NCSEA Structural Engineering Exam

Review Course

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Concrete Lateral Design

By

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REINFORCED CONCRETE DESIGN

OVERVIEW

Reinforced Concrete Design is governed by Chapter 19 of IBC. IBC refers to ACI 318 for the design of reinforced concrete, with amendments listed in §1905. The following codes are referenced in this concrete design review manual:

1. IBC 2012: Section 1613 & Chapter 19
2. ASCE 7-10: Excluding Chapter 14 & Appendix 11A (IBC 1613.1)
3. ACI 318-11 (with appropriate amendments per IBC 1905)

Load Combinations:

Concrete design is to be per the load combinations listed in IBC §1605.2 (see section 'Chapter 16' for a full list of load combinations). Since concrete design is typically based on strength level demands, the following load combinations apply:

IBC 1605.2

1. $1.4(D + F)$
2. $1.2(D + F) + 1.6(L + H) + 0.5(L_r \text{ or } S \text{ or } R)$
3. $1.2(D + F) + 1.6(L_r \text{ or } S \text{ or } R) + (f_1 L + 0.5W) + 1.6H$
4. $1.2(D + F) + 1.0W + f_1 L + 0.5(L_r \text{ or } S \text{ or } R) + 1.6H$
5. $(1.2 + 0.2S_{DS})D + 1.2F + \rho Q_E + f_1 L + 0.2S + 1.6H$
6. $0.9D + 1.0W + 1.6H$
7. $(0.9 - 0.2S_{DS})D + 0.9F \pm \rho Q_E + 1.6H$

Notes: 1. Load combinations 5 and 7 are from ASCE 7, §12.4.2.3

Additional special seismic load combinations with over-strength (ASCE 7, §12.4.3.2), when required by the code:

1. $(1.2 + 0.2S_{DS})D + \Omega_o Q_E + f_1 L + 0.2S$
2. $(0.9 - 0.2S_{DS})D \pm \Omega_o Q_E$

Where,
D, L = Dead, live loads
W, Q_E = Wind, horizontal seismic loads (E_h)
F, H, T= Flood, hydrostatic (e.g. earth pressure) and self-straining (e.g. temperature) loads
R,S = Rain, snow loads
f_1 = 1.0 for assembly areas, garage loads or if L > 100psf
= 0.5 for all other live loads
f_2 = 0.7 for roofs that do not shed snow (saw tooth roofs etc.)
= 0.2 for all other roofs

The combinations listed above shall be used in conjunction with the appropriate ϕ factors described in the following sections.

Selection & Design of Lateral System

References: IBC Chapter 19 & ACI 318

The IBC places limits on the type of structural systems that can be used for lateral design based on Seismic Design Category (SDC)--see section '*Chapter 16*' and '*Earthquake Loads*' for more information.

The brief list below specifies the minimum concrete lateral system requirements for a given SDC. It is always permitted to provide a better lateral system and take advantage of the lower seismic design forces (ACI 318, R21.1.1)

For a detailed listing of lateral systems and associated limitations, see ASCE 7 Table 12.2-1. All reference in the following are to IBC, unless noted otherwise.

Seismic Design Categories A & B (Low Seismic Risk)

- <u>Ordinary Shear Walls</u>

  Designed using Chapters 1 through 18 of ACI 318.

  Note: For SDC A, shear walls can be ordinary plain concrete walls per Chapter22 of ACI 318 or detailed plain concrete structural walls per IBC §1905.1.7.

- <u>Ordinary Precast Concrete Shear Walls</u>

  Designed using Chapters 1 through 18 of ACI 318.

- <u>Ordinary Moment Frames</u>

  Designed using Chapters 1 through 18 of ACI 318.

  For SDC B, ACI 318 §21.2.2 – Provide at least two reinforcing bars continuously at top and bottom in beams and develop at the face of the support.

  For SDC B, ACI 318 §21.2.3 – Columns with clear height to maximum dimension ratio of five or less shall also be designed for shear per ACI §21.3.3.2.

Seismic Design Category C (Intermediate or Moderate Seismic Risk)

- Ordinary Shear Walls:

  Designed using Chapters 1 through 18 of ACI 318.

  Note: Plain concrete shear walls not permitted except as basement (retaining) walls for one or two family dwellings with stud framing above (§1905.1.8).

- Intermediate Precast Concrete Shear Walls:

  Designed using Section 21.4 in addition to Chapters 1 through 18 of ACI 318.

- Intermediate Moment Frames:

  Designed using Section 21.3 in addition to Chapters 1 through 18 of ACI 318.

- Discontinuous Members (ACI 318 §21.3.5.6): Columns supporting discontinuous lateral systems above (such as shear walls), and with factored axial force exceeding $A_g f'_c/10$ (or $A_g f'_c/4$ for load combinations with Ω_o), shall be provided with traverse reinforcement with spacing per 21.3.5.2 over the full height as well as above and below as required.

Seismic Design Categories D, E, F (High Seismic Risk)

- Special Shear Walls:

  Cast-in-place walls designed using Sections 21.1.3 through 21.1.7 and 21.9, in addition to Chapters 1 through 18 of ACI 318.

  Precast walls shall also satisfy §21.10 of ACI 318 in addition to the above.

- Special Moment Frames:

  Cast-in-place frames designed using Sections 21.1.3 through 21.1.7 and Sections 21.5 through 21.8 in addition to Chapters 1 through 18 of ACI 318.

  Precast frames designed per Section 21.8 of ACI 318.

- Diaphragms and Foundations:

  Design using Section 21.11 for diaphragms and 21.12 for foundations, in addition to Chapters 1 through 18 of ACI 318.

- Frame members not part of the lateral system:

  Design/check per section 21.13 to ensure that they can continue to carry the gravity loads at the maximum lateral displacements corresponding to the design level seismic forces.

The following tables summarize the design requirements for the different lateral systems for different Seismic Design Categories.

Moment Frames:

TYPE	**DESIGN REQUIREMENTS**	**SDC**
Ordinary MF	ACI 318, Chapters 1-18 + Section 21.2	A, B
Intermediate MF	ACI 318, Chapters 1-18 + Section 21.3	A, B, C
Special MF	ACI 318, Chapters 1-18 + Section 21-5 to 21-8	ALL

Structural Walls:

TYPE	**DESIGN REQUIREMENTS**	**SDC**
Ordinary RC Shear Walls	ACI 318, Chapters 1-18	A, B, C
Special RC Shear Walls	ACI 318, Chapters 1-18 + Sections 21.1.3 to 21.1.7, 21.9	ALL
Ordinary Precast Shear Walls	ACI 318, Chapters 1-18	A, B
Intermediate Precast Shear Walls	ACI 318, Chapters 1-18 + Section 21.4	A, B, C
Special Precast Shear Walls	All Requirements for Special RC Walls + Section 21.10	ALL

Notes:

1. Refer to ASCE 7, Table 12.2-1 for walls types, R & Ω_o values, height limitations etc.
2. Also see ACI 318 Section 2.2 & IBC Section 1905

REINFORCED CONCRETE

DEVELOPMENT & SPLICING OF REINFORCEMENT

DEVELOPMENT OF REINFORCEMENT

Reference: ACI 318 Chapter 12

Development in Tension (§12.2)

Development length is a function of clear spacing (s_c), clear cover (c) and/or tie spacing (s). The basic equation of development length is:

$$l_d = \left(\kappa \frac{f_y \psi_t \psi_e}{\lambda \sqrt{f'_c}} \right) d_b \;\; \& \;\; l_d \geq 12" \qquad (f_y \;\&\; f'_c \text{ in psi}) \qquad \S 12.2.2$$

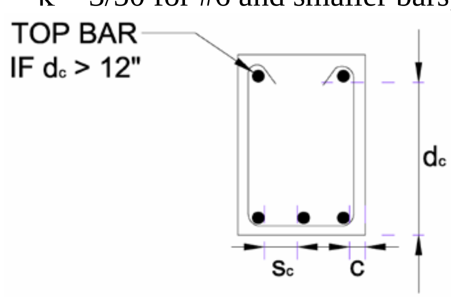
& $\psi_t \psi_e$ need not exceed 1.7

$$\& \qquad \sqrt{f'_c} \leq 100\, psi \qquad \S 12.1.2$$

where, ψ_t = 1.0 for $d_c \leq$ 12in; 1.3 for d_c >12in (i.e. top reinforcement)
ψ_e = 1.0 for uncoated reinforcement (§12.2.4 for coated reinforcement)
λ = 1.0 for normal weight concrete; 0.75 for all light weight concrete
κ = based on clear spacing, cover, stirrup or tie extent over lapped bars (s) etc.:

Case I - $s_c \geq d_b$; $c \geq d_b$; stirrups or ties over lap comply with code minimum OR $s_c \geq 2d_b$ & $c \geq d_b$:
κ = 1/25 for #6 and smaller bars; 1/20 for larger bars

Case II – Other conditions
κ = 3/50 for #6 and smaller bars; 3/40 for larger bars



Notes: 1. See Table 1(Sheet RDL3) for l_d for f_y = 60ksi
2. For Special Structural Walls and Coupling Beams, at locations of bar yielding the development length shall be 1.25 times that calculated above for f_y (§21.9.2.3).

Development in Compression (§12.3)

$$l_d = \frac{0.02 d_b f_y}{\lambda \sqrt{f'_c}} \quad ,\text{but} \quad l_d \geq 0.0003 d_b f_y \;\; \& \quad l_d \geq 8" \text{ ACI 318 §12.3.2}$$

Note: compression development length can be reduced to 75% of above for appropriately confined bars. See Sections ACI 318 12.3.3 for requirements.

Standard Hooks in Tension (§12.5)

Development length for a standard hook, l_{dh}, shall not be less than $8d_b$, 6in or from the equation below:

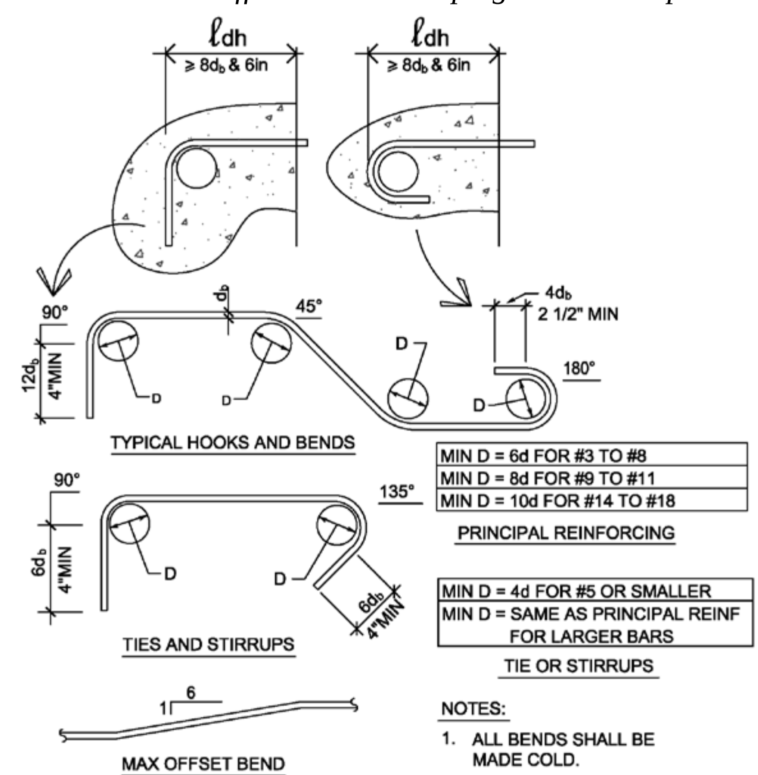
$$l_{dh} = \frac{0.02 \psi_e f_y}{\lambda \sqrt{f'_c}} d_b \qquad (f_y, f'_c \text{ in psi}) \qquad §12.5.2$$

where, ψ_e = 1.0 for regular reinforcement; 1.2 for epoxy-coated reinforcement
λ = 1.0 for normal weight concrete; 0.75 for lightweight concrete

Scale factors that can be applied:

- For #11 bar or smaller with side cover ≥ 2.5in & cover on 90º hook extension ≥ 2in — 0.7
- For #11 bar or smaller if 90° or 180° hook and ℓ_{dh} is enclosed by stirrups spaced at ≤ $3d_b$. — 0.8

Note: Hooks are not considered effective in developing bars in compression.



Hooks & Development Lengths for Joints of Special Moment Frames (§21.7.5)

Development length for a standard hook (§21.7.5.1): l_{dh} shall not be less than the largest of $8d_b$, 6in or from the equation below:

$$l_{dh} = \frac{f_y d_b}{65\sqrt{f'_c}} \qquad (f_y \ \&\ f'_c \text{ in psi}) \qquad \text{(Eqn 21-6)}$$

For lightweight concrete scale all above limits by 1.25. The 90° hook shall be within the confined core of a column or boundary element.

Development length for straight bars, #11 or smaller, (§21.7.5.2):

Top bars: $l_d \geq 3.25 l_{dh}$ (see RDL1 for definition of top bars, i.e. $d_c \geq 12$in)

Other bars: $l_d \geq 2.5 l_{dh}$

Note: The development length for standard hooks for joints of special frames is based on §12.5.2 and incorporates the reduction factors for adequate cover (0.7) and confinement (0.8) as well as an increase to account for load reversal. For example for f'_c = 4000psi and f_y = 60ksi, §12.5.2 gives an $l_{dh} = 18.97d_b$ while §21.7.5.1 gives $l_{dh} = 14.6d_b$.

	f'_c = 3000psi, f_y = 60,000psi				
	#6 or Smaller		#7 or Larger		Joints of SMF
	Case I	Case II	Case I	Case II	
Top bars	$57d_b$	$86d_b$	$72d_b$	$107d_b$	$55d_b$
Other Bars	$44d_b$	$66d_b$	$55d_b$	$83d_b$	$43d_b$

	f'_c = 4000psi, f_y = 60,000psi				
	#6 or Smaller		#7 or Larger		Joints of SMF
	Case I	Case II	Case I	Case II	
Top bars	$50d_b$	$74d_b$	$62d_b$	$93d_b$	$48d_b$
Other Bars	$38d_b$	$57d_b$	$48d_b$	$72d_b$	$37d_b$

	f'_c = 5000psi, f_y = 60,000psi				
	#6 or Smaller		#7 or Larger		Joints of SMF
	Case I	Case II	Case I	Case II	
Top bars	$45d_b$	$67d_b$	$56d_b$	$83d_b$	$43d_b$
Other Bars	$34d_b$	$51d_b$	$43d_b$	$64d_b$	$33d_b$

Notes: 1. Values based on §12.2.2 and §21.7.5.2 for: ψ_e =1.0, λ = 1.0

Table 1 – Typical Development lengths in Tension for Straight Bars

REINFORCEMENT SPLICES

Reference: ACI 318 §12.14

Lap Splices (§12.14.2)

- Lap splice lengths are specified in terms of development length, l_d.
- Minimum splice length = 12in.
- Center-to-center distance between lapped bars shall not exceed $\frac{lap\,length}{5}$ & 6in.
- Lap splices typically not permitted for bars larger than #11 (except at footing to column joints §12.14.2.1, §12.16.2 & §15.8.2.3).

Tension Lap Splices (§12.15)

Two ***Splice Classes*** are defined:

Class A	$1.0l_d$	ACI 318 12.15.1
Class B	$1.3l_d$	

Type of Lap splice required (ACI 318 §12.15.2)

$\frac{A_s\,Provided}{A_s\,Required}$	OR	$\frac{f_s}{f_y}$	Max. % of A_s spliced within required lap length	
			50%	100%
≥ 2		≤ 0.5	Class A	Class B
< 2		> 0.5	Class B	Class B

Note: At column mid-height & beam mid-span Class A splices are permitted.

Compression Lap Splices (§12.16)

- For $f_y \leq 60$ksi $\quad lap\,length = 0.0005 f_y d_b \quad$ (f_y in psi)
- For $f_y > 60$ksi $\quad lap\,length = (0.0009 f_y - 24) d_b \quad$ (f_y in psi)
- Minimum splice length = 12in.

Note: If f'_c < 3000psi, scale lap length by 1.33

Mechanical & Welded Splices (§12.14.3)

- Full mechanical or welded splices shall develop $1.25f_y$ of the bar
- Development of a lower stress in the bar shall be permitted only for #5 or smaller and with the following conditions (§12.15.5):
    - Splices are staggered 24"
    - Stress in bar is limited to splice strength and less than f_y

- Tensile force required at splice exceeds that requires by analysis and is at least 20,000psi time total reinforcement area.

Splice Requirements for Columns (ACI 318 §12.17)

If the bar stress due to factored loads is compressive:

- Use compression lap slice length from above (§12.17.2.1).

- When bars of different sizes, use lap splice length for larger diameter bar (§12.16.2).

- Lap splices between #14 & #18 bars with #11 or smaller bars are permitted (§12.16.2)

- In tied columns, if ties over the lap length satisfy $A_s \geq 0.0015hs$, scale lap length by 0.83, with a minimum lap length of 12in. (§12.17.2.4).
  Note: Use tie legs perpendicular to each column dimension and use critical value.

- For columns with spirals, scale lap lengths by 0.75, with a minimum lap length of 12in. (§12.17.2.5).

If the bar stress due to factored loads is tensile:

- If the bar stress, $f_s > 0.5f_y$, use Class B splice (§12.17.2.3).

- If the bar stress, $f_s \leq 0.5f_y$ (§12.17.2.2):
  - If half the bars or less are spliced at any section, and the splices are staggered by ℓ_d, use class A splice.
  - If more than half the bars are spliced at any section, use class B splice.

Splices for Seismic Design

Flexural members of special moment frames (§21.5.2.3):

- Lap splices are only permitted where confinement is provided by hoop or spiral transverse reinforcement.

- Lap slices are not permitted in joints, within a distance of twice the member dimension from the face of the joint and within regions of flexural yielding (i.e. plastic hinges).

- Only Class B splices shall be provided (§12.15.2)

Shear walls:

It is often unavoidable to provide lap splices at the top of the foundation for shear walls. In this case, the lap splices should be Class B with a development length for the dowels calculated using $1.25f_y$ (§21.9.2.3).

Mechanical & Welded Splice (§21.5.2.4):

For special moment frames and shear walls (§21.1.6 & 21.):

- Mechanical splices (§21.1.6)
  - Shall be either Type 1 (i.e. develop $1.25f_y$ of the bar per §12.14.3.2) or Type 2 (develop the full tensile strength of the bar).
  - Type 2 splices are permitted at any location. Type 1 splices cannot be used within a distance of twice the member depth from the joint face for frames or within areas of yielding of reinforcement due to inelastic displacements.
- Welded splices (§21.1.7)
  - Shall develop 1.25fy of the bar per §12.14.3.4.
  - Shall not be used within a distance of two times the member depth from the joint face for frames or within areas of yielding of reinforcement due to inelastic displacements (§21.1.7.1).
  - Welding of stirrups, ties, inserts etc to longitudinal reinforcement is not permitted (§21.1.7.2).

