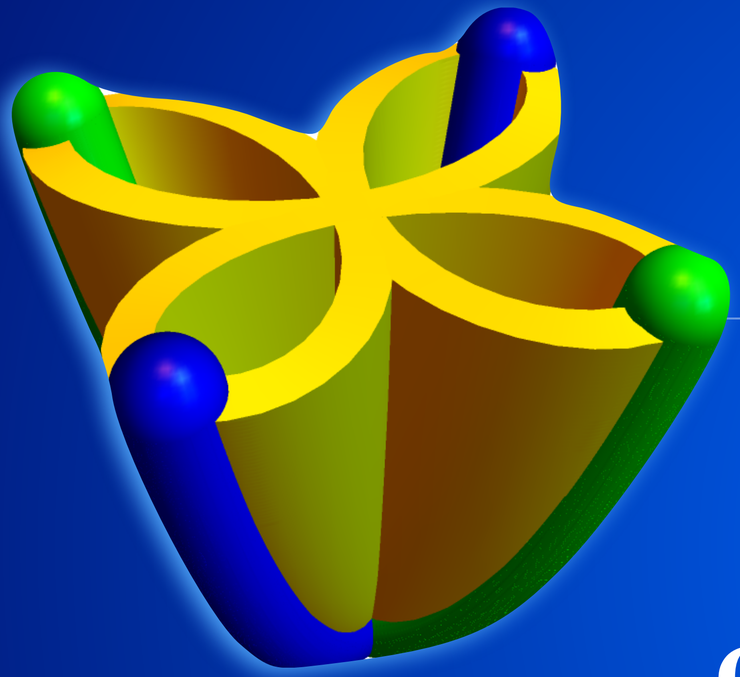
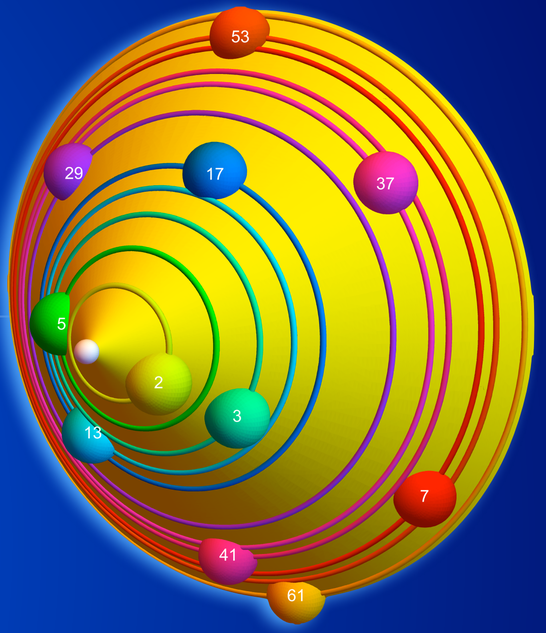
3D printing Experiments in Mathematics





Oliver Knill, 9/30/2016

2016 SIAM Conference on Applied Mathematics Education

Main Points of this talk

Main Points of this talk

New Technology invigorates teaching math

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It changes how we think about mathematics

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Exciting technology helps to motivate students

Main Points of this talk

New Technology invigorates teaching math

It changes how we think about mathematics

Exciting technology helps to motivate students

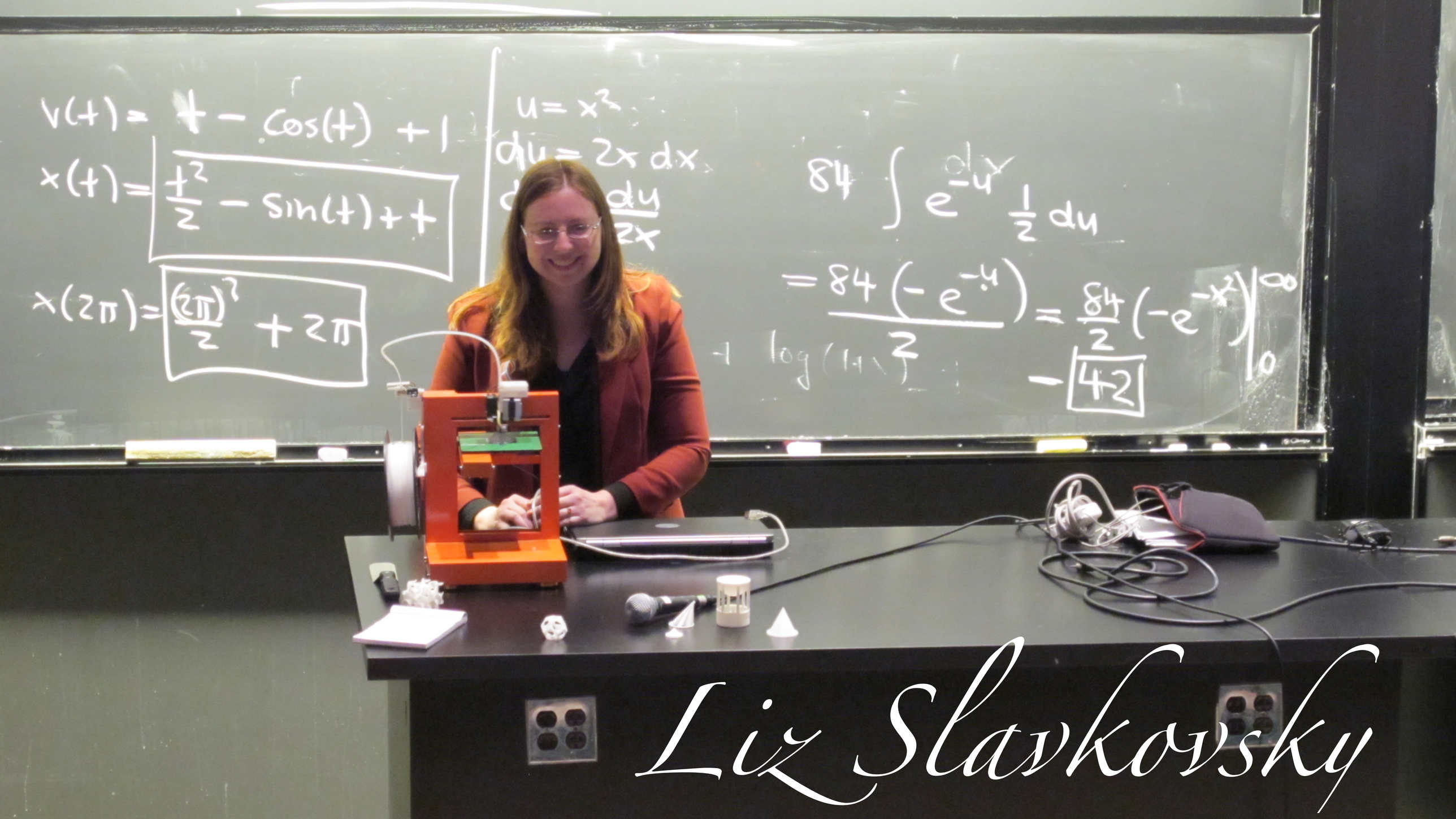
The choice and right use of tools is important

Thanks

John Zweck

for organizing this meeting and the invigtation



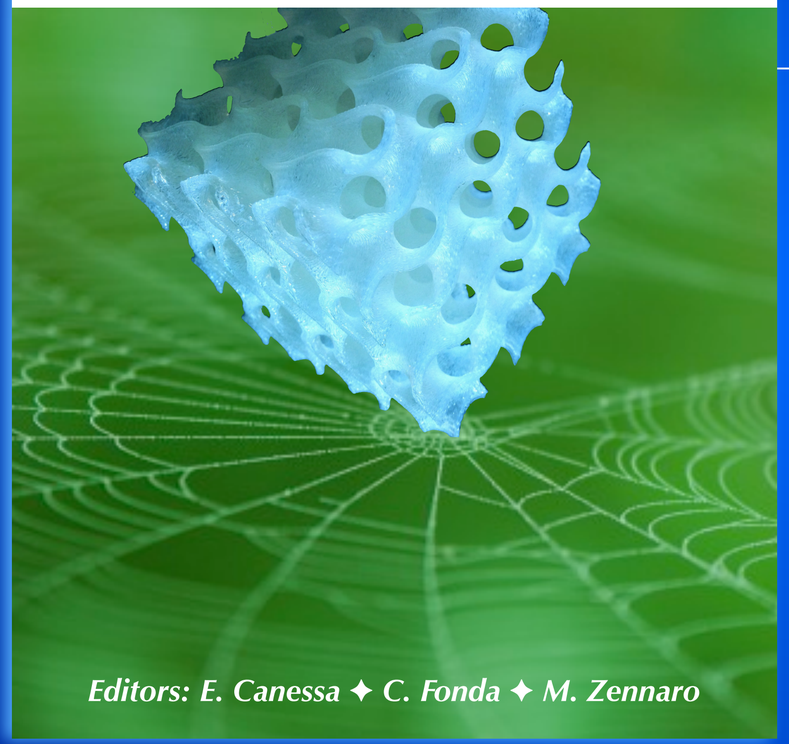

Liz Slavkovsky


Liz
Sophie
Jamel

Enrique Canossa, Carlo Fonda, Marco Zennaro

LOW-COST 3D PRINTING

FOR SCIENCE, EDUCATION & SUSTAINABLE DEVELOPMENT



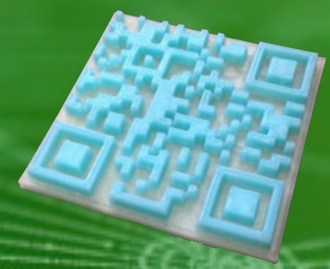
Editors: E. Canessa ✦ C. Fonda ✦ M. Zennaro

The Abdus Salam International Centre for Theoretical Physics

Low-cost 3D Printing for Science, Education & Sustainable Development

Low-cost, three-dimensional (3D) desktop printing, although still in its infancy, is rapidly maturing, with seemingly unlimited potential. The hope is that this cutting-edge 3D technology will open new dimensions to science and education, and will make a marked impact in developing countries.

This book gives a reasonable, first overview of current research on 3D printing. It aims to inspire curiosity and understanding in young scholars and new generations of scientists to motivate them to start building up their own 3D printing experiences and to explore the huge potential this technology provides –*with the final goal of putting learning literally in their hands.*



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Triest Workshop

Oliver

Marius Kintel

2013




Gregor Luetolff
2013

Elson Family Grant

Proposal for the Elson Family Arts Initiative **Oliver Knill, May 4, 2012**

Abstract

In a creative final project, multivariable calculus students will create an art sculpture and have the opportunity to have their "sculpture" project printed on a 3D printer.

Course enhancement

The multivariable calculus projects in Math21a are traditionally creative. Figure 1 shows two examples from the past. Our Math21a course is a standard multivariable course which extends single variable calculus to higher dimensions. It not only provides vocabulary and background for understanding fundamental processes and phenomena in economics, life sciences, finance or chemistry but also builds tools for describing geometrical objects like curves, surfaces, and solids. This helps to form intuition which is needed in other fields like computer vision or data visualization or medical sciences. In 21a, students are also exposed to a powerful computer algebra system which allows exploration of space without any programming experience. It helps to sharpen visualization skills. An important part of multivariable calculus has always been link to artistic components like architecture or music. These connections have been lived traditionally in our courses for decades.

This summer, I will use a 3D printing printing component the first time in the summer school. I will not have the budget to print all of the student creations but print the best ones. In the summer course of 2012, the project already will be **"build an art sculpture"**. I will just select a winner and print that one. In the fall project, students will be asked to build a sculpture for display, either inspired by previous art or a new creation by the student. With a grant, we would be able to print them. In this project, **art and mathematics will meet and come to live** with the exciting **emerging technology of 3D printing**.

Budget

Printing a single object costs about 25 dollars in Shapeways which produces high quality 3D printouts. This is easier and much better than any consumer 3D printer can do. Objects can be submitted electronically and will be sent back by mail. With an estimated 200 student enrollment, we can build 200 creative objects, mathematical sculptures and make an exhibit. The following line item details are chosen to match the 5000 dollar grant. Depending on material this can be adapted:

Prize for one Printout	25 Dollars estimate (http://www.shapeways.com/materials)
Number of Students	200 Students estimate
Total	5000 US Dollars

Class exhibit examples

Student gallery exhibits have tradition in math21a, math21b. Examples:

http://www.math.harvard.edu/~knill/galleries

http://www.courses.fas.harvard.edu/~math21a/exhibits/garden

http://www.math.harvard.edu/archive/21a_fall_06/marblebook

http://www.math.harvard.edu/archive/21b_spring_08/exhibits/songs

Previous grants

Here are previous projects crossing disciplines. Examples:



FIGURE 1. A garden project with more than 200 objects built by our Harvard Math21a class in Fall 2011. The task had been to build a fruit, plant or garden object. The right figure illustrates "marblebook" a spoof on Facebook. This project was built by the Harvard Math21a class in Fall 2006. More than 300 marbles were rendered from pictures submitted by the class. It illustrates the parametrization of surfaces.

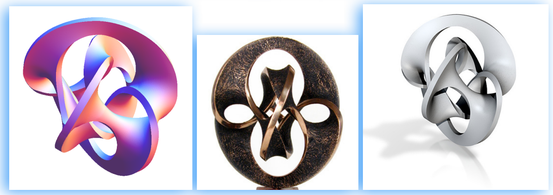


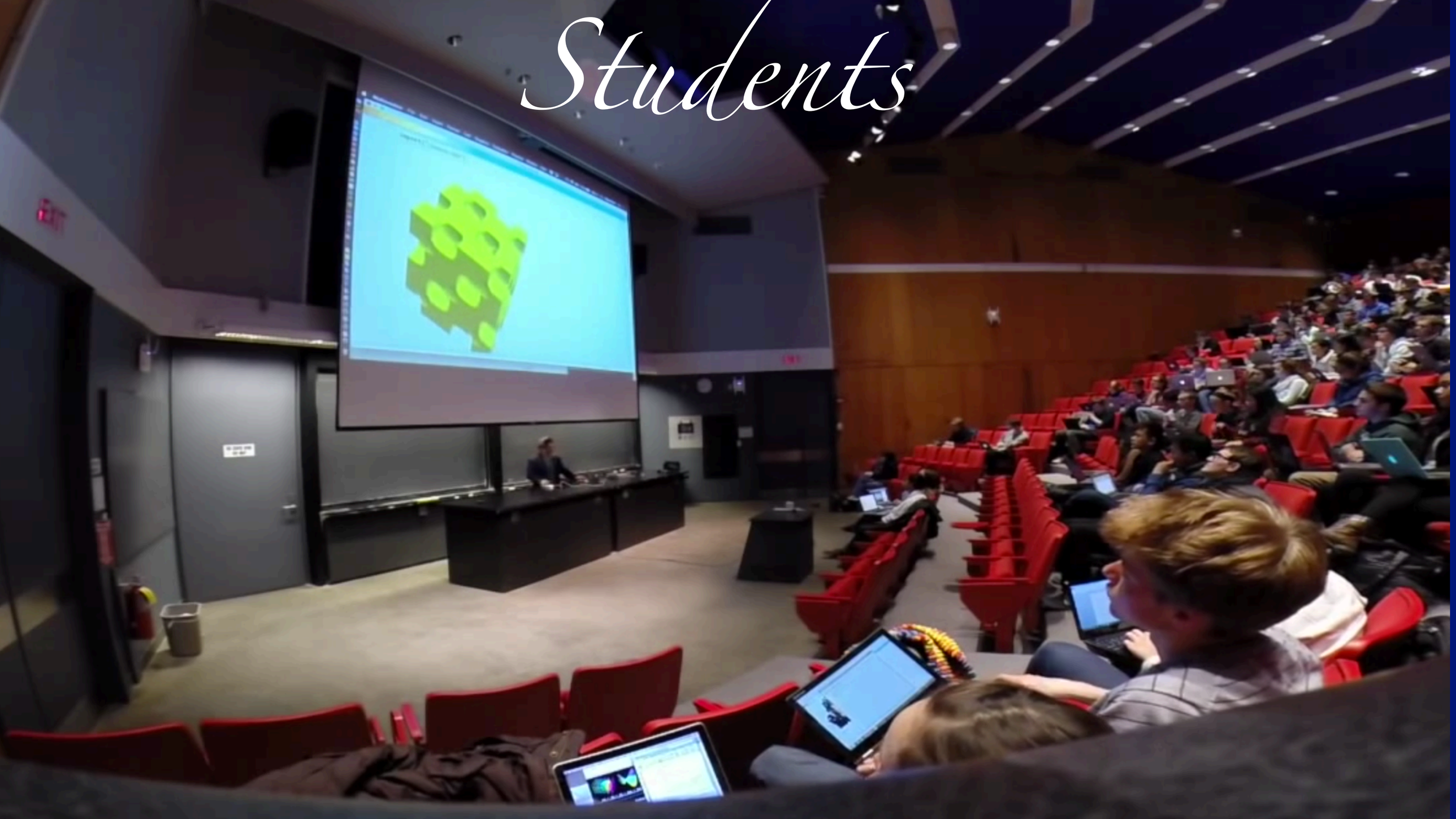
FIGURE 2. The sculpture "anachron" by Artist **Bathsheba Grossman**, rendered in the computer algebra system Mathematica, realized in bronze by the artist and printed by a 3D printer in shapeways.

- One is a PITF project on exploring vector fields
  http://www.math.harvard.edu/~knill/pitf
  from summer 2004 with the Harvard presidental fellow **David Mahfouda** who concentrated in the department of visual and environmental studies.
- A visually centered project was a year long HCRP project of 2009 with **Michael Teodorescu**
  http://www.math.harvard.edu/~knill/caustic/exhibits
  http://www.math.harvard.edu/~knill/conchoid/exhibits
- The extension school thesis project of **Jose Ramirez** was artistic in some way because the developed software allows image analysis and redrawing. The exhibit page is here:
  http://www.math.harvard.edu/~knill/3dscan2/exhibits
- Finally, I direct an extension school thesis project of **Elizabeth Slavkovsky** which deals with 3D printing in education. I have learned a lot during this project about the technology of 3D printing. Liz also has her own printer and we have used the technology to demonstrate volume computation by slicing. Here is the exhibit which is built up:
  http://www.math.harvard.edu/~knill/3dprinter/exhibit.html

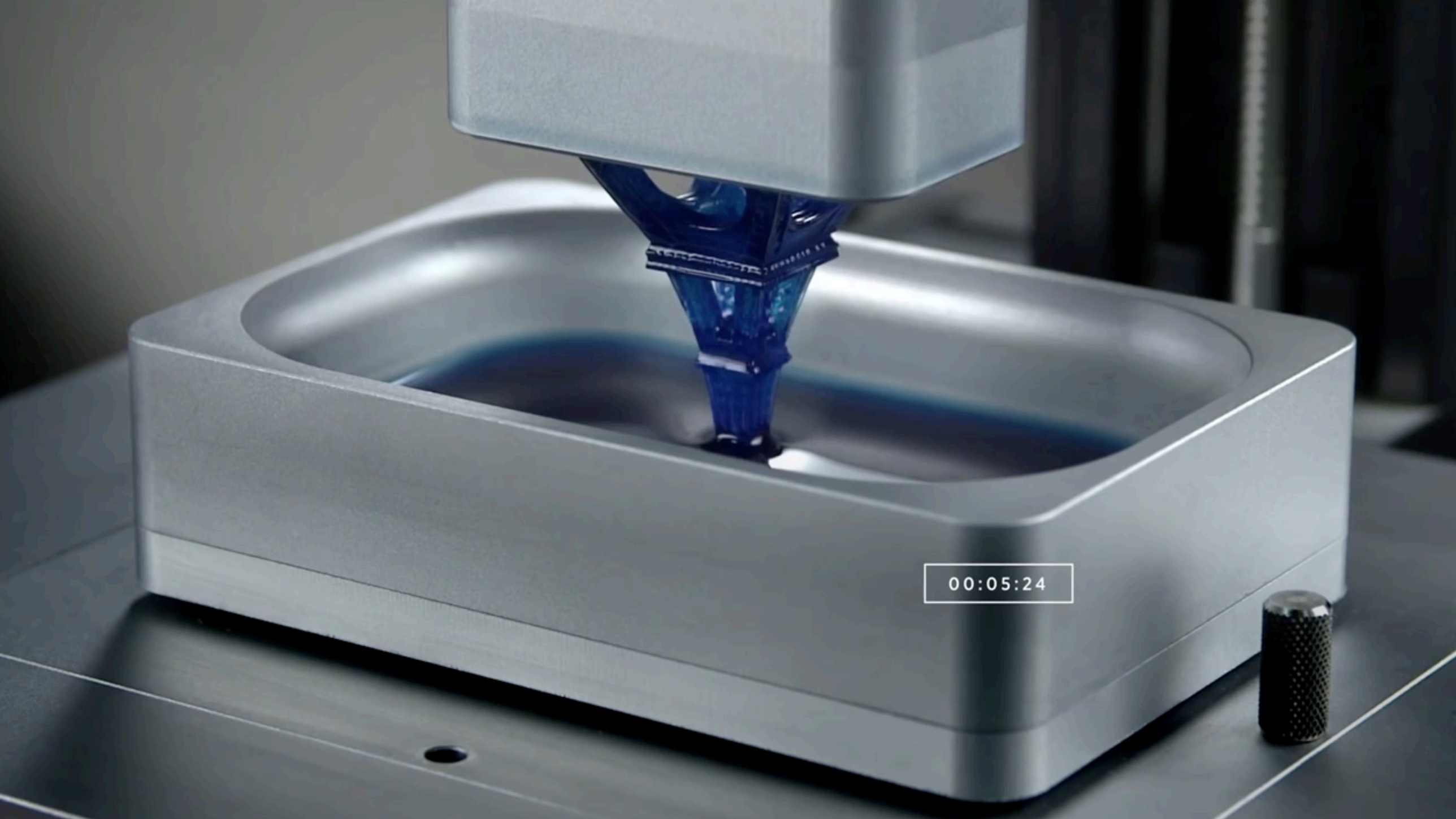
Oliver Knill, Preceptor, Department of Mathematics, Harvard, knill@math.harvard.edu

