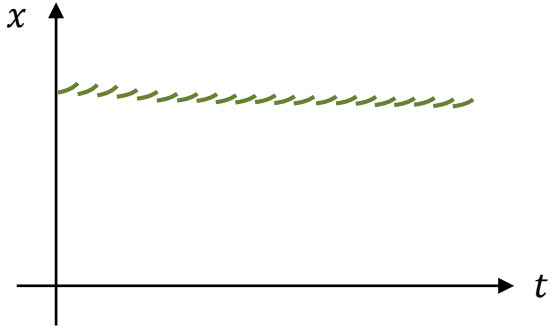


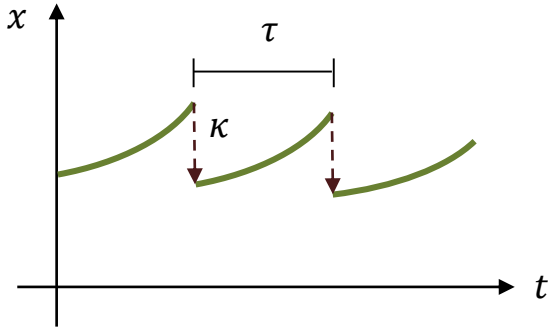
http://ocean-acidification.net

continuous ⟵ ⟶ **discrete**

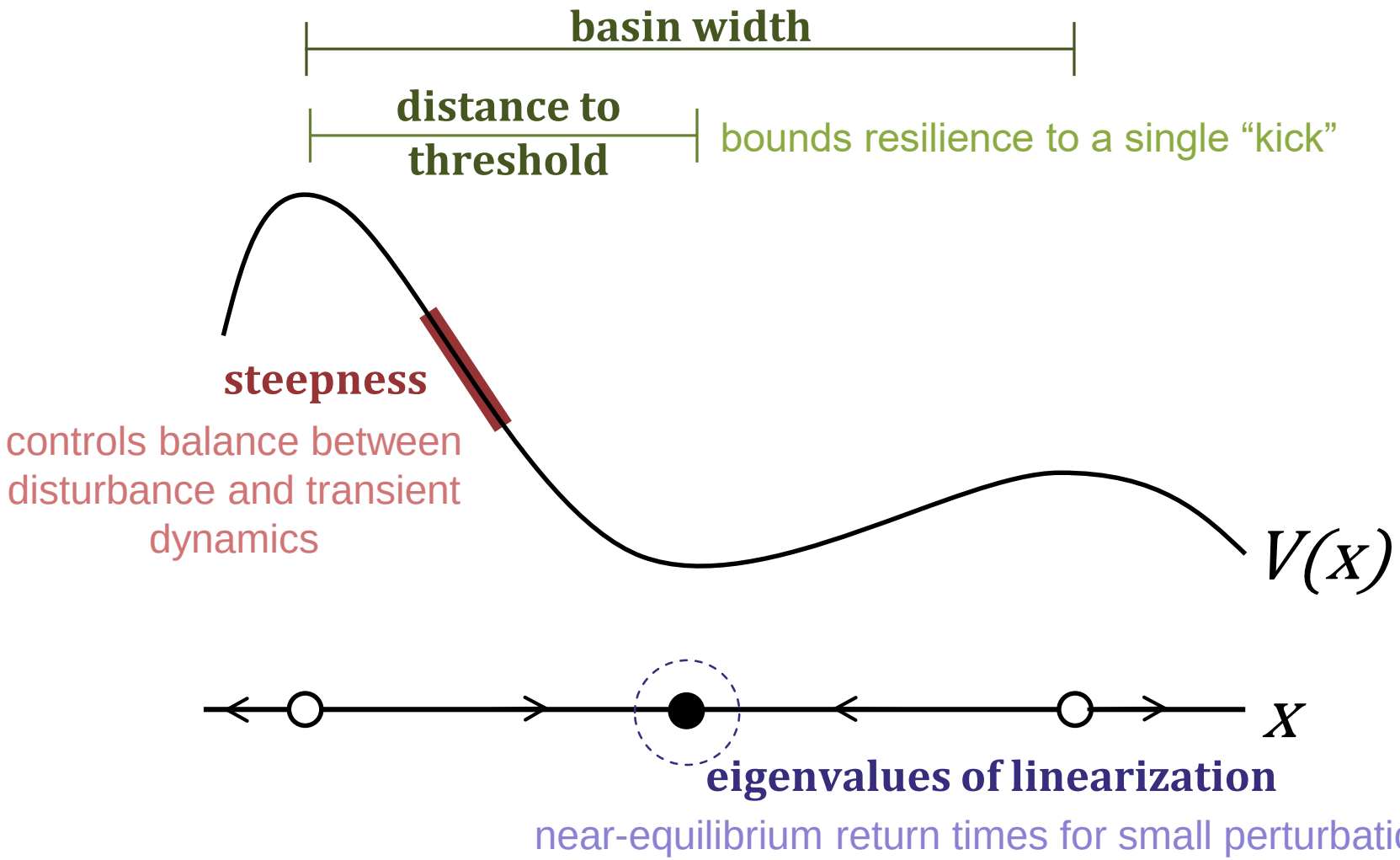
$$x' = f(x) + \kappa/\tau \qquad \xleftarrow[\text{let } \tau, \kappa \to 0]{\text{fix } \frac{\kappa}{\tau} = r} \qquad x \mapsto \varphi_\tau(x) + \kappa$$