REINFORCED CONCRETE

BEAM DESIGN

Reference: ACI 318

General Design Provisions for Beams:

Analysis

Reinforcement Limits:

- **Check ρ**

$$A_{s\,min} \geq \begin{cases} \rho_{\min} = \dfrac{3\sqrt{f'_c}}{f_y} b_w d & \text{(Eqn 10-3)} \\ & \text{§10.5.1} \\ \rho_{\min} = \dfrac{200}{f_y} b_w d & \end{cases}$$

$$\left(A_{s\min} = \frac{6\sqrt{f'_c}}{f_y} b_w d \text{ for T-beams with flanges in tension} \right)$$

The above limits need not apply if A_s provided, at each section, exceeds by 1/3rd the steel area required by analysis (§10.5.3).

In ACI 318, section strength is governed by available ductility (i.e. amount of reinforcement at a given section and tensile stress in the reinforcement) and the strength-reduction factor, ϕ, e.g. the higher the ductility, the smaller the strength reduction and vice-versa.

Each section is classified as either compression-controlled, tension-controlled or in transition. These categories are based on the net tensile strain (ε_t) in the extreme tension steel and are defined at a concrete ultimate strain (ε_{cu}) of 0.003.

- Balanced strain conditions: @ $\varepsilon_{cu} = 0.003$, $\varepsilon_{sb} = f_y/E_s$ §10.3.2

  $\varepsilon_{sb} = 0.002$ for $f_y = 60$ksi
- Compression-controlled: @ $\varepsilon_{cu} = 0.003$, $\varepsilon_t \leq \varepsilon_{sb}$ §10.3.3
- Tension-controlled: @ $\varepsilon_{cu} = 0.003$, $\varepsilon_t \geq 0.005$ §10.3.4
- Transition-range: @ $\varepsilon_{cu} = 0.003$, $0.002 < \varepsilon_t \geq 0.005$ §10.3.4

The strength reduction factor for both ***axial load and flexure*** is a function of the section classification (§9.3.2) and is calculated as below:

Tension-controlled sections:	$\phi = 0.9$	
Compression-controlled sections:	$\phi = 0.75$	Spiral reinforcement
	$\phi = 0.65$	Other sections
Transition-range sections:	Linear interpolation between above	

For beams (and lightly loaded columns with axial load less than $0.1f'_c A_g$), §10.3.5 requires that $\varepsilon_t \geq 0.004$, which typically leads to $\phi = 0.9$.

In previous versions of ACI 318, the section was ensured to be tension-controlled by limiting the area of steel (or steel ratio). This approach is still valid and is described below.

$$\rho < \rho_{max} = 0.75\rho_{bal} = \left[0.64\beta_1 \frac{f'_c}{f_y} x \frac{87}{87 + f_y}\right] (f_y \text{ in ksi}) \qquad (8\text{-}1)$$

where, $\beta_1 = 0.85$ for f'_c of 4000psi or lower.

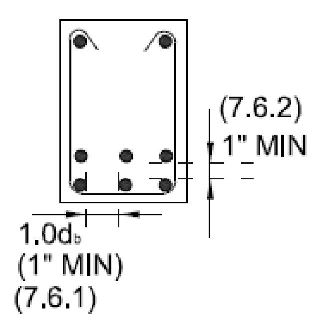
& $\rho = \dfrac{A_s}{b_w d}$

Table of ρ_{min}, ρ_{bal}, ρ_{max} for f_y = 60ksi

f'_c (ksi)	ρ_{min} (%)	ρ_{bal} (%)	ρ_{max} (%)
2.5	0.33	1.78	1.34
3.0	0.33	2.15	1.61
4.0	0.33	2.85	2.14
5.0	0.33	3.39	2.54
6.0	0.33	3.77	2.83

Spacing Limits for reinforcement:

The clearances shown also apply to contact lap splices & adjacent splices or bars (where d_b = bar diameter).



- <u>Check reinforcement spacing for crack control</u>

§10.6.4 Spacing of reinforcement closest to the tension face shall not exceed:

$$S = 15\left(\frac{40{,}000}{f_s}\right) - 2.5c_c \qquad \text{(Eqn 10-4)}$$

Bar spacing shall also not exceed:

$$S = 12\left(\frac{40{,}000}{f_s}\right)$$

where, c_c = least distance from rebar surface to tension face
f_s = Service level stress (psi) in reinforcement closest to tension face (**permitted to use $f_s = 2/3f_y$**)

- **Flexural Capacity**

Given A_s, find flexural capacity:

Depth of compression block, $a = \dfrac{A_s f_y}{0.85 f'_c b_w}$

Flexural capacity, $\phi M_n = A_s f_y \left(d - \dfrac{a}{2}\right)$

Note: See sheet CBD2 for load-reduction factor, ϕ.

- **Shear (§11.3)**

$$\phi V_n = \phi(V_c + V_s) \qquad \text{(Eqn 11-2)}$$

$$\phi V_c = \phi(2\lambda\sqrt{f'_c}\, b_w d) \qquad \text{(Eqn 11-3)}$$

$\phi = 0.75$ §9.3.2.3
$\lambda = 1.0$ for normal-weight concrete §8.6.1

If the above capacity is not adequate for members subject to flexure and shear only, try the following equation:

$$V_c = \left(1.9\lambda\sqrt{f'_c} + 2500\rho_w \frac{V_u d}{M}\right) b_w d \qquad \text{(Eqn 11-5)}$$

but not greater than $3.5\lambda\sqrt{f_c'}\, b_w d$.

where, $\rho_w = \dfrac{A_s}{b_w d}$ & $V_u d/M_u \le 1.0$ where M_u and V_u occur simultaneously at the section considered.

For members subjected to axial compression see §11.2.2.2.

If $V_u > ½\, \phi V_c$ => $A_{v\,min}$ required (except for slabs etc. §11.4.6.1)

$$A_{v\min} = 0.75\sqrt{f'_c}\,\frac{b_w s}{f_{yt}} \qquad \text{(Eqn 11-13)}$$

$A_{v\,min}$ shall not be less than $A_{v\min} = \dfrac{50 b_w s}{f_{yt}}$

where, b_w = beam width (in)
s = shear reinforcement spacing (in)
f_{yt} = specified yield strength of transverse reinforcement

$$\phi V_s = \phi\left(\frac{A_v f_y d}{s}\right) \qquad \text{§11.4.7.2}$$

For circular columns:

$$\phi V_c = \phi\left(2\sqrt{f'_c}\,0.8A_g\right)$$

$$\phi V_s = \phi\frac{\pi A_b f_{yt} D'}{2s}$$

where, D’ = Hoop diameter
A_b = Area of hoop bar with f_{yt}
s = Pitch

$$V_s \le 8\sqrt{f'_c}\,b_w d \qquad \text{§11.4.7.9}$$

$$V_n \le 10\sqrt{f'_c}\,b_w d$$

Spacing of Stirrups:

- MIN(d/2 , 24”) §11.4.5.1
- If $V_s > 4\sqrt{f'_c}\,b_w d$, Spacing shall be MIN(d/4 , 12”) §11.4.5.3

- **Beam Deflection (§9.5.2)**

Per Table 9.5(a), deflections need not be computed if the following minimum depths are provided (for normal weight concrete & 60ksi reinforcement):

	Beams	Slabs
- Simple span:	L/16	L/20
- Cantilever:	L/8	L/10
- One end continuous:	L/18.5	L/24
- Both ends continuous:	L/21	L/28

Deflection Calculation (§9.5.2.3):

Deflection is to be based on beam formulas and I_e shown below,

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \qquad \text{(Eqn 9-8)}$$

$$M_{cr} = \frac{f_r I_g}{y_t} \quad \text{where, } f_r = 7.5\lambda\sqrt{f'_c} \text{ \& } y_t = \frac{D}{2} \qquad \text{(Eqn 9-9)}$$

M_a is the service level moment (if $M_a < M_{cr}$=> No cracking =>$I_e = I_g$)

$$\left.\begin{aligned} I_{cr} &= \frac{b(kd)^3}{3} + nA_s(d-kd)^2 \\ kd &= \frac{\left(\sqrt{2\left(\frac{b}{nA_s}\right)d + 1} - 1\right)}{\left(\frac{b}{nA_s}\right)} \end{aligned}\right\}$$

Using transformed section from working stress design.
PCA Notes on ACI 318-95, pg. 8-3

<u>Long-term deflection factors (§9.5.2.5)</u>:

$$\lambda = \frac{\xi}{1 + 50\rho'} \qquad \text{(Eqn 9-11)}$$

where, ρ' = compressive steel ratio
ξ = 2.0 for 5 years or more, 1.4 for 12 months or more etc.

Notes: 1. See ACI 318, Table 9.5(a) for quick estimates of slab/beam thickness.
2. See ACI 318, Table 9.5(b) for permissible deflections (typically use $\Delta_{D+L} \le L/240$ & $\Delta_L \le L/360$).

Design

Determination of A_s:

Given beam size, span and loads, compute A_s.

Assumption: Neglect all compression reinforcement.

- Approach 1: Basic Principles

  1. Assume jd = 0.925d (for slabs jd = 0.95d)
  2. $A_s = \dfrac{Mu}{\phi f_y\, jd}$ => 1st estimate of tension reinforcement.
  3. $a = \dfrac{A_s f_y}{0.85 f'_c b_w}$ => Depth of compression block for A_s.
  4. $jd = d - \dfrac{a}{2}$ => calculate lever arm
  5. if actual jd matches assumption or slightly exceeds it => done.
  6. Iterate a couple of times at most to converge.
  7. Check reinforcement ratio versus ρ_{min} and ρ_{max} (see page CBD1)
  8. Design for shear (see page CBD2)
  9. Check deflection (see page CBD4)

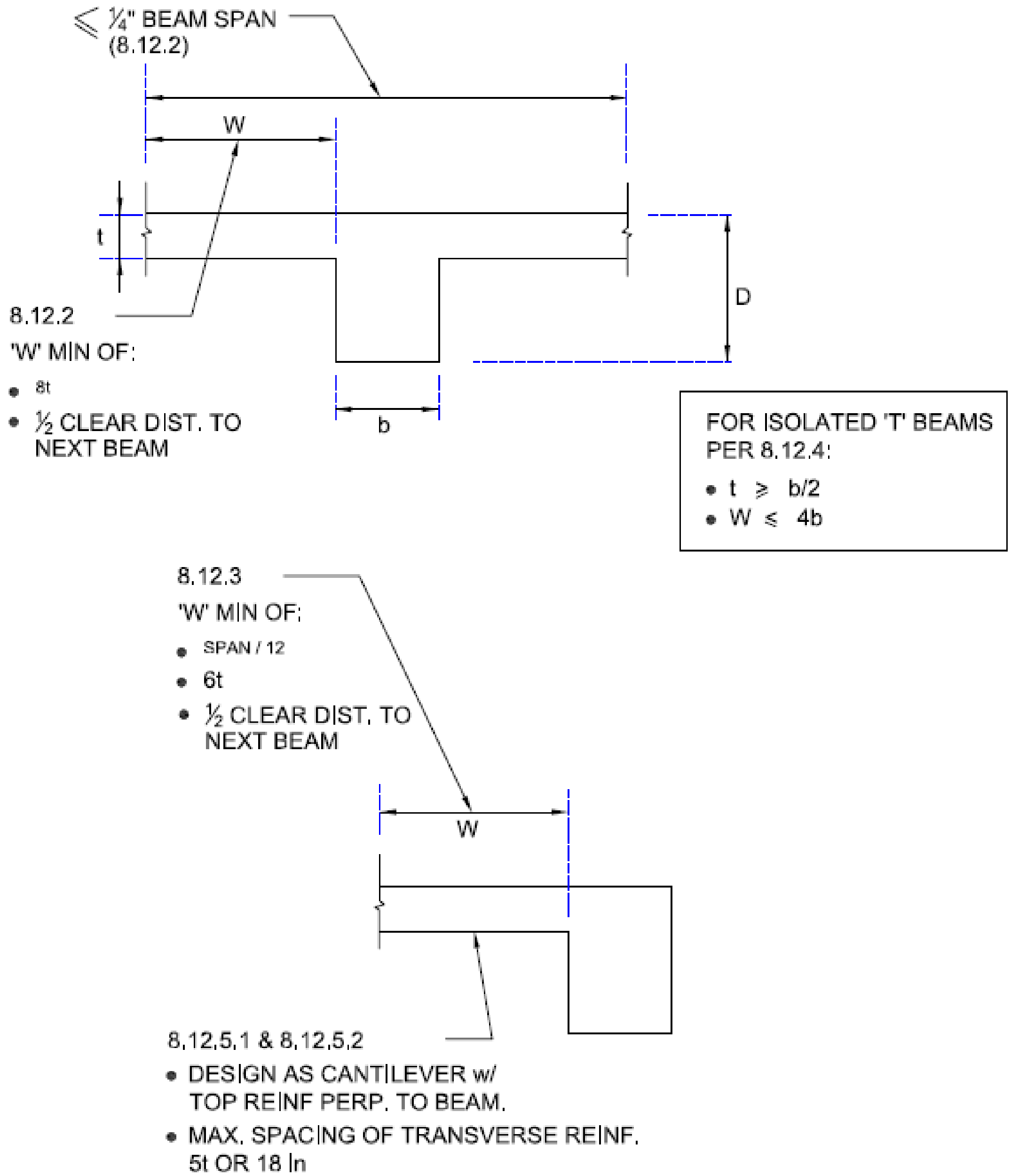
- Approach 2: Direct Relations

  1. $\rho = \dfrac{1}{m}\left(1 - \sqrt{1 - \dfrac{2mR_u}{\phi f_y}}\right)$ => Reinforcement ratio.
  2. $m = \dfrac{f_y}{0.85 f'_c}$ & $R_u = \dfrac{M_u}{bd^2}$ => M_u in lb-in
  3. Check reinforcement ratio versus ρ_{min} and ρ_{max} (see page CBD1)
  4. Design for shear (see page CBD2)
  5. Check deflection (see page CBD4)

Note: If d > 36", provide skin reinforcement @ sides per ACI-318 10.6.7.

T-Beam Provisions

A portion of the slab can be used to provide a flange for additional compression area.



Note: All other design and detailing similar to rectangular beams.

Non-Seismic Beam Reinforcement Detailing (§12.10, 12.11, & 12.12):

Positive Moment Reinforcement:

- Distance to extend reinforcement past where it is no longer required = MAX(d,12d_b) – *except at supports of simple spans and ends of cantilevers*.

- Extend bars at least ℓ_d past the critical section.

- Minimum reinforcement to be extended at least 6" into support (Bars 'B'):

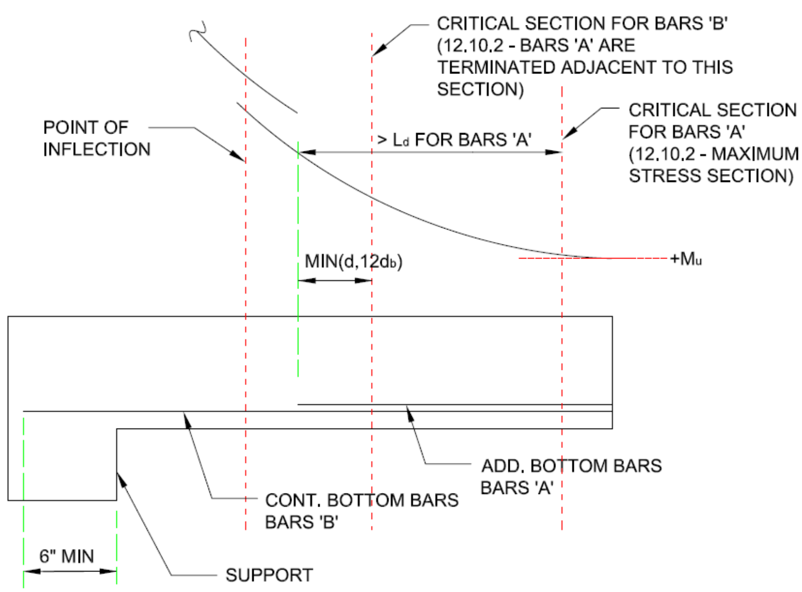
$$\left.\begin{array}{l}\text{Simple span beams: } \frac{1}{3}A_{s+} \\ \text{Continuous beams: } \frac{1}{4}A_{s+}\end{array}\right\} \qquad \text{§12.11.1}$$

- At simple supports & points of inflection, bar size shall be limited such that ℓ_d satisfies the following:

$$L_d \le \frac{M_n}{V_u} + l_a \qquad \text{§12.11.3}$$

M_n = Nominal flexural capacity of beam
V_u = Factored shear at the section
l_a = Embedment length beyond center of support or maximum of beam effective depth & 12d_b at an inflection point.

Notes: 1. This provision limits the bar size to ensure adequate L_d is available.
2. Use 1.3(M_n/V_n) in above equation if a compressive reaction confines the end of the bars.