2012

Students



An Industrial Revolution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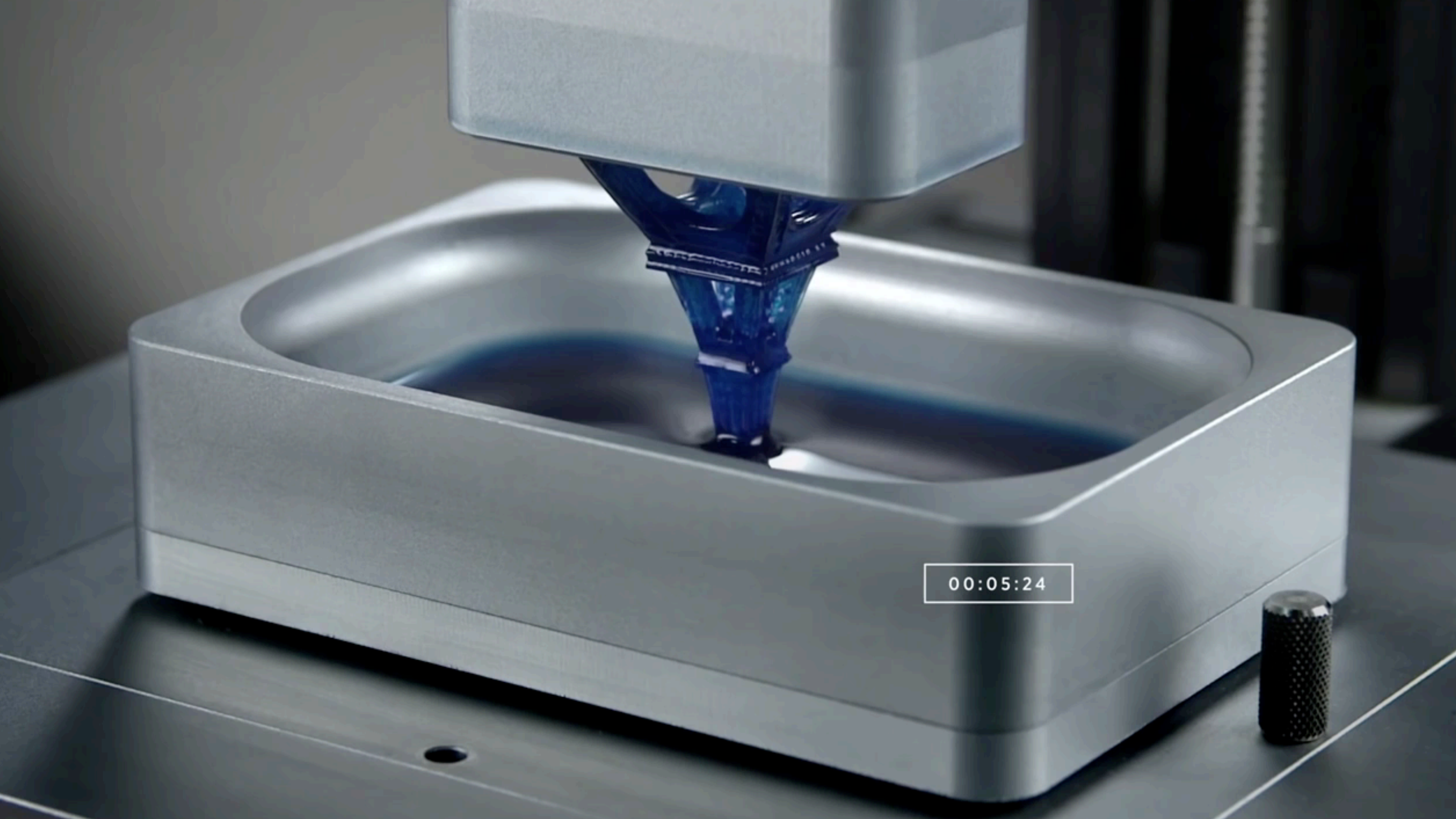
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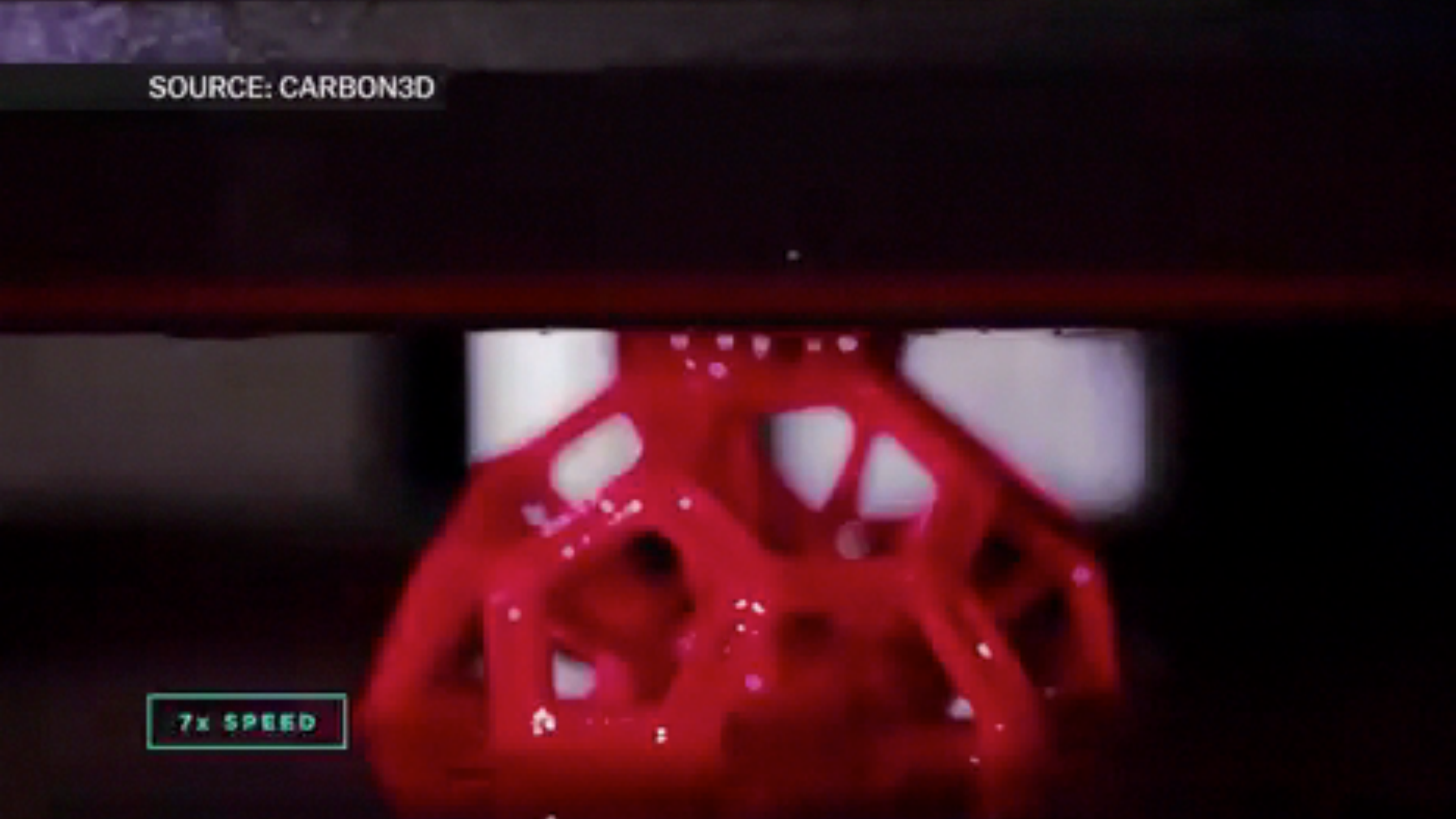
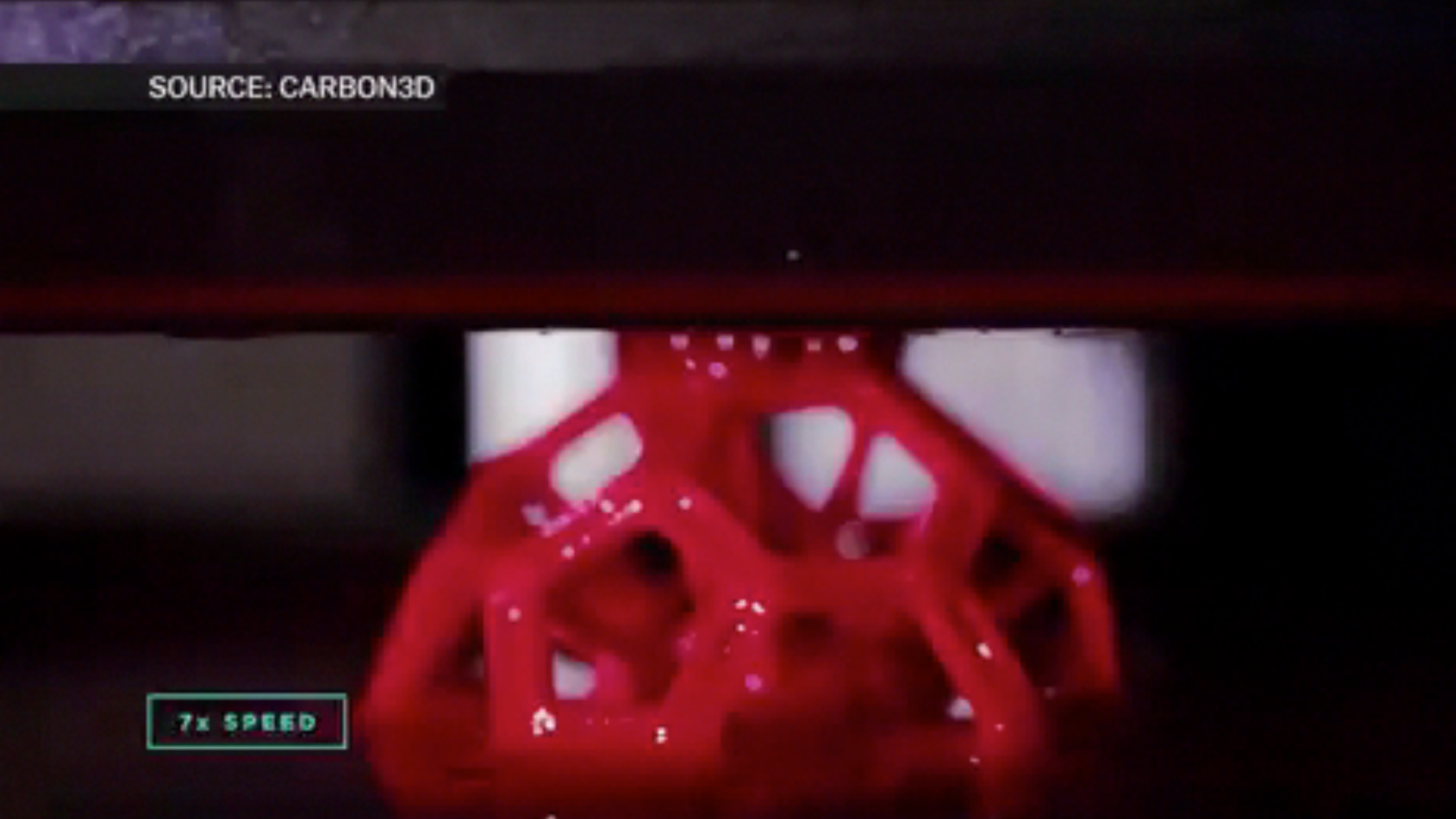