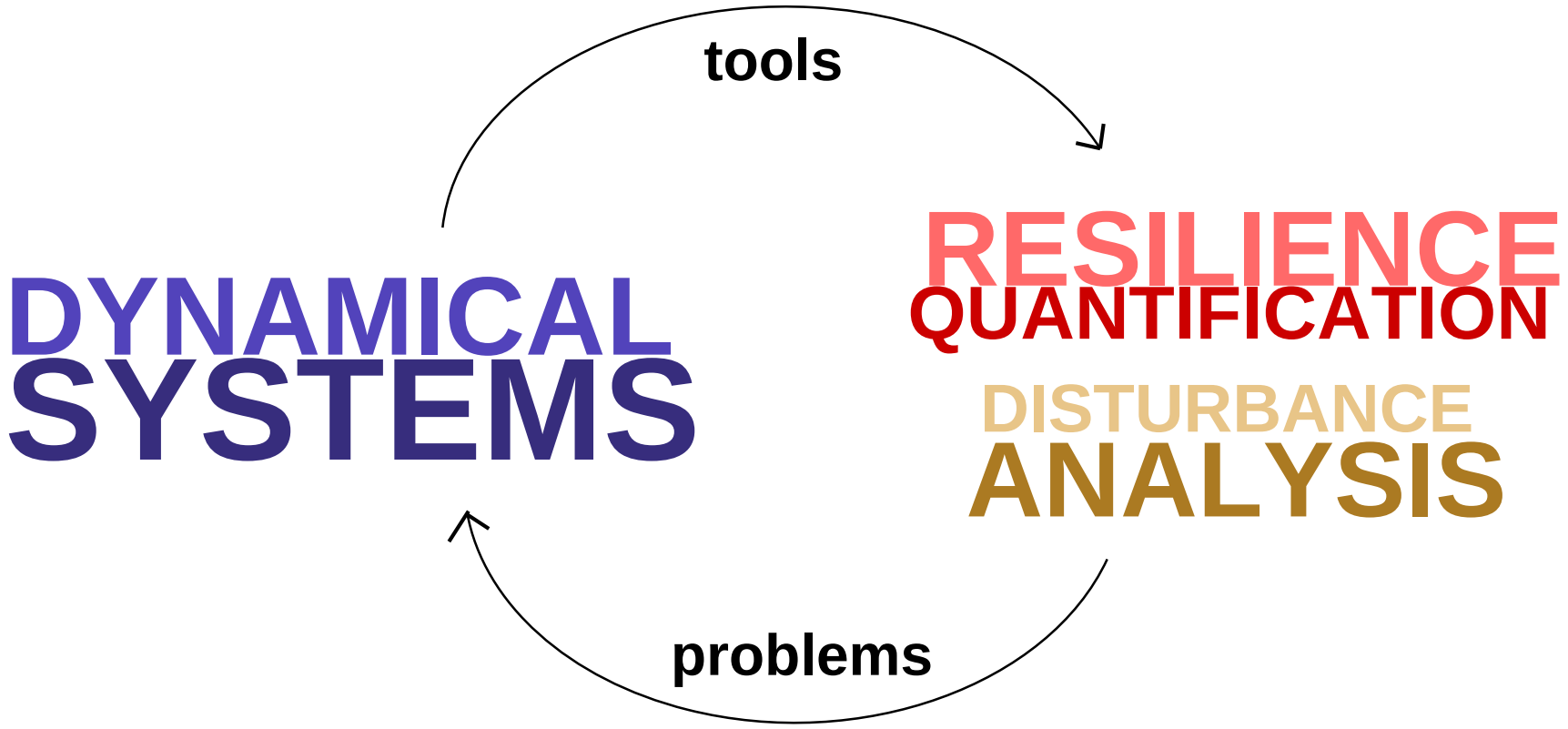


Common Resilience Indicators



Meyer, *Natural Resource Modeling,* 2016

An Opportunity



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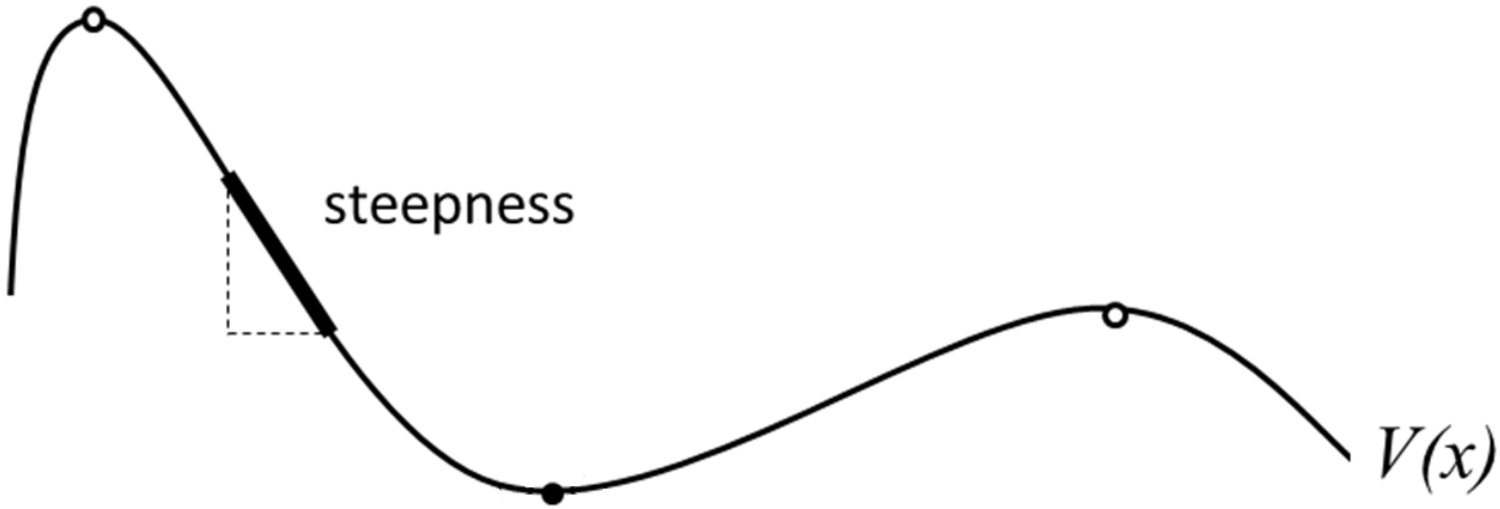
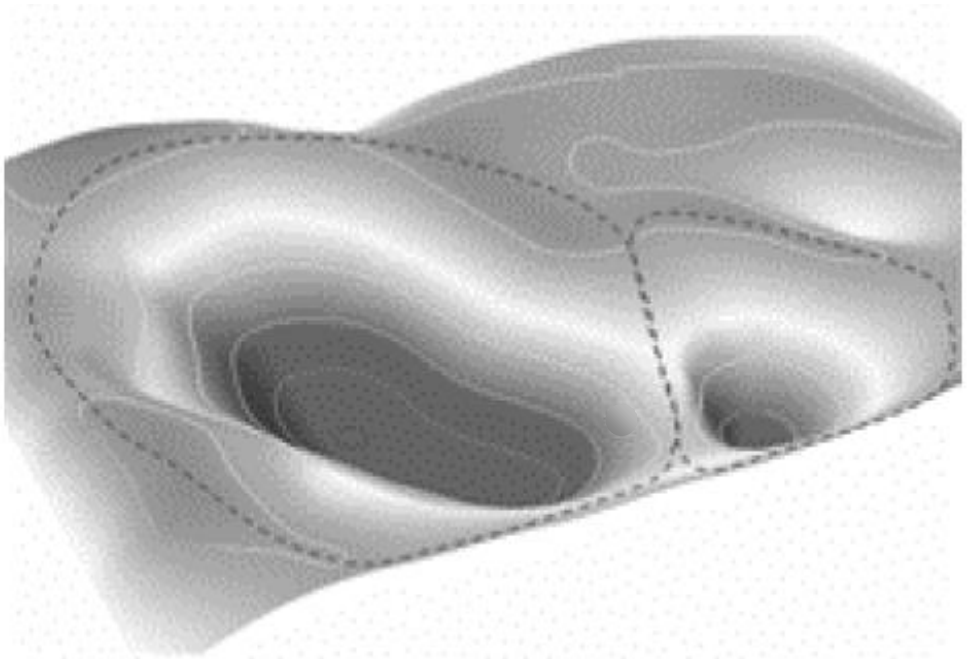
3. An example

What does “steepness” really measure?



What does “steepness” really measure?



Walker et al. (2004)