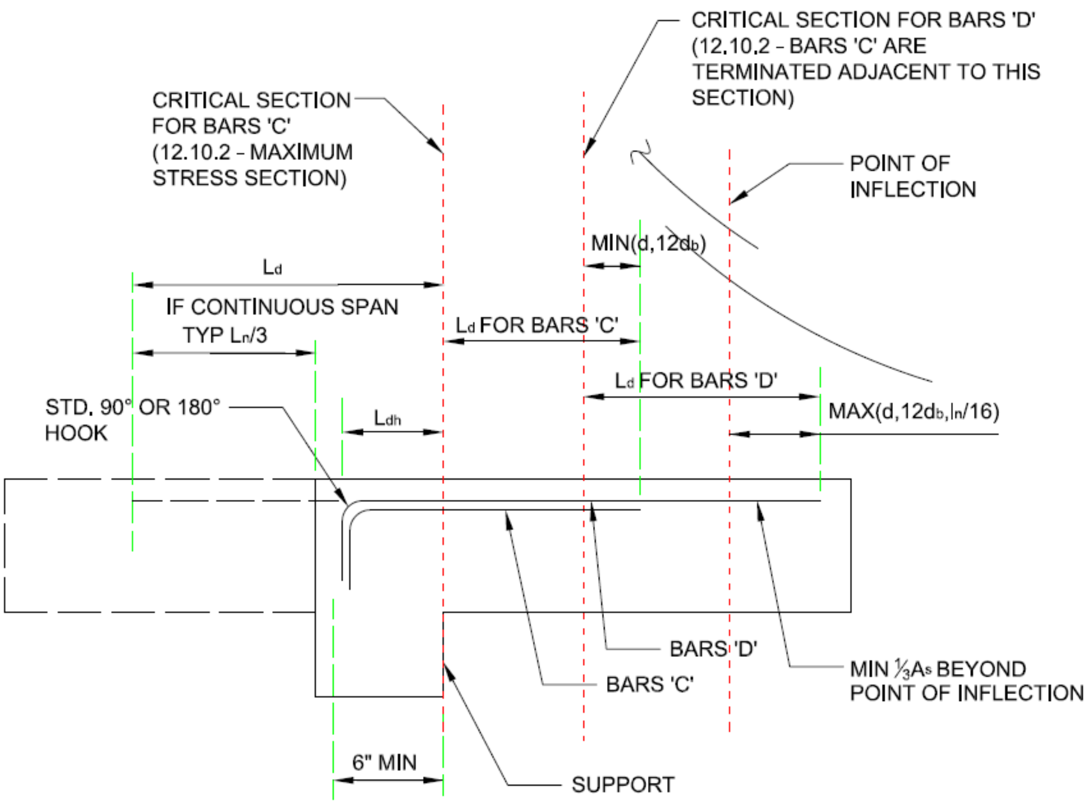


Negative Moment Reinforcement:

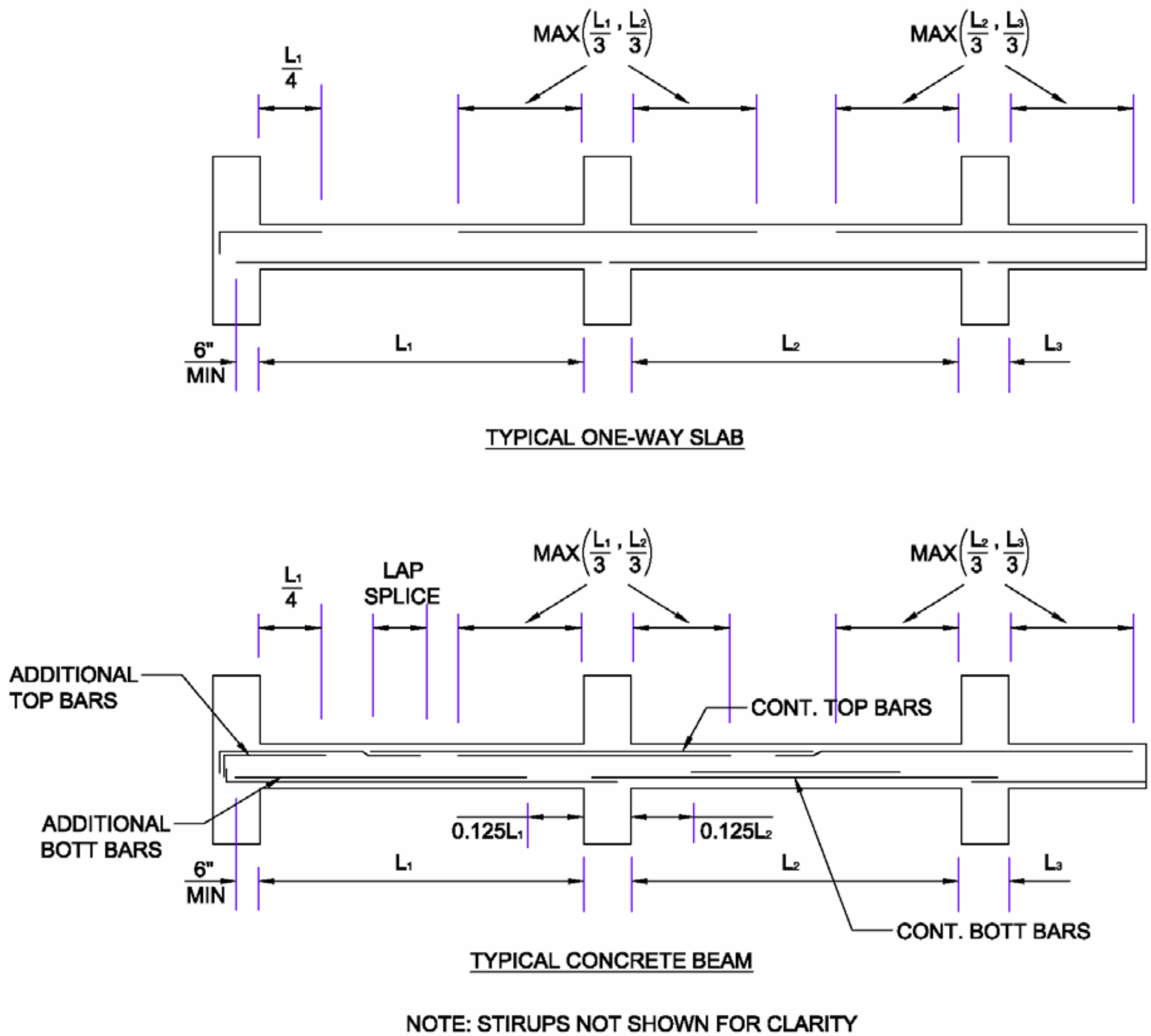
- At least 1/3 the total negative moment at a support shall extend past the inflection point by the maximum of $12d_b$, d, or $l_n/16$ (l_n is the clear span) – *accounts for shifting of the moment diagram at a point of inflection*.

- Negative reinforcement shall extend into the span a distance MAX(d,$12d_b$) past the point where it is no longer required. The provision above may govern. Continue all reinforcement a minimum of L_d past where it is not longer resists flexure.

- At the support all negative reinforcement shall be anchored by either l_d, hooks or mechanical anchorage.



Note: For development lengths, splices etc. use the ACI Rebar Design & Detailing Charts.



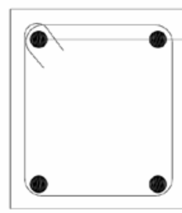
TYPICAL CONCRETE FLEXURAL DETAILING

Seismic Provisions for Special Moment Frame (SMRF) Beams:

- $P_u \le 0.10 A_g f'_c$ §21.5.1.1
- $\dfrac{ClearSpan}{depth} \ge 4$ §21.5.1.2
- $\dfrac{Width}{Depth} \ge 0.3$ §21.5.1.3
- $Width \ge 10''$ §21.5.1.3
- $A_{s\ top}$ & $A_{s\ bot} \ge \dfrac{3\sqrt{f'_c}}{f_y} b_w d$ & $\ge \dfrac{200 b_w d}{f_y}$ §10.5 & 21.5.2.1

  $\rho_{\max} = 0.025$, where $\rho = \dfrac{A_s}{b_w d}$

  Provide 2 continuous bars top and bottom, typical.



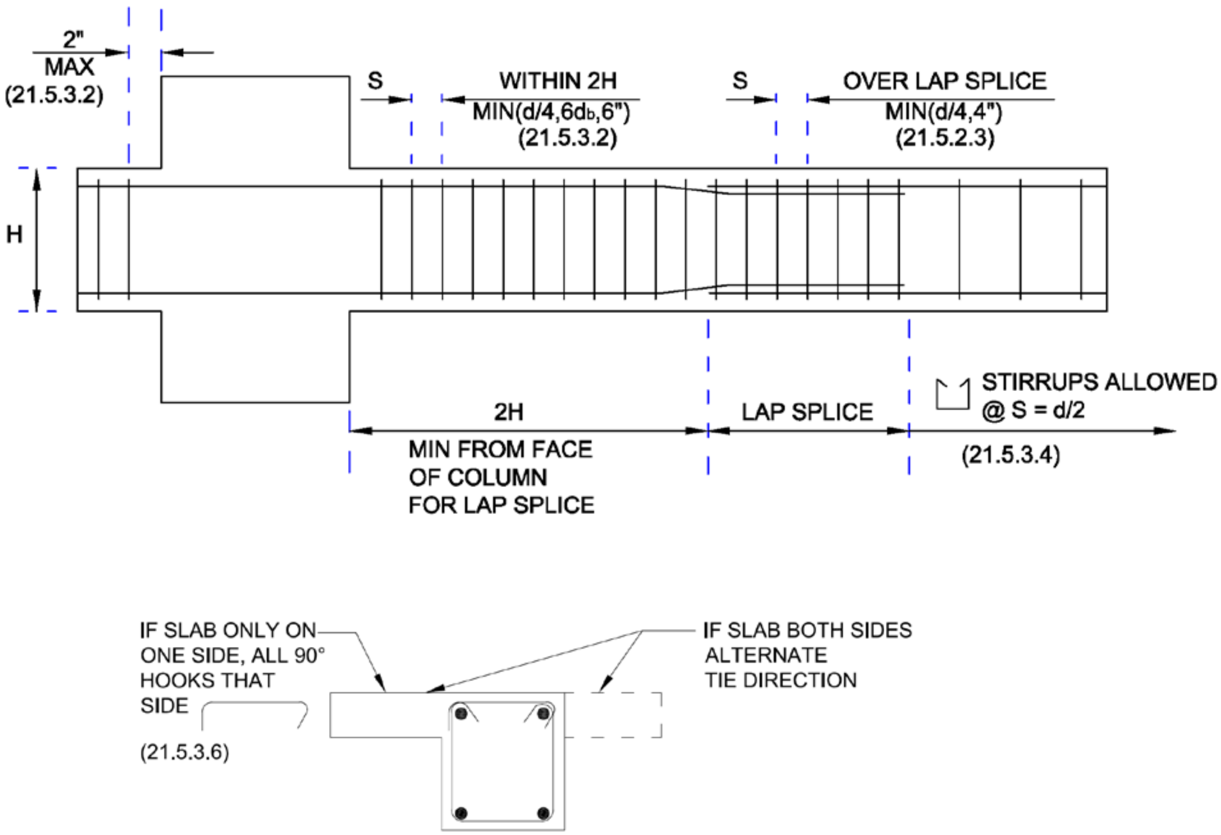
- At face of joint, $M_{n+} \ge \frac{1}{2} M_{n-}$ §21.5.2.2
- Anywhere along the beam length, $M_{n\ min}$ at top & bottom $\ge \frac{1}{4} M_{n\ max\ @\ joint}$.
- Lap splices permitted only if confined over full length by hoop or spiral reinforcement (see figure below). No lap splices within joints or locations of flexural yielding. §21.5.2.3
- Beam shear demand, $V_e = \dfrac{(M_{prA} + M_{prB})}{L_{clr}} + \dfrac{w_g L_{clr}}{2}$ §21.5.4.1

  Where, M_{prA} & M_{prB} = Moment capacities @ beam ends using $1.25f_y$ & ϕ=1.0
  W_g = Factored gravity load
  L_{clr} = Clear span

- If $V_e \ge \dfrac{V_u}{2}$ & $P_u < A_g f'_c / 20$ §21.5.4.2

  V_e = seismic shear demand
  Assume $V_c = 0$ & design stirrups to carry entire shear demand, V_e, within a distance of 2H (see 21.5.3.1).

Since V_e approximates the maximum shear that can develop in a member, use $\phi = 0.75$ (§21.5.4.1, 9.3.2.3 & 9.3.4).



<u>Detailing Requirements for SMRF Beams</u>

REINFORCED CONCRETE

COLUMN DESIGN

Reference: ACI-318-11

General Design Provision for Columns:

- **§10.3.6.1**

  Columns with Spirals:

  $$\phi P_n = \phi\{0.85[0.85 f'_c (A_g - A_{st}) + f_y A_{st}]\}, \textit{ where } \phi\textit{=0.75} \qquad (10\text{-}1)$$

  Columns with ties:

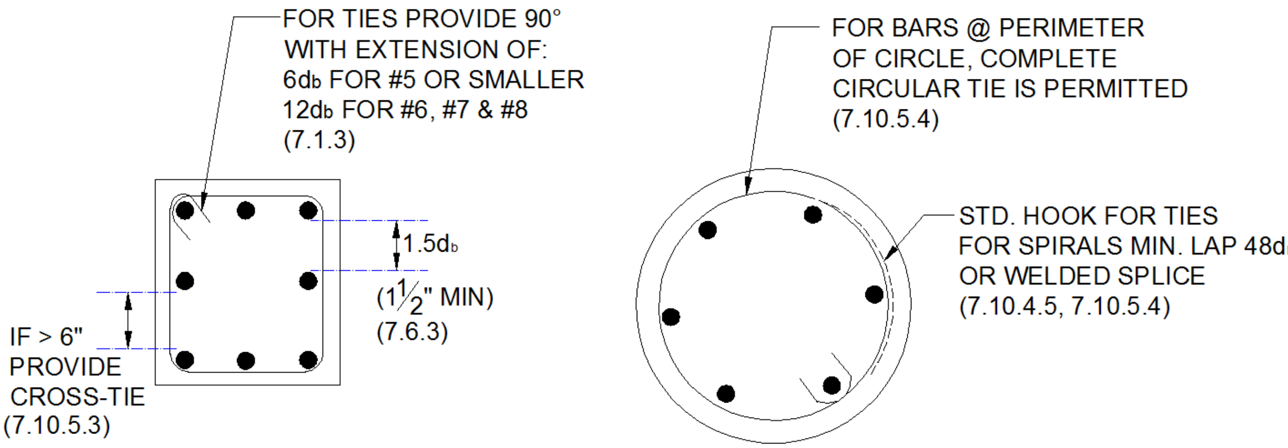
  $$\phi P_n = \phi\{0.80[0.85 f'_c (A_g - A_{st}) + f_y A_{st}]\}, \textit{ where } \phi\textit{=0.65} \qquad (10\text{-}2)$$

  Note: The same ϕ factor applies to axial load and flexure and it is a function of whether the section is compression or tension-controlled. For columns with little flexure, the above values can be used. For columns that will experience large flexure, e.g. columns in lateral frames, ϕ can be calculated per CBD2.

- **Reinforcement (§10.9.1, §10.9.2, §7.10.4, & §7.10.5):**

  Longitudinal: $0.01A_g \leq A_{st} \leq 0.08A_g$ §10.9.1

Section Type	Minimum Number of Bars (§10.9.2)	Transverse Reinforcement (§7.10.4 & 7.10.5)
Rectangular	4	Ties: #3 ties for bars up to #10 & #4 above
Circular	4 6 (8 recommended for symmetry)	Ties: #3 ties for bars up to #10 & #4 above or Spirals: #3 minimum
Other Shapes	Longitudinal bar in each corner/apex	Appropriately shaped ties with above limits.

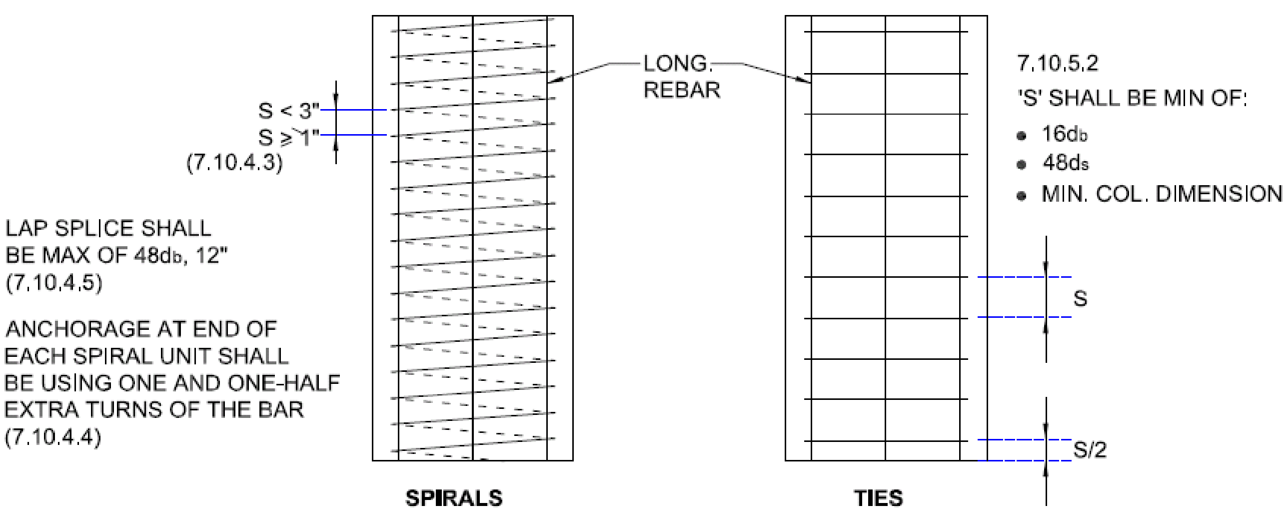


Note: For development lengths, splices etc. use the ACI Rebar Design & Detailing Charts.

- **Spiral Reinforcement (§10.9.3):**

For spirals, $$\rho_{s\,min} = 0.45\left(\frac{A_g}{A_{ch}} - 1\right)\frac{f'_c}{f_{yt}} \qquad (10\text{-}5)$$

Where, A_g = Gross area of column
A_{ch} = Area of core from out-to-out of spirals
$f'_c \leq$ 100,000psi (for f_y > 60ksi, use lap splices per 7.10.4.5(a))



- **Moment Magnification for Slenderness Effects (§10.10)**

Unsupported length of a column = clear length of the column.

For biaxial columns, magnify moment in each direction separately.

Classification of non-sway & sway columns:

Columns are non-sway if:

- §10.10.5.1 **-** Increase in end moments due to second order effects does not exceed 5% of the first order end moments.
- §10.10.5.2 **–** An entire story within a structure can be classified as non-sway if:

$$Q = \frac{\Sigma P_u \Delta_o}{V_{us} l_c} \leq 0.05 \qquad (10\text{-}10)$$

where, ΣP_u and V_{us} = >total vertical load & story shear
Δ_o = first order story drift

All other columns are to be considered as sway columns.

- \$10.10.2.1 – Total moment (including second-order effects) in compression members, restraining beams etc. shall not exceed 1.4 times the moment due to first-order effects. If this is satisfied, structural stability need not be checked.

Kl/r limits for slender columns:

Slenderness can be neglected for columns that meet the criteria below:

- Non-sway Columns, i.e. braced against side-sway (§10.10.1):

$$\frac{kl_u}{r} \leq 34 - 12\frac{M_1}{M_2} \leq 40 \qquad \textit{(Use k = 1.0, §R10.10.1)} \qquad (10\text{-}7)$$

where, ℓ_u = clear distance between capitals, slabs or beams
M_1/M_2 is positive for single curvature

Notes: *1. For a 12"x12" column, $kl_u/r \leq 40$, gives $l_u \leq 116'' = 9'\text{-}8''$.*
2. For most typical applications, columns will not be slender.

- Sway Columns, i.e. not braced against side-sway (§10.10.1):

$$\frac{kl_u}{r} < 22 \qquad (10\text{-}6)$$

Note: §10.10.1 - for moment frame columns, k shall be computed based on I_{cr} (§10.10.1) & E and shall be ≥ 1.0. Use the chart in Figure R10.10.1. Radius of gyration, r, can be taken as 0.3 time column dimension of rectangular column in direction of interest and 0.25 times diameter for circular columns (10.10.1.2).