SOURCE: CARBON3D
7× SPEED


SOURCE: CARBON3D
7× SPEED

Printing Cars

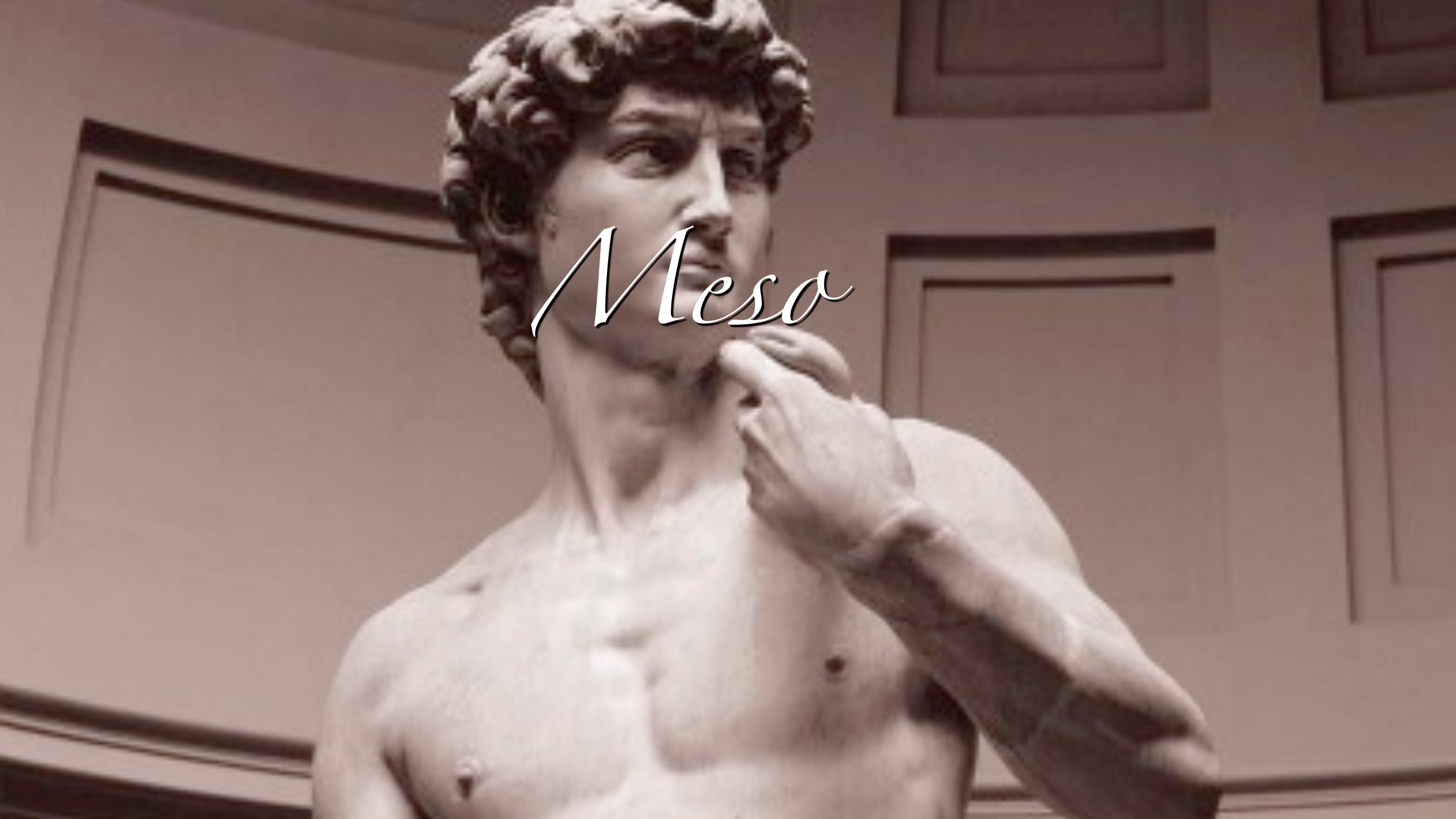


Printing Cars



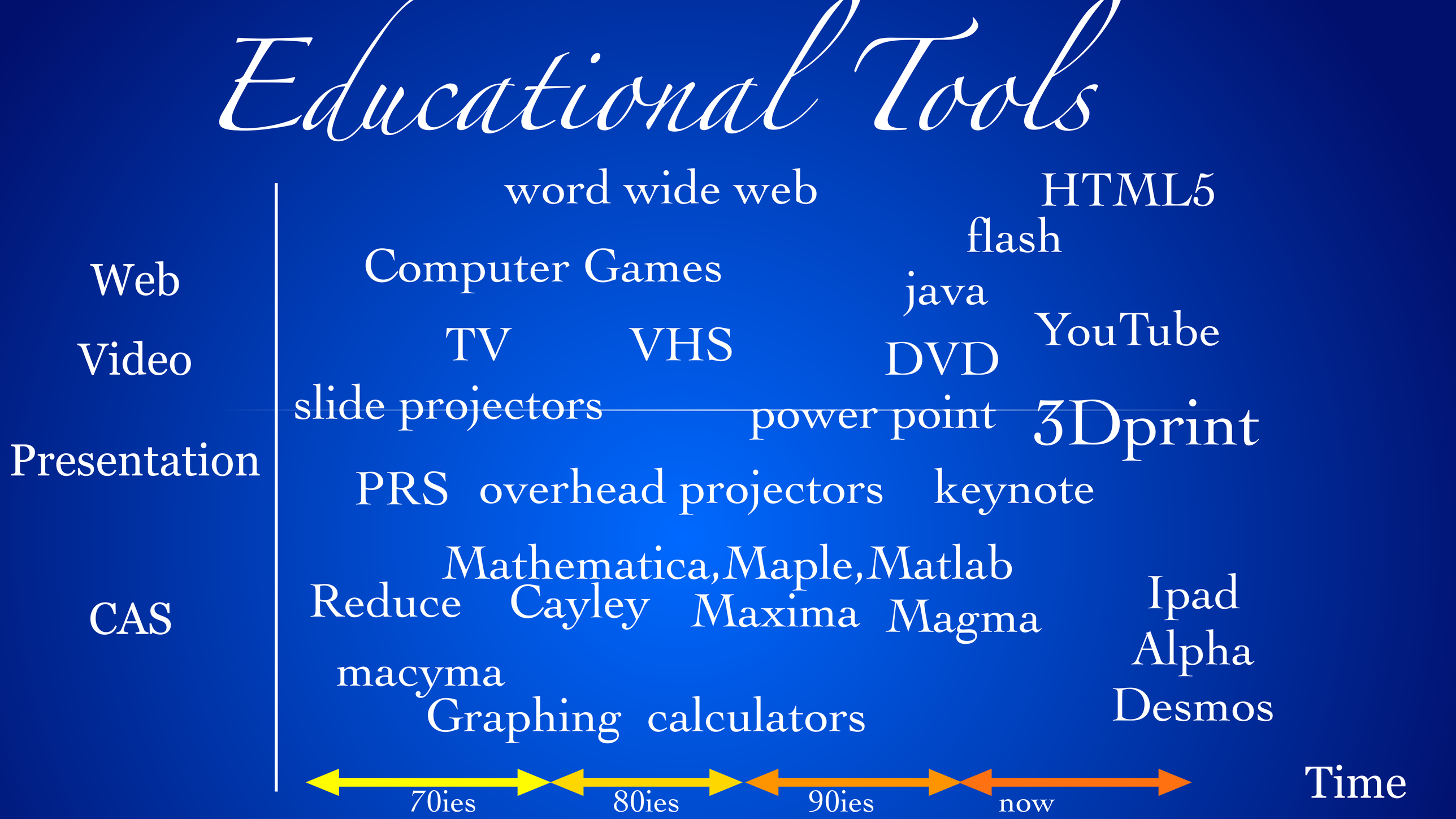

Micro


Macro


Meso

A tool in Education

Educational Tools



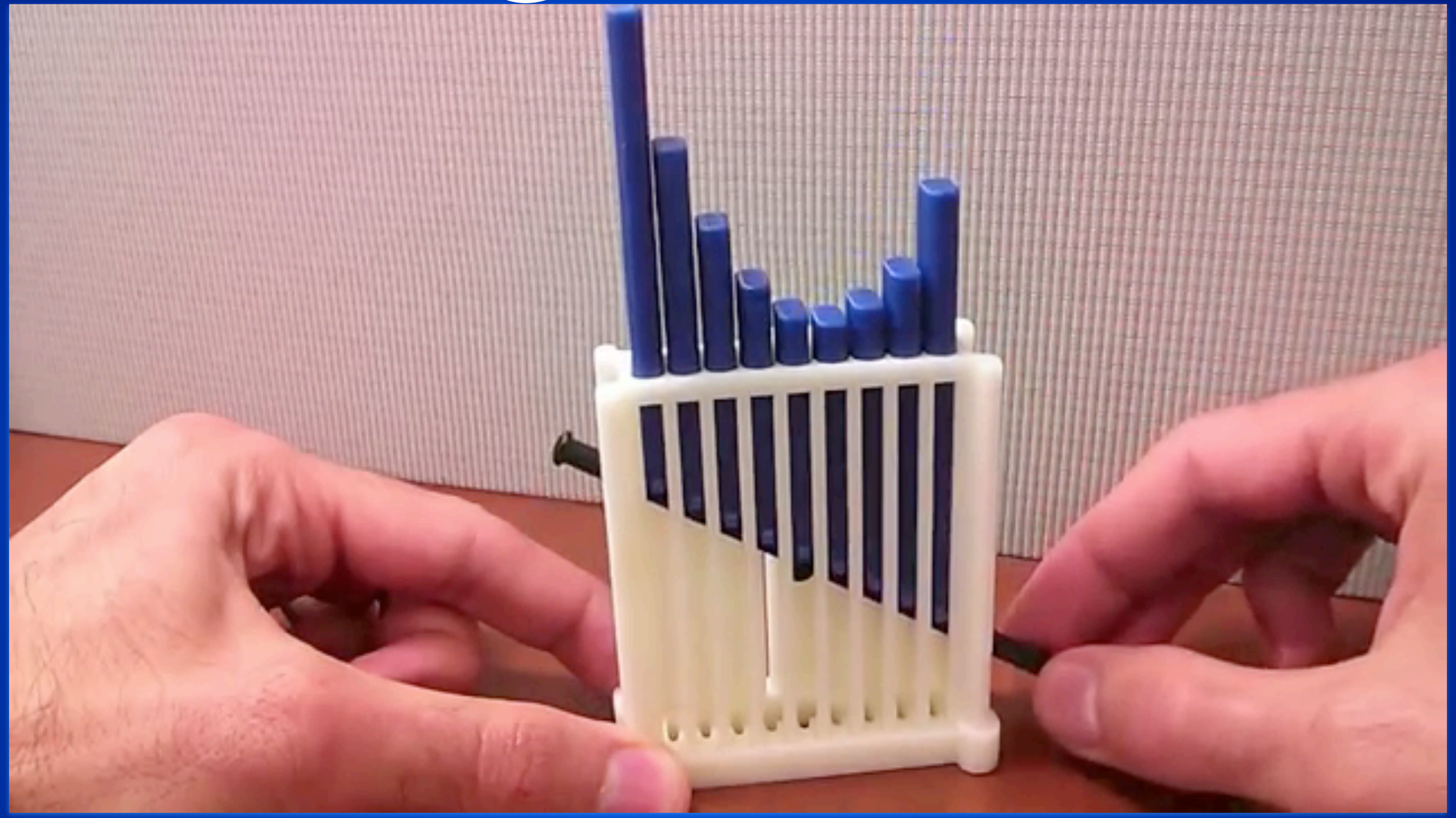
Laura Taalman



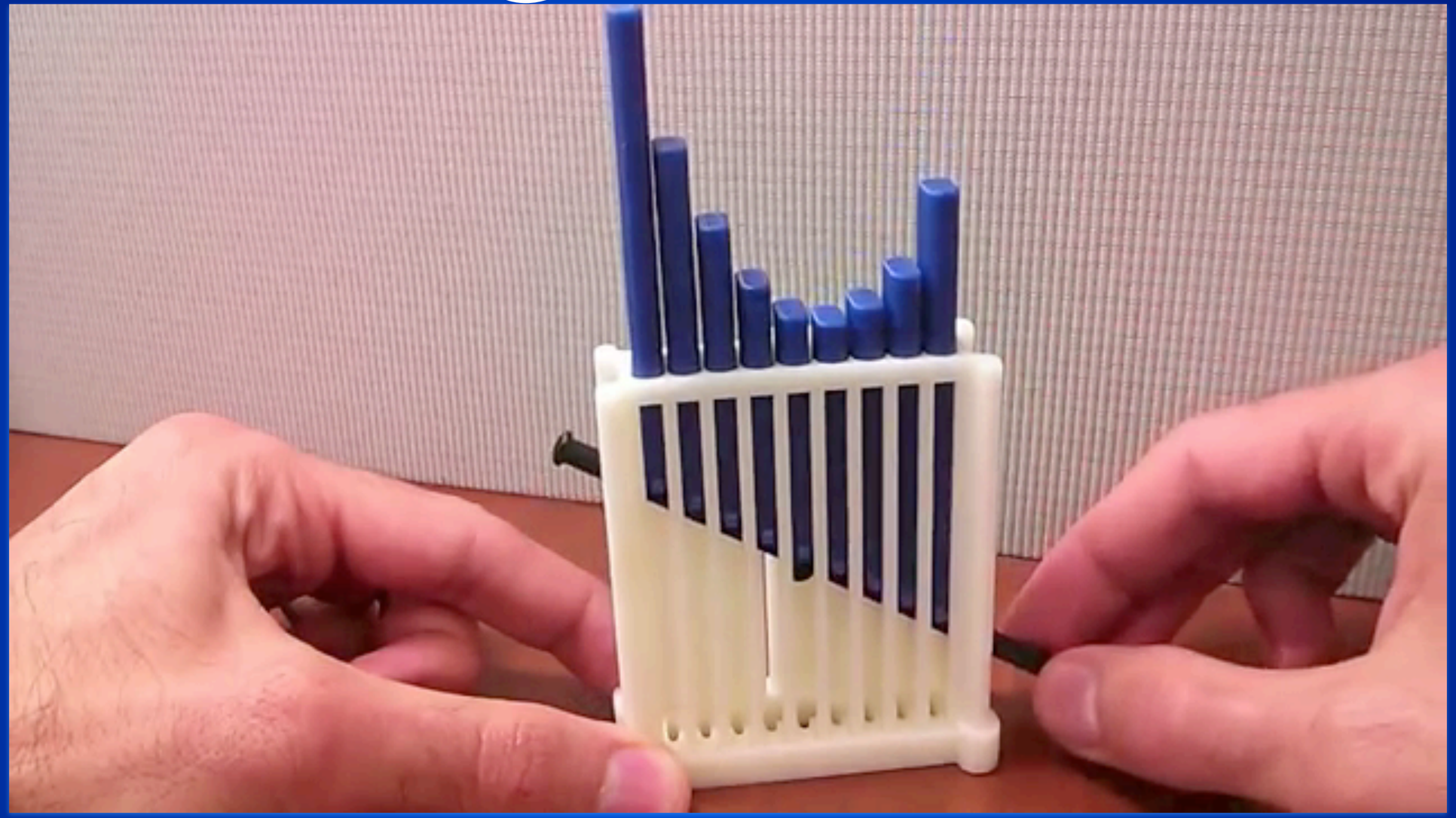
Laura Taalman



Matthew Petterson



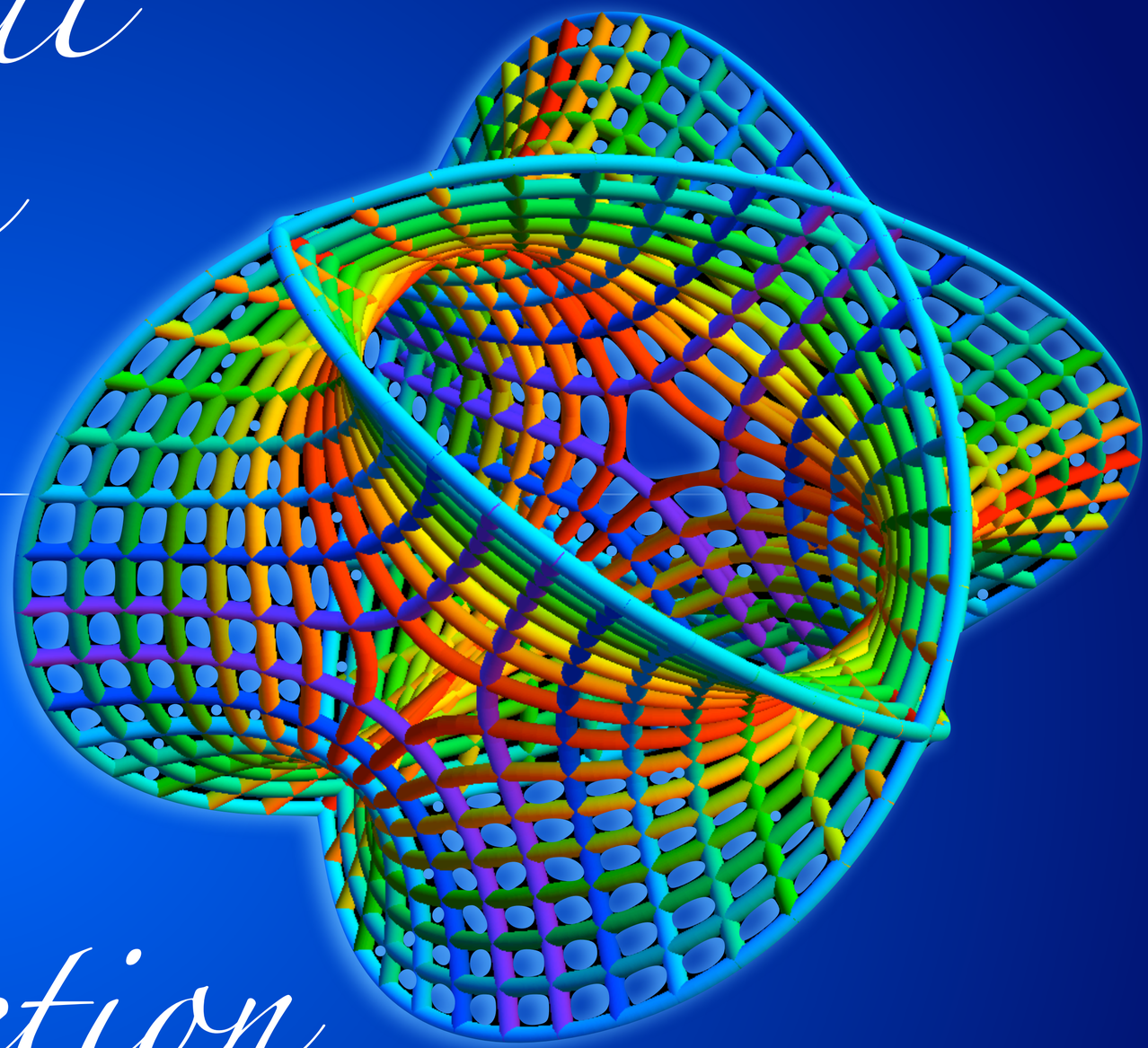
Matthew Petterson



Choice of Technology

Mathematical Description

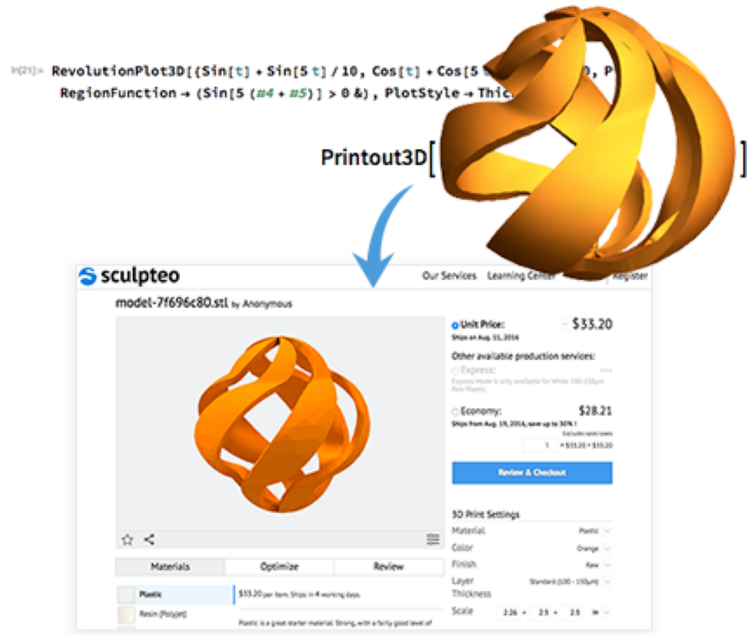
```
CalabiYau[z_, k1_, k2_] := Module[{
  z1 = Exp[2 Pi I k1/5] Cosh[z]^(2/n),
  z2 = Exp[2 Pi I k2/5] Sinh[z]^(2/5)},
N[{Re[z1], Re[z2], Cos[3] Im[z1] + Sin[3] Im[z2]}]];
```



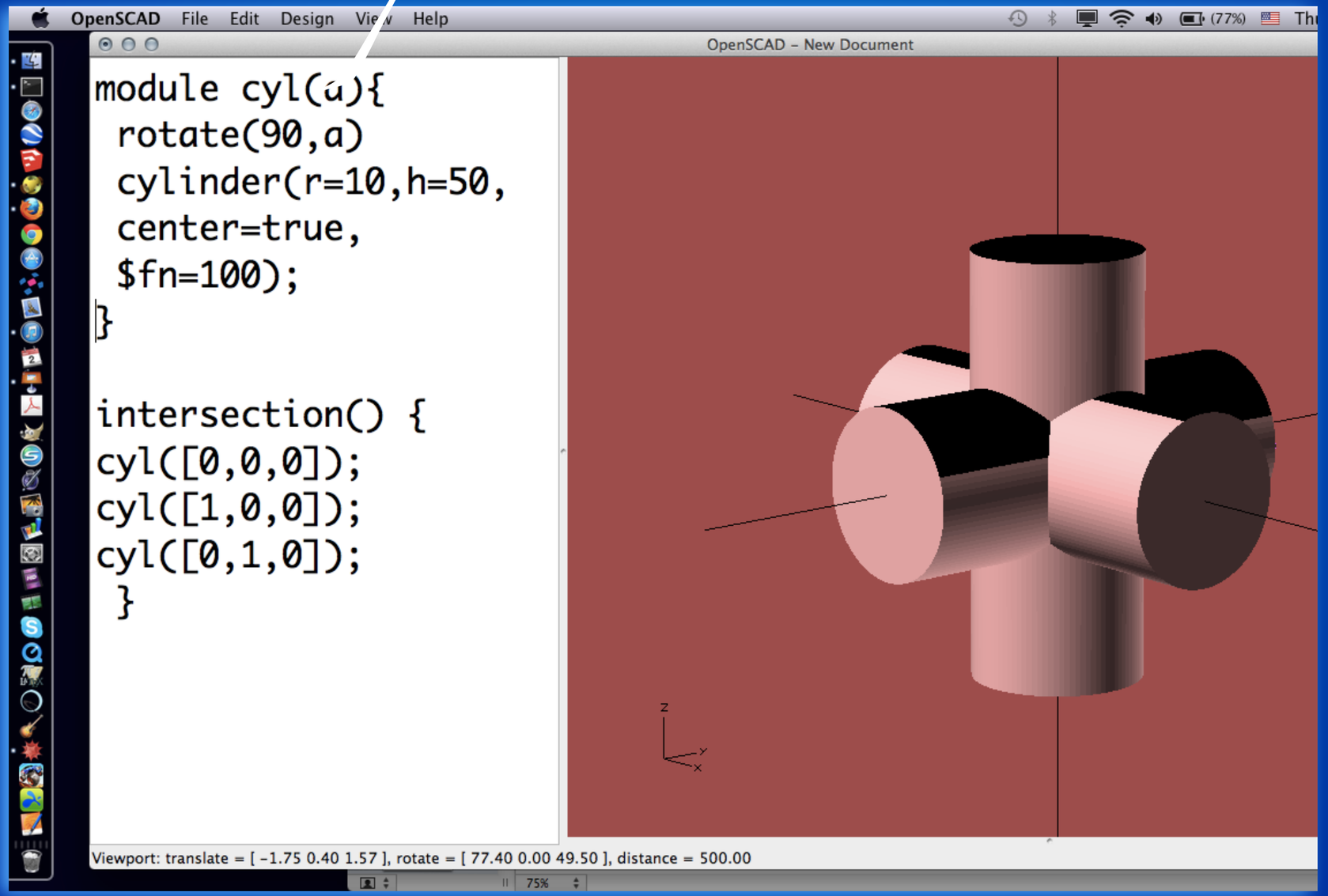
Mesh Collection

Computer Algebra

3D Printing There are lots of big, meaty new areas in Version 11. But let's jump right into one of them: 3D printing. I made my first 3D printout (which didn't last long before disintegrating) back in 2002. And we've had the ability to export to STL for years. But what's new and exciting in Version 11 is that we've built a complete pipeline that goes from creating 3D geometry to having it printed on your 3D printer (or through a printing service).



OpenScad


OpenSCAD File Edit Design View Help
OpenSCAD – New Document
module cyl(a){
rotate(90,a)
cylinder(r=10,h=50,
center=true,
$fn=100);
}
intersection() {
cyl([0,0,0]);
cyl([1,0,0]);
cyl([0,1,0]);
}
Viewport: translate = [-1.75 0.40 1.57], rotate = [77.40 0.00 49.50], distance = 500.00
75%

Mesh Lab

solid Processed by ADMesh version 0.95

facet normal 2.45300293 E - 01 - 3.88517678 E - 02 9.68668342 E - 01

outer loop

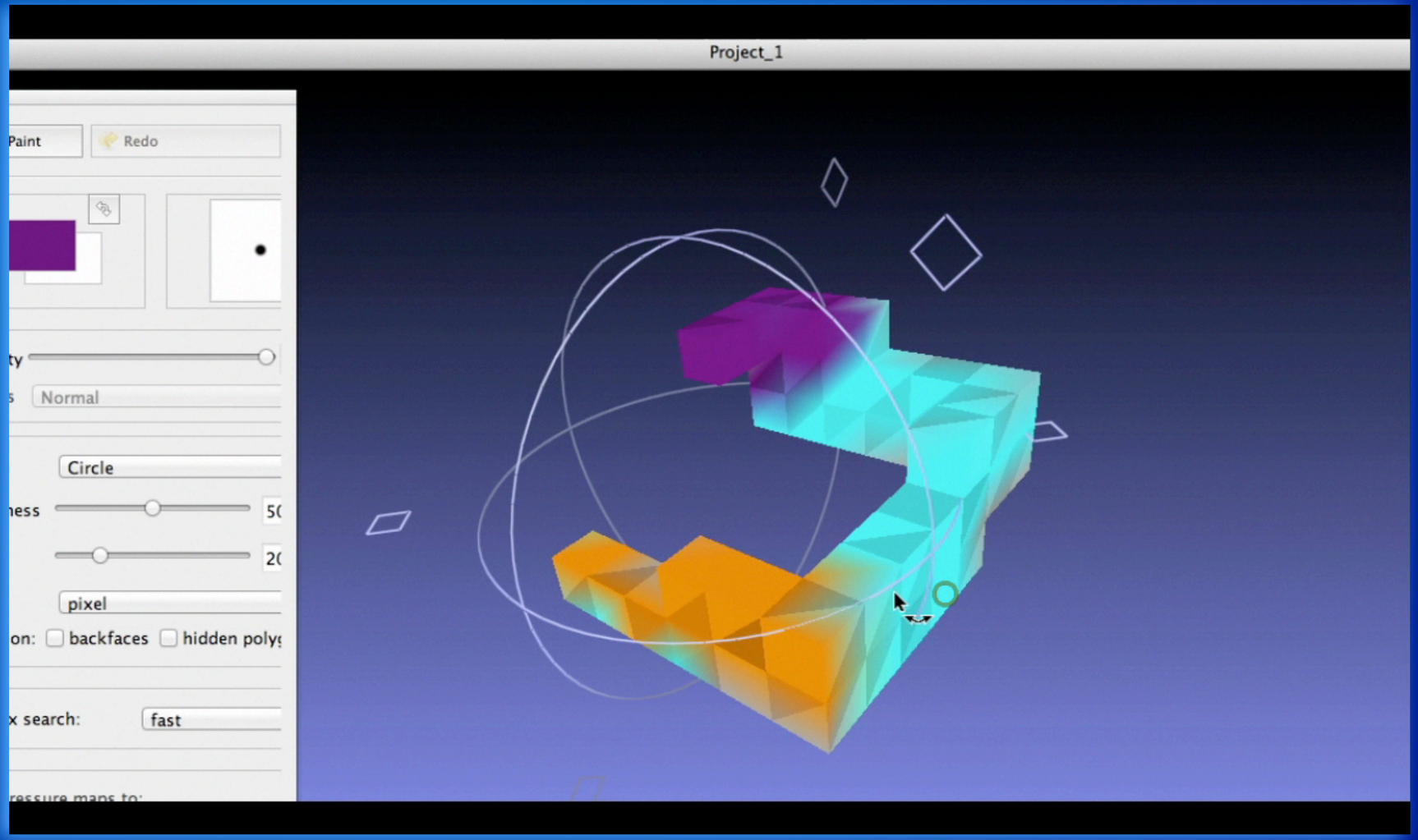
vertex 1.64594591 E - 01 0.00000000 E + 00 9.86361325 E - 01

vertex 1.56538755 E - 01 - 5.08625247 E - 02 9.86361325 E - 01

vertex 3.08807552 E - 01 - 1.00337654 E - 01 9.45817232 E - 01

endloop

endfacet



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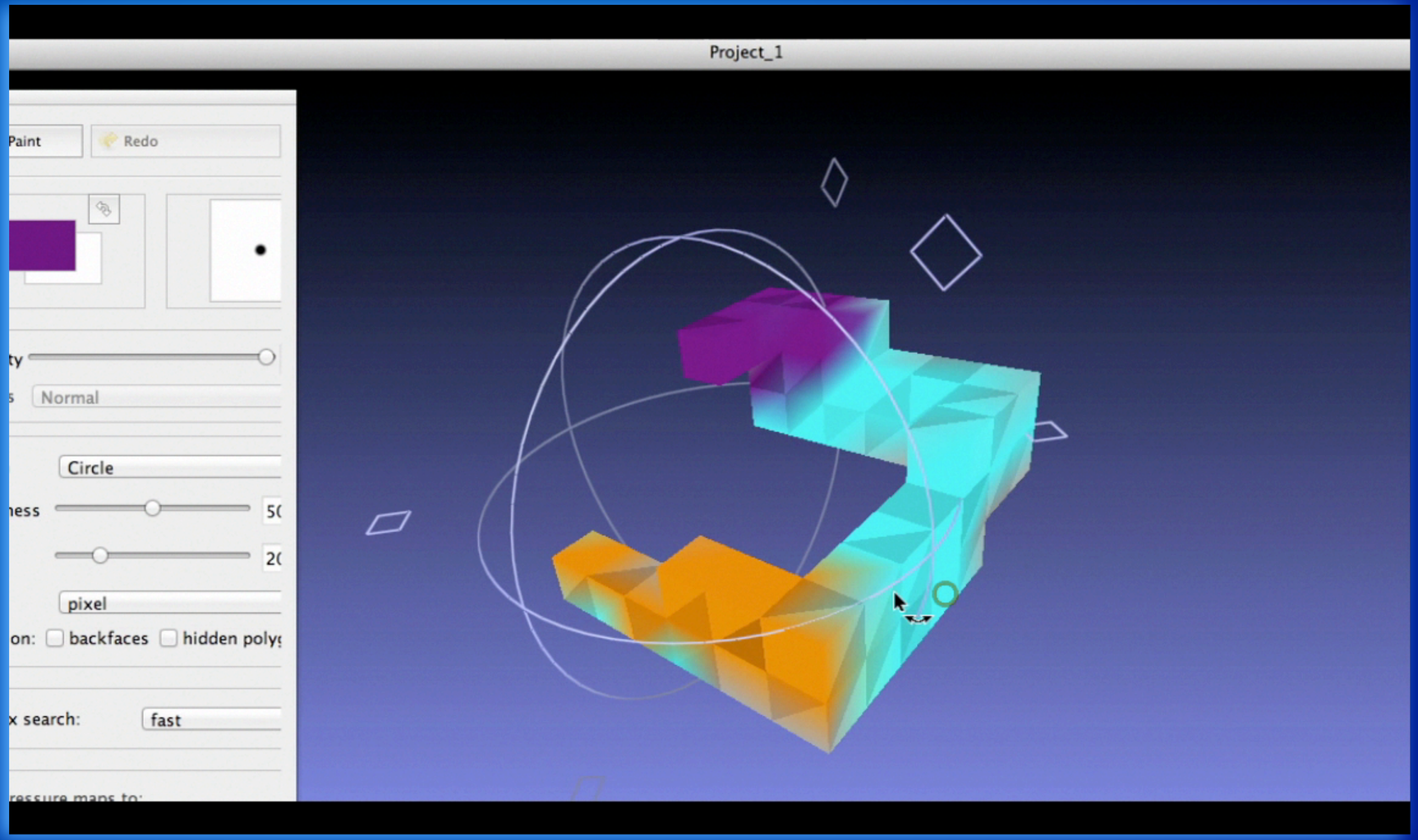
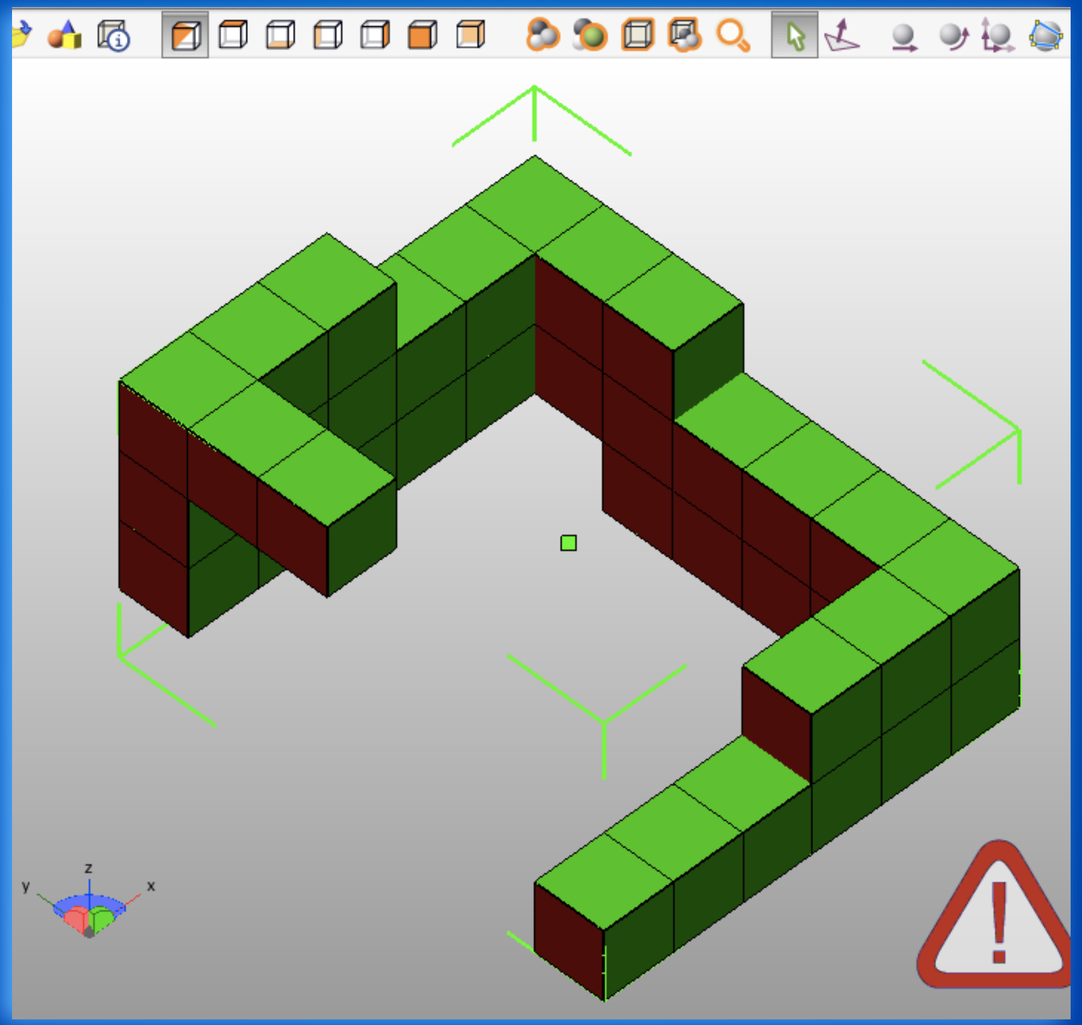