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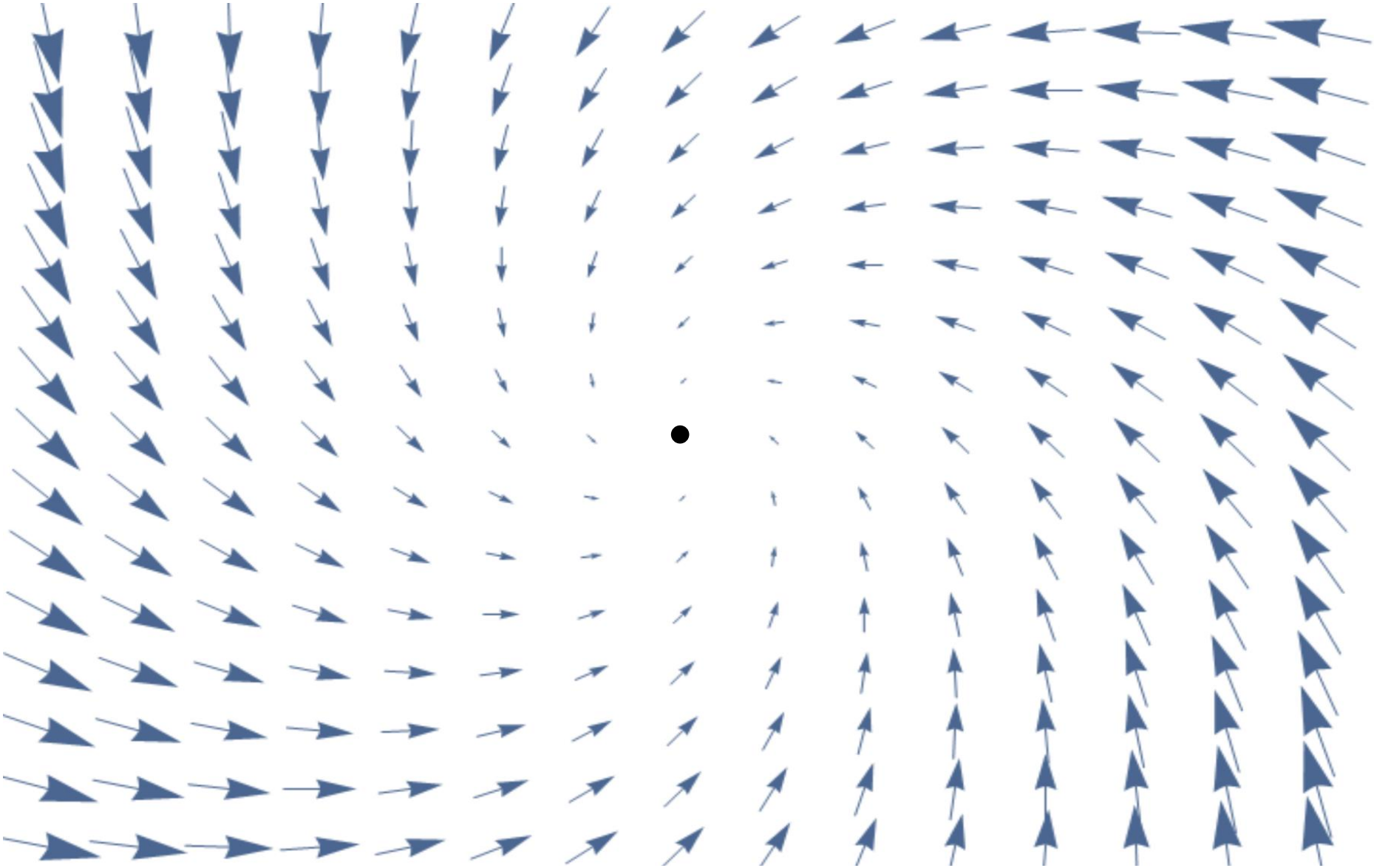
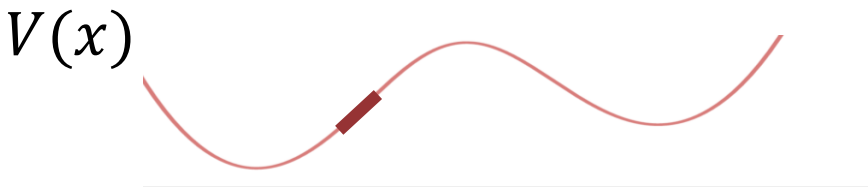
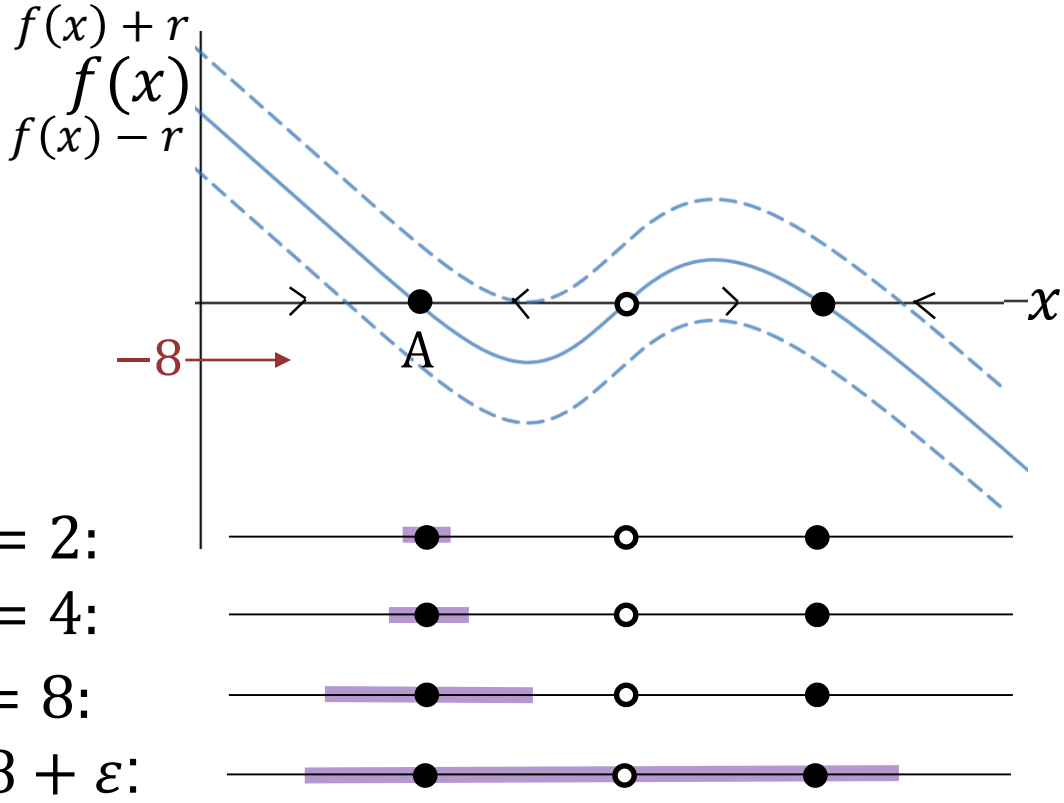


A possible definition of steepness

$$x' = f(x) + g(t)$$

$$|g|_{\sup} \le r$$





$P_r(A)$

points accessible from A with $|g| \le r$

A possible definition of steepness

$$x' = f(x) + g(t)$$
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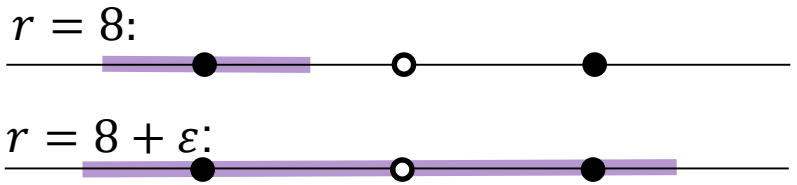
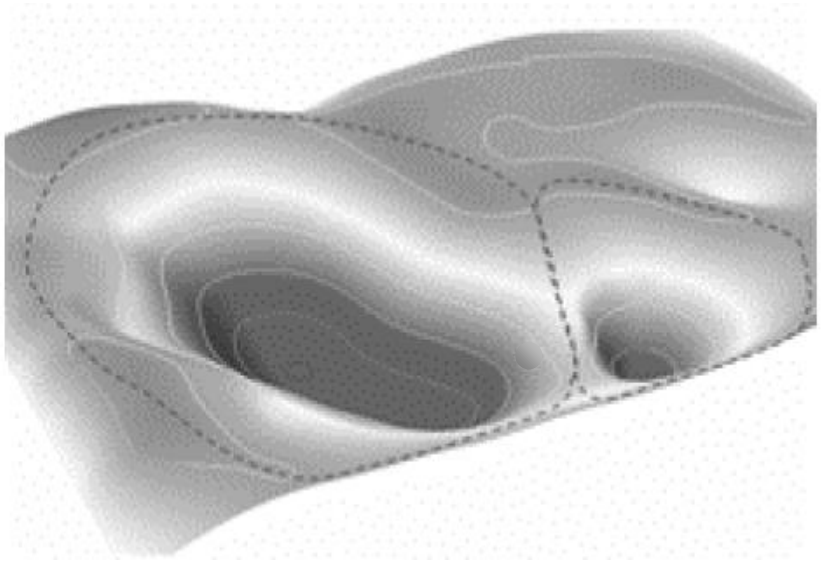


In analogy to McGehee (1988):

Chain Intensity of Attraction of A $\equiv \sup \{r \in \mathbb{R}^+ \mid P_r(A) \subset \mathcal{D}(A)\}$