Moment Magnification (§10.10.5, 10.10.6 & 10.10.7):

Some relevant points are presented below. Usually moment magnification is not required due to the low kl/r values of typical columns.

- Non-Sway Columns:

  - The moment magnification is to account for member curvature (out-of-straightness etc.) causing maximum moment near the mid-height of the column. The C_m factor is a correction factor if the maximum moment occurs at the ends.
  - If the factored column end moments are low, the code prescribes a lower bound moment based on minimum eccentricity, given by:

    $$M_{2\,min} = P_u(0.6 + 0.003h) \qquad (10\text{-}17)$$

    This moment is applied to the direction under consideration only and h = overall column dimension (in inches) in that direction.

- Sway Frames

  - The end moment is given as: $M = M_{ns} + \delta_s M_s$ (10-18)(10-19)
    M_{ns} is the moment due to loads that do not cause sidesway (e.g. axial load on the column), and M_s is the moment due to the loads that do cause sidesway (e.g. lateral load).
  - §10.10.4 – If a second order (P-Δ) analysis is performed taking into account cracked section properties, δM_s can be assumed to be adequately considered.
  - §10.10.7.1 – In sway frames, the beams should be designed for the magnified end moments of the columns at the joints.

Seismic Design Provision for Special Moment Frame (SMRF) Columns:

- Treat as a column if $P_u > 0.1A_g f'_c$ §21.6.1
  (where, P_u includes earthquake effects)

  Compute ϕM_n at P_u.

- §21.6.1.1 & §21.6.1.2 Minimum dimension shall be 12" & ratio of short to long dimension shall not be less than 0.4.

  §21.7.2.3 Where beam longitudinal reinforcement extends through the beam-column joint, column dimension parallel to the reinforcement shall be:

  - 20 times largest bar diameter for normal-weight concrete.
  - 26 times largest bar diameter for light-weight concrete.

- §21.6.2.2 At a joint, check:

$$\Sigma M_{nc} \geq \frac{6}{5}\Sigma M_{nb} \qquad (21\text{-}1)$$

  Where, ΣM_{nc} = Sum of nominal flexural strengths of the columns.
  ΣM_{nb} = Sum of nominal flexural strengths of the girders.

  Note: §21.6.2.3 If Eqn. 21-1 is not satisfied, the lateral strength and stiffness of the columns framing into the joint shall be ignored in the structural stiffness/strength. These columns shall be designated as not part of the lateral system and conform to 21.13.

- Longitudinal Reinforcement:

  - $0.01A_g \leq A_{st} \leq 0.06A_g$ (Applies to splice as well) §21.6.3.1
  - In columns with circular hoops, 6 longitudinal bars minimum §21.6.3.2

  - Lap splices only within center half of column & confined per 21.6.4.2. and 21.6.4.3 §21.4.3.2

- Transverse Reinforcement (§21.6.4.4):

Spirals: Volumetric ratio shall be maximum of:

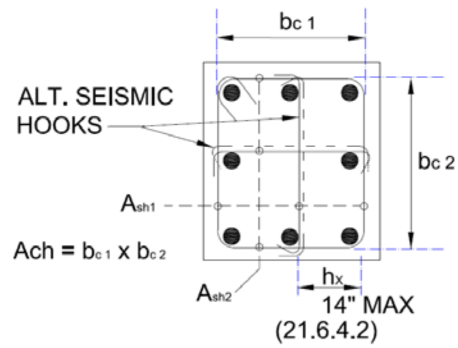
$$\rho_s = 0.45\left(\frac{A_g}{A_{ch}} - 1\right)\frac{f'_c}{f_{yt}} \qquad (10\text{-}5)$$

$$\rho_s = 0.12\left(\frac{f'_c}{f_{yt}}\right) \qquad (21\text{-}3)$$

<u>Hoops</u>: Total area shall be maximum of:

$$A_{sh} = \left(0.3sb_c\frac{f'_c}{f_{yt}}\right)\left(\frac{A_g}{A_{ch}}-1\right) \qquad (21\text{-}4)$$

$$A_{sh} = \left(0.09sb_c\frac{f'_c}{f_{yt}}\right) \qquad (21\text{-}5)$$

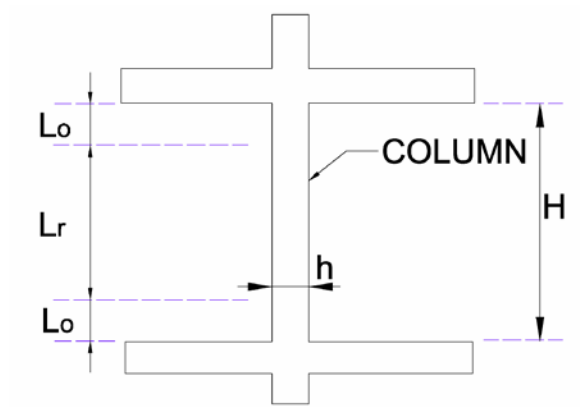


§21.6.4.1 The ties/spirals computed above are required within L_o as shown in the figure:

L_o is greater of $\begin{cases} h \\ H/6 \\ 18" \end{cases}$

Two zones are defined for column lateral for column lateral reinforcement:

- *L_o for confinement reinforcement*
- *L_r for ties based on shear demand*



§21.6.4.3 Spacing of transverse reinforcement within L_o shall be the minimum of:

- h/4
- $6d_b$
- $s_o = 4+\left(\frac{14-h_x}{3}\right)$

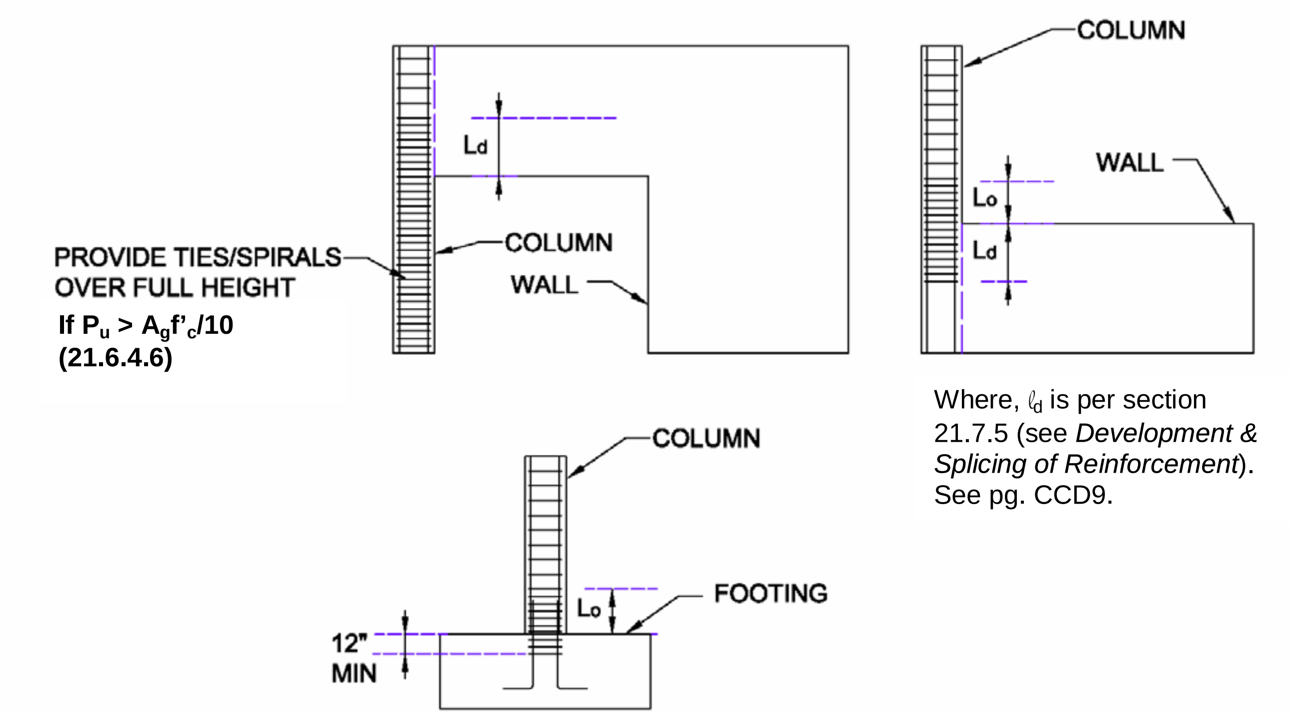
  $s_o \leq 6$in & need not be less than 4in.

  where, h_x = maximum horizontal spacing of cross ties or legs of overlapping hoops (14in max).

§21.6.4.5 Beyond length Lo, spiral or hoop reinforcement per pages CCD1 and CCD2 at spacing not exceeding $6d_b$ or 6" (except if there is a lap splice to be confined).

Other situations where ties/spirals per §21.6.4.4 are required:

Columns supporting reactions from discontinued stiff members like walls (see pg. CCD12), &



Tie/spiral spacing at other locations (§21.6.4.5):

In regions where ties/spirals are NOT required per above (e.g. L_r), the spacing of shall be minimum of:

- $6d_b$
- 6”

Check seismic shear strength (21.6.5) to compute if a larger amount of transverse reinforcement is required.

- Longitudinal Reinforcement Lap Splices:

  - Column reinforcement lap splices shall be located at column mid-height.
  - Confine lap splices with ties/spirals per §21.6.4.4. (pg. CCD5).
  - Spacing of ties/spirals per §21.6.4.3. (pg. CCD6).

- Column Shear Strength Requirements (§21.6.5):

§21.6.5.1 Columns shall be designed for the shear (V_e) due to the probable moment strengths (M_{pr}) of the column.

$M_{pr} = \frac{1.25(\phi M_n)}{\phi}$, where the 1.25 factor reflects expected yield strength.

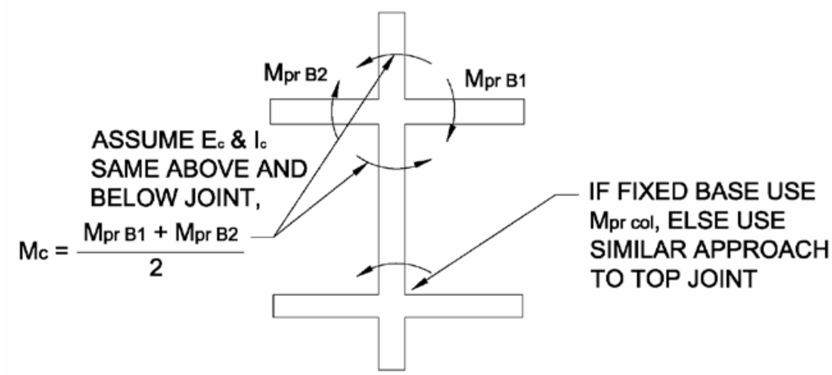
For a typical SMRF column:

$$V_e = \frac{\left(M_{pr\,top} + M_{pr\,bot}\right)}{H}$$

V_e need not exceed shear due to M_{pr} of the beams framing into the joint:

$$V_e = \frac{\left(M_{c\,top} + M_{c\,bot}\right)}{H}$$

$V_e \geq$ Factored shear from analysis.



§21.6.5.2 The transverse reinforcement within distance L_o (see page CCD5) shall be designed to resist the entire shear demand (i.e. $V_c = 0$) if:

1. $V_e > 0.5V_u$
2. $P_u < A_g f'_c/20$

<u>Shear capacities (§11.2.1.1)</u>:

- Use $\phi V_c = \phi\left(2\lambda\sqrt{f'_c}\,b_w d\right)$ Eqn (11-3)
  (For circular columns, $b_w d = D(0.8D)$ *§11.2.3*)

- Since V_e corresponds to the probable moment strength of the column, use $\phi = 0.75$ (§9.3.2.3 & 9.3.4)

Note: Although Eqn. 11-8 provides a different formula, the above equation for shear capacity is recommended for conservatism.

- Compute A_s and compare with 21.6.4.4 (page CCD5) & Eqn. 10-5 (page CCD5).
- Provide spacing per §21.6.4.3 (page CCD6).

- Column Lap Splices:

  If factored stress in the bar > $0.5f_y$ (tension) use Class B splice (§12.17.2.3).

  For all moment frame columns, it is recommended that a Class B splice be provided. Class B lap splice length is given as:

  $$L_s = 1.3 l_d \qquad \text{§12.15.1}$$

  $$l_d = 3.25 \frac{f_y}{65\sqrt{f'_c}} d_b \quad (f_y \;\&\; f'_c \text{ in psi}) \text{ for top bars.} \qquad \text{§21.7.5.2}$$

  $$l_d = 2.5 \frac{f_y}{65\sqrt{f'_c}} d_b \quad \text{for other bars.}$$

  $$l_d \geq 12'' \qquad \text{§12.15.1}$$

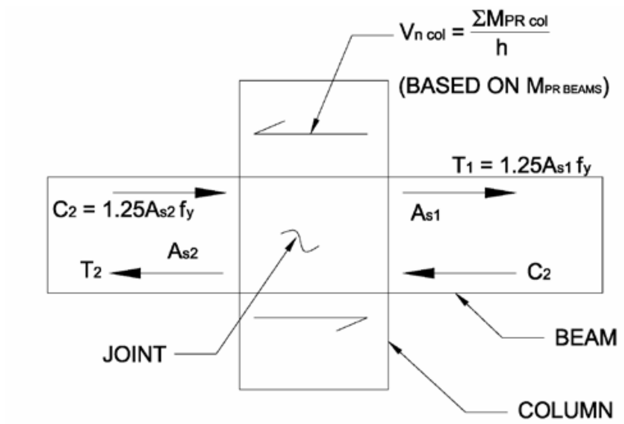
  Lap splice lengths can be modified by the following factors (see RDL5):

  - In tied columns, where ties satisfy $A_s \geq 0.0015hs$, factor the lap length by 0.83.
  - In columns with spirals, factor lap length by 0.75.

Note: See page CCD11 for typical column & beam-column joint seismic detailing.

- Beam Column Joints in Special Moment Frames (§21.7):

§21.7.2.1: Joint shear demand shall be determined as shown below:



See page CCD8 for $M_{PR\ col}$.

$$V_{u\ jnt} = T_1 + C_2 - V_{n\,col}$$

§9.3.4(c): Use $\phi = 0.85$

§21.7.3.1: Joint transverse reinforcement shall be per 21.6.4 (page CCD5 & CCD6) and shall confine the column reinforcement through the joint. If beams from all four sides confine the column, reduction is permitted in the amount of transverse reinforcement—see §21.7.3.2.

Column transverse reinforcement shall confine the beam longitudinal reinforcement if it is outside the column core, unless a perpendicular beam provides the necessary confinement (§21.7.3.3).

§21.7.4: Nominal shear strength of the joint shall not be taken as greater than the following for normal-weight concrete:

$20\sqrt{f'_c}A_j$ Joints confined on all four faces.

$15\sqrt{f'_c}A_j$ Joints confined on three or two opposite faces.

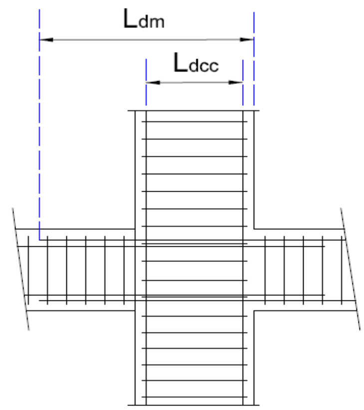
$12\sqrt{f'_c}A_j$ All other joints.

Confinement on a face is assumed if the member that frames into that face covers at least ¾ of the joint face (§21.7.4.1 & 21.7.3.2).

Development lengths (§21.7.5):

Beam straight bars terminating @ a joint:
(§21.5.4.2)

For top bars: $l_d = 3.25\left(\frac{f_y d_b}{65\sqrt{f'_c}}\right)$



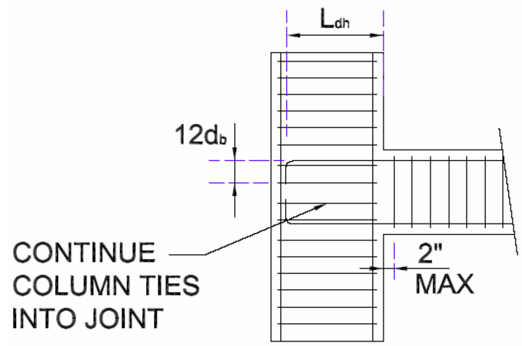
For bottom bars: $l_d = 2.5\left(\frac{f_y d_b}{65\sqrt{f'_c}}\right)$

Where, f_y & f'_c are in psi.