NetFab

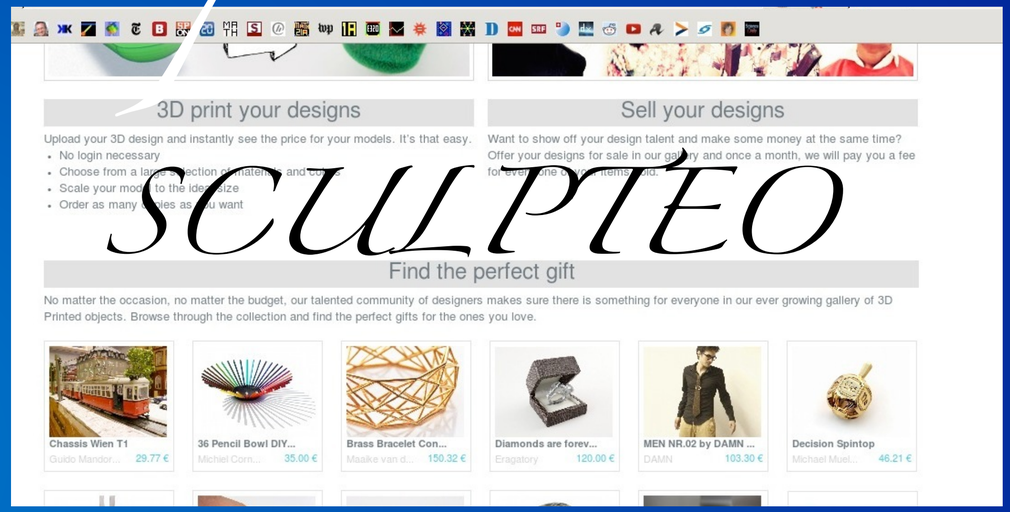


Printing Shops

SHAPEWAYS



SCULPTEO



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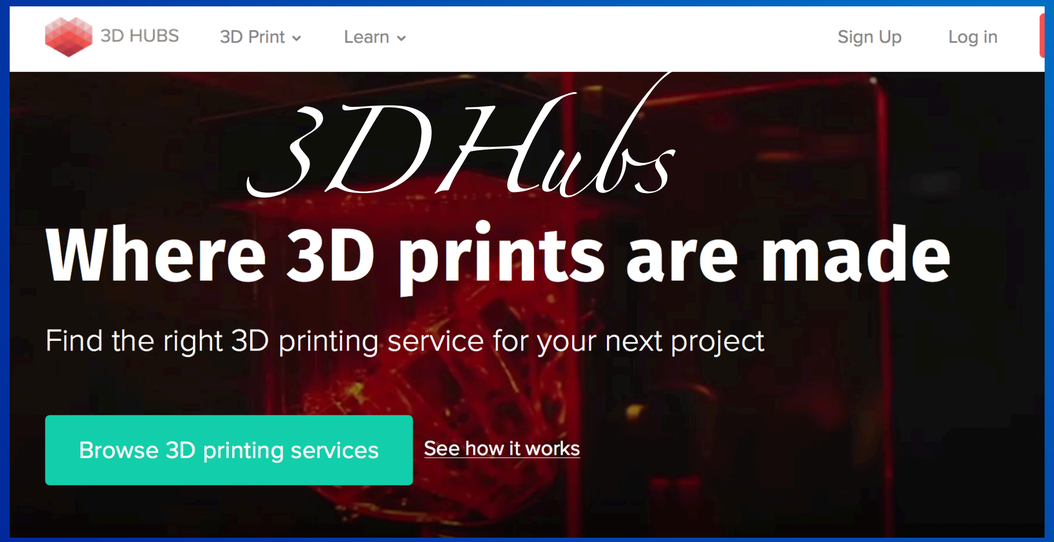
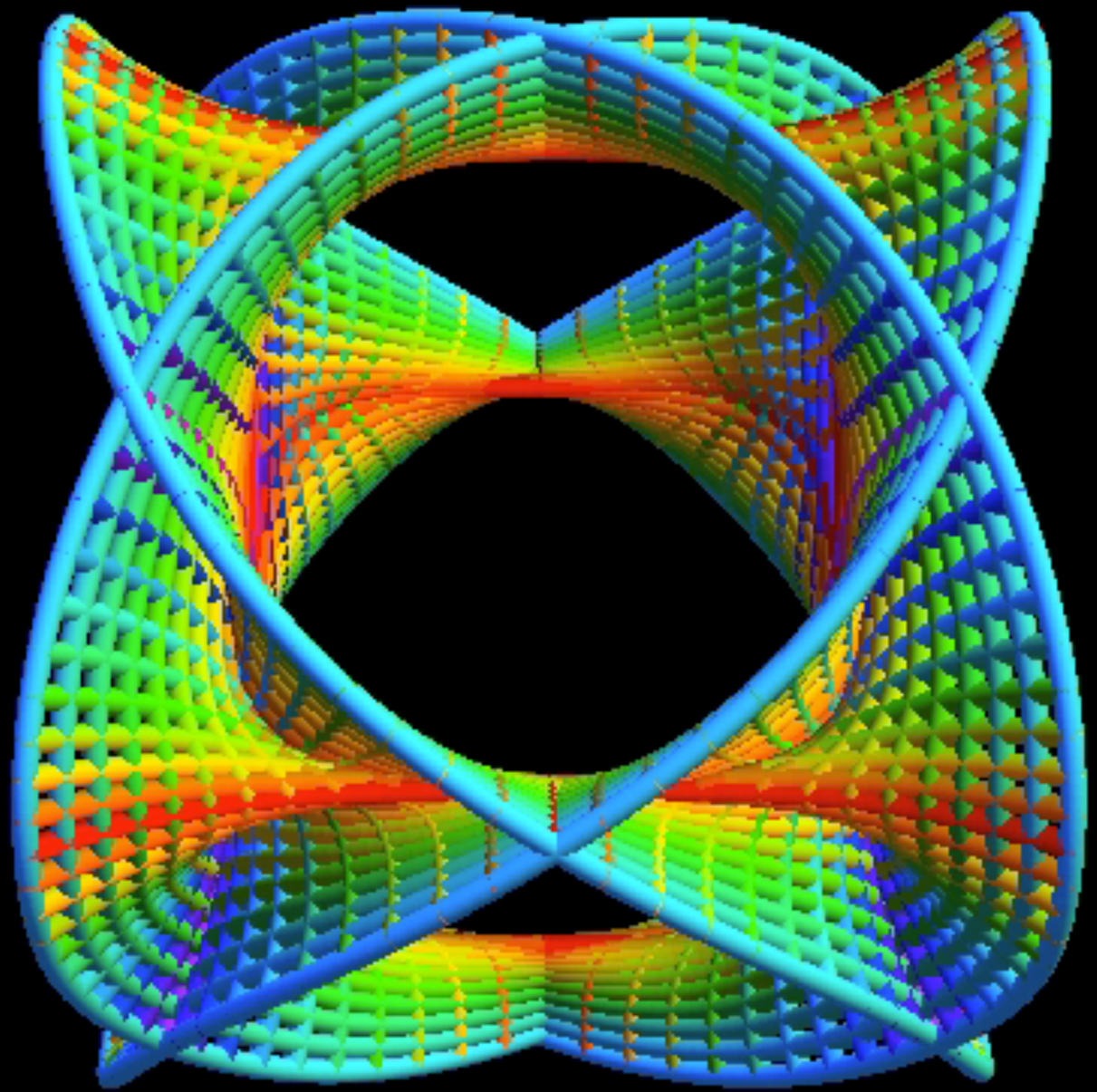
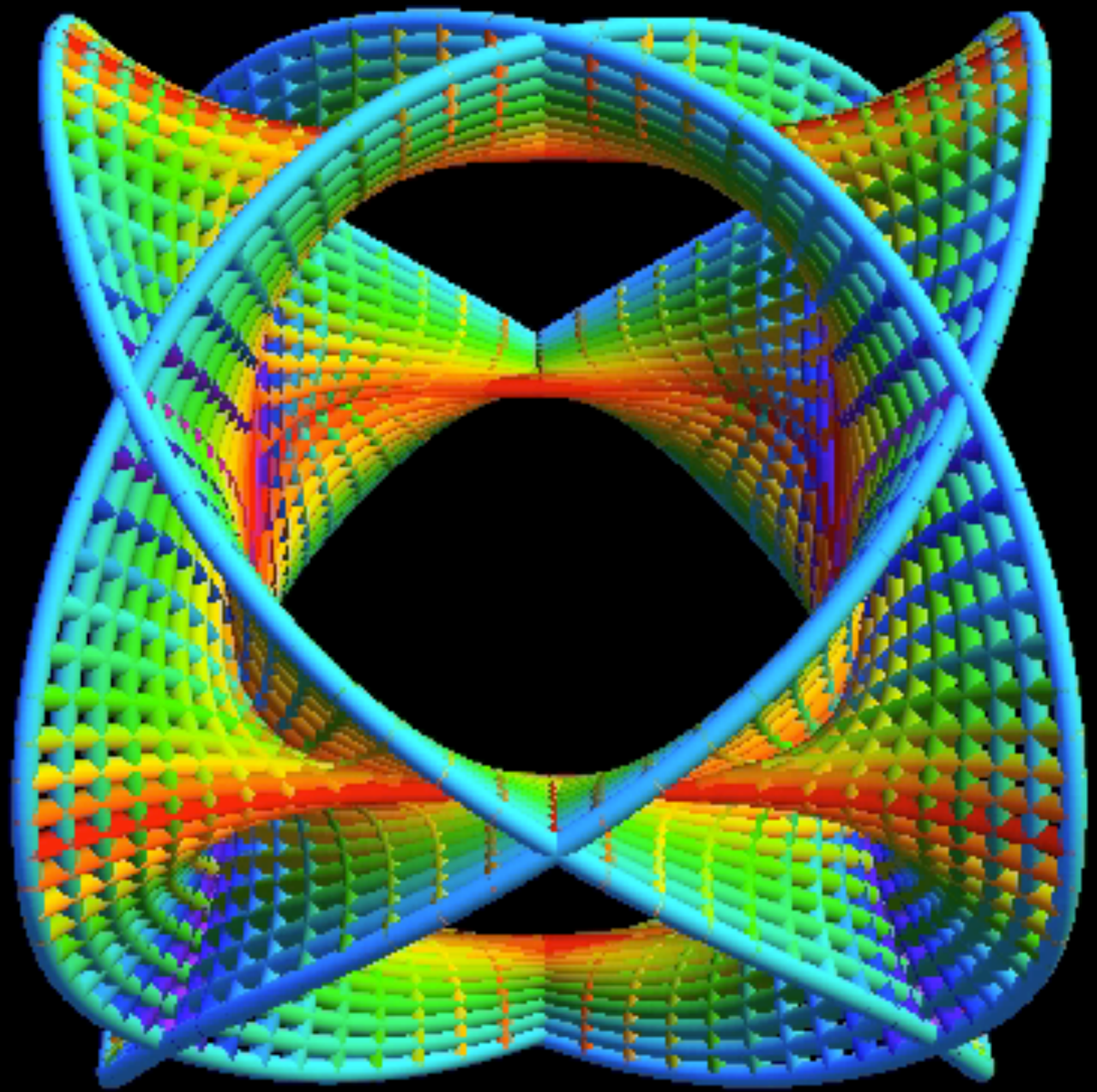


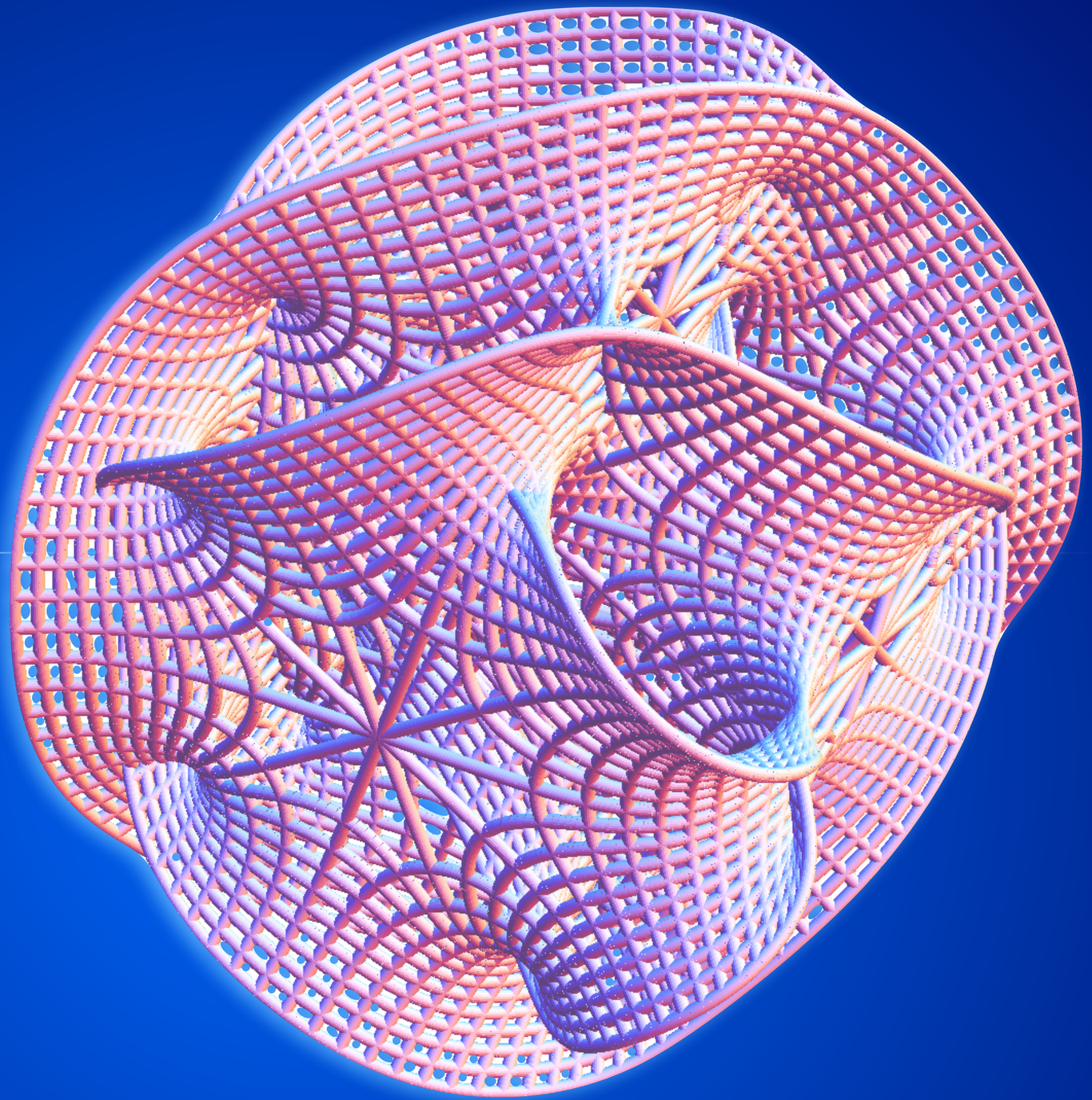
iMaterialize







STL



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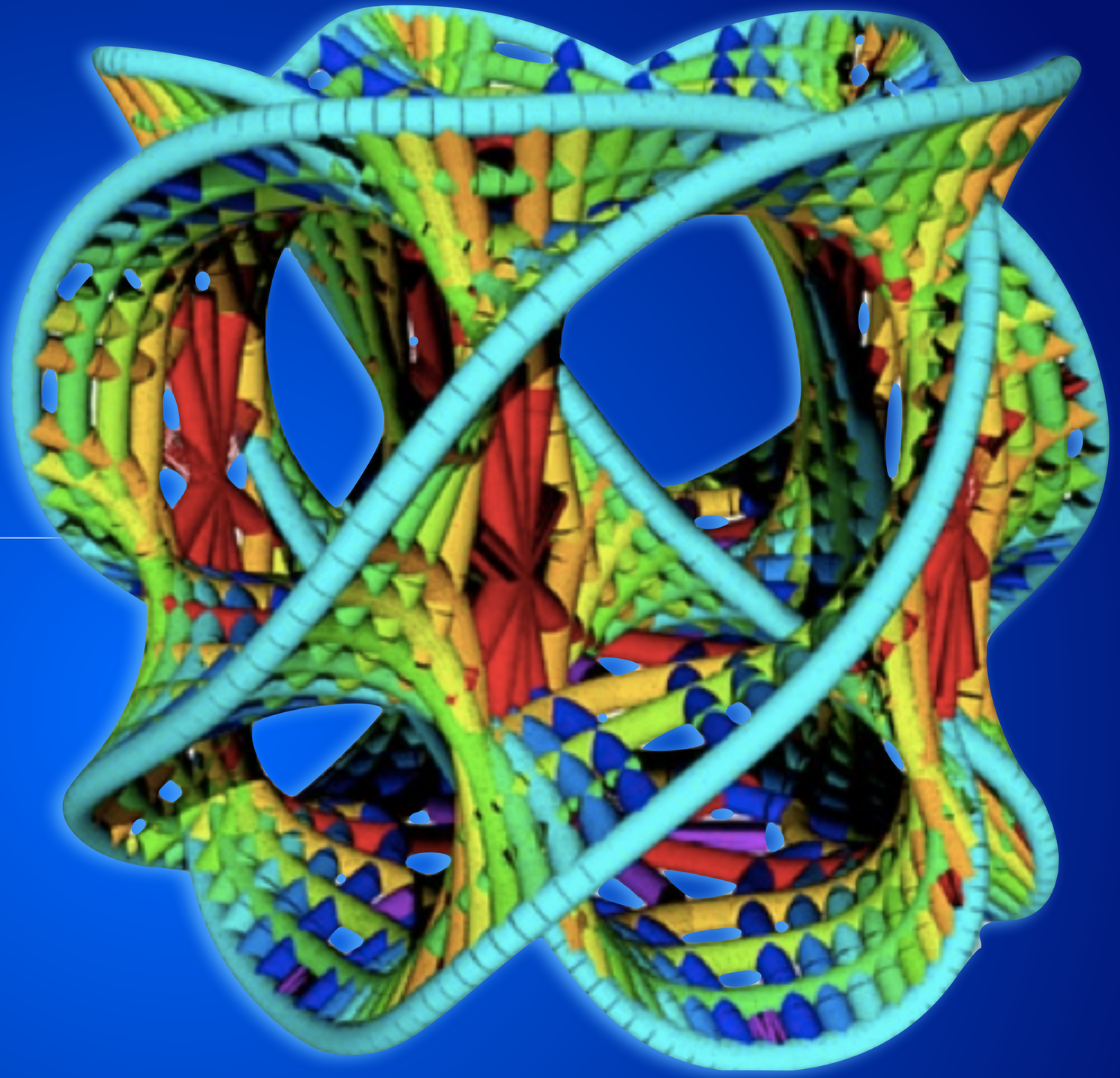
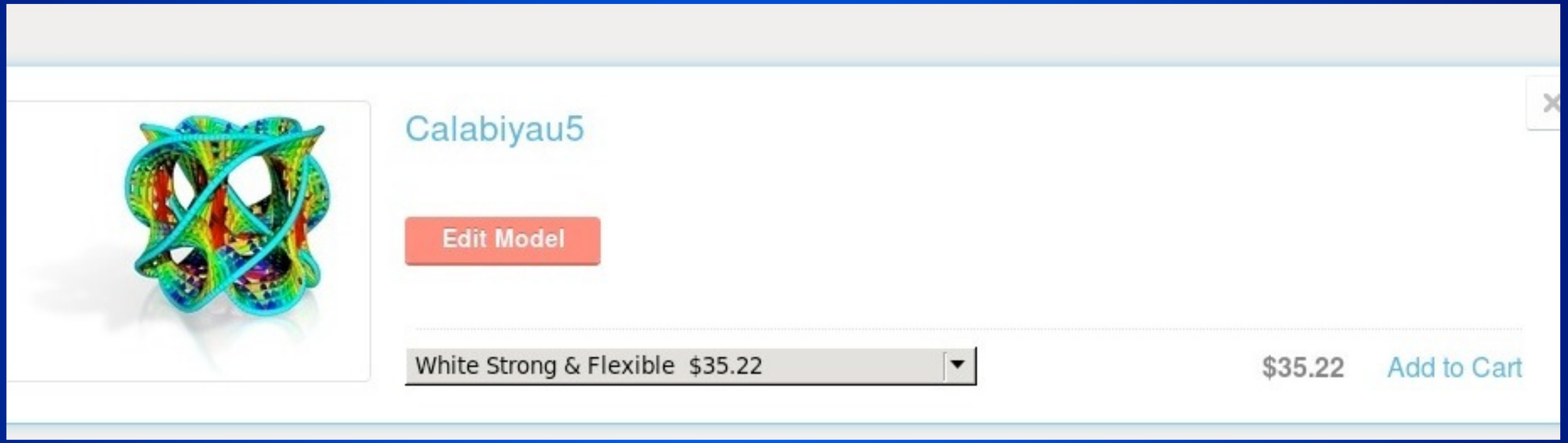


In shapeways



Scanning





3D Sense

Makerbot

Phone

Kinect



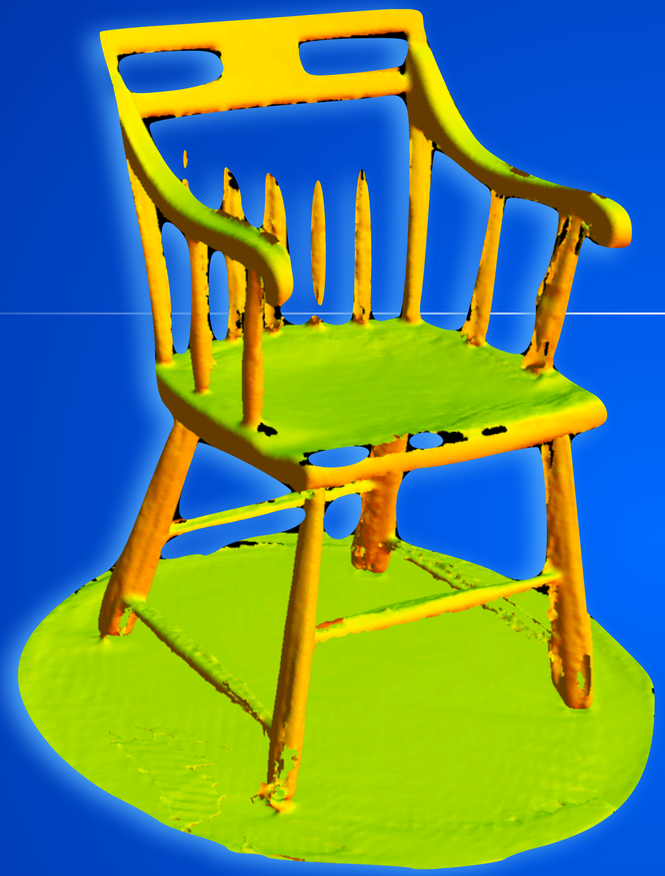


With 3D Sense





With Kinect





With Kinect







With Kinect







With Kinect




OVERSEER OF HARVARD COLLEGE 1825–1836
FELLOW 1834–1853 PROFESSOR 1839–1853
PRESIDENT 1853–1860 OVERSEER 1864–1874
WHERE IS WISDOM WHERE IS STRENGTH WHERE IS UND
THOU MAYEST KNOW ALSO WHERE IS LENGTH OF DAYS
WHERE IS THE LIGHT OF THE EYES AND PEACE


With Phone RISD


Chimera



Digitized with 123D Catch





Printed



More Printouts

partly from Elson Grant









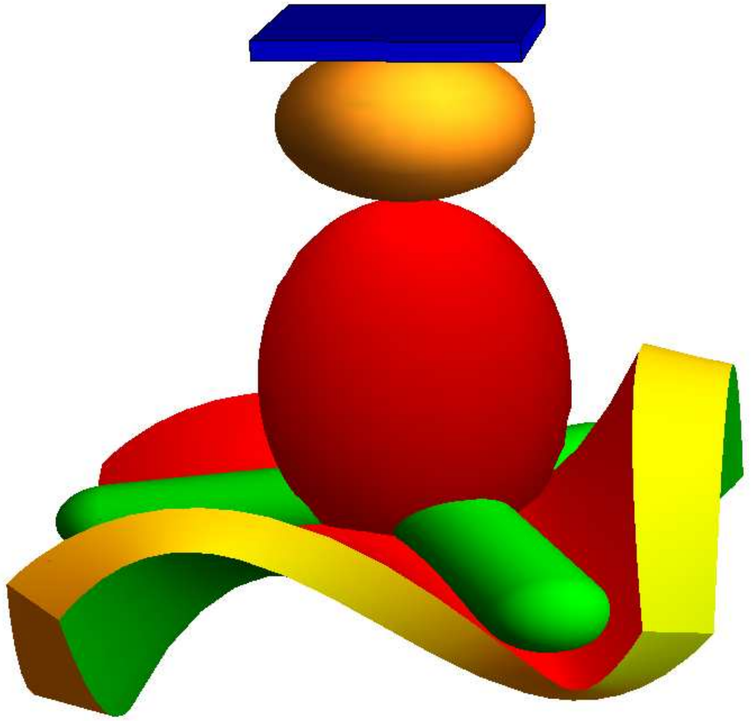






Problem 9) (10 points)

As a souvenir for this exam, we build a Monkey riding a "Monkey saddle" and 3D print it. No explanations are necessary.



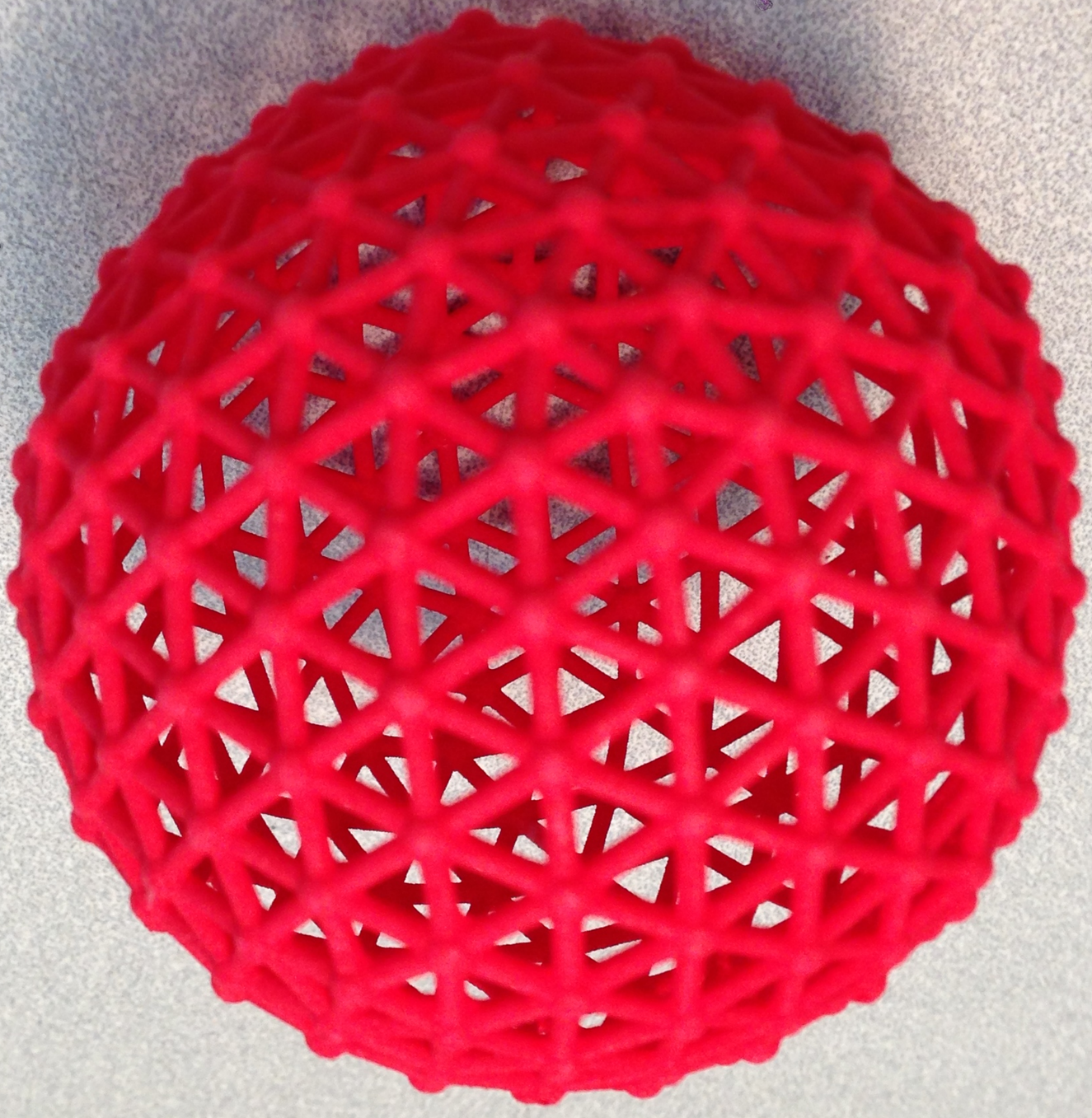
a) (2 points) Parametrize the hat $z = 5$.