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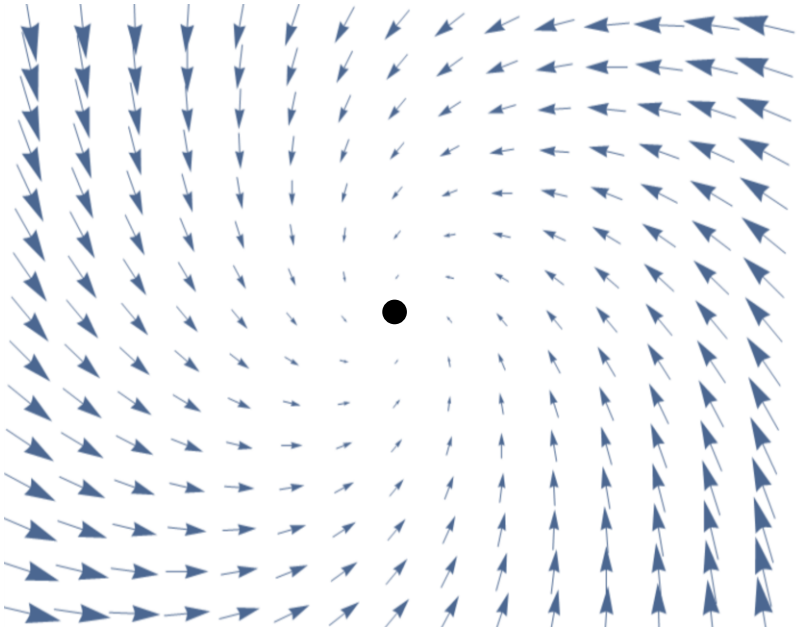
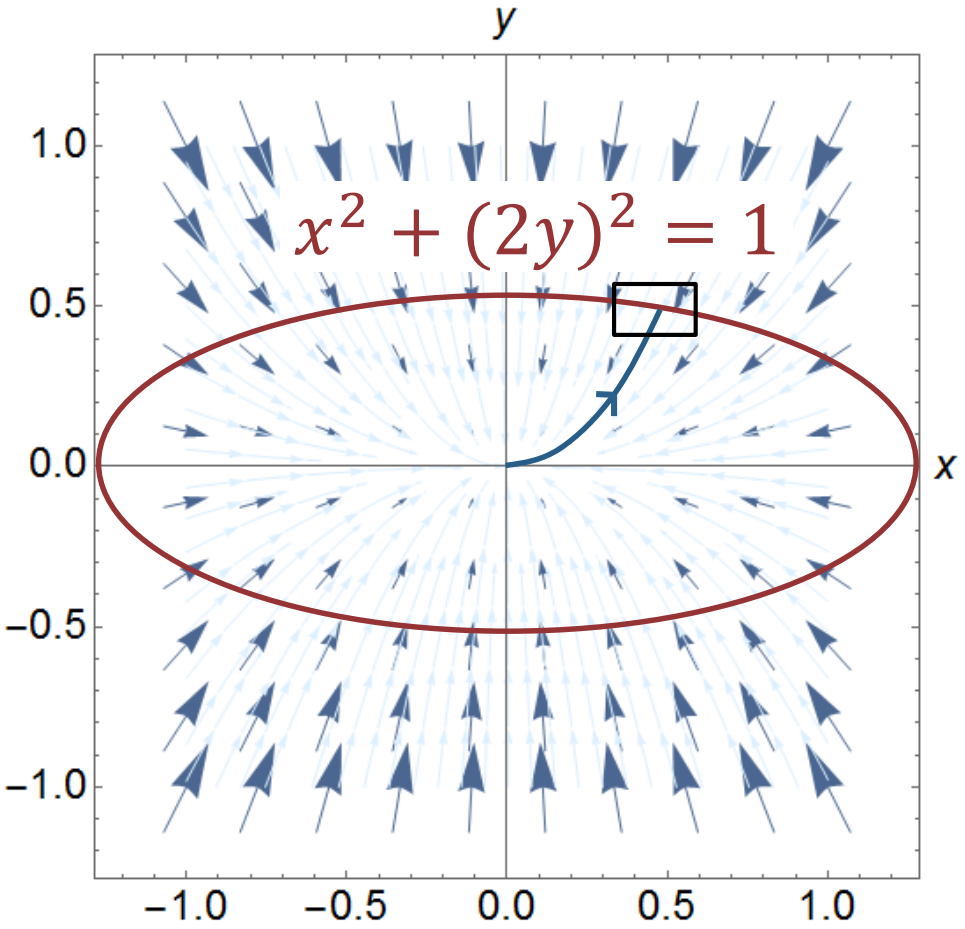
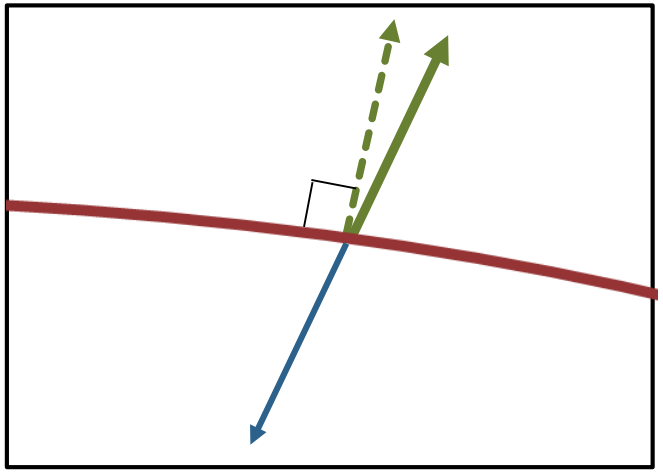