Increase in development length for portion of ℓ_d not within confined core:

$\ell_{dm}=1.6(\ell_d-L_{dcc}) + L_{dcc}$
where, L_{dcc} is development length within confined core.
(§21.7.5.3)

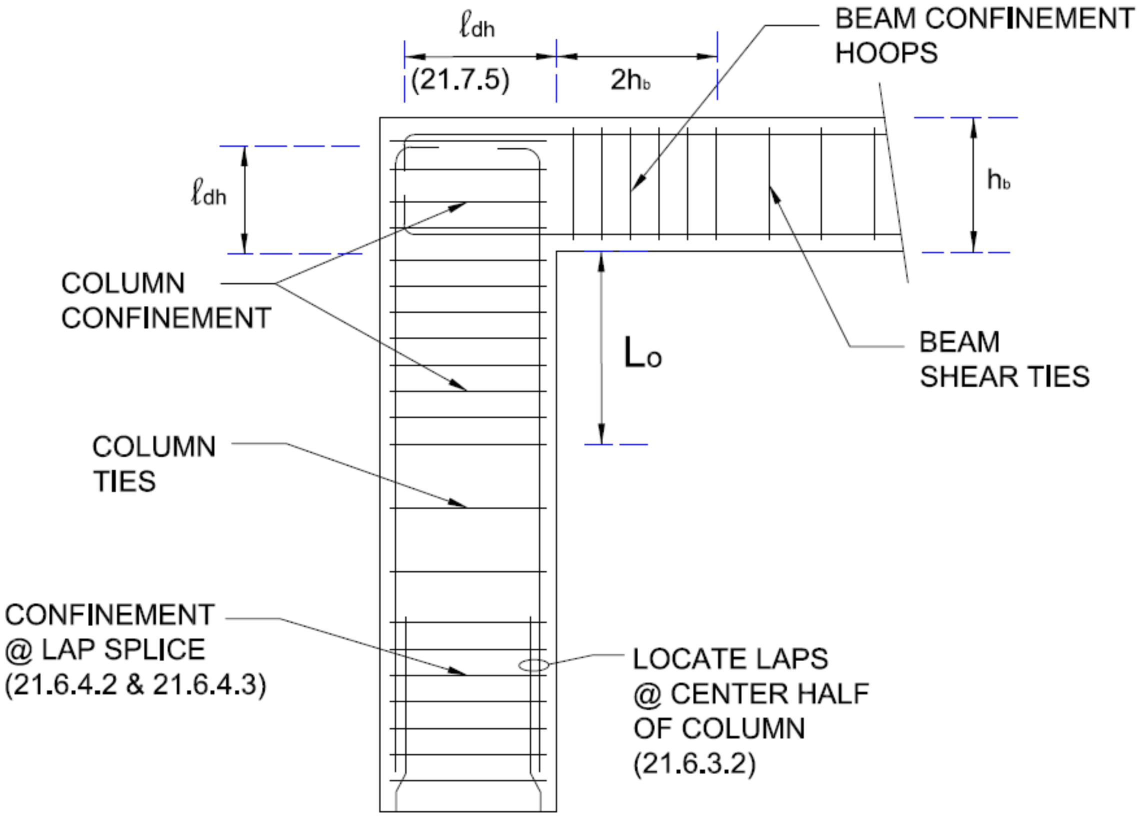
Beam bars terminating in a column with standard hook:
(§21.7.5.1)



ℓ_{dh} = hooked bar development length and shall be the maximum of:

- $8d_b$
- 6”
- $\frac{f_y d_b}{65\sqrt{f'_c}}$ (21-6)

Typical Column & Beam-Column Joint Seismic Detailing

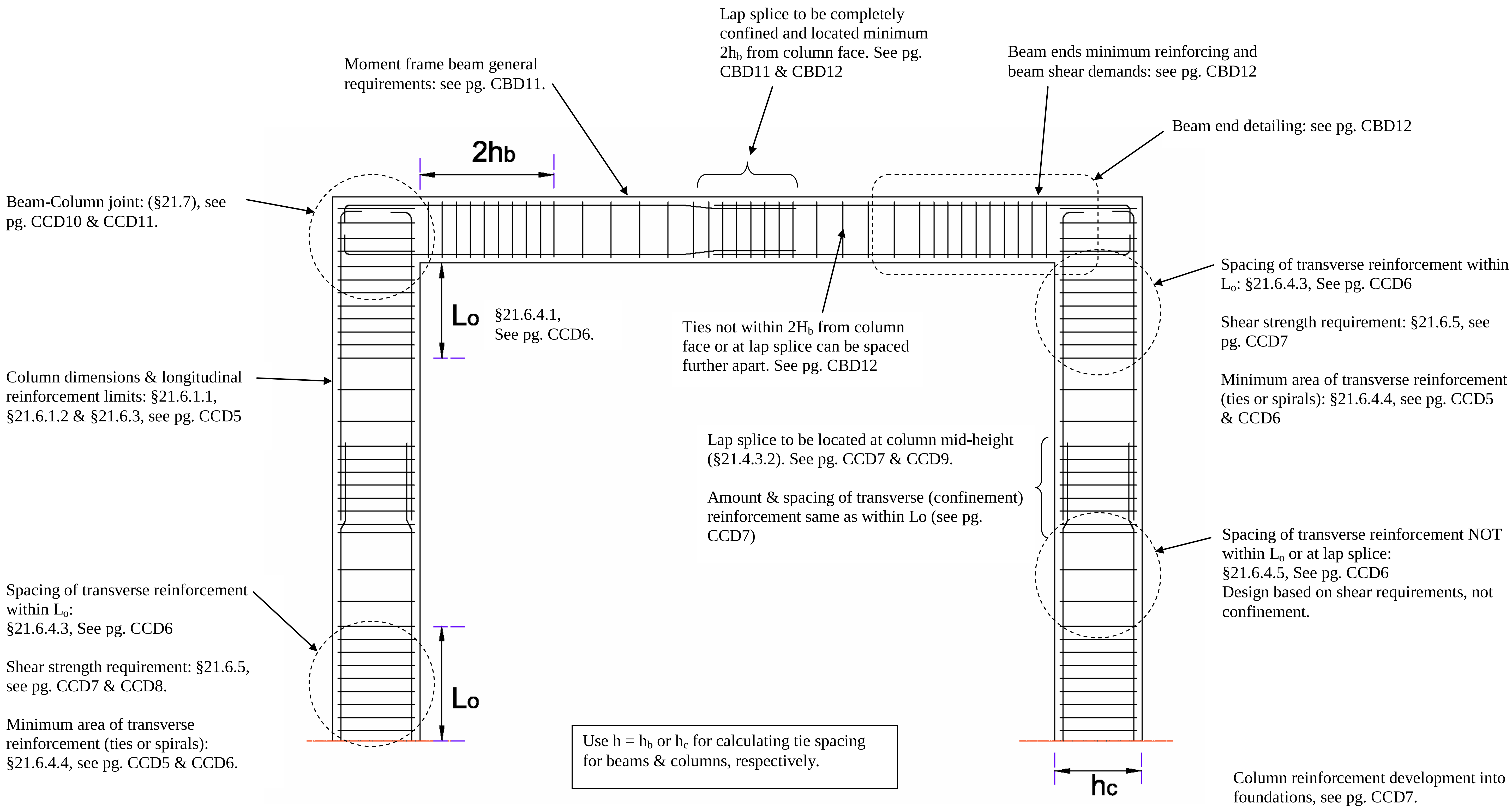


Columns Supporting Reactions from Discontinuous Stiff Members (§21.6.4.6)

For columns supporting discontinuous members such as shear walls:

Provide transverse reinforcement *over the entire height of the column* below the discontinuity if factored axial compressive force exceeds $A_g f'_c/10$. If the factored load is based on a special load combination (i.e. including Ω_o), increase the limit to $A_g f'_c/4$.

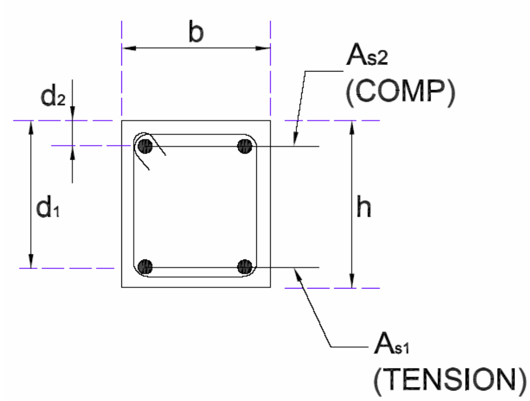
- The transverse reinforcement shall be as described on pages CCD5 & CCD6.
- The reinforcement shall be continued above and below as shown on CCD7.



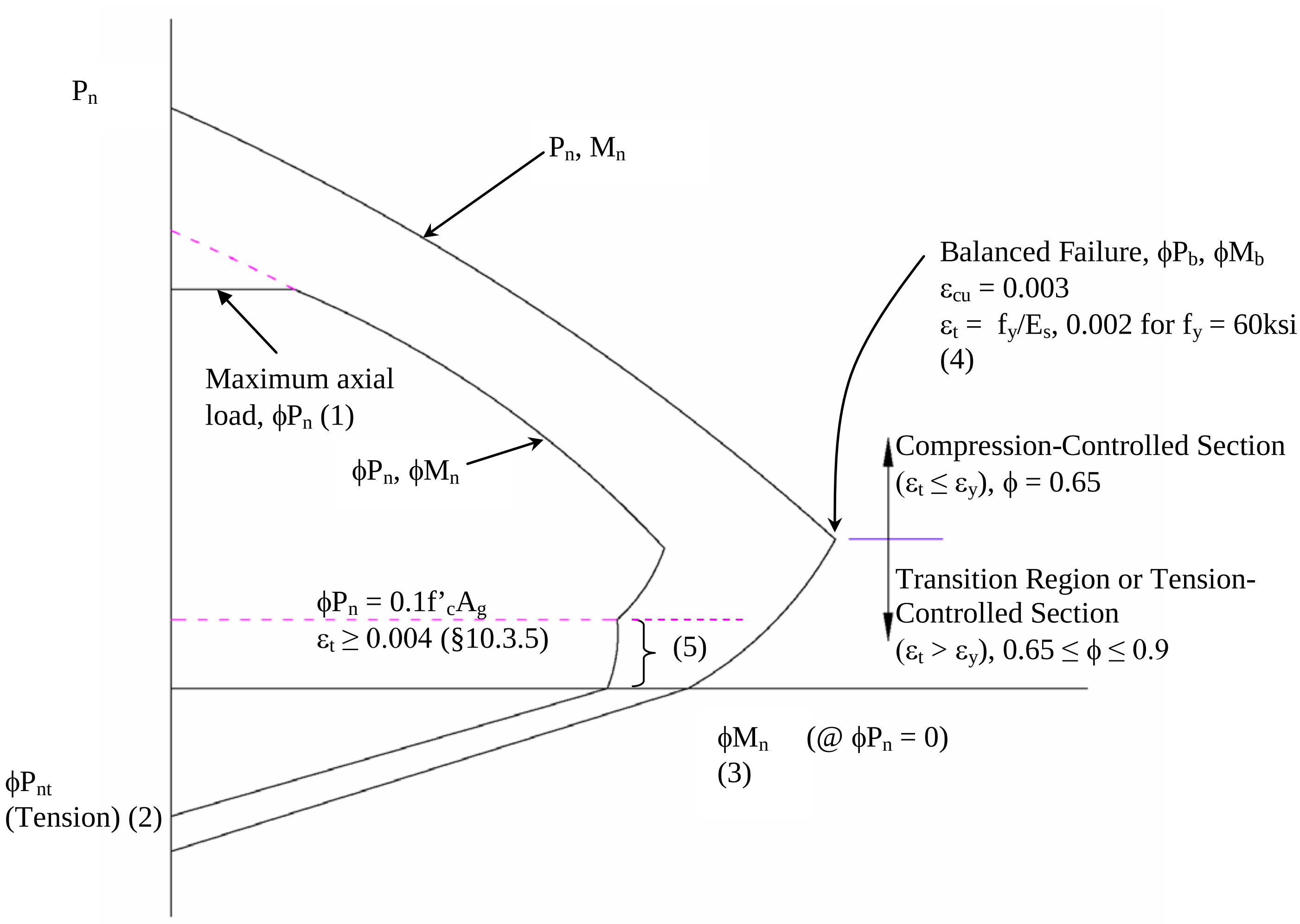
CONCRETE SPECIAL MOMENT RESISTING FRAMES
$R = 8, \Omega_o = 3, C_d = 5.5$

REINFORCED CONCRETE

SIMPLIFIED COLUMN AXIAL-FLEXURE INTERACTION



Typical Column Section



(1) ϕP_n:

For typical column with ties, $\phi = 0.65$ §9.3.2.2

$$\phi P_n = \phi\{0.80[0.85 f'_c (A_g - A_{st}) + f_y A_{st}]\} \quad (10\text{-}2)$$

(2) ϕP_{nt}:

$$\phi P_{nt} = \phi f_y (A_{s1} + A_{s2})$$

$\phi = 0.9$ §9.3.2.1

(3) ϕM_n:

Neglecting the contribution of the compression (top) reinforcement.

$$a = \frac{A_{s1} f_y}{0.85 f'_c b}$$ - Depth of the compression block.

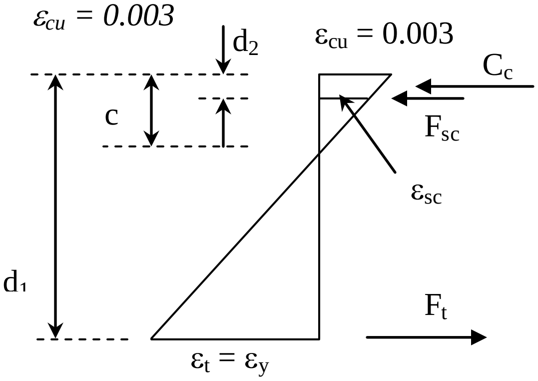
$$jd = \left(d - \frac{a}{2}\right)$$ - Lever arm.

$$\phi M_n = 0.9 A_{s1} f_y (jd)$$

$\phi = 0.9$ §9.3.2.1

(4) Balanced Condition, ϕP_b & ϕM_b:

At balanced failure, $\varepsilon_t = \varepsilon_y$ and $\varepsilon_{cu} = 0.003$



$$c = \frac{0.003}{0.003 + \varepsilon_y} d_1$$

$\varepsilon_y = 0.0020$ for f_y=60ksi. (§10.3.3)

$a = \beta_1 c$ (depth of compression block)

$\beta_1 = 0.85$ for 2,500psi $\le f'_c \le$ 4000psi; 0.05 less for each 1000psi above 4000psi but not less than 0.65 (§10.2.7)

$$\varepsilon_{sc} = \left(\frac{c - d_2}{c}\right) 0.003$$

$$f_{sc} = E_s \varepsilon_{sc}$$

Force Components

$C_c = 0.85 f'_c\, ab$ - Compression in concrete, $f'_c \leq 4000$psi assumed)

$F_t = A_{s1} F_y$ - Tension in bottom bars.

$F_{sc} = A_{s2} f_{sc}$ - Tension in top bars.

Capacities @ balanced failure

$$P_n = C_c + \Sigma F_s = C_c - F_t + F_{sc}$$

$$\phi P_b = \phi P_n$$

Using moment summation about the centroid:

$$M_n = C_c\left(\frac{h}{2} - \frac{a}{2}\right) + F_t\left(d_1 - \frac{h}{2}\right) + F_{sc}\left(\frac{h}{2} - d_2\right)$$

$$\phi M_b = \phi M_n$$

$\phi = 0.65$ for both axial and flexure §9.3.2.2

(5) ϕ Variation between $P_n = 0$ and Balanced Condition:

The value of ϕ for both the axial and flexural capacities is the same and varies as shown below:

$\phi = 0.90$ for tensioned controlled sections ($\varepsilon_{si} \geq 0.005$) §9.3.2.1

$\phi = 0.65$ for compression-controlled sections ($\varepsilon_t \leq f_y/E_s$) §9.3.2.2

Linear interpolation between the above limits (i.e within the Transition Region):

$\phi = 0.65 + (\varepsilon_t - 0.002)(250/3)$ §R9.3.2.2

For $0 \leq P_n < 0.1 f'_c A_g$:

For columns lightly loaded axially (and for beams), the following approach applies:

$$a = \frac{A_{s1} f_y}{0.85 f'_c b}$$ - Depth of the compression block.

$$c = \frac{a}{0.85}$$ - Depth of neutral axis from compression face

$$\varepsilon_t = \left(\frac{d_1 - c}{c} \right) 0.003$$ - Strain in tension face steel

The code requires that for sections with axial load less than $0.1f'_c A_g$, the strain in the tension face steel shall not be less than 0.004 (§10.3.5).

$$\varepsilon_{sc} = \left(\frac{c - d_2}{c} \right) 0.003$$ - Strain in compression face steel

$$F_t = A_{s1} E_s \varepsilon_t$$ - Force in tension face steel

$$F_{sc} = A_{s2} E_s \varepsilon_{sc}$$ - Force in compression face steel

$$C_c = F_t + P_n - F_{sc}$$ - Compressive force for equilibrium

$$F_{sc} = A_{s2} f_{sc}$$ - Tension in top bars.

$$M_n = C_c \left(\frac{h}{2} - \frac{a}{2} \right) + F_t \left(d_1 - \frac{h}{2} \right) + F_{sc} \left(\frac{h}{2} - d_2 \right)$$

Compute ϕ based ε_t => typically $\phi = 0.9$.

<u>For P_n Between $0.1f'_c A_g$ & Balanced Condition</u>:

$$a = \frac{(A_{s1} f_y + P_n - A_{s2} f_y)}{0.85 f'_c b}$$ - Depth of the compression block.

The remaining steps are the same as for the earlier case of $P_n \leq 0.1f'_c A_g$.

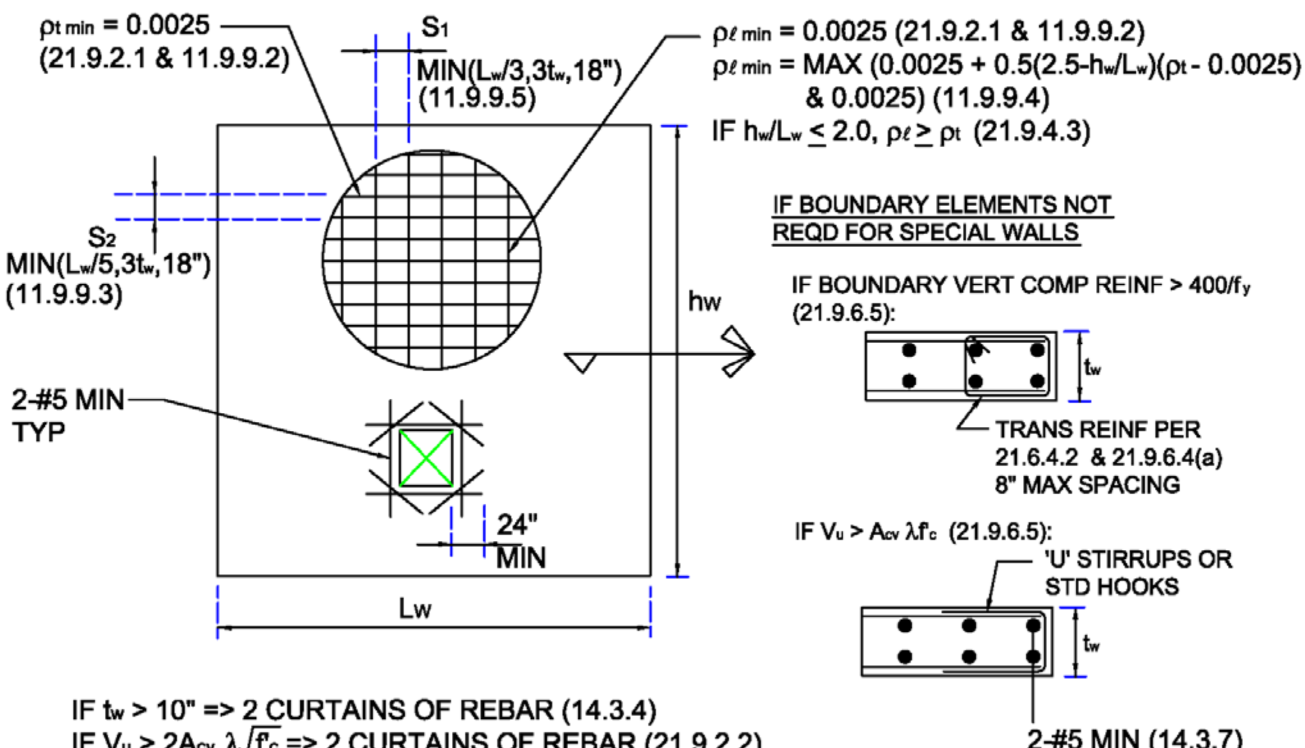
REINFORCED CONCRETE

SHEAR WALL DESIGN

References: ACI-318-11 & IBC 2012

Wall Type	SDC	Reinforcement Limits	Shear Design	Axial & Flexural Design	Other
Ordinary Shear Wall	A, B, C	14.3 11.9.8, 11.9.9	11.9	14.2, 14.3 (10.2, 10.3)	-
Special Shear Wall	A, B, C, D, E, F	21.9.2	21.9.4	21.9.5 (10.2, 10.3)	Boundary Elements 21.9.6

Reinforcement Limits



Note: 1. *For shear walls, reinforcement development lengths (& splices) shall be per Chapter 12 (§21.9.2.3). See 'Reinforcement Development & Lap Splices', pp. RDL1-RDL2. At locations where yielding of longitudinal reinforcement can occur (base of walls etc), use development lengths 1.25 times that for f_y in tension.*

2. *Reinforcement for shear walls, coupling beams and moment frames shall comply with ASTM A706. ASTM A615 Grades 40 and 60 are permitted under certain conditions. See ACI 318, Section 21.1.5.2.*

Shear Strength (§11.9.5, 11.9.9 & 21.9.4):

For Ordinary Shear Walls (§11.9.5, 11.9.9):

$$\phi V_n = \phi\left(2\lambda\sqrt{f'_c}(t_w)(0.8L_w) + \frac{A_v f_y (0.8L_w)}{s_2}\right) \qquad \text{(§11.9.5 \& 11-29)}$$

where, $0.8L_w$ represents effective depth ‘d’ (§11.9.4) & A_v is horizontal reinforcement.