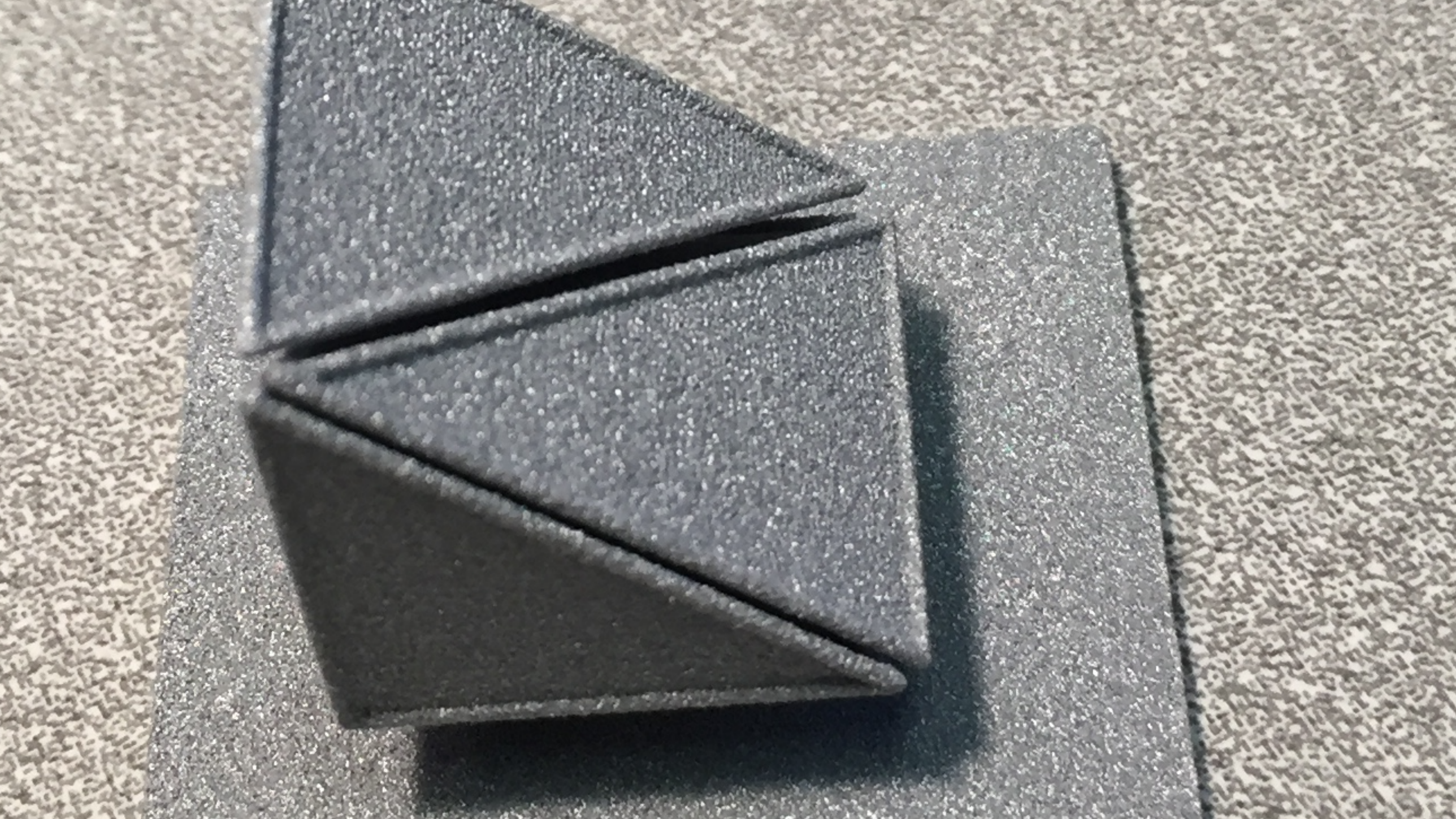
b) (2 points) Parametrize the saddle $z = yx^2 - x^3$.

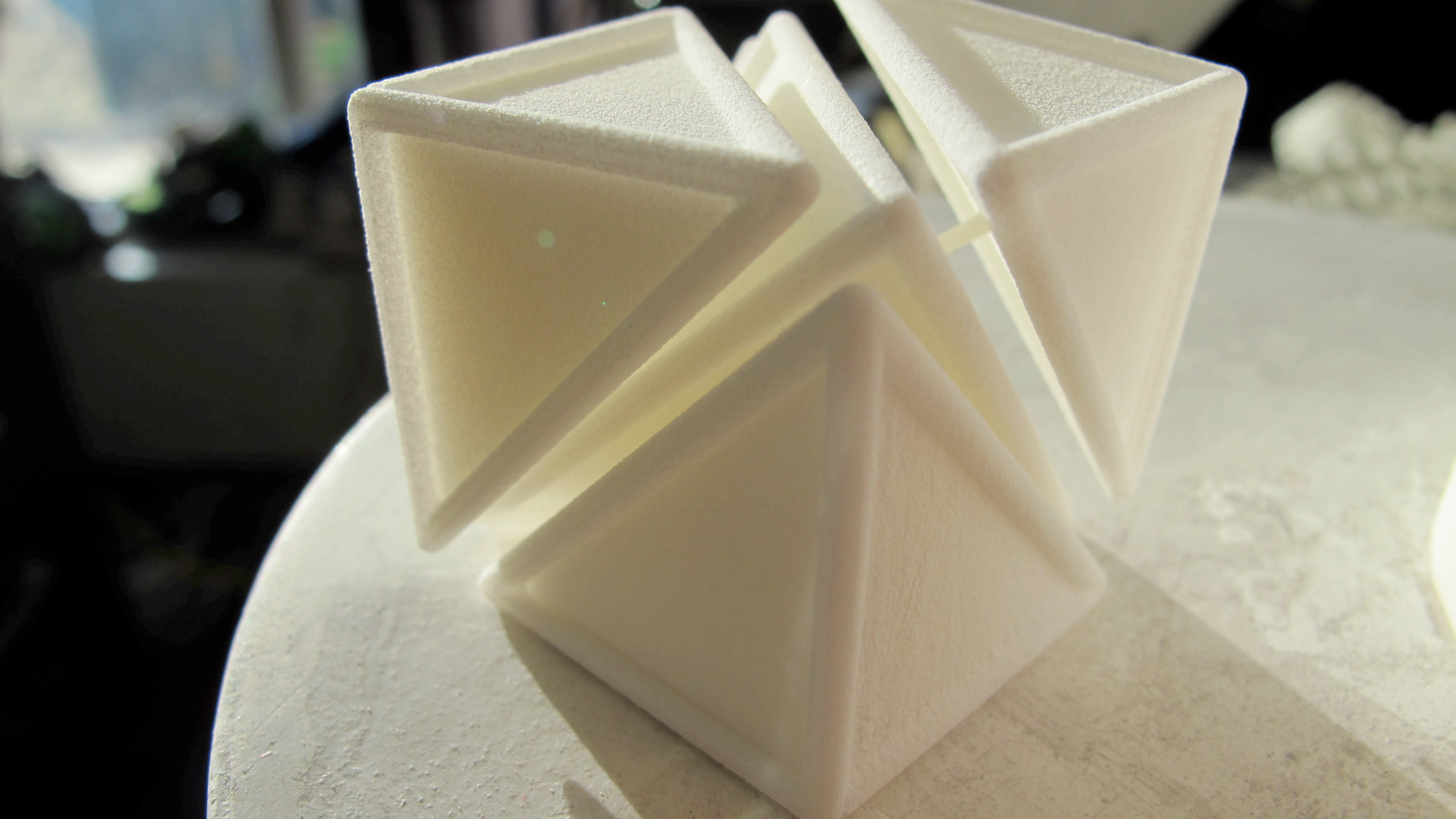
c) (2 points) Parametrize the torso $x^2 + y^2 + \frac{(z-1)^4}{4} = 1$.

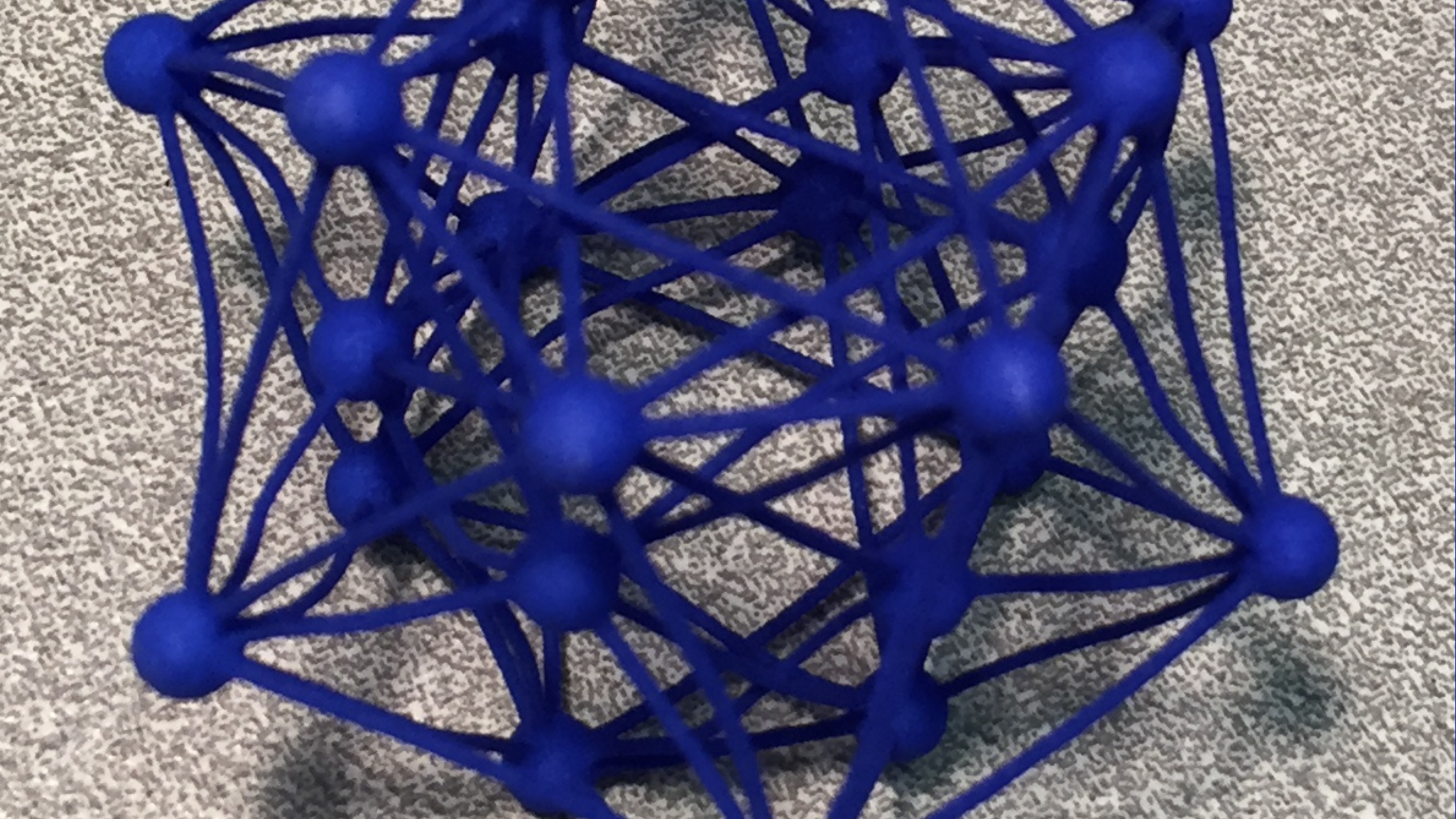
d) (2 points) Parametrize the head $4x^2+y^2+(z-4)^2 = 1$.

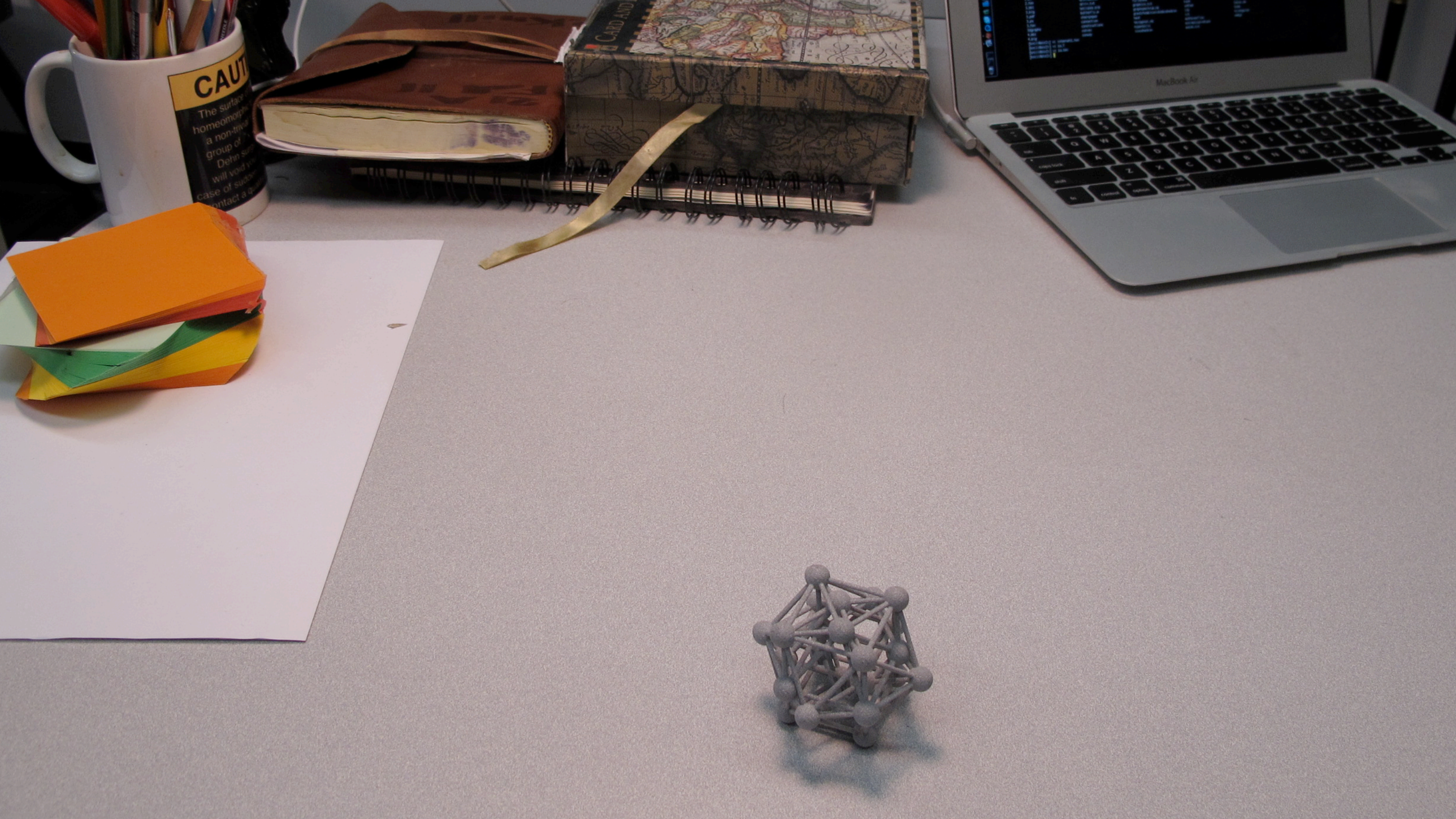
e) (2 points) Parametrize the monkey tail $x^2 + z^2 = \frac{1}{4}$.