Computing accessible regions

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \vec{g}(t)$$

$$P_1(\mathbf{0}) = ?$$





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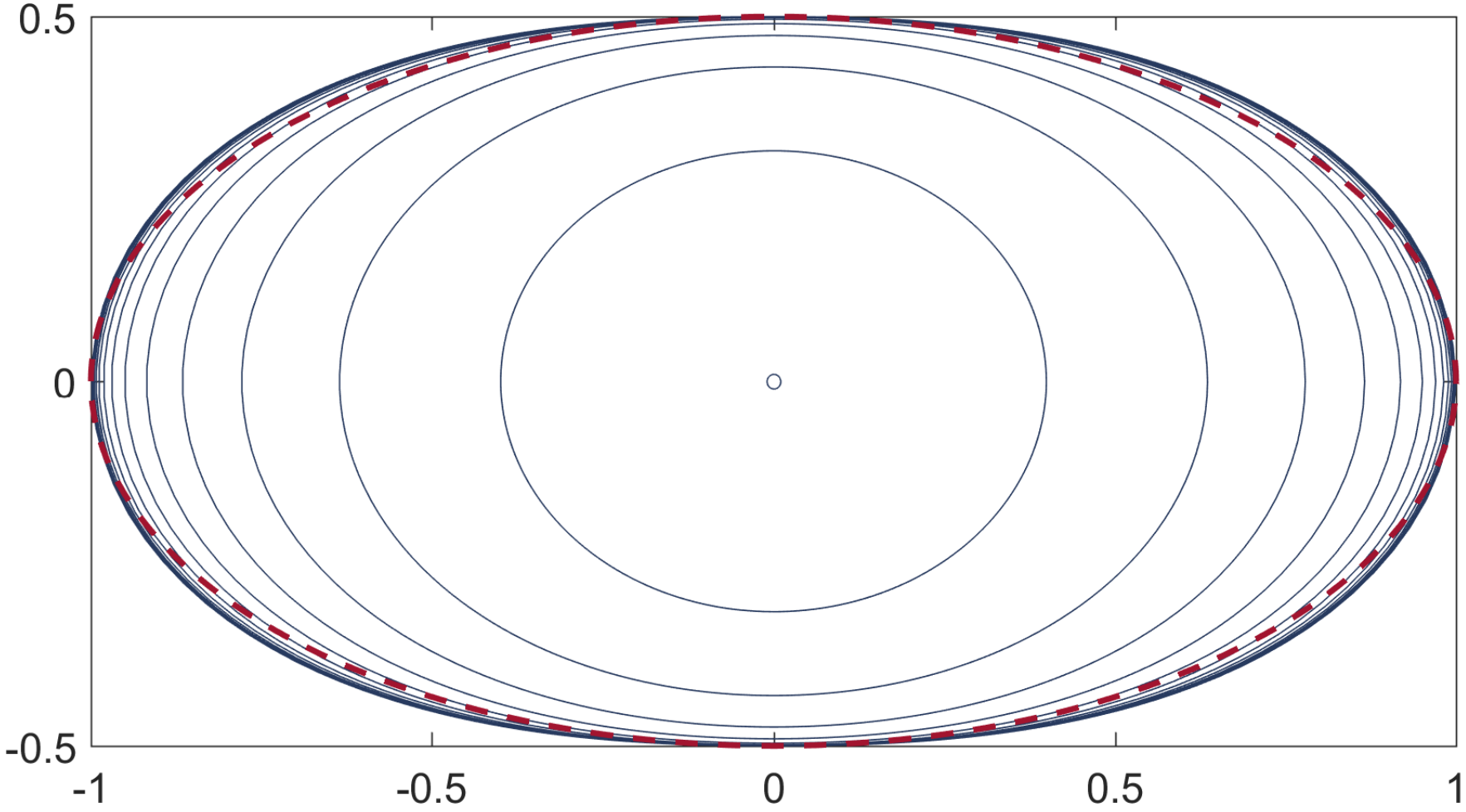
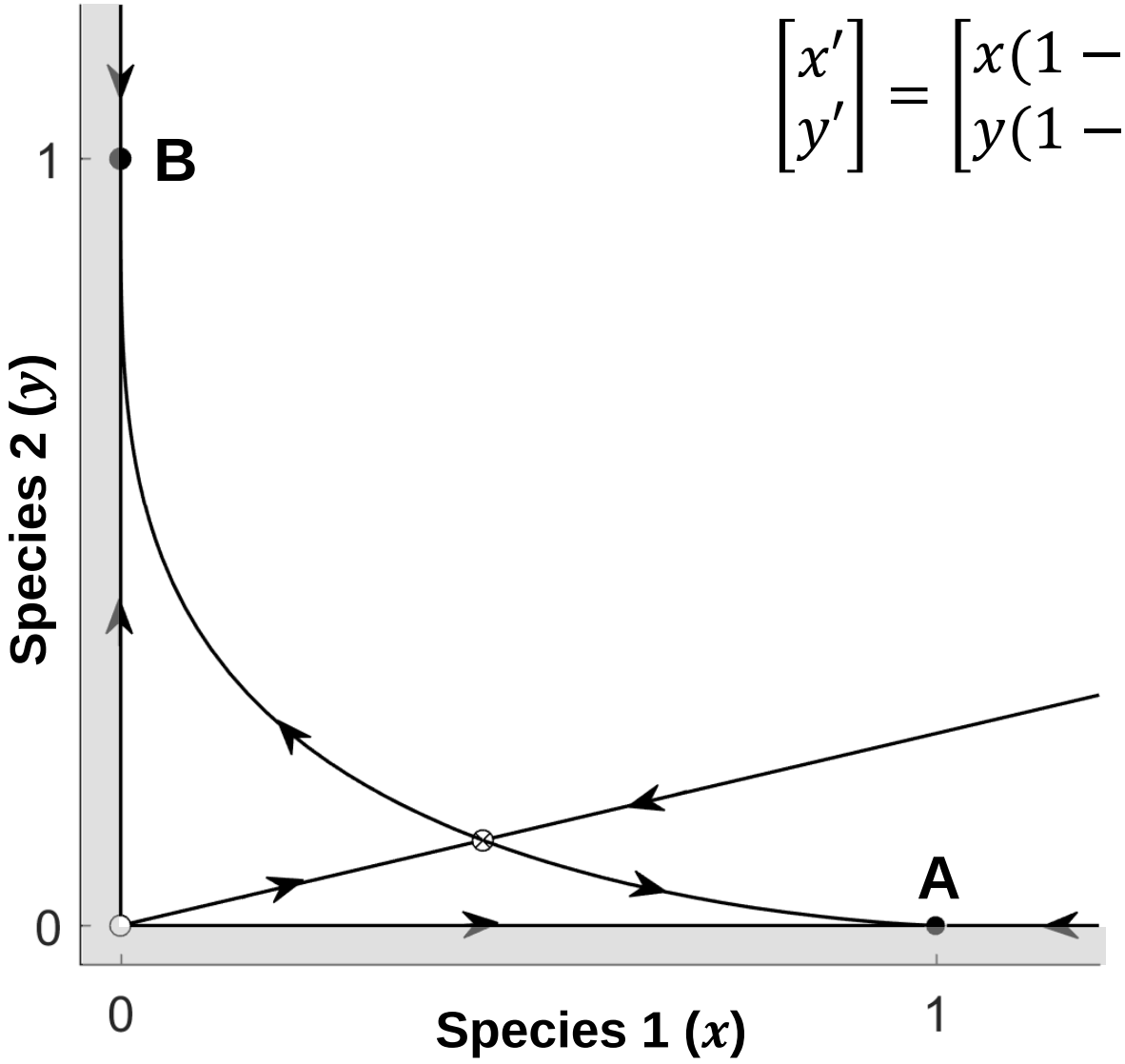


Steepness / Intensity in Lotka-Volterra

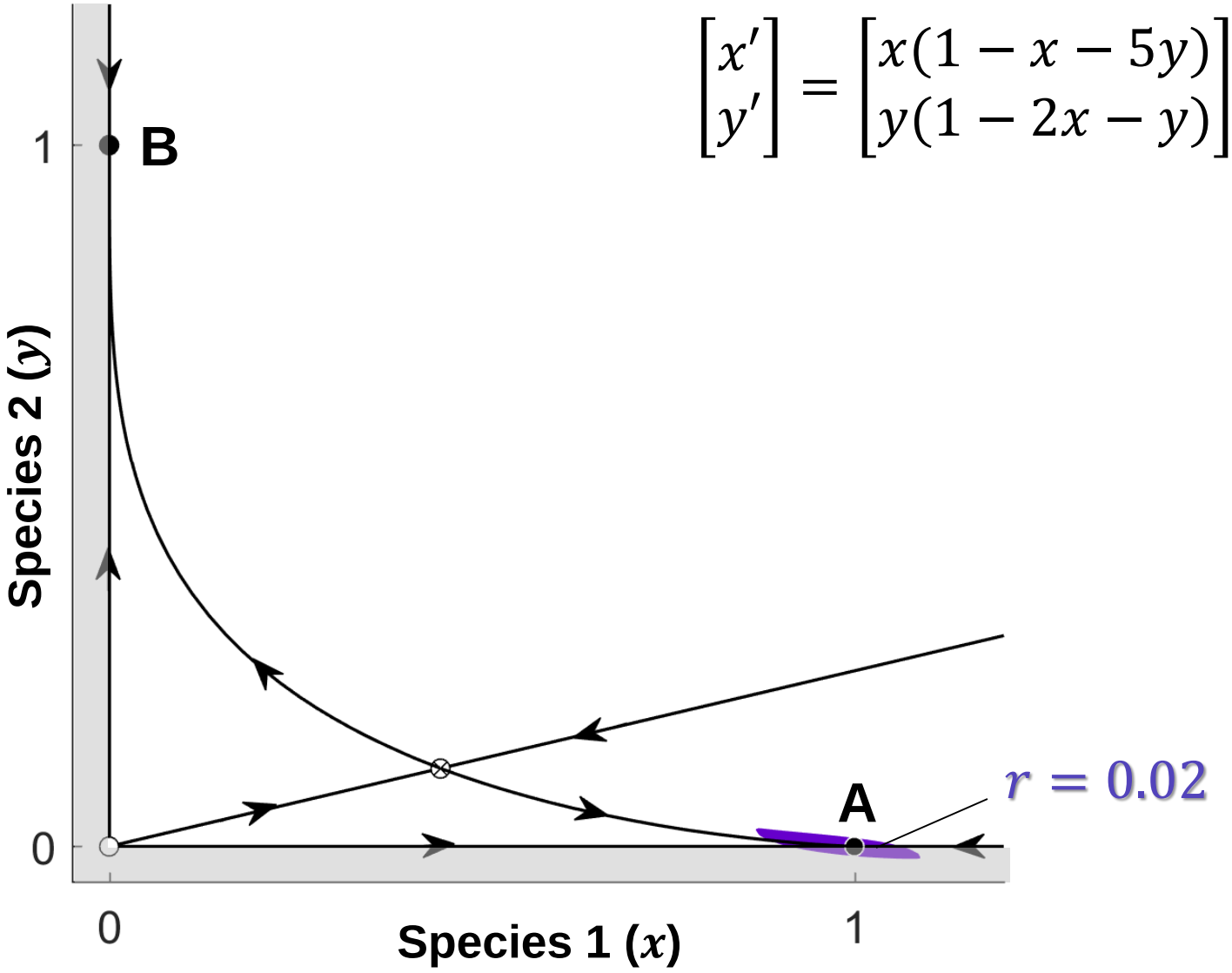
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x(1-x-5y) \\ y(1-2x-y) \end{bmatrix}$$