For Special Shear Walls (§21.9.4):

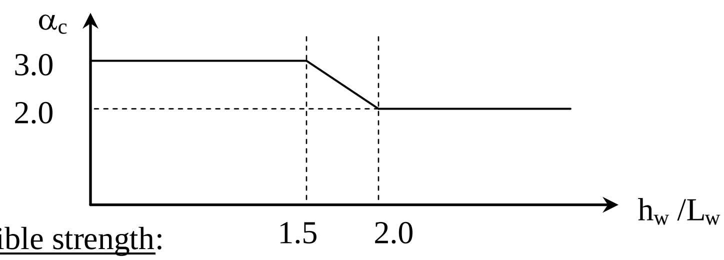
$$\phi V_n = \phi\, A_{cv}\left(\alpha_c \lambda \sqrt{f'_c} + \rho_t f_y\right) \;=\; \phi\, A_{cv} \alpha_c \lambda \sqrt{f'_c} + \phi A_v f_y \qquad \text{(21-7)}$$

Where, A_{cv} = Concrete shear area, L_w x t_w.

ϕ = 0.6 (if certain that flexure governs, ϕ=0.75) §9.3.2.3 & 9.3.4

ρ_t = Horizontal shear reinforcement ratio.

α_c = factor that varies linearly between h_w/L_w of 1.5 and 2. §21.9.4.1



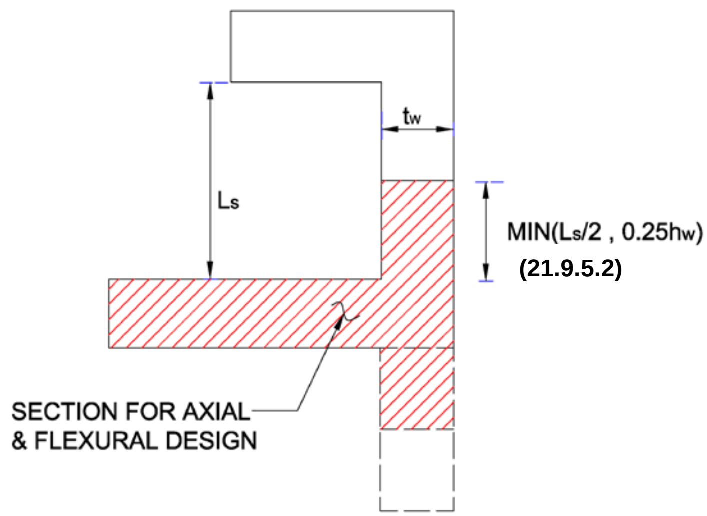
Maximum permissible strength:

$V_n \le 10\sqrt{f'_c}A_{cv}$ Horizontal wall, individual piers & coupling beams. §21.9.4.5

$V_n \le 8\sqrt{f'_c}A_{cv}$ Where A_{vc} is the gross area of all wall segments or piers sharing common lateral load. §21.9.4.4

Flexural & Axial Strength (§10.2, 10.3, 21.9.5):

Effective section of T, L, U etc. shaped walls (§21.9.5.2):



Load Combinations:

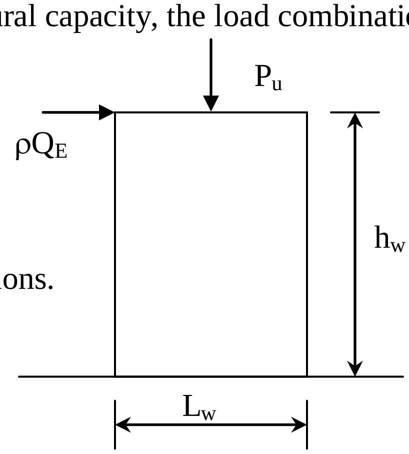
Shear walls are typically designed for the following load combinations:

- 1.4(D + F)
- 1.2(D + F + T) + 1.6(L + H) + 0.5(L_r or S or R)
- 1.2D + 1.6(L_r or S or R) + (f_1L + 0.8W) + 1.6H
- (1.2 + 0.2S_{DS})D + ρQ_E + f_1L + 0.2S + 1.6H
- (0.9 - 0.2S_{DS})D ± ρQ_E + 1.6H

Simplified calculation of wall flexural strength for a given axial demand (P_u):

Since axial load (up to an extent) contributes to flexural capacity, the load combination that typically governs the design is,

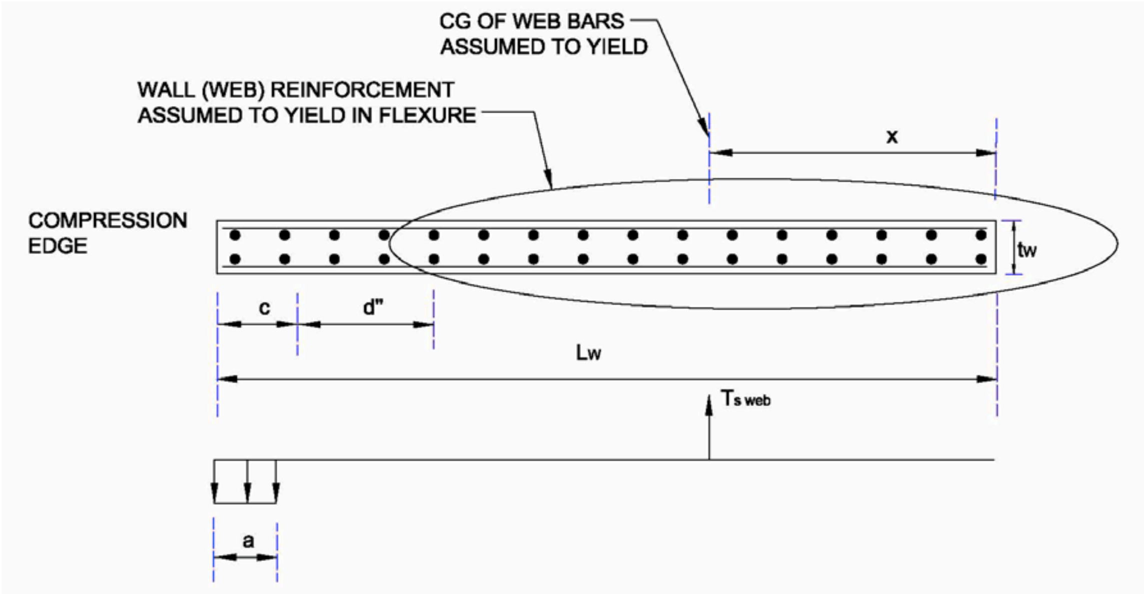
(0.9 - 0.2S_{DS})D ± ρQ_E



The moment capacity of the wall should also be computed for (1.2 + 0.2S_{DS})D + ρQ_E + f_1L for use with boundary element and maximum shear calculations.

(Hydrostatic, soil, snow loads are neglected here.)

See wall section shown below.
The reinforcement shown is for flexural strength only.



Assume that a portion of the vertical web reinforcement yields (see figure above). For simplification of the calculation, assume that all the web reinforcement yields (SEAOC Blue Book 1999). This has been verified in cyclic tests. This assumption implicitly means that no web bars lies within the compression zone.

The depth of the compression block is:

$$a = \frac{A_{sw} f_y + P_u}{0.85 f'_c t_w}$$ where A_{sw} = Area of vertical web reinf. assumed to yield.

The ϕ factor for both axial and flexural capacity is the same and can be computed as follows:

- For shear-controlled shear walls, i.e. where nominal shear strength is less than shear corresponding to nominal flexural strength, $\phi = 0.6$ (§9.3.4)
- For flexure-controlled walls:
  - $\phi = 0.90$ for tensioned controlled sections ($\varepsilon_{si} \geq 0.005$) (§9.3.2.1)
  - $\phi = 0.65$ for compression-controlled sections ($\varepsilon_t \leq f_y/E_s$) (§9.3.2.2)
  - Linear interpolation between the above limits (Transition Region):

$$\phi = 0.65 + (\varepsilon_t - 0.002)\left(\frac{250}{3}\right)$$ §R9.3.2.2

$$\phi M_{n\ web+P_u} = \phi\left[\left(A_{sw} f_y\right) + P_u\right]\left(x - \frac{a}{2}\right)$$

Where $x = L_w/2$ if all the web steel is assumed to yield.

$\phi M_{n\ web+Pu} \geq M_u$ => Wall flexural capacity does not require boundary reinforcement.

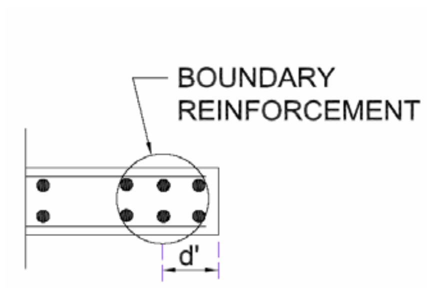
$\phi M_{n\ web+Pu} \leq M_u$ => Boundary reinforcement required. See below.

$\phi M_{n\ bnd} = M_u - \phi M_{n\ web+Pu}$

Assume $M_{n\,bnd} = A_{s\,trim} f_y \left(L_w - 2d'\right)$

Assume d'.

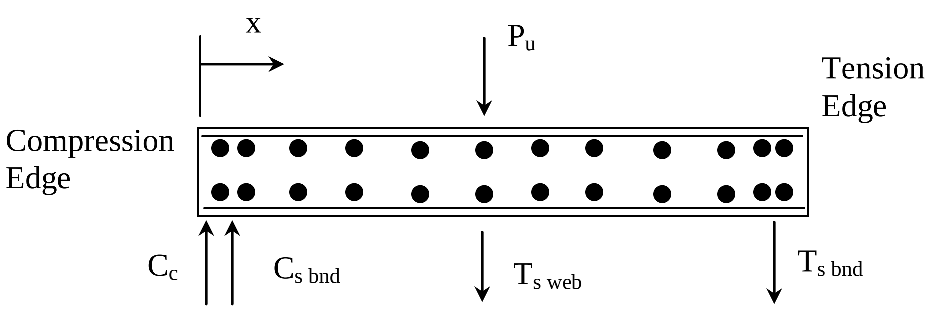
$$A_{s\,bnd} = \frac{M_{n\,bnd}}{f_y \left(L_w - 2d'\right)}$$



Check actual d' based on $A_{s\ bnd}$ and revise $A_{s\ bnd}$ – Iterate as required.

Provide 2-#5 minimum as trim bars at wall ends.

Overall flexural strength of the wall can be computed as shown below:



Complete the table below and sum to obtain M_n.

Force Component	$\mathbf{A_s}$	**C or T (C –ve, T +ve)**	**x**	**C*x or T*x**
$C_{s\ bnd}$	$A_{s\ bnd}$	$- A_{s\ bnd} f_y$	d'	$(- A_{s\ bnd} f_y)\ d'$
$T_{s\ web}$	A_{sw}	$A_{sw} f_y$	$L_w/2$	$(A_{sw} f_y)\ (L_w/2)$
$T_{s\ bnd}$	$A_{s\ bnd}$	$A_{s\ bnd} f_y$	$L_w - d'$	$(A_{s\ bnd} f_y)\ (L_w - d')$
P_u	-	P_u	$L_w/2$	$(P_u)(\ L_w/2)$
C_c	-	$-C_{c\ reqd}$ *See Note*	$a/2$	$(-C_{c\ reqd})(a/2)$
				$\mathbf{\Sigma = M_n}$

Note: 1. $C_{c\ reqd} = (-C_{s\ bnd} + T_{s\ web} + T_{s\ bnd} + P_u)$
2. *Check* $a = C_c/0.85f'_c t_w$ *–> Depth of compression block.*
3. $c = a/0.85$ *–> Depth to neutral axis.*

If the location of the neutral axis refutes the assumption that all the web reinforcement yields, iterate till agreement is reached.

<u>Boundary element requirements for Special Walls (§21.9.6)</u>:

Boundary element requirements can be evaluated by either one of the two methods described below:

a) For walls that are effectively continuous from the base to the top with a single critical section for axial and flexural loads (§21.9.6.2):

Provide boundary elements where: $$c \geq \frac{l_w}{600\left(\delta_u / h_w\right)} \qquad (21\text{-}8)$$

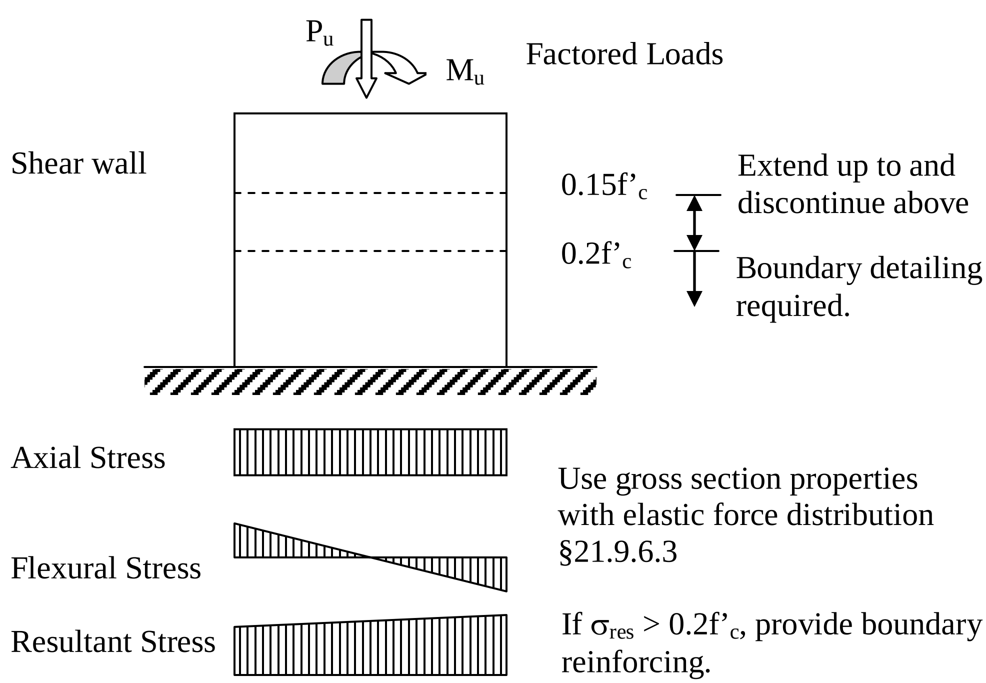
where, c = neutral axis depth for the factored axial force and nominal moment strength consistent with δ_u. Typically the load combination $(1.2 + 0.2S_{DS})D + \rho Q_E + f_1 L$ applies.
δ_u = design displacement at top of wall (i.e. $C_d\Delta_x/I$)
$\delta_u/h_w \geq 0.007$ (§21.9.6.2(a))

For $\delta_u/h_w = 0.007$, $c \geq 0.24\ell_w$; higher the δ_u, lower the limit for c.

At some height along the wall, the above requirement will not be applicable

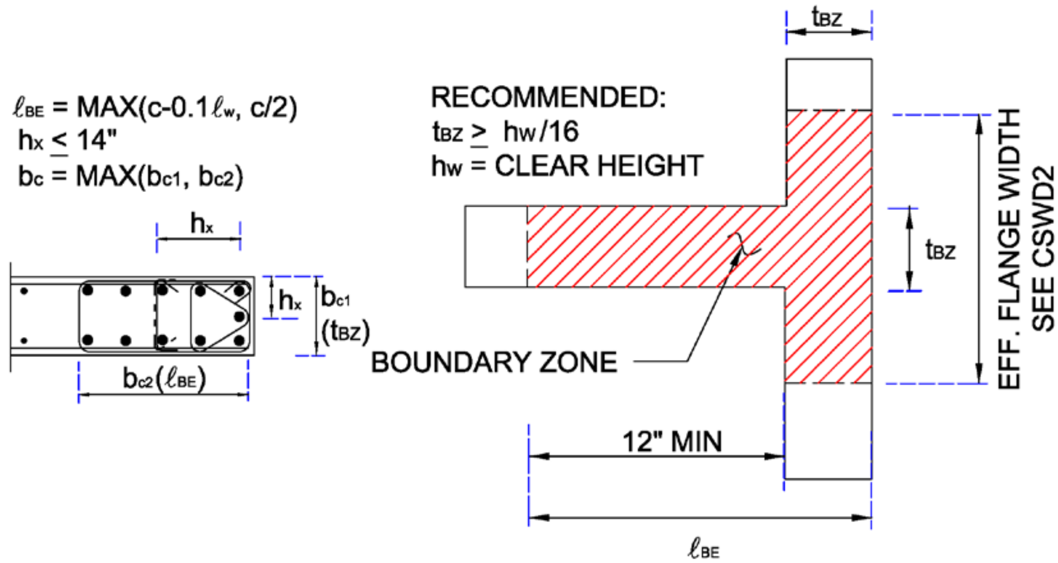
Extend the boundary element reinforcing vertically above the critical section by a distance not less than the larger of ℓ_w or $M_u/4V_u$ (§21.9.6.2(b)).

b) For walls not designed per above, provide boundary elements at wall boundaries, and edges of openings where maximum compressive stress exceeds $0.2f'_c$. Discontinue boundary detailing where the stress is less than $0.15f'_c$ (§21.9.6.3).