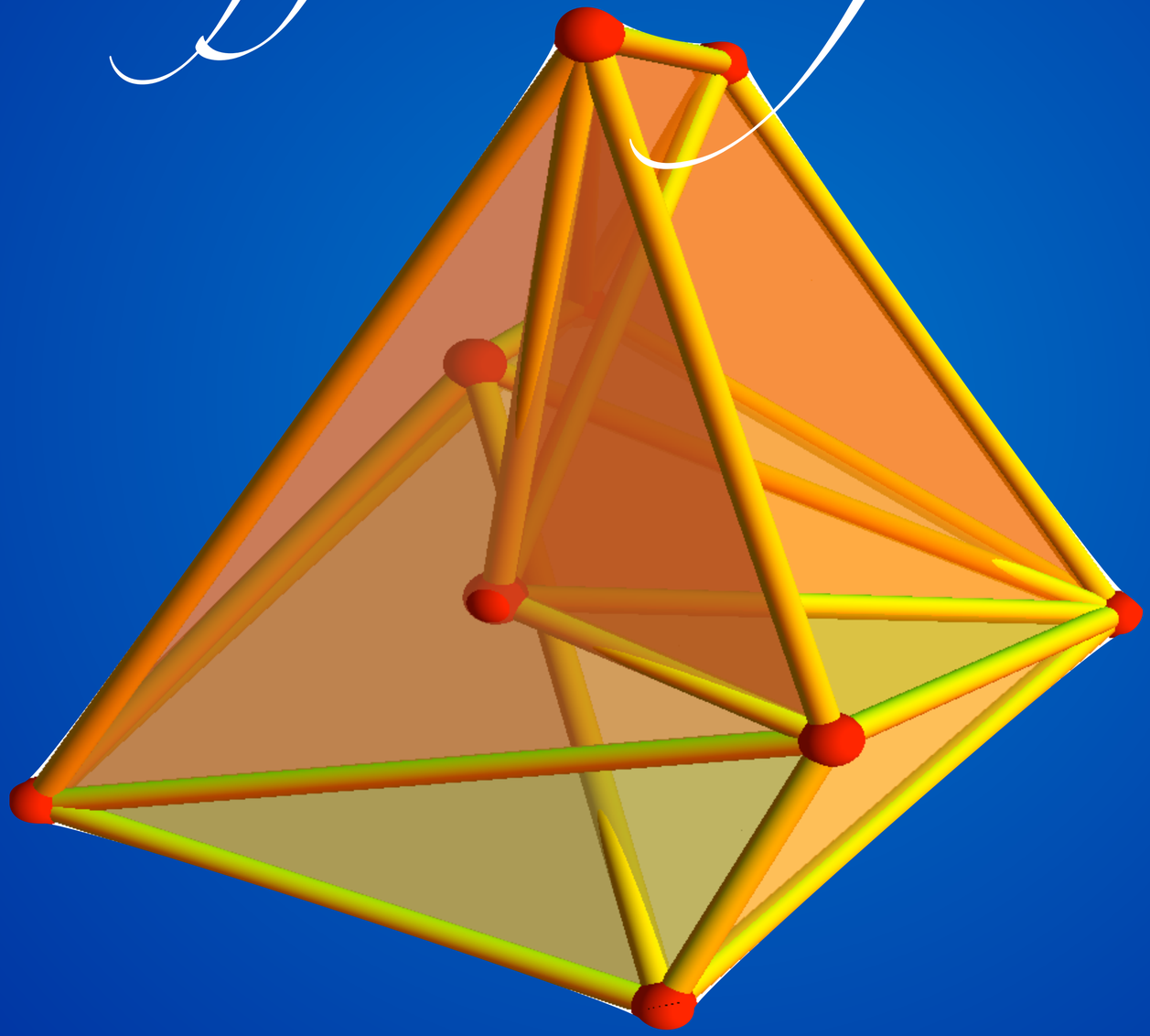





CAUT
MacBook Air



Steffen Surface

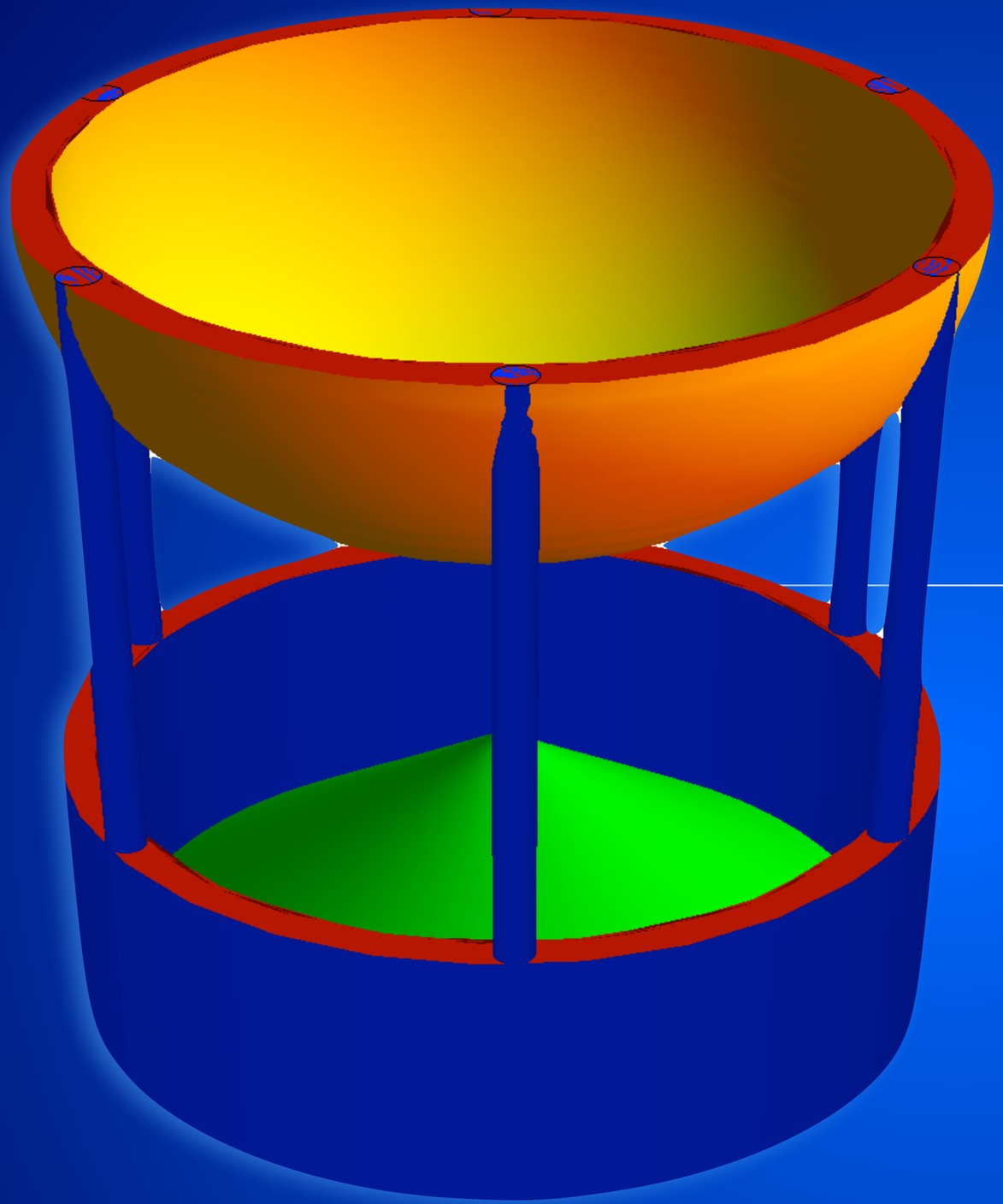




Sody Hexlet 3D

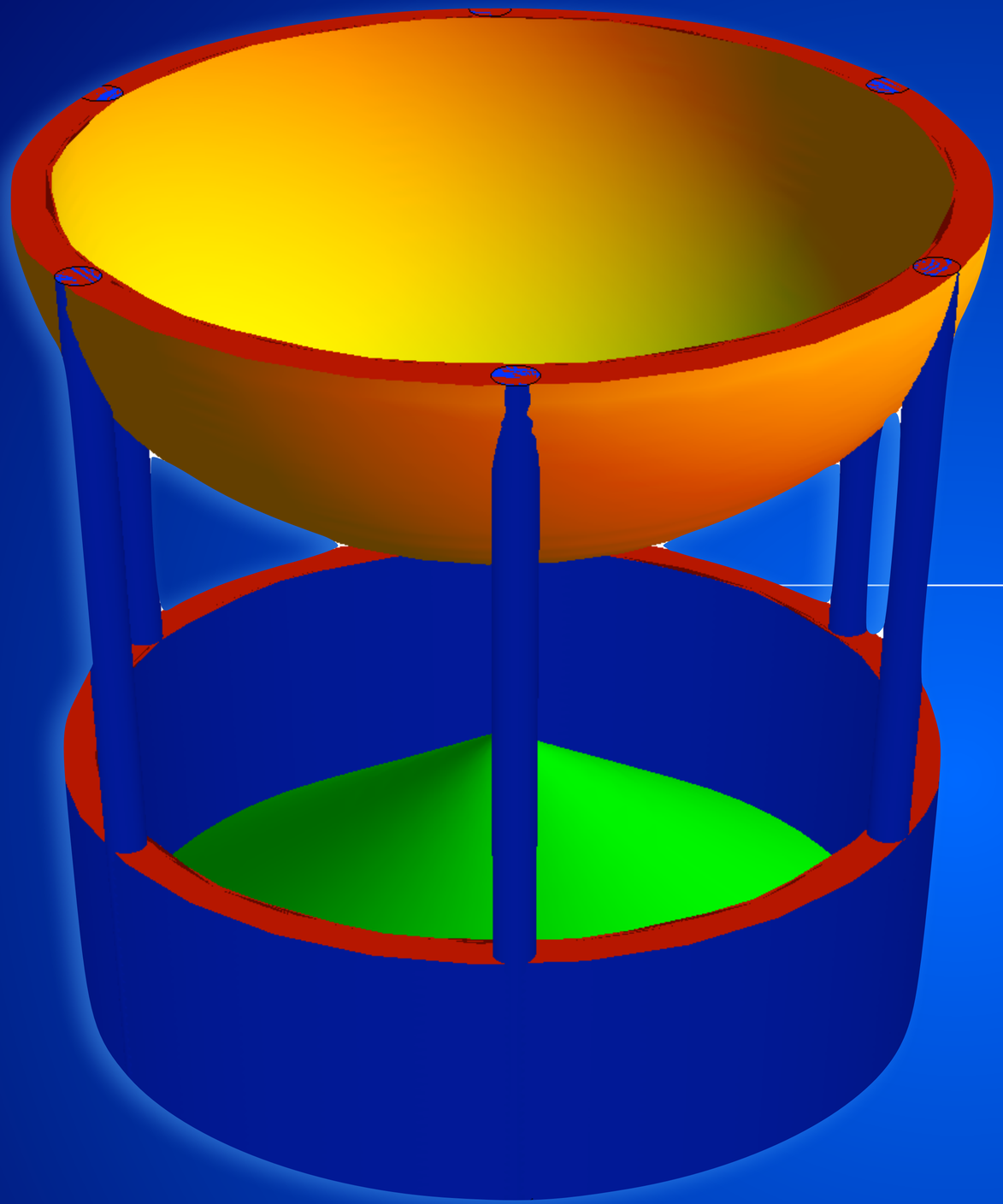


Proofs



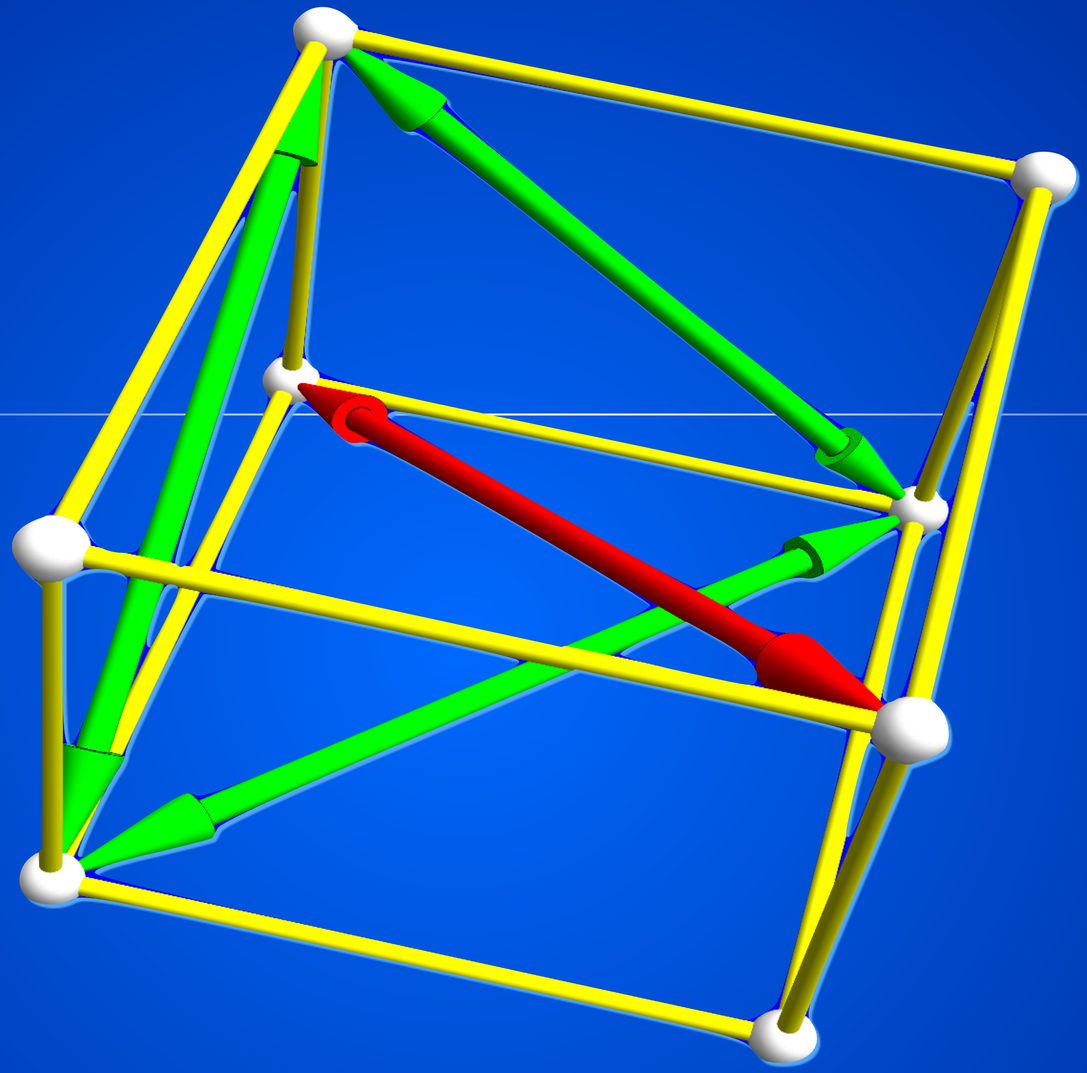


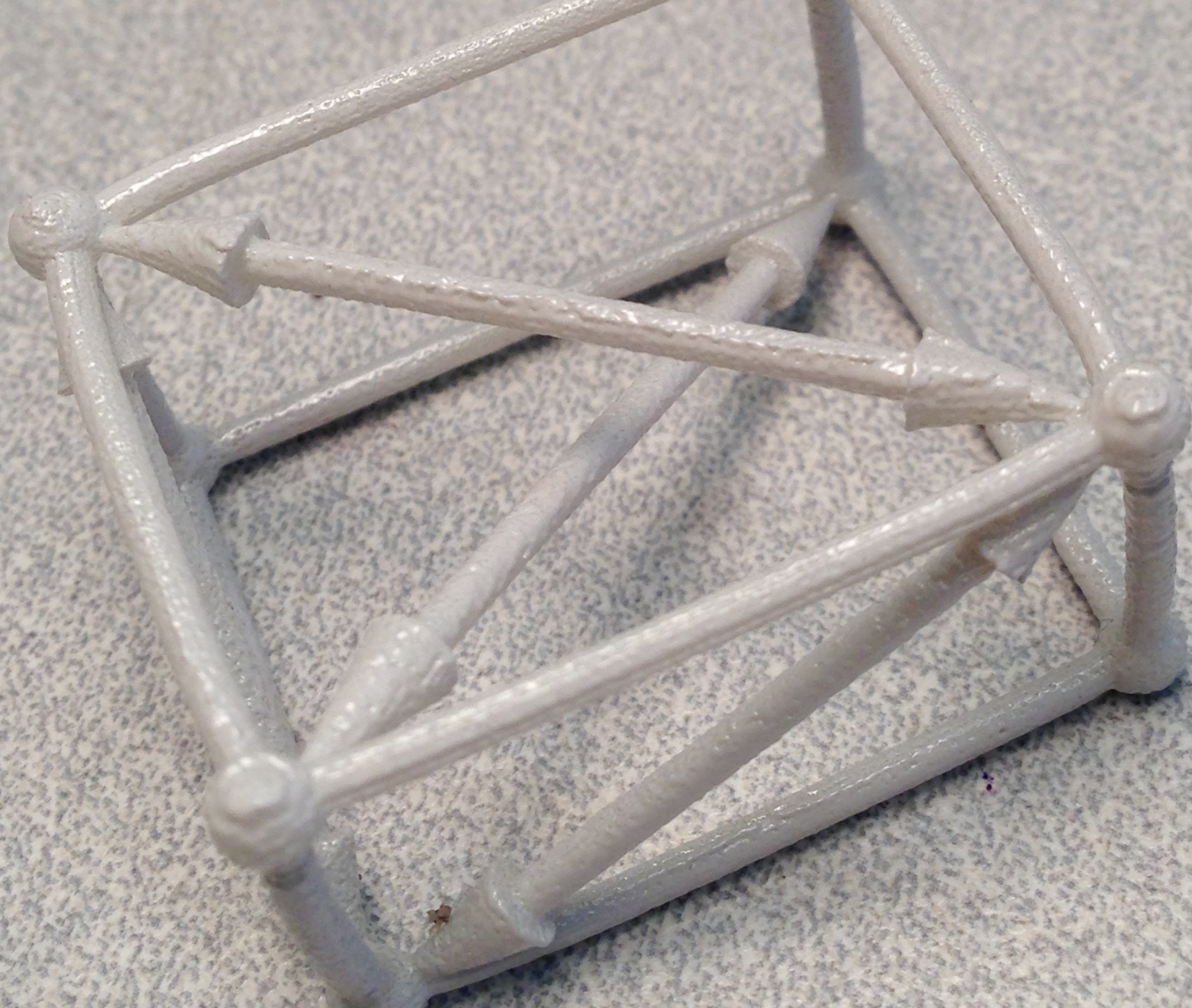
Proofs



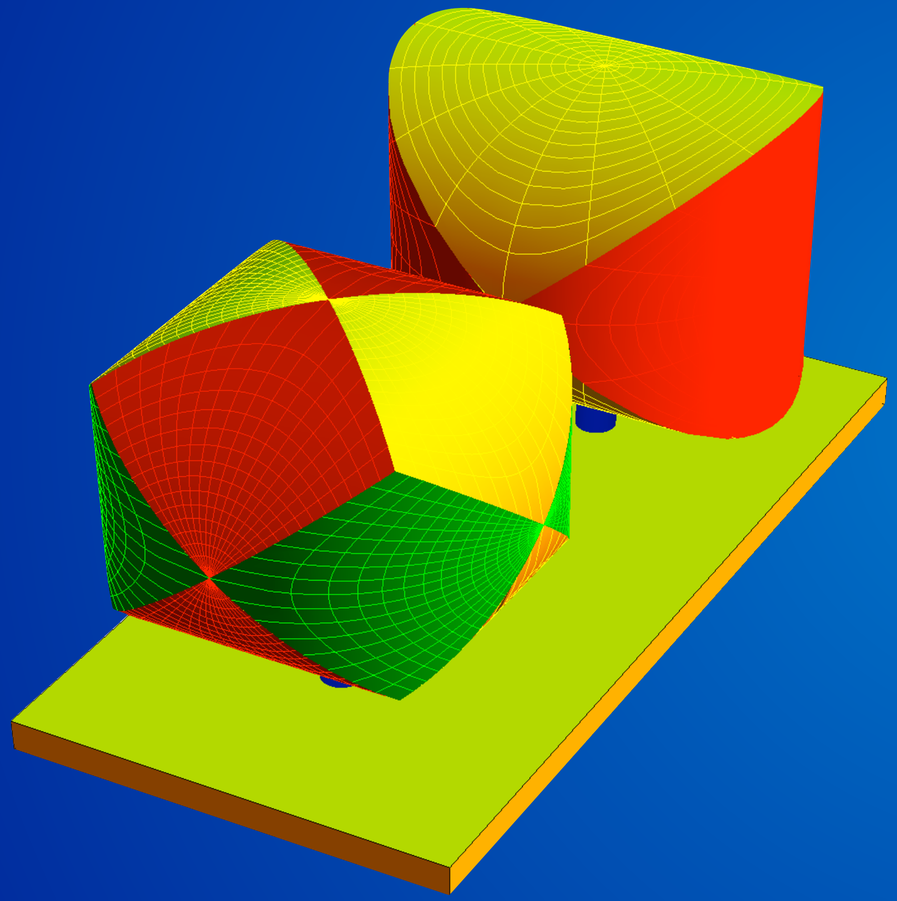


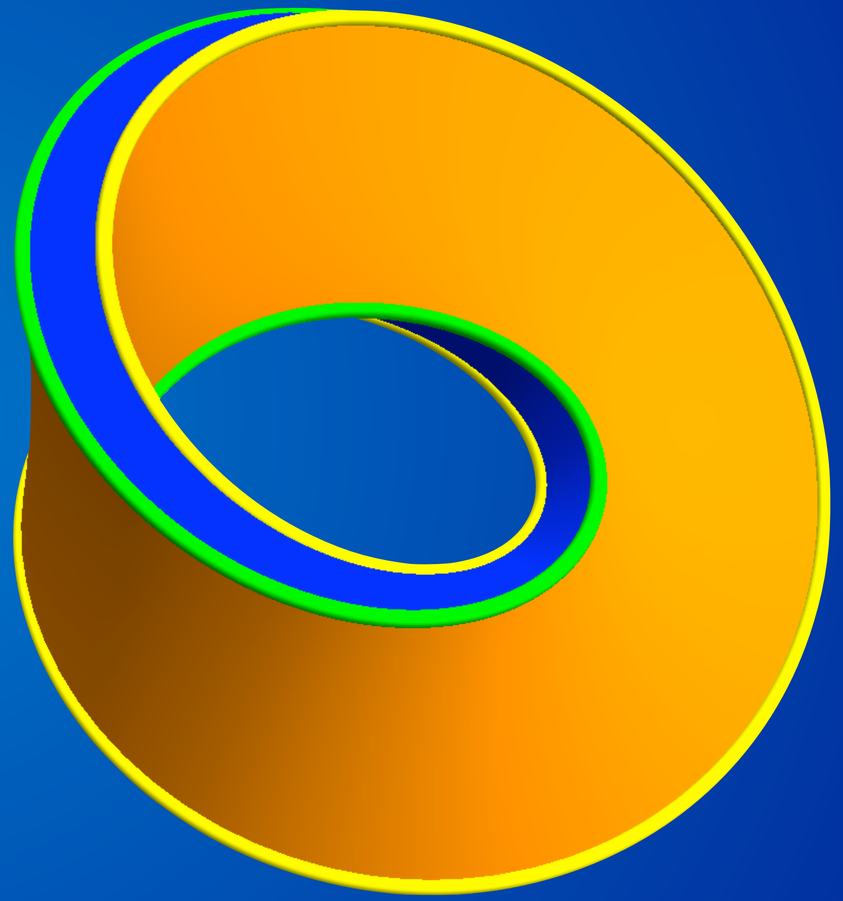
Euler Brick













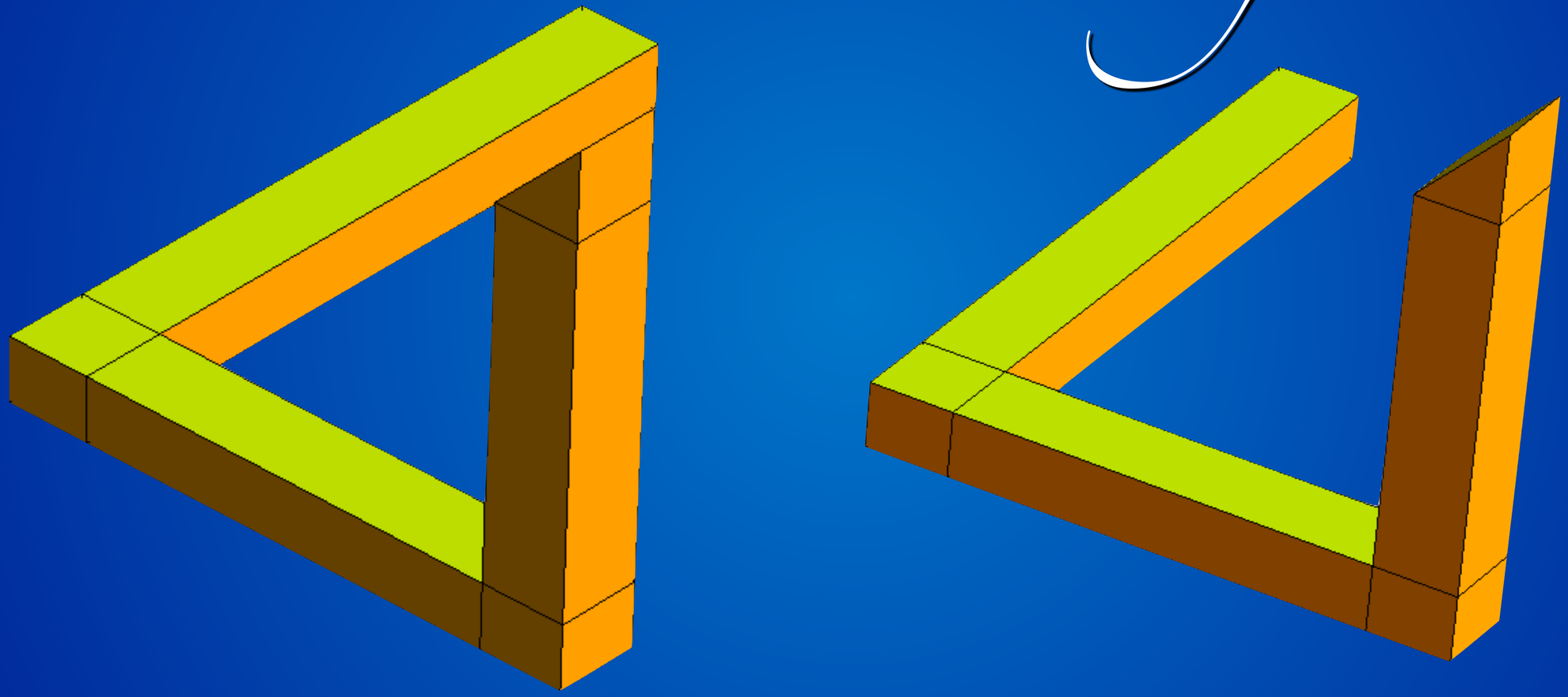
Archimedean Solids



Gregor Luetolf



Penrose Triangle



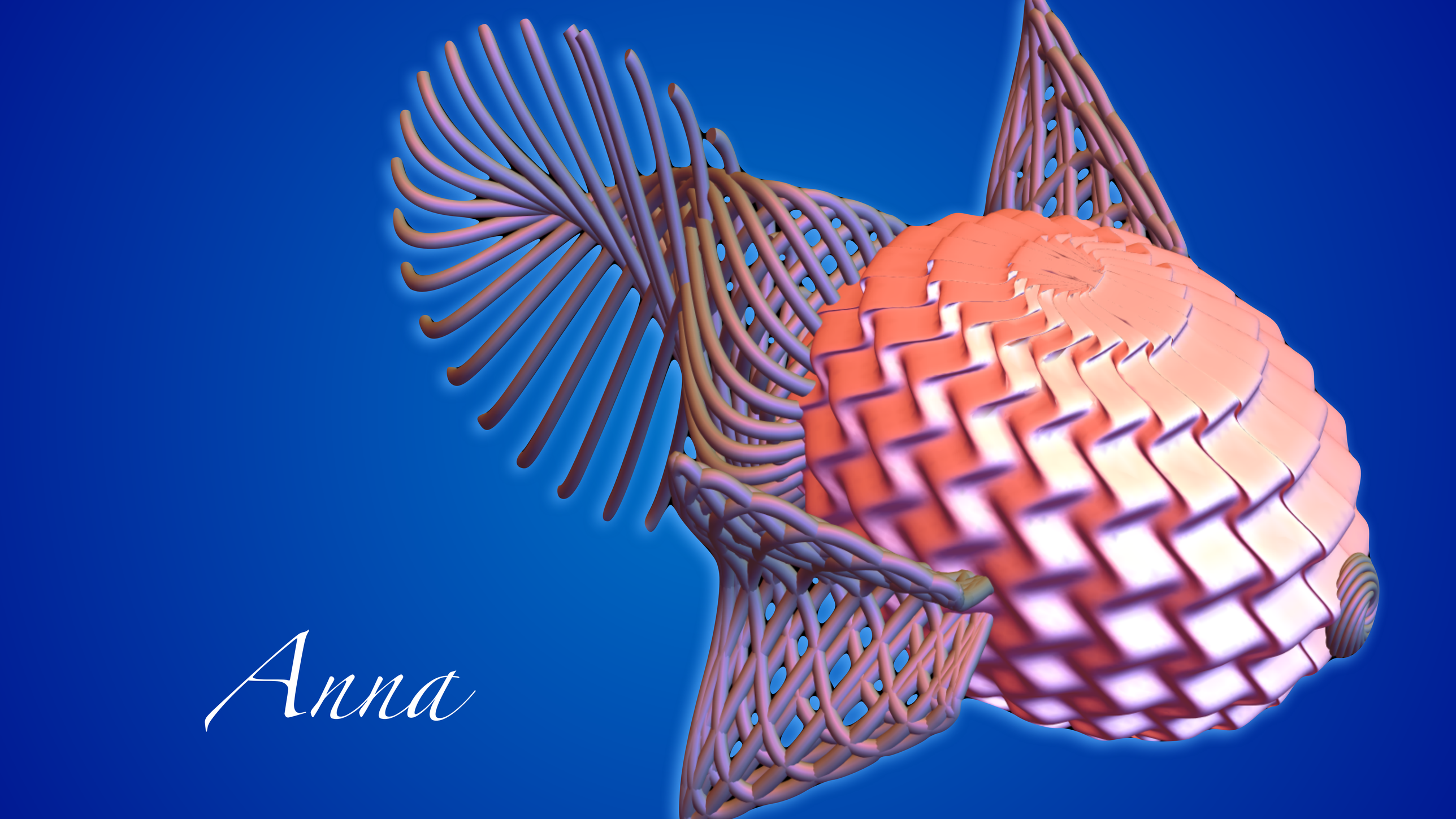

It becomes an impossible figure
2015 Google
Image Landsat


It becomes an impossible figure
2015 Google
Image Landsat

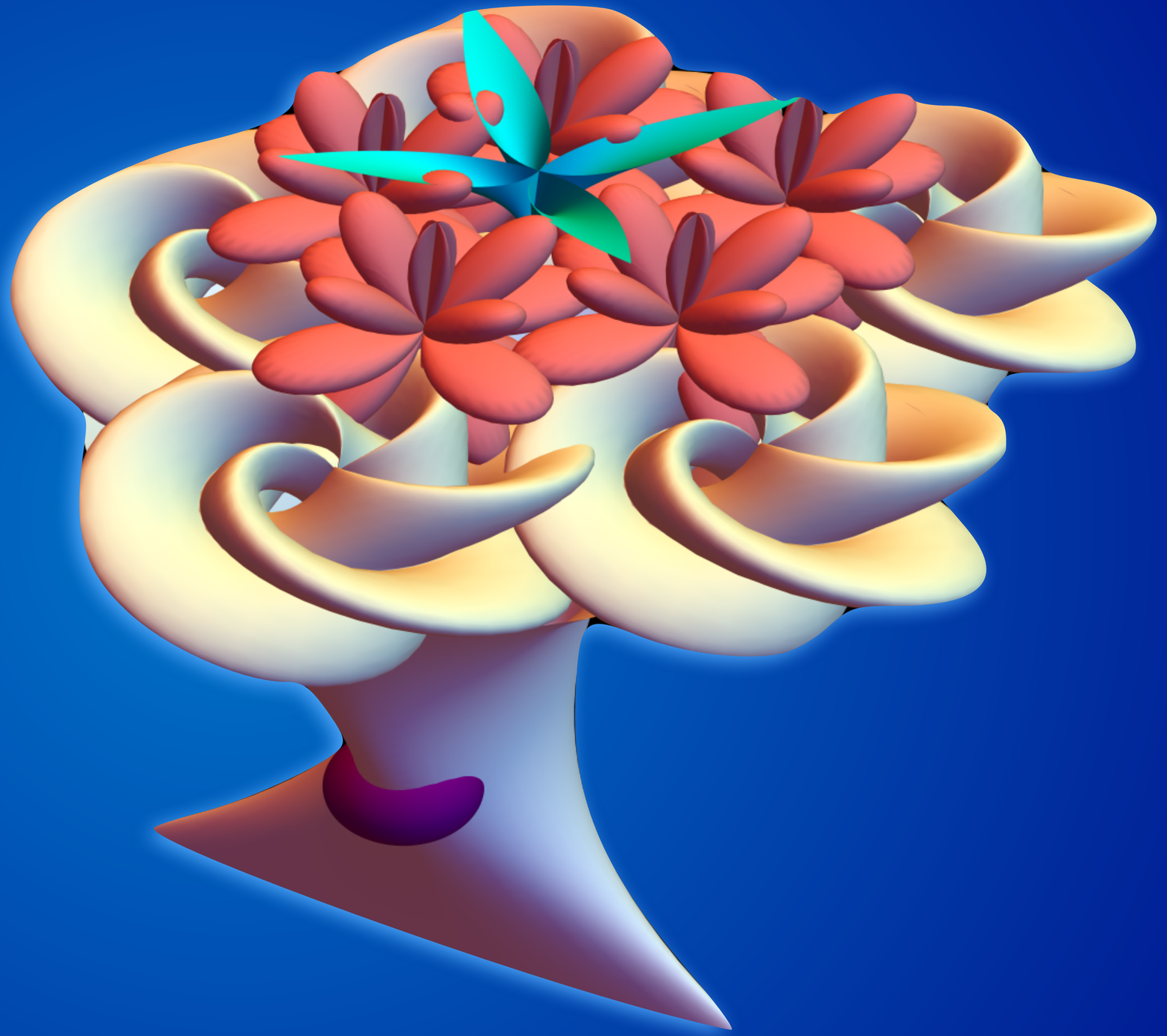
Student Projects


Summer 2016




Anna

Jordan




2015





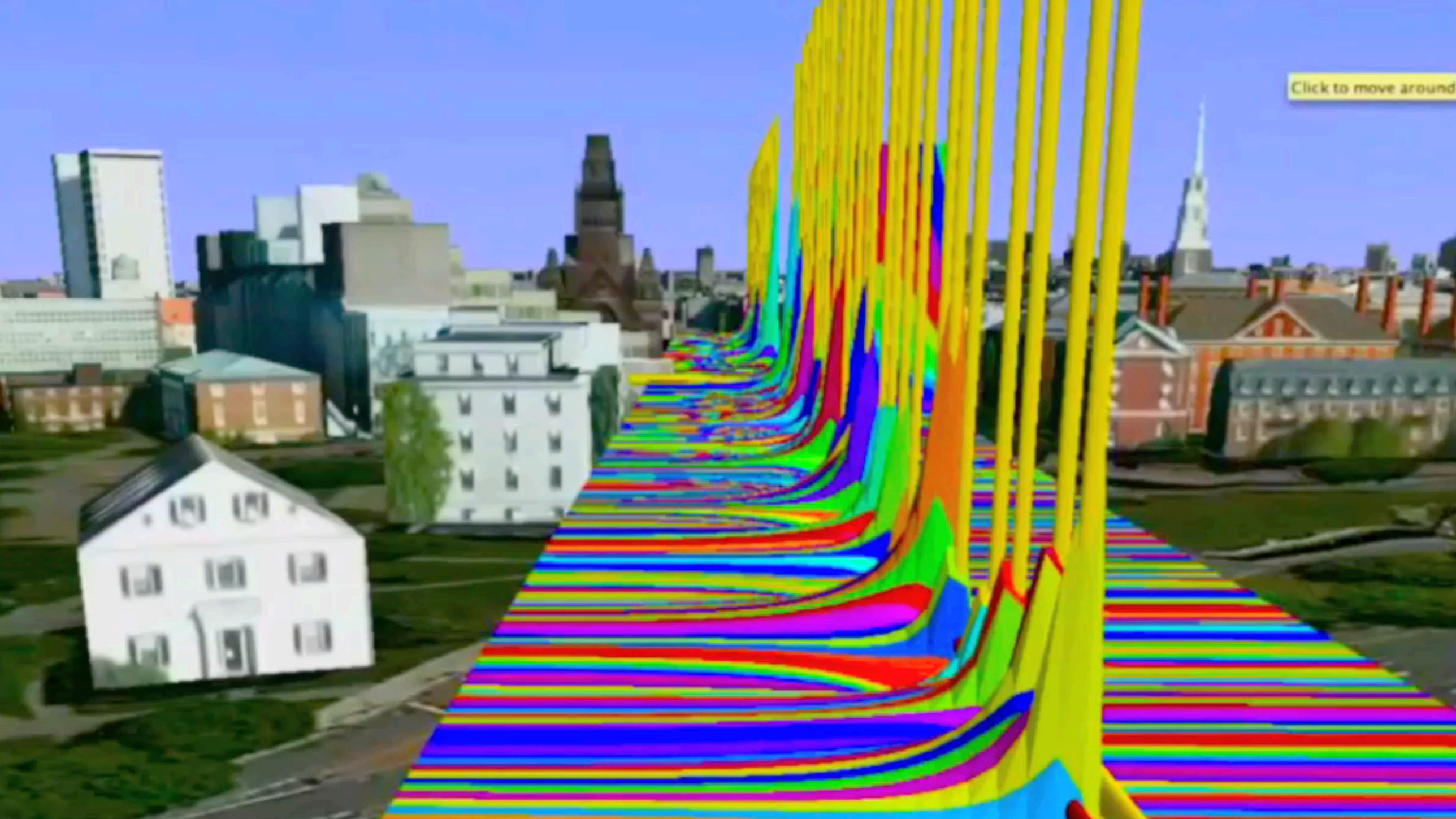


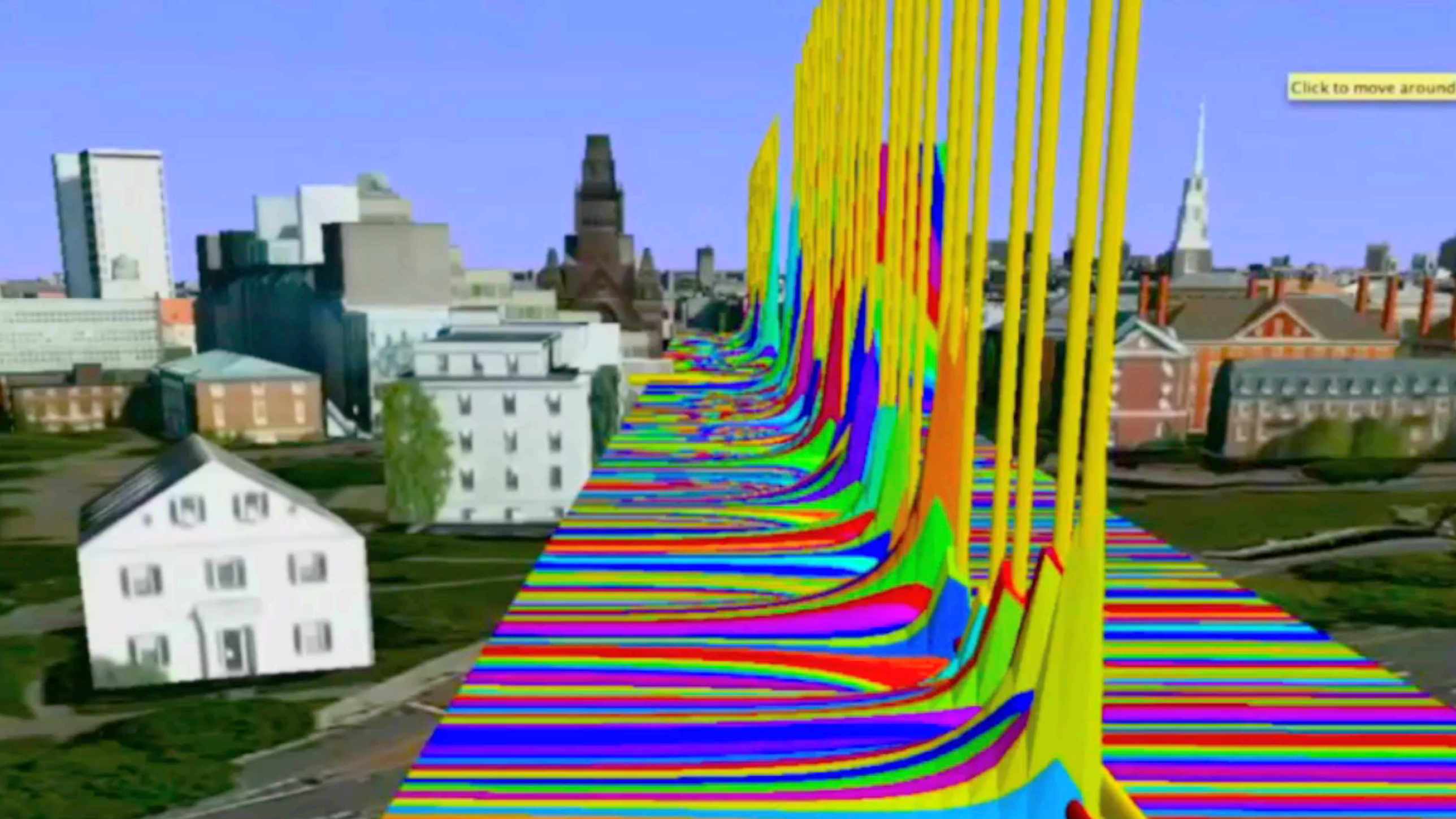


From 3D printing to virtual realizations


Image Landsat


Image Landsat