Steepness / Intensity in Lotka-Volterra

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x(1-x-5y) \\ y(1-2x-y) \end{bmatrix} + \vec{g}(t)$$

$$|g| \leq r$$

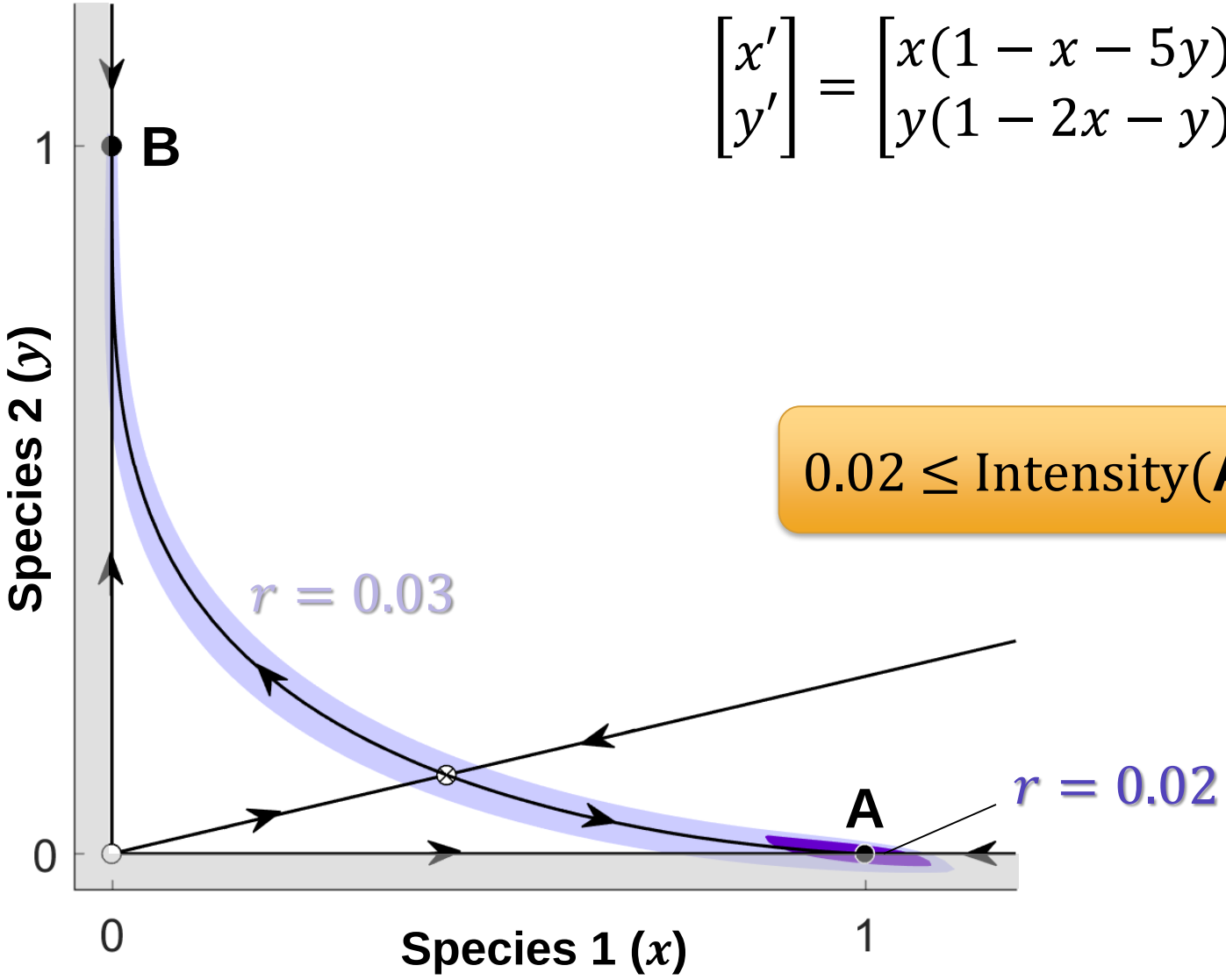


Steepness / Intensity in Lotka-Volterra

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x(1-x-5y) \\ y(1-2x-y) \end{bmatrix} + \vec{g}(t)$$