<u>If Boundary Elements Are Required (§21.9.6.4)</u>:

Boundary element detailing:



FOR T, L, C ETC. SHAPED WALLS

- Transverse reinforcement for the boundary element (§21.6.4.4 except Eqn 21-4):

Rectangular hoop reinforcement $$A_{sh} = 0.09 s b_c \frac{f'_c}{f_y} \qquad (21\text{-}5)$$

Spiral or circular hoop $$\rho_s = 0.12 \frac{f'_c}{f_y} \qquad (21\text{-}3)$$

- Maximum spacing of hoops/ties shall be (§21.6.4.3 except (a) is modified):

s = Minimum of (min(b_{c1}, b_{c2})/3, 6d_b , s_o)

where, $$s_o = 4 + \left(\frac{14 - h_x}{3}\right)$$

$s_o \leq$ 6in & need not exceed 4in.

s ≤ 4" (at lap splices of vertical bars).

Where d_b = diameter of the largest vertical bar.

- Use single or overlapping hoops. Each end of a crosstie shall engage a peripheral longitudinal bar and consecutive crossties shall be alternated end for end.

- The longitudinal (vertical) boundary element reinforcement shall extend into a support at least equal to the development length of the largest bar. If yielding of the longitudinal reinforcement is expected, the development length shall be taken as 1.25 times the value calculated for f_y. The transverse reinforcement shall also be extended accordingly.

  If the boundary element terminates at a footing/mat, extend the transverse reinforcement a minimum of 12in into the footing/mat.

- All lap splices in vertical reinforcement shall be confined by hoops or crossties at spacing ≤ 4 in.

- All horizontal reinforcement shall extend into the boundary element to within 6in of the end of the wall. It shall be anchored into the confined core of the boundary element to develop yield strength of the bar with standard hooks or heads at the end of the bar. It is possible to terminate the horizontal steel without hooks or heads if there is adequate development length within the boundary element and if transverse reinforcement in the boundary element exceeds the horizontal reinforcement, i.e boundary element has more shear capacity than the web. See §21.9.6.4(e).

<u>If Boundary Elements Are Not Required (§21.9.6.5)</u>:

- If the longitudinal (vertical) reinforcement ratio at the wall boundary is greater than $\rho = 400/f_y$:

  - A boundary element shall be created such that it extends from the extreme compression edge into the wall by a distance no less than the maximum of $(c - 0.1\ell_w)$ and c/2, where c = neutral axis depth for $(1.2 + 0.2S_{DS})D + \rho Q_E + f_1 L$.

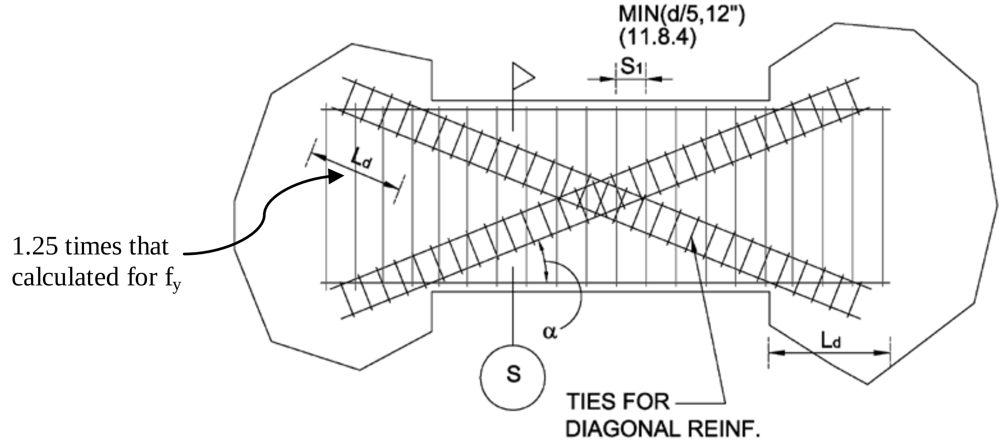
  - Spacing of transverse reinforcement shall not exceed 8in.

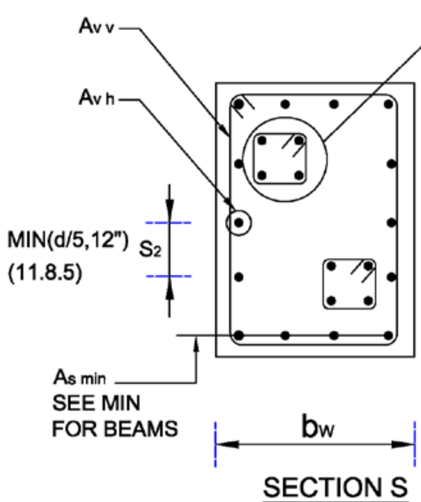
  - Transverse reinforcement shall be per Section 21.6.4.2 and shall be provided with either single or overlapping hoops. Each end of a crosstie shall engage a peripheral longitudinal bar and consecutive crossties shall be alternated end for end. Horizontal spacing of crossties or legs of overlapping hoops shall not exceed 14in.

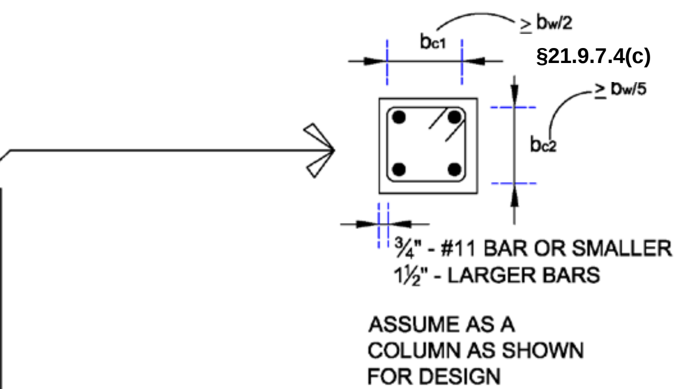
- If $V_u \geq A_{cv}\lambda\sqrt{f'_c}$ the horizontal reinforcement shall have a standard hook engaging the edge vertical reinforcement or the edge reinforcement shall be enclosed in U-stirrups spaced like the horizontal reinforcement and spliced to it (see figure on page CSWD1).

Coupling Beam Design (§21.9.7)

If $L_n/h \geq 4$ *Where,* L_n *= clear span* *h = clear depth*	Design as a flexural element §21.5 See seismic design of beams. Requirements of 21.5.1.3 & 21.5.1.4 for beam width can be waived by showing lateral stability as not being an issue, i.e. beam braced adequately.
$V_u > 4\lambda\sqrt{f'_c}A_{cw}$ & If $L_n/h < 2$	where, $A_{cw} = b_w d$. Reinforce with two intersecting groups of symmetrical diagonal bars (4 bars minimum) per Section 21.9.7.4. See figure below.
If $2 \leq L_n/h < 4$	Reinforce with either symmetrical diagonal bars per 21.9.7.4 or per 21.5.2 or 21.5.4.







Note: See next sheet for requirements for $A_{v\,v}$, $A_{v\,v}$, S_1, S_2, $A_{s\,min}$ *etc.*

Ties for Diagonal Reinforcement (§21.6.4.2 through 21.6.4.4):

Treated as confinement for columns—the total area of hoop reinforcement shall not be less than either of the following:

$$A_s = \left(0.3 s b_c \frac{f'_c}{f_y}\right)\left(\frac{A_g}{A_{ch}} - 1\right) \quad (21\text{-}4)$$

$$A_s = \left(0.09 s b_c \frac{f'_c}{f_y}\right) \quad (21\text{-}5)$$

Where, A_{ch} = area of core, out-to-out of ties.

Spacing of the ties, s = minimum ($b_c/4$, $6d_b$, s_o) §21.6.4.3

For more than 4 bars per diagonal, spacing between hoops or crossties shall not exceed 14”. §21.9.7.4(c)

Design Shear Strength of Coupling Beams (§21.9.7.4):

Using both sets of diagonal reinforcement:

$$\phi V_n = 2\phi\, f_y\, A_{vd} \sin\alpha \quad (\le 10\phi\sqrt{f'_c}A_{cw}) \quad (21\text{-}9)$$

Where, ϕ = 0.85 (ACI 318 §9.3.4)
A_{vd} = Total area of all the bars in each diagonal group.
α = Angle of the diagonal bars with the longitudinal axis.
A_{cw} = Concrete section area of coupling beam, $b_w d$.

Note: The diagonal bars shall be embedded in the wall beyond the beam no less than 1.25 times the development length for f_y in tension.

Flexural Strength of Coupling Beam:

In addition to the bottom/top steel provided, the contribution of the diagonal bars to the flexural strength, M_n, shall be considered.

Other Reinforcing in Coupling Beams:

Bottom steel in the beam §10.5

$$A_{s\ \min} = \frac{3\sqrt{f'_c}}{f_y} b_w d \quad \text{, but} > \frac{200}{f_y} b_w d$$

Vertical shear reinforcement, $A_{v\,v}$ §21.9.7.4(c)

$A_{v\,v} = 0.002 b_w s_1$ where s_1 = Minimum (d/5,12")

Horizontal shear reinforcement, $A_{v\,h}$ §21.9.7.4(c)

$A_{v\,h} = 0.002 b_w s_2$ where s_2 = Minimum (d/5,12")

Note: The above procedure is per ACI 318 Section 21.9.7.4(c) wherein the diagonal bars are confined with transverse reinforcement. An alternate procedure is provided in Section 21.9.7.4(d) for confining the entire coupling beam section rather than the diagonal bars only.

Wall Piers (§21.9.8)

Wall piers often occur in shear wall buildings due to the presence of door and window openings.

A wall pier is defined as a piece of wall wherein (§2.2):

$$\frac{Horizontal\ Length, l_w}{Wall\ Thickness, b_w} \leq 6.0$$

&

$$\frac{Clear\ Height, h_w}{Horizontal\ Length, l_w} \geq 2.0$$

- Wall piers behave like columns if $l_w/b_w \leq 2.5$ and as a wall if $l_w/b_w > 6.0$)
- Wall piers shall satisfy the special moment frame requirements of §21.6.3, §21.6.4 and §21.6.5, with joint faces taken as top and bottom of the clear height of the pier.
- If $l_w/b_w > 2.5$, the wall pier could alternatively satisfy:
  - *Design shear* forces shall be per 21.6.5.1 at the top and bottom on the clear height
  - *Design shear* force need not exceed Ωo times that from a seismic analysis
  - Transverse reinforcement shall be either hoops or single curtain horizontal reinforcement with 180° hooks at each end
  - Spacing of transverse reinforcement $\leq$ 6"
  - Extend transverse reinforcement above and below pier clear height by at least 12".
  - Provide special boundary elements if required by 21.9.6.3.

- For all wall piers sharing a common lateral force:
  $V_n \leq 8A_{cv}\sqrt{f_c^{'}}$,where A_{cv} is the gross area of all piers

- For any individual pier:
  $V_n \leq 10A_{cw}\sqrt{f_c^{'}}$, where A_{cw} is the area of the concrete section of the pier

For wall piers at the end of a wall, provide adequate shear reinforcement in the wall above and below the pier to transfer the *design shear* into the adjacent wall segments.

Notes:

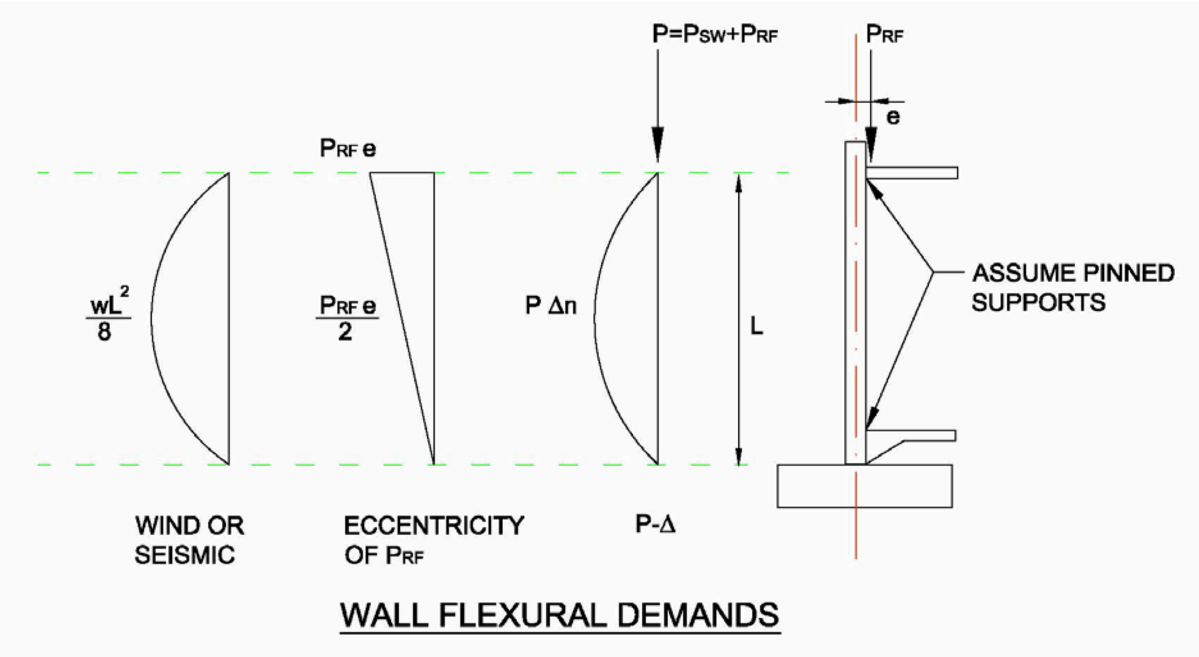
1. *Wall piers not designated as part of the lateral system shall be designed per Section 21.13.*

REINFORCED CONCRETE

SLENDER WALL OUT-OF-PLANE DESIGN

References: ACI 318-11 §14.8, ASCE 7-10

Applies to Pre-Cast, Tilt-Up Walls etc.



All the following wall calculations are performed on a 1’-0” wall strip.

<u>Load Combinations</u>:

- $1.2D + 1.0(E_h + E_v) + f_1L + 0.2S$

  Use => ***(1.2+0.2S_{DS})D + 1.0E_h + f_1L + 0.2S***

Typically solve for this load combination. The following combination should also be checked.

- $(0.9\text{-}0.2S_{DS})D \pm 1.0E_h$ => Usually leads to smaller M_u, (but also smaller M_n).

Note: The procedure described in this section satisfies the requirements of §10.10 for slenderness effects in compression members. In high seismic areas, wind forces do not typically govern the out-of-plane design of slender walls.

Flexural Demands on Wall:

Seismic Moment:

For Structural Walls (ASCE 7 §12.11.1):

$$F_p = 0.4 S_{DS} k_a I_e W_p \tag{12.11-1}$$

$$F_p \geq 0.2 k_a I_e W_p$$

where, $k_a = 1.0 + \frac{L_f}{100}$ (12.11-2)

k_a = amplification factor for diaphragm flexibility
L_f = Span of flexible diaphragm providing support to the wall between lateral system elements; Use zero for rigid diaphragms.

For Non-Structural Walls (ASCE 7 §13.3):

$$F_p = \frac{0.4 a_p S_{DS} I_p}{R_p}\left(1 + 2\frac{z}{h_r}\right)W_p \tag{13.3-1}$$

$F_p \leq 1.6 S_{DS} I_p W_p$ & $F_p \geq 0.3 S_{DS} I_p W_p$ (13.3-2 & 13.3-3)

Per Table 13.5-1, $\boldsymbol{a_p = 1.0}$ & $\boldsymbol{R_p = 2.5}$.

Note: See Section '*Seismic Loads for Non-Structural Component Design*'.

Compute F_p at the roof ($z = h_r$) and at the base ($z = 0$) and compute average F_p at mid-height:

$$F_{p\,avg} = \frac{F_{p\,roof} + F_{p\,base}}{2}$$

Note: An alternative method is to compute F_p at $z_{x\,avg} = h_r/2$.

Wall seismic moment, $M_{us} = \frac{F_p L^2}{8}$

Wall Moment Due to Eccentricity of Roof Load:

Wall moment due to eccentricity, $M_{ue} = \frac{1}{2} P_{u\,RF}(e)$

Where, t = wall thickness

e = Eccentricity of roof load (includes ½ wall thickness).
$P_{u\,RF}$ = Factored axial load on wall due to roof loads.

Wall Moment Due to P-Δ (ACI §14.8.3):

$M_{P\text{-}\Delta}$ is calculated at the maximum potential deflection Δ_n.

$$M_{u\,P-\Delta} = P_u\,\Delta_u$$

where, $P_u = P_{u\,RF} + P_{u\,SW}$

$$\Delta_u = \frac{5}{(0.75)48}\frac{M_u L_c^{\;2}}{E_c I_{cr}} \qquad (14\text{-}5)$$

where, L_c = clear height of wall.

Total Design Moment Due to Factored Loads (ACI §14.8.3):

$$M_u = M_{ua} + P_u\,\Delta_u \qquad (14\text{-}4)$$

where, $M_{ua} = M_{us} + M_{ue}$

$$M_u = \frac{M_{ua}}{1 - \dfrac{5P_{u\,total}\,l_c^2}{(0.75)48E_c I_{cr}}} \qquad (14\text{-}6)$$

where, M_{ua} and $P_{u\,total}$ are knowns, I_{cr} and M_u are unknowns.

$$I_{cr} = \frac{E_s}{E_c}A_{se}(d-c)^2 + \frac{l_w c^3}{3} \qquad (14\text{-}7)$$

where, $\dfrac{E_s}{E_c} \geq 6$, $c = \dfrac{a}{0.85}$, l_w = 12”

A_{se} = Effective area of steel, $A_{se} = \left(A_s + \dfrac{P_u}{f_y}\dfrac{h}{2d}\right)$ ACI 318 §R14.8.3

For walls with a single curtain of reinforcement: $d = \dfrac{h}{2}$ & $A_{se} = \left(A_s + \dfrac{P_u}{f_y}\right)$

Solving for Mu:

There are three approaches for solving for M_u:

1. Asssume Δ_u and Iterate as follows:

a. Using trial value of Δ_u, find M_u from Eqn. 14-4.
b. Using M_u, compute $A_{s\,reqd}$:

- Assume $jd = d - a/2 = 0.925d$
- $A_{s\,reqd} = \left(\dfrac{M_u}{0.9 f_y\ jd}\right) - \dfrac{P_u}{f_y}$
- Compute A_{se} and $a = \dfrac{A_{se} f_y}{0.85 f'_c\,(12")}$
- Check assumption for *jd* and iterate till convergence.

c. Knowing A_s, find A_{se} and compute I_{cr} from Eqn. 14-7.
d. Check Δ_u from Eqn. 14-5.
e. Recompute M_u and repeat steps b through e till convergence.

2. Assume A_s and iterate as follows:
   a. Compute M_u using Eqn. 14-6.
   b. Check if A_s is adequate for M_u. Revise A_s if necessary.
   c. Iterate.