Click to move around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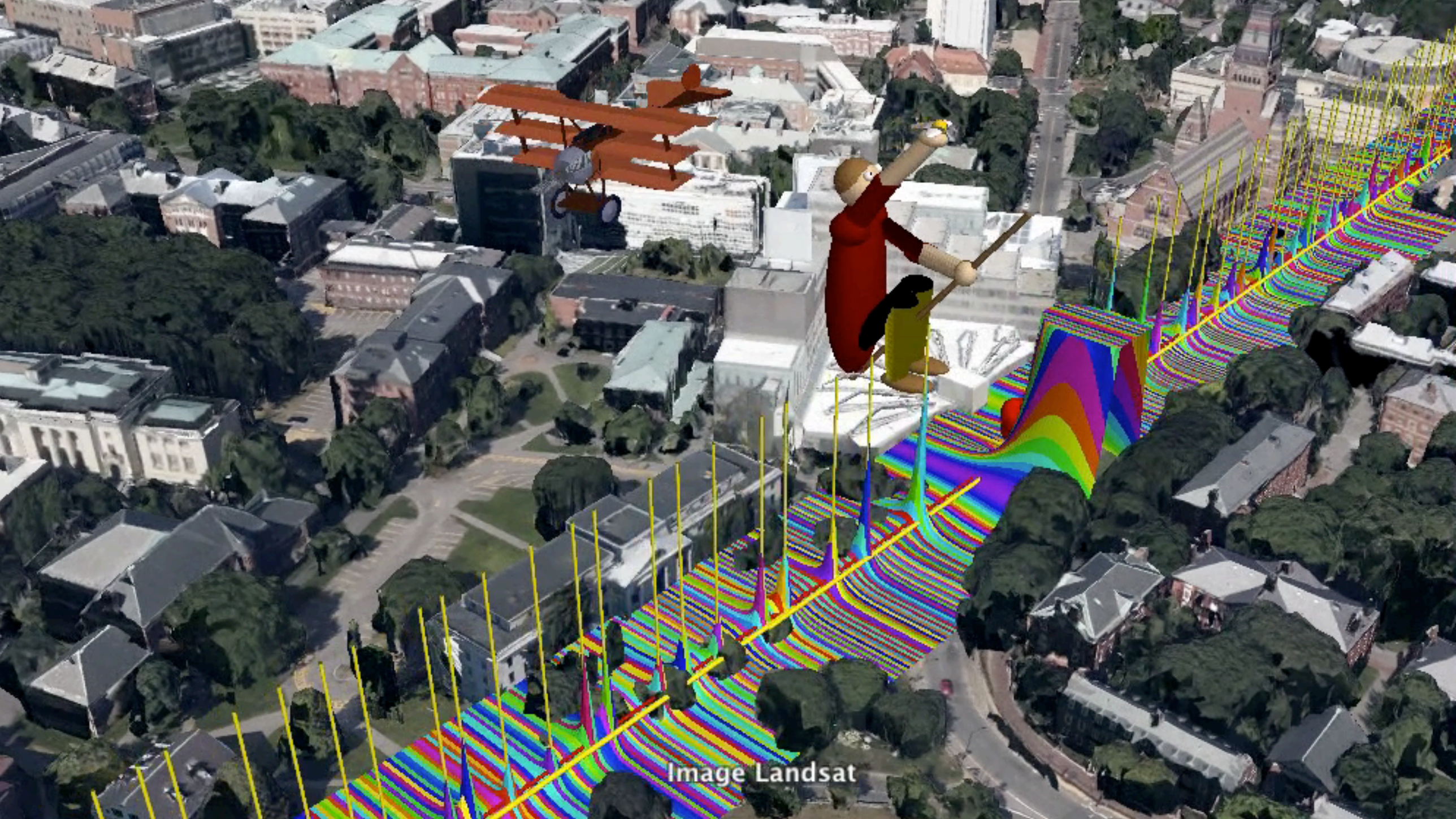
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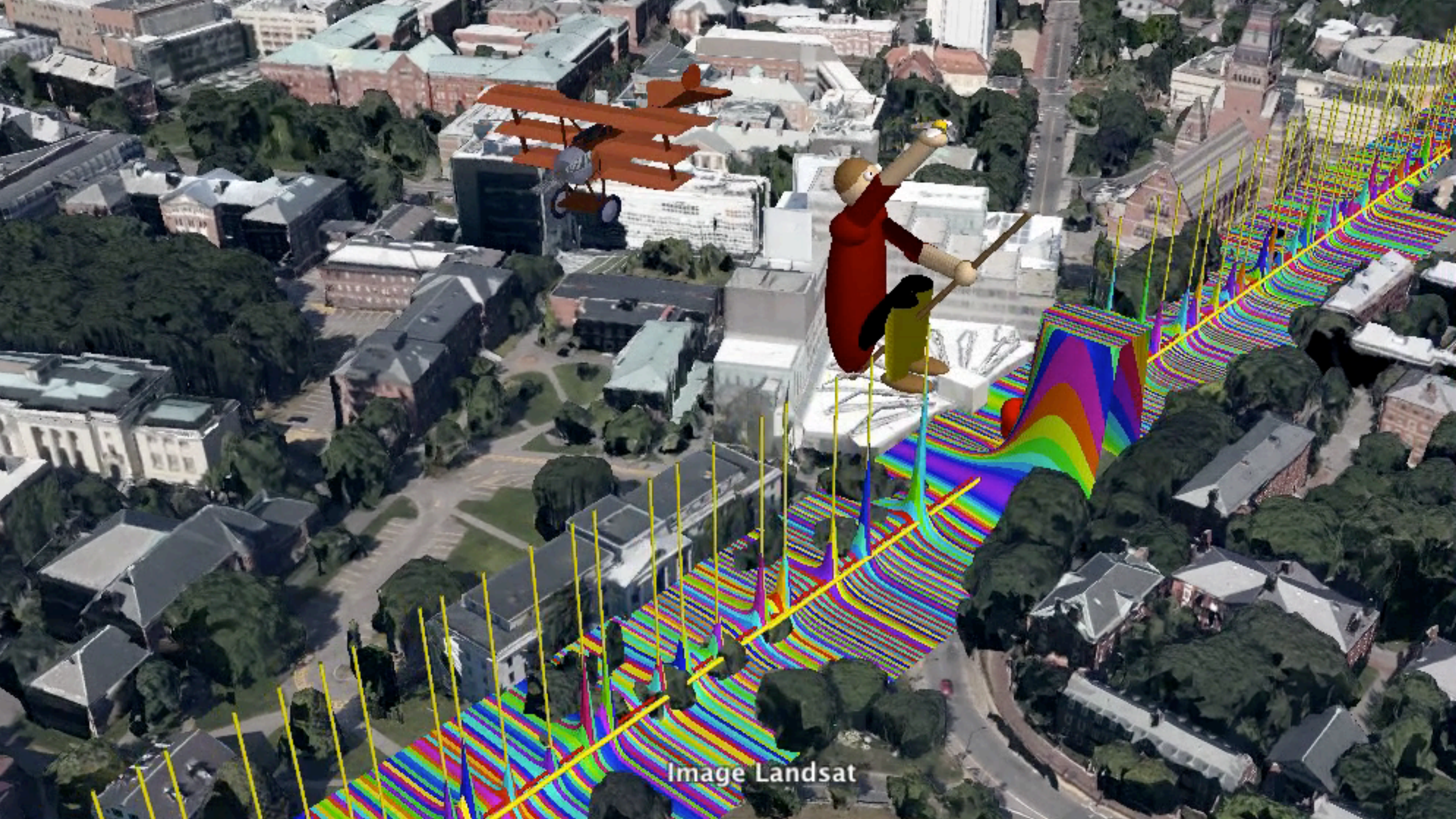
Image Landsat






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W
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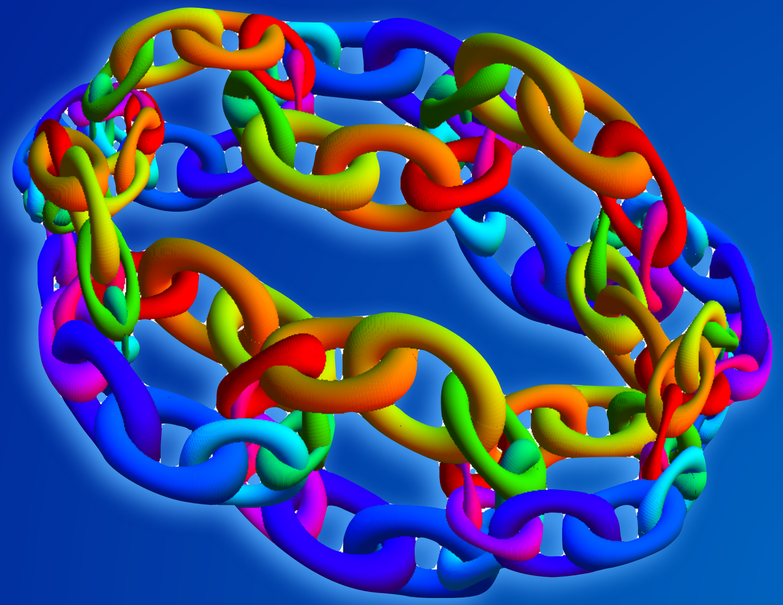
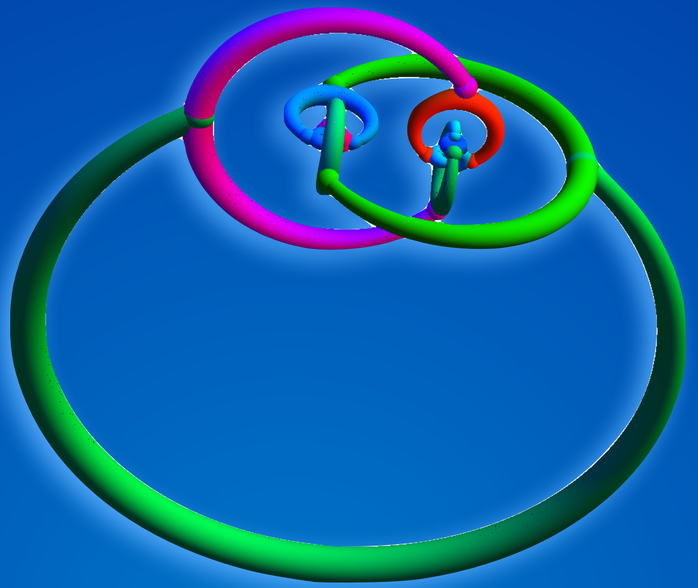
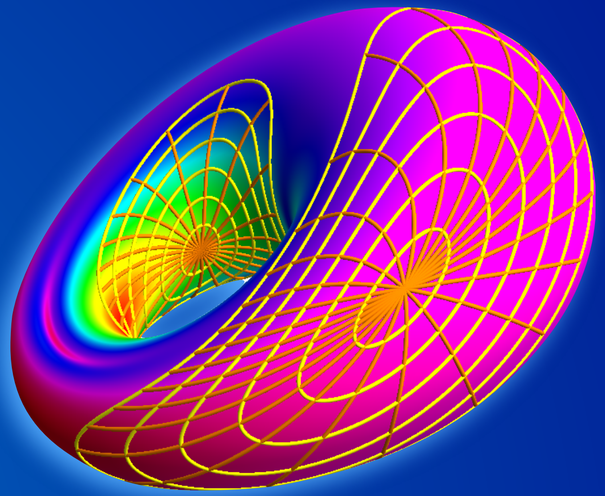
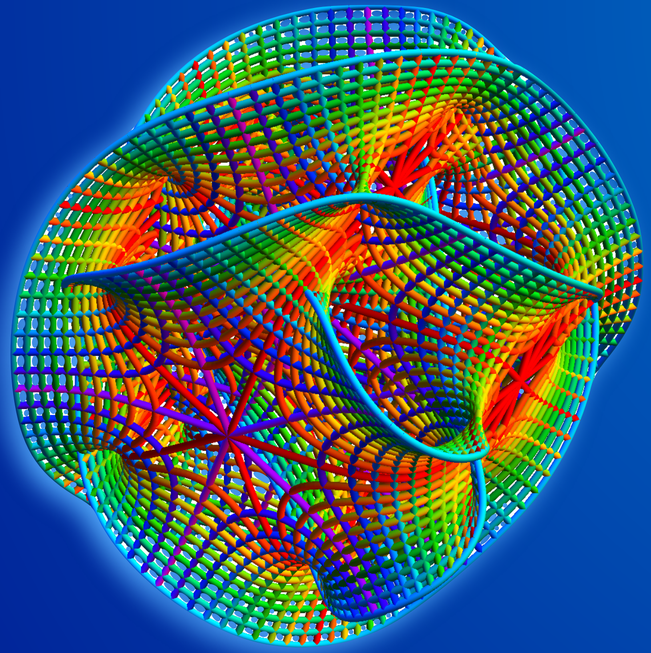
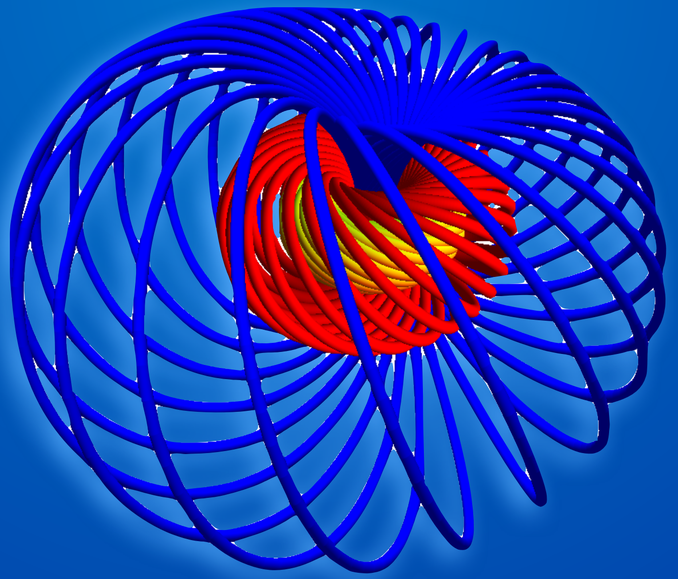
More to come

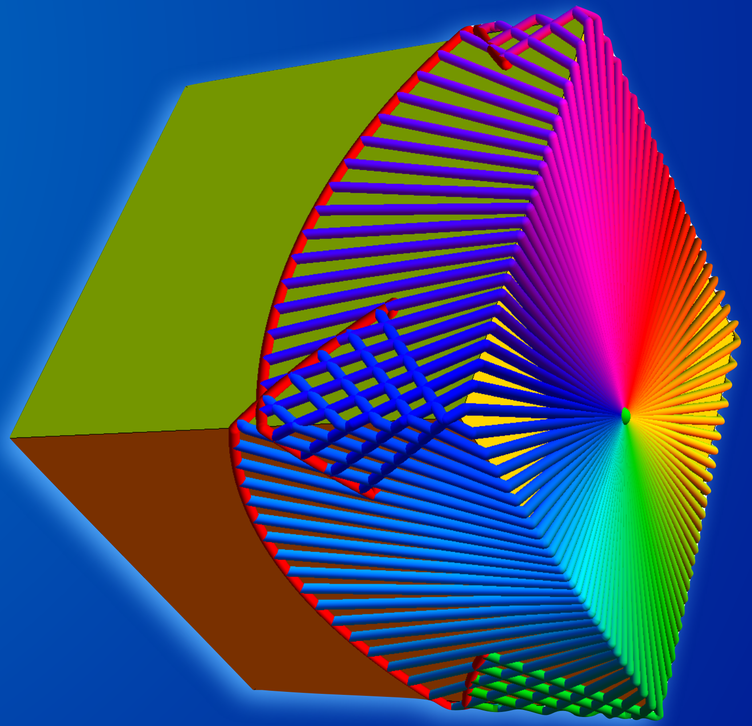








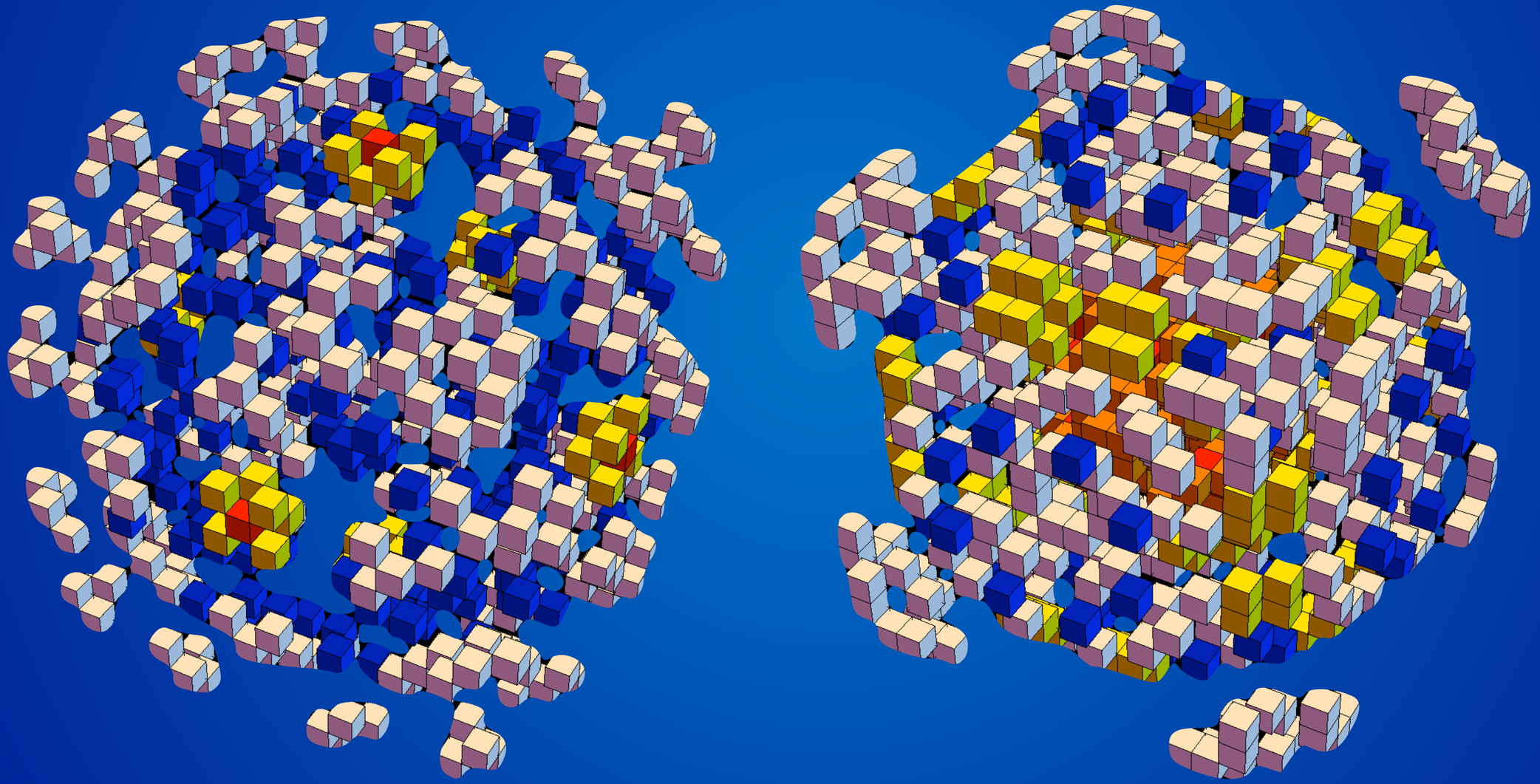




Primes



3D Primes



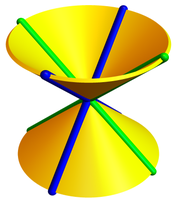
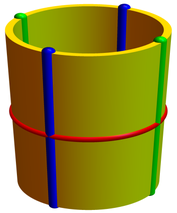
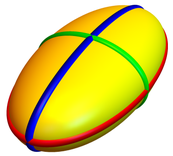
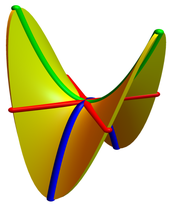
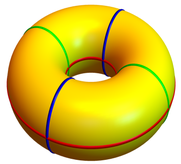
Thinking 3D

Here are exam problems given two days ago to 320 students.