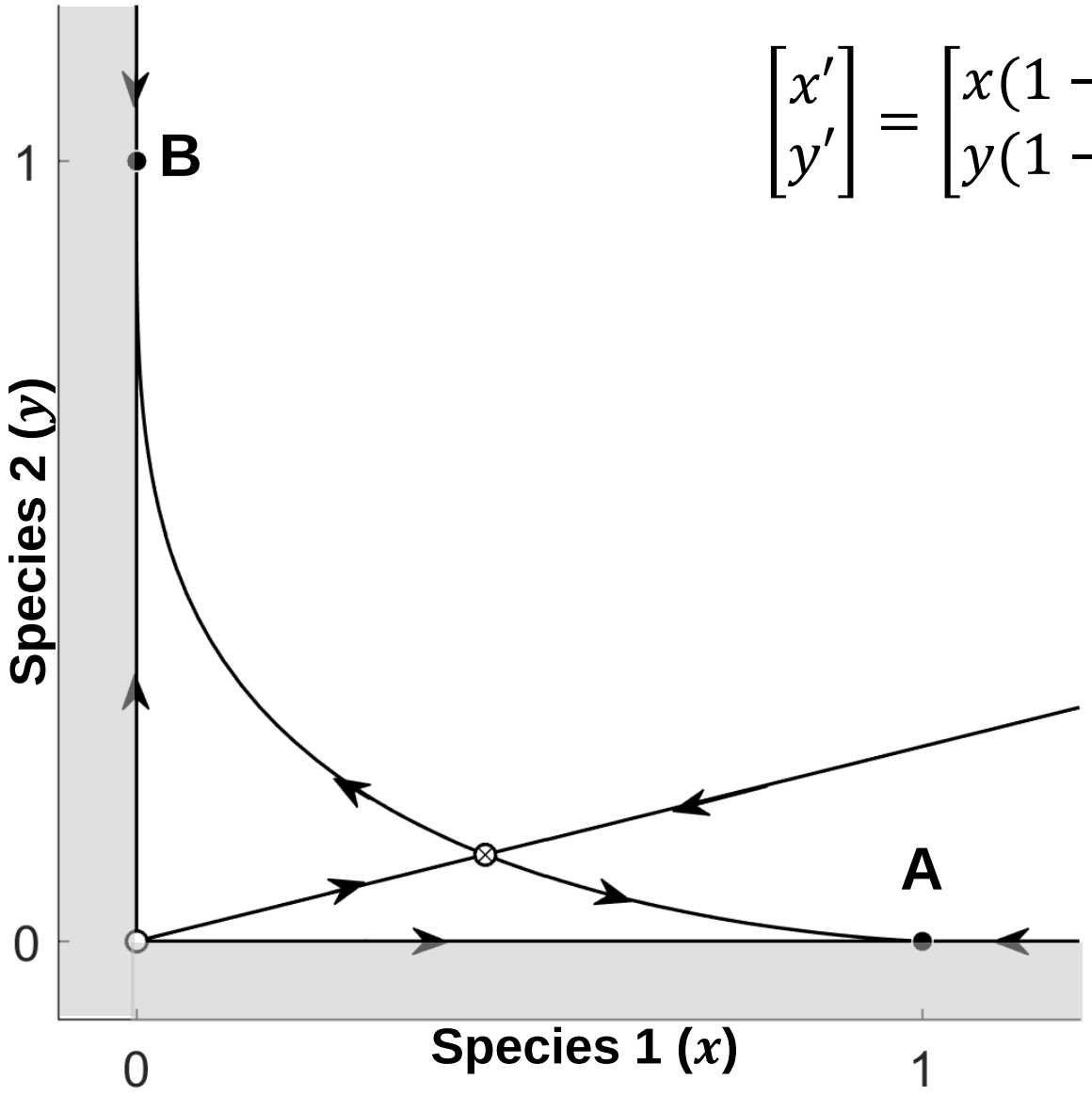
$$|g| \leq r$$

$0.02 \leq \text{Intensity}(\mathbf{A}) \leq 0.03$

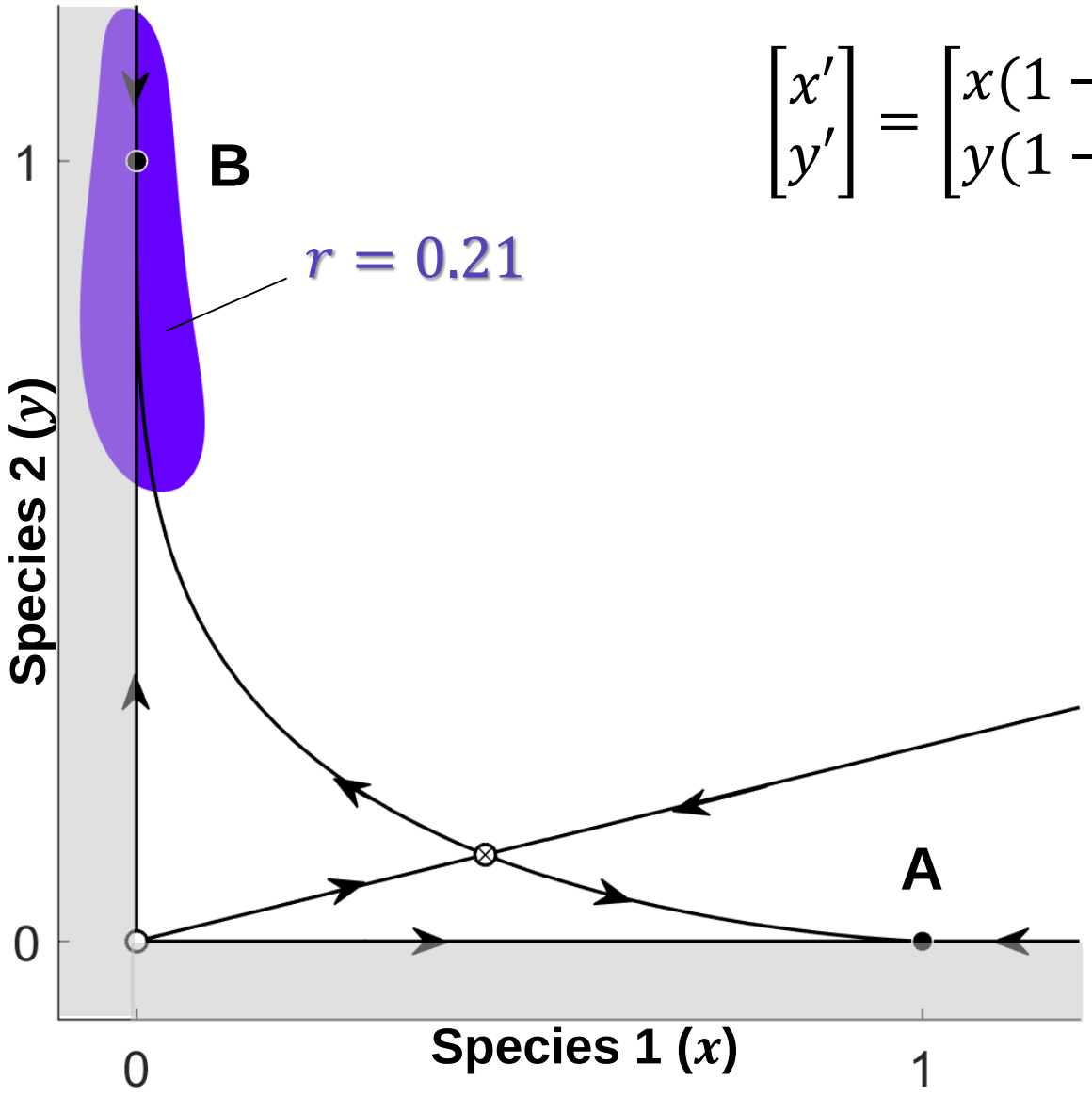


Steepness / Intensity in Lotka-Volterra

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x(1-x-5y) \\ y(1-2x-y) \end{bmatrix}$$