3. Assume that $\Delta_u = (x)\dfrac{L_c}{150}$

   Where x = 1.4, is the equivalent load factor to convert the service level deflection to the factored level.

   Compute M_u from Eqn. 14.4.

   Note: This is obviously an approximation for a quick design and not recommended for practice. Another quick approximation would be to assume $I_{cr} = 0.25I_g$.

Design of Slender Wall:

Knowing P_u (i.e. $P_{u\,total}$) and M_u, the wall can be designed as a lightly axially loaded column element as follows:

$a = \dfrac{A_s f_y}{0.85 f'_c\, b}$ - Depth of the compression block.

$c = \dfrac{a}{0.85}$ - Depth of neutral axis from compression face

$$\varepsilon_s = \left(\frac{t/2 - c}{c} \right) 0.003$$ - Strain in tension steel

The code requires that for sections with axial load less than $0.1f'_c A_g$, the strain in the tension face steel shall not be less than 0.004 (§10.3.5).

$$F_s = A_s E_s \varepsilon_s$$ - Force in tension steel

$$C_c = F_s + P_u$$ - Compressive force for equilibrium

$$M_n = C_c \left(\frac{h}{2} - \frac{a}{2} \right) + F_s \left(\frac{t}{2} - \frac{h}{2} \right)$$

Compute ϕ based ε_s => typically $\phi = 0.9$.

For walls with two curtains of reinforcement, see page SCPMI3 of '*Simplified Column Axial-Flexure Interaction*'.

<u>The following have to be checked</u>:

The above procedure can be used if the following conditions are met:

1. Factored level axial stress at location of maximum moment $\leq 0.06f'_c$.

$$\frac{P_{u\,RF} + P_{u\,SW\ of\ half\ height}}{A_g} \leq 0.06 f'_c$$

2. The wall is tension controlled, i.e. $\varepsilon_s \geq 0.005$ @ $\varepsilon_{cu} = 0.003$.
   This is typically true for lightly axially loaded elements (item 1 above).

3. $\phi M_n > M_{cr}$ where, $M_{cr} = \dfrac{7.5\sqrt{f'_c}\, I_g}{(t/2)}$ §14.8.2.4

   See §9.5.2.3 for lightweight concrete.

<u>Service Level Deflection Check</u>:

Service level deflection, $\Delta_s \leq \dfrac{L_c}{150}$ §14.8.4

Service level moment, $M_s = \dfrac{(F_p/1.4)L^2}{8} + \dfrac{P_{sRF}(e)}{2} + P_{sWS}\Delta_n$

If $M_s \le M_{cr}$ => $\Delta_s = \dfrac{5}{48}\dfrac{M_s L_c{}^2}{E_c I_g}$

If $M_s > M_{cr}$ => $\Delta_s = \dfrac{5}{48}\dfrac{M_{cr} L_c{}^2}{E_c I_g} + \left(\dfrac{M_s - M_{cr}}{M_n - M_{cr}}\right)\left(\dfrac{5L_c{}^2}{48E_c}\left(\dfrac{M_n}{I_{cr}} - \dfrac{M_{cr}}{I_g}\right)\right)$

REINFORCED CONCRETE

FOOTING DESIGN

Reference: IBC & ACI-318 (Chapter 15)

Determining Footing Size:

§15.2.2 Footing Area – Based on unfactored forces and moments to be transferred to the soil.

§15.2.1 Footing Design – Based on factored forces and moments.

Consider an isolated footing supporting a column with unfactored axial load, P, moment, M, and shear, V.

<u>Design for axial compressive load</u>:

Area of footing required, $A_{ftg} = \dfrac{P'}{q_{all}}$

where, P' = Unfactored column load + approximate footing weight.
q_{all} = Allowable bearing pressure.

Compute Footing size => Width, B x Length, L

Note: For axial load only square footings are most economical.

<u>Design for moment</u>:

Assume that the footing width is B, based on the area required for vertical load. The moment acting on the footing can be considered equivalent to the vertical load at an eccentricity 'e' from the footing centroid (see figure on pg. CFD2).

$$e = \frac{M}{P'}$$

Case 1: e < L/6

In this case, the footing continues to stay in contact with the soil. The maximum soil pressure is given by:

$$q_{\max} = \frac{P'}{BL} + \frac{6M}{BL^2} \qquad q_{\max} \le q_{all}$$

Where, q_{max} = maximum allowable soil bearing pressure.
Knowing q_{max} & B, the required length, L, can be determined.

Case 2: e = L/6

In this case, the effective length of the footing is equal to the actual length of the footing. The maximum soil pressure is given by:

$$q_{\max} = \frac{2P'}{BL} \qquad\qquad q_{\max} \le q_{all}$$

Knowing q_{max} & B, the required length, L, can be determined.

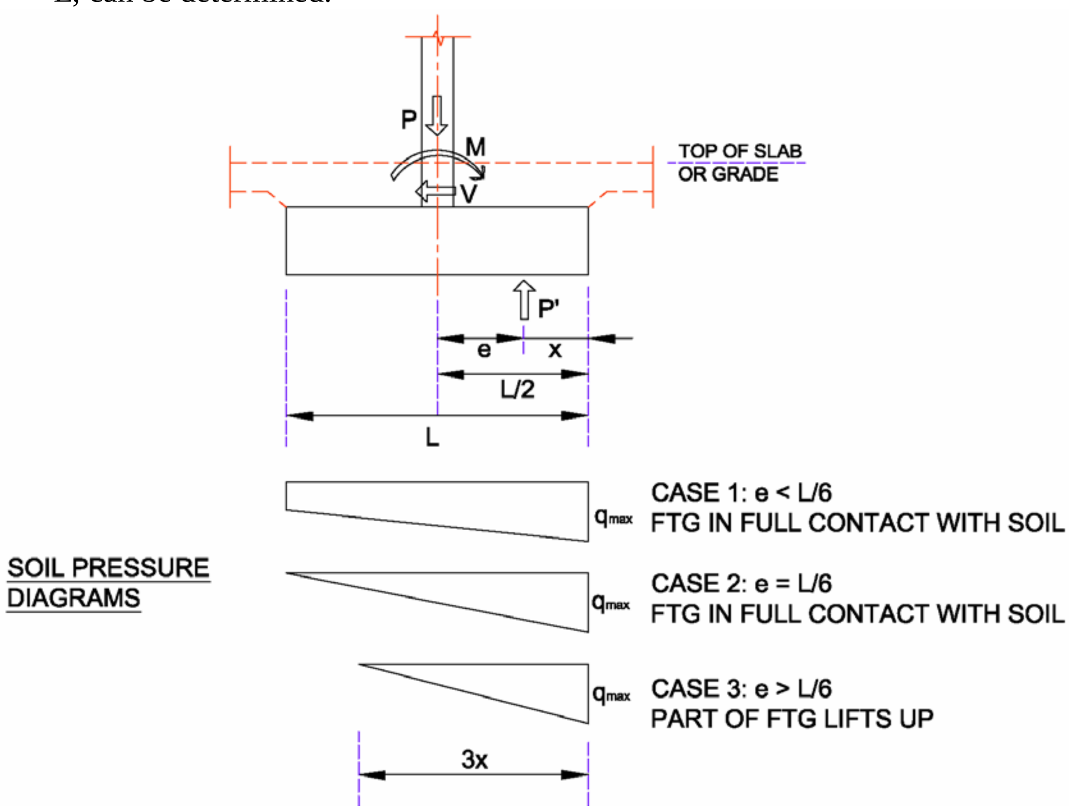
Case 3: e > L/6

In this case, the resultant vertical reaction under the footing is outside the kernel and a portion of the footing loses contact with the soil. The maximum soil pressure is given by:

$$q_{\max} = \frac{2P'}{B(3x)} \qquad\qquad q_{\max} \le q_{all}$$

where, $x = \frac{L}{2} - e$

Knowing q_{max} & B, the effective length (3x) and knowing x, the required length, L, can be determined.

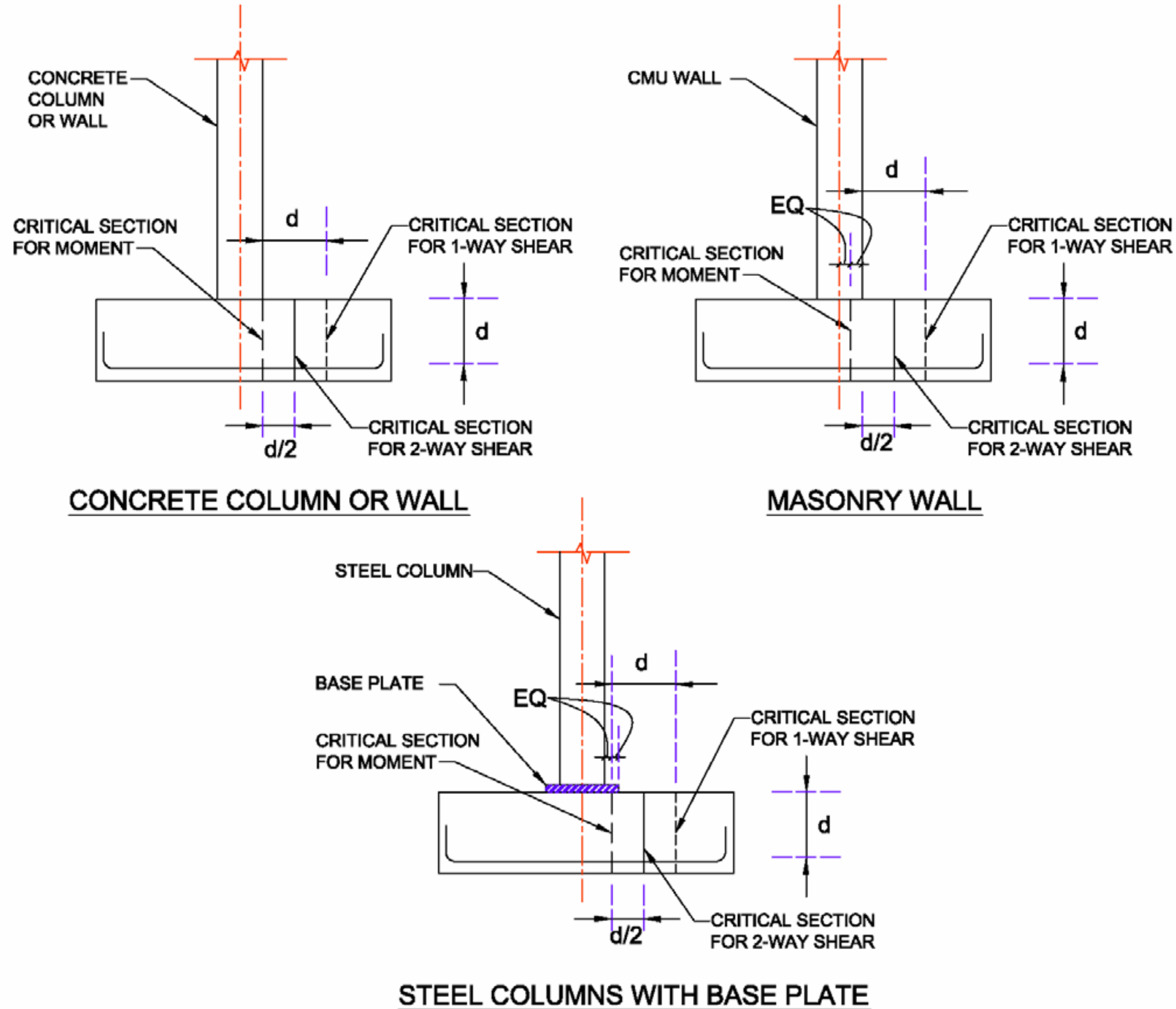


Footing Design:

Design Forces & Critical Locations (§15.4.2 & 15.5.2):

Using the maximum soil pressure, q_{max}, determined above, compute the factored design forces. The simplest approach is to apply an 'equivalent' load factor the maximum pressure, e.g. for axial load only, load factor = $(1.2P_{DL} + 1.6P_{LL})/(P_{DL}+P_{LL})$.

The factored design demands shall be computed at 'critical locations' as shown below:



CRITICAL LOCATIONS FOR MOMENT & SHEAR

<u>Reinforcement in Footings</u>:

- Minimum (temperature & shrinkage) reinforcement in any direction (§10.5.4 & 7.12.2.1). See also flexural design of footings on sheet CFD6.

  1. For grade 40 or 50 deformed bars – $0.0020A_c$.
  2. For grade 60 deformed bars, welded wire fabric (deformed or smooth) – $0.0018A_c$.
  3. for bars with $f_y > 60$ksi with a yield strain of 0.35% - $\dfrac{0.0018x60{,}000}{f_y}$

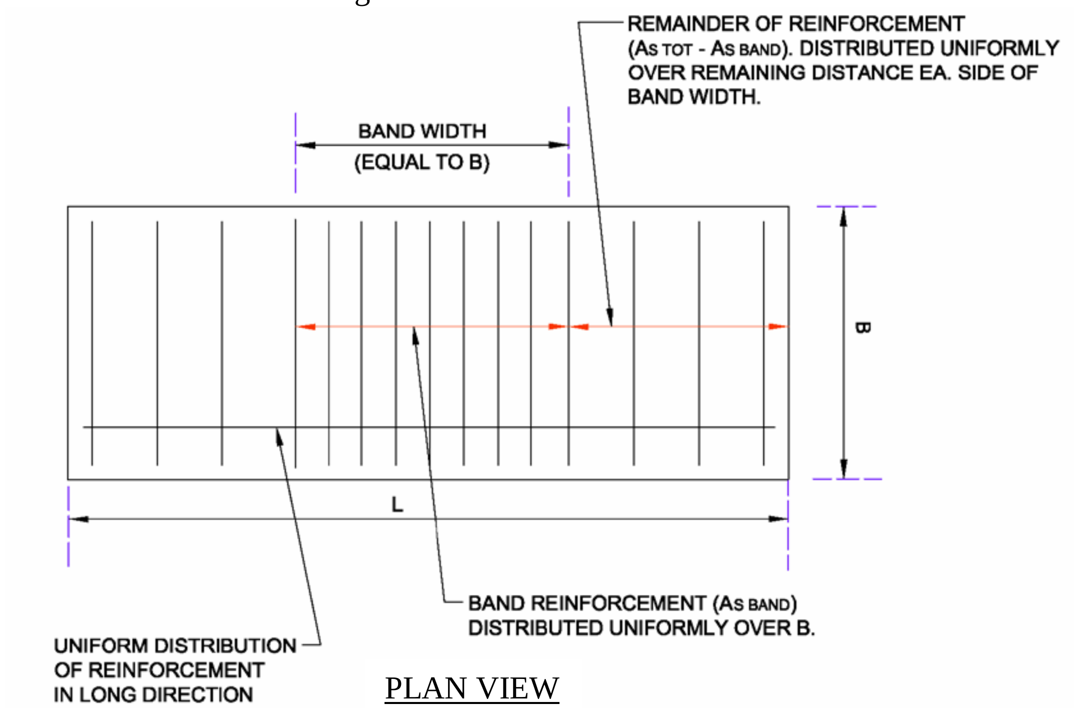
- Maximum spacing – 3 times thickness of footing or 18in.

- One-way (continuous) footings & two-way square footings – Reinforcement shall be distributed uniformly across entire width of footing (§15.4.3).

- In two-way rectangular footings:

  - Long Direction – Uniformly distributed across width (§15.4.4.1).

  - Short Direction – Per figure below:



<u>PLAN VIEW</u>

$$\frac{A_{s\ BAND}\ \ in\ Band\ Width}{A_{s\ TOT}\ \ Total\ \ For\ \ Short\ \ Direction} = \frac{2}{(\beta + 1)} \qquad (15\text{-}1)$$

$$\text{where, } \beta = \frac{Long\ \ Dimension\ (L)}{Short\ \ Dimension\ (B)}$$

Footing Shear Design:

One-way shear (§11.2):

Use one of the equations below:

$$\phi V_c = \phi 2\lambda\sqrt{f'_c}b_w d \qquad (11\text{-}3)$$

$$\phi V_c = \phi\left(1.9\lambda\sqrt{f'_c} + 2500\rho\frac{V_u d}{M_u}\right)b_w d \qquad (11\text{-}5)$$

where, $\phi = 0.75$
$\rho = A_s/b_w d$ (A_s is flexural reinforcement for M_u, see CFD6)
$V_u d/M_u \leq 1.0$ & M_u is factored moment at section where V_u is being considered.
$\lambda = 1.0$ for normal-weight concrete

Note: Use the second equation only if the first doesn't give adequate capacity.

Two-way shear (§11.11.2):

$$\phi V_c = MIN\begin{cases}\phi\left(2+\dfrac{4}{\beta}\right)\lambda\sqrt{f'_c}b_o d & (11\text{-}31)\\[2ex] \phi\left(\dfrac{\alpha_s d}{b_o}+2\right)\lambda\sqrt{f'_c}b_o d & (11\text{-}32)\\[2ex] \phi 4\lambda\sqrt{f'_c}b_o d \quad \textit{(This equation typically governs)} & (11\text{-}33)\end{cases}$$

$\beta = \left(\dfrac{Long\ Side}{Short\ Side}\right)$ of column, concentrated load/reaction.

α_s = 40 (interior columns), 30 (edge columns), 20 (corner columns)

where, $\phi = 0.75$
b_o = Critical perimeter for two-way shear; usually taken at d/2 from face of column (see sheet CFD3).

Notes:
1. *Shear reinforcement is typically not provided in footings. Use adequate depth to achieve the desired capacity. Minimum depth of footings shall be 6in above bottom reinforcement for footings on soil and 12in above bottom reinforcement for footings on piles.*
2. *Provide minimum 3" cover for all reinforcement in footings.*

Footing Flexural Design:

Required flexural reinforcement can be computed as:

$$\rho_{reqd} = \frac{1}{m}\left(1-\sqrt{1-\frac{2mR_u}{\phi f_y}}\right),$$ check $\rho_{reqd} \geq \rho_{min}$ per sheet CFD4.

where, $m = \dfrac{f_y}{0.85 f'_c}$

$R_u = \dfrac{M_u}{bd^2}$, M_u in lb-in.

$\phi = 0.9$

$$\rho_{\max} = 0.75\rho_{bal} = 0.75\left(0.64\beta_1 \frac{f'_c}{f_y} x \frac{87}{87+f_y}\right)$$ *(f'_c and f_y in ksi)*

where, $\beta_1 = 0.85$ for $2{,}500\text{psi} \leq f'_c \leq 4000\text{psi}$;

For $f'_c > 400\text{psi}$: $\beta_1 = 0.85 - 0.05\left(\dfrac{f'_c - 4000}{1000}\right)$ but ≥ 0.65.

Check that reinforcement is adequately developed on either side of all flexural critical sections (§15.6). See section '*Reinforcement Development & Lap Splices*'.