Problem 9) (10 points)

In each of the 5 following parametrizations, there are two entries still to be added. Add them in each case (2 points each).

a)		The surface $\boxed{x^2+y^2=z^2}$ is parametrized by $\vec{r}(\theta,z)=\langle \ldots\ldots\ldots,\ldots\ldots\ldots,z\rangle$
b)		The surface $\boxed{x^2+y^2=1}$ is parametrized by $\vec{r}(\theta,z)=\langle \ldots\ldots\ldots,\ldots\ldots\ldots,z\rangle$
c)		The surface $\boxed{2(x-1)^2+(y-5)^2+4z^2=1}$ is parametrized by $\vec{r}(\theta,\phi)=\langle \ldots\ldots\ldots,\ldots\ldots\ldots,\cos(\phi)/2\rangle$
d)		The surface $\boxed{x^2-y^2=z^2}$ is parametrized by $\vec{r}(x,y)=\langle \ldots\ldots\ldots,\ldots\ldots\ldots,x^2-y^2\rangle$
e)		The surface $\boxed{(\sqrt{x^2+y^2}-2)+z^2=1}$ is parametrized by $\vec{r}(\theta,\phi)=\langle(2+\cos(\phi))\cos(\theta),\ldots\ldots\ldots,\ldots\ldots\ldots\rangle$

Problem 2) (10 points) No justifications are needed in this problem.

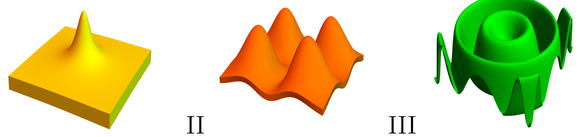
a) (2 points) Match the contour surfaces $g(x,y,z)=1$. Enter O, if there is no match.



I II III

Function $g(x,y,z)=1$	Enter O,I,II or III
$x^2+y^2-z^2$	
x^2y^2	
$x-y$	
x^2+z^2	

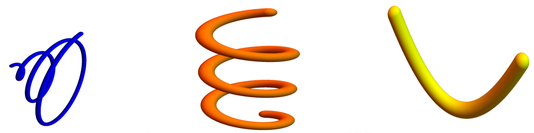
b) (2 points) Match the graphs of the functions $f(x,y)$. Enter O, if there is no match.



I II III

Function $f(x,y)=$	Enter O,I,II or III
$1/(1+x^2+y^2)$	
$x^2y^2e^{-x^2-y^2}$	
$\sin(x^2+y^2)$	
$\cos(y)$	

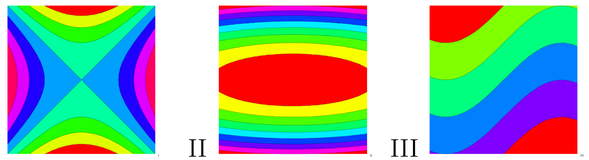
c) (2 points) Match the space curves with their parametrizations $\vec{r}(t)$. Enter O, if there is no match.



I II III

Parametrization $\vec{r}(t)=$	Enter O, I,II or III
$\langle t, t\sin(3t), t\cos(3t)\rangle$	
$\langle \cos(t), \sin(t), \cos(t)\rangle$	
$\langle \sin(t), \cos(t), t\rangle$	
$\langle t, t, t^2\rangle$	

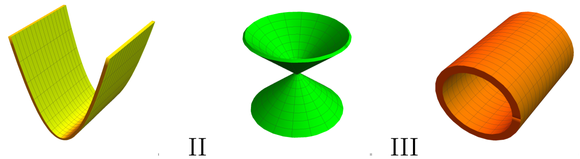
d) (2 points) Match functions g with contour plots in the xy-plane. Enter O, if there is no match.



I II III

Equation	Enter O, I,II or III
$\sin(x)-y$	
x^2-y^2	
$10x^2+y^2$	
x^2+10y^2	

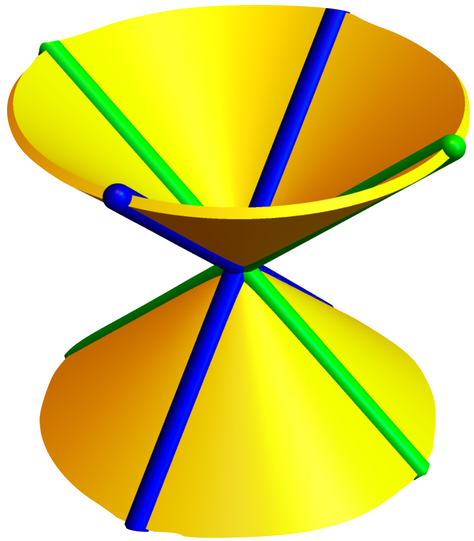
e) (2 points) Match the quadrics. Enter O if no match.



I II III

Quadric	Enter O,I,II or III
$x^2=z$	
$x^2+y^2=1$	
$xy=1$	
$y=1$	
$x^2+y^2=z^2$	

a)



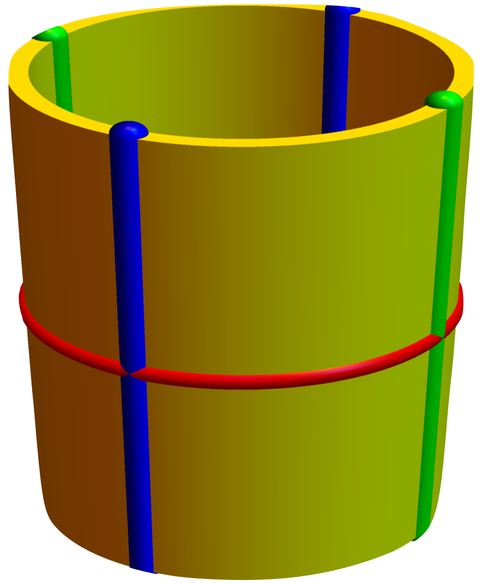
The surface

$$\boxed{x^2 + y^2 = z^2}$$

is parametrized by

$$\vec{r}(\theta, z) = \langle \ldots\ldots\ldots, \ldots\ldots\ldots, z \rangle$$

b)

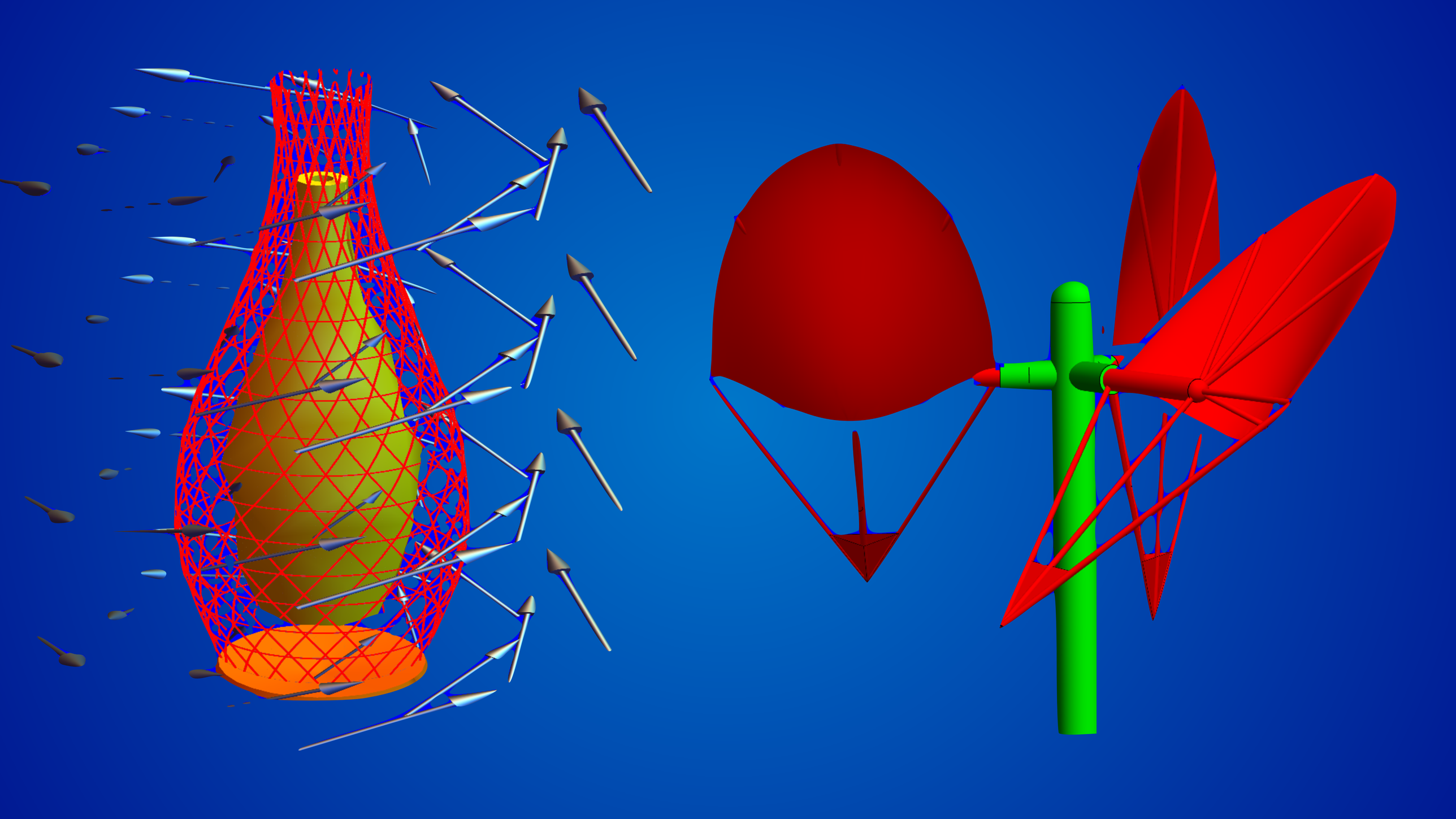


The surface

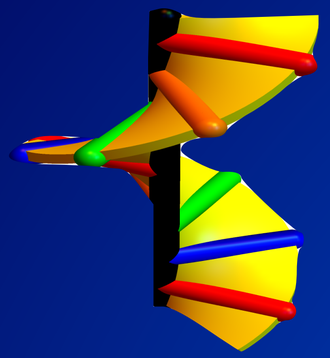
$$\boxed{x^2 + y^2 = 1}$$

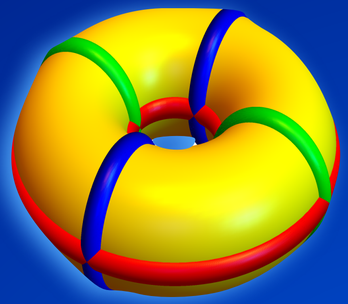
is parametrized by

$$\vec{r}(\theta, z) = \langle \ldots\ldots\ldots, \ldots\ldots\ldots, z \rangle$$



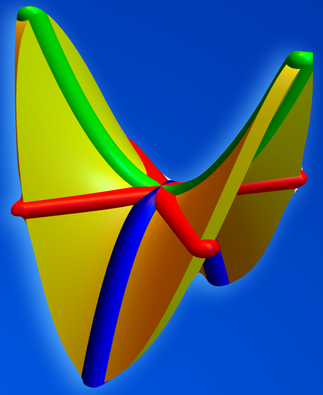
For presentations

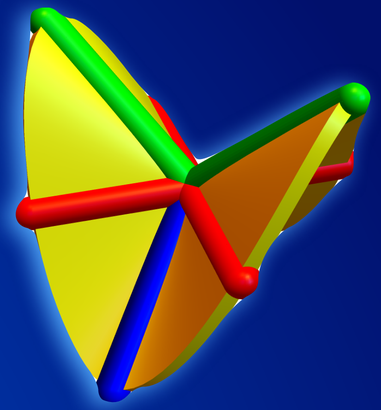




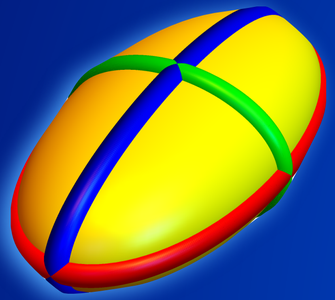
Parametrization

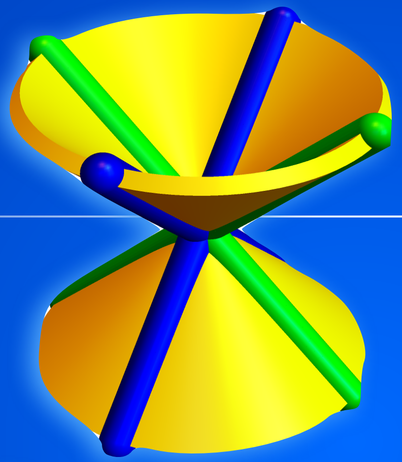
Arc Length



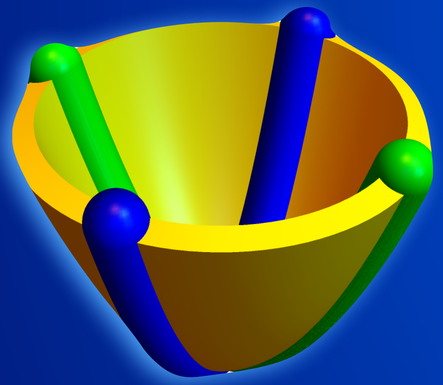


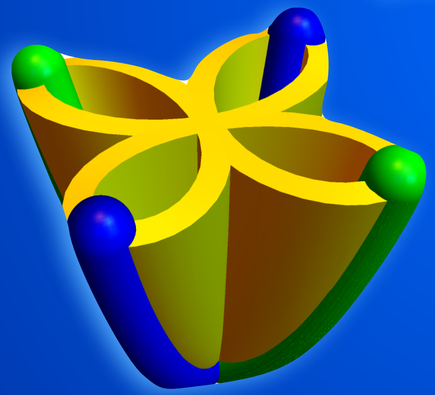
Distances

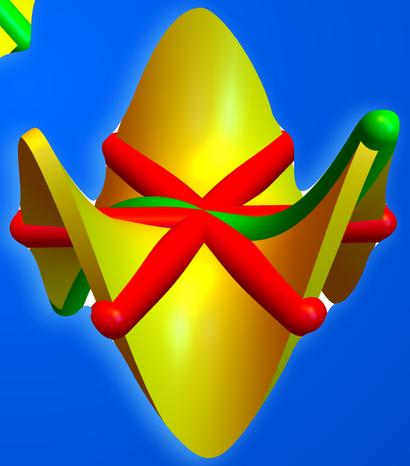


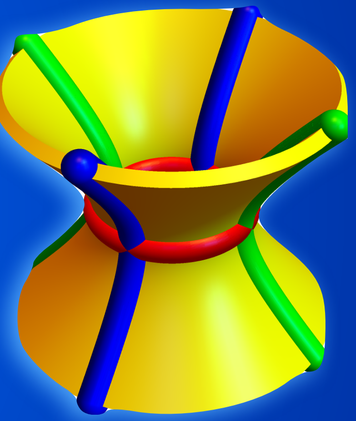


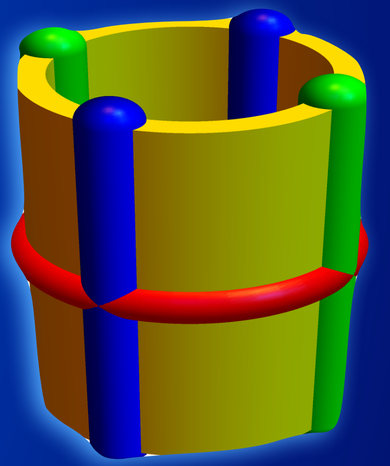
Curves



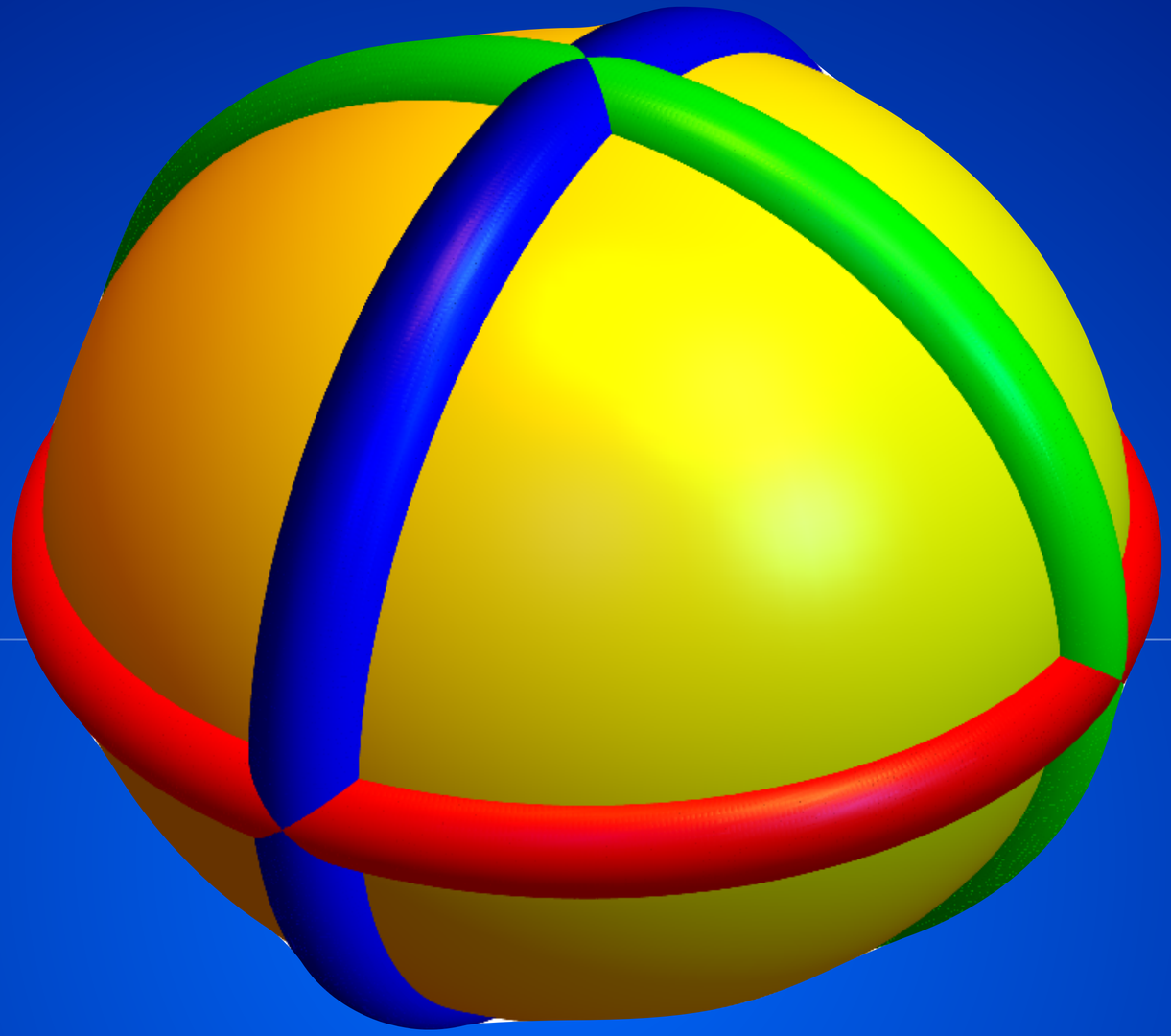




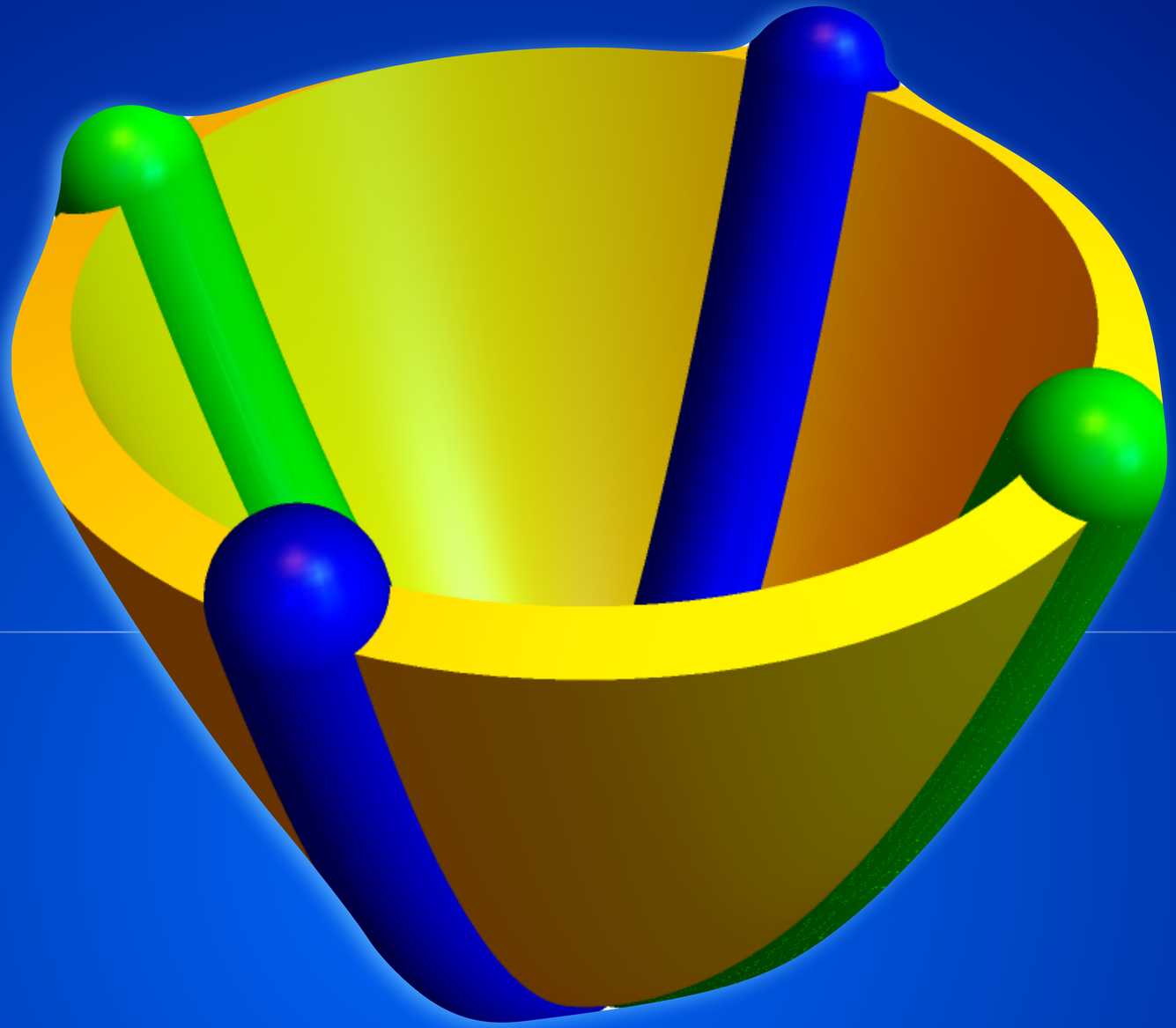