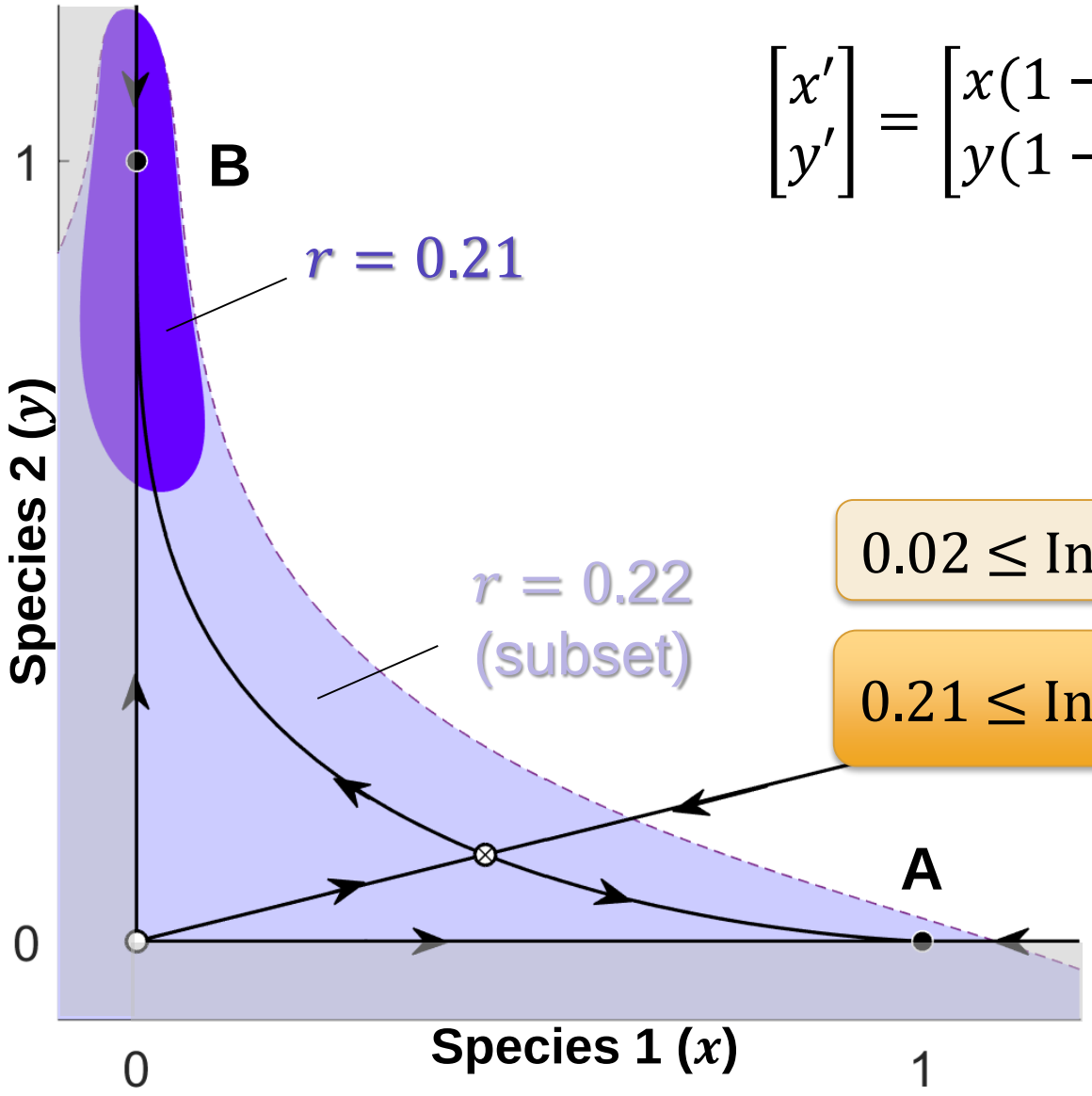
Steepness / Intensity in Lotka-Volterra



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x(1-x-5y) \\ y(1-2x-y) \end{bmatrix} + \vec{g}(t)$$

$$|g| \leq r$$

Steepness / Intensity in Lotka-Volterra



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x(1-x-5y) \\ y(1-2x-y) \end{bmatrix} + \vec{g}(t)$$

$$|g| \leq r$$

$0.02 \leq \text{Intensity}(\mathbf{A}) \leq 0.03$

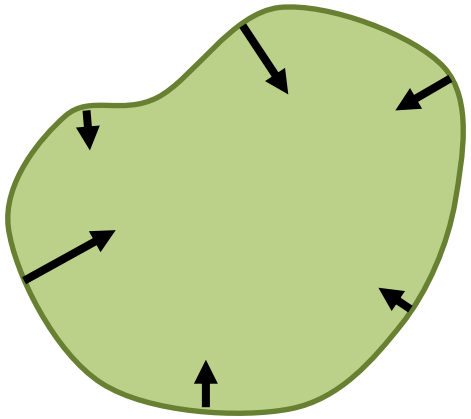
$0.21 \leq \text{Intensity}(\mathbf{B}) \leq 0.22$

Current Directions

Attractor block formulation (as in McGehee 1988)

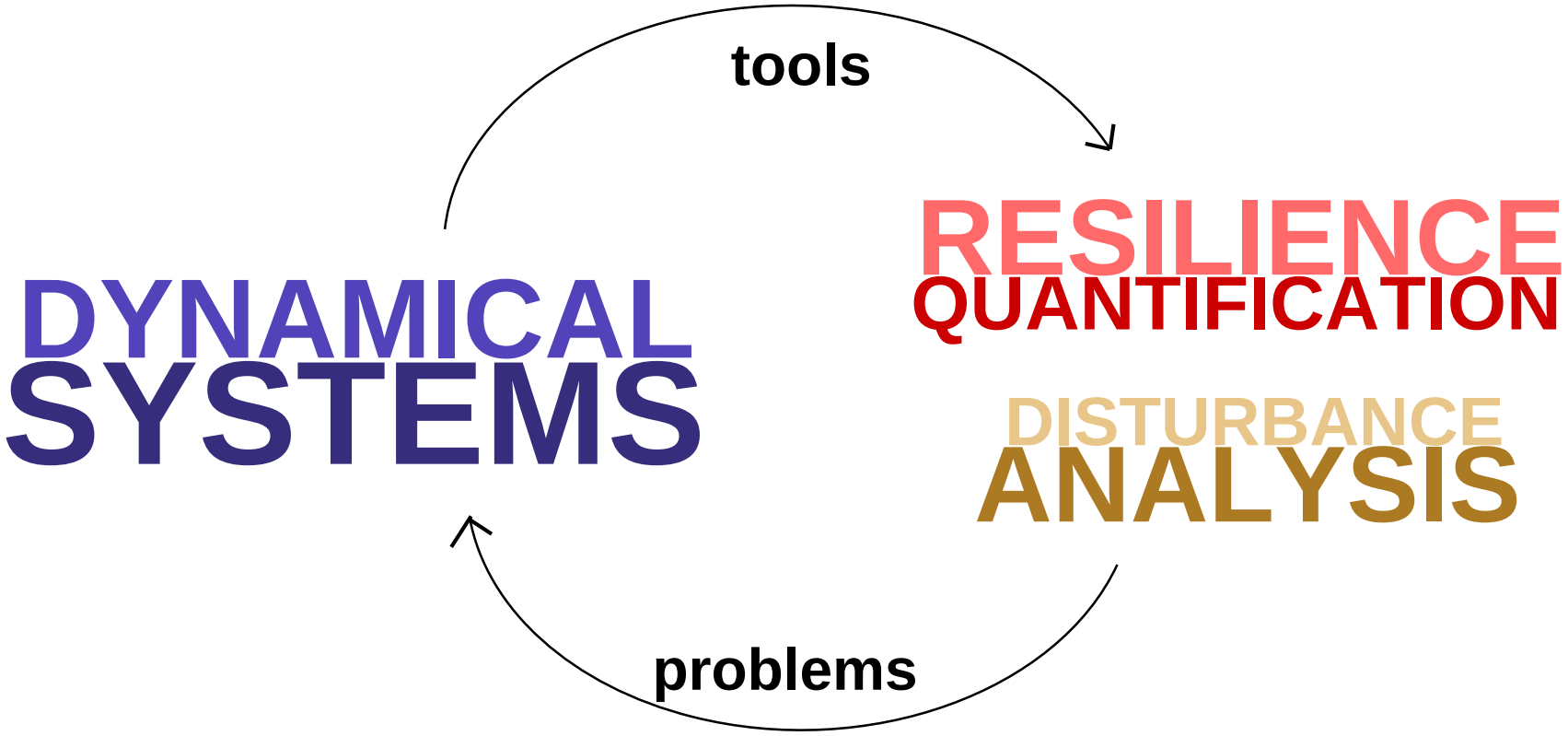
Block intensity of attraction:
$\sup\{\beta(B): B \text{ an attractor block associated with } A\}$

Conjecture:
Chain intensity = Block intensity



Computational methods for finding accessible regions

To Be Continued…



References

Resilience Concept

Walker B and Salt D. (2006) *Resilience thinking: sustaining ecosystems and people in a changing world*. Washington DC: Island Press. Print.

Carpenter S, Walker B, Anderies, and Abel N. (2001) From metaphor to measurement: resilience of what to what? Ecosystems 4: 765-781

Lake Eutrophication

Carpenter SR, Ludwig D, and Brock WA (1999) Management of eutrophication for lakes subject to potentially irreversible change. Ecological Applications 9:751-771.

Common Resilience Indicators

Meyer K. (2016) A mathematical review of resilience in ecology. Natural Resource Modeling 29: 339-352.

Intensity of Attraction

McGehee R. (1988) Some metric properties of attractors with applications to computer simulations of dynamical systems. Unpublished report. *http://www-users.math.umn.edu/~mcgehee/publications/McGehee1988/index.html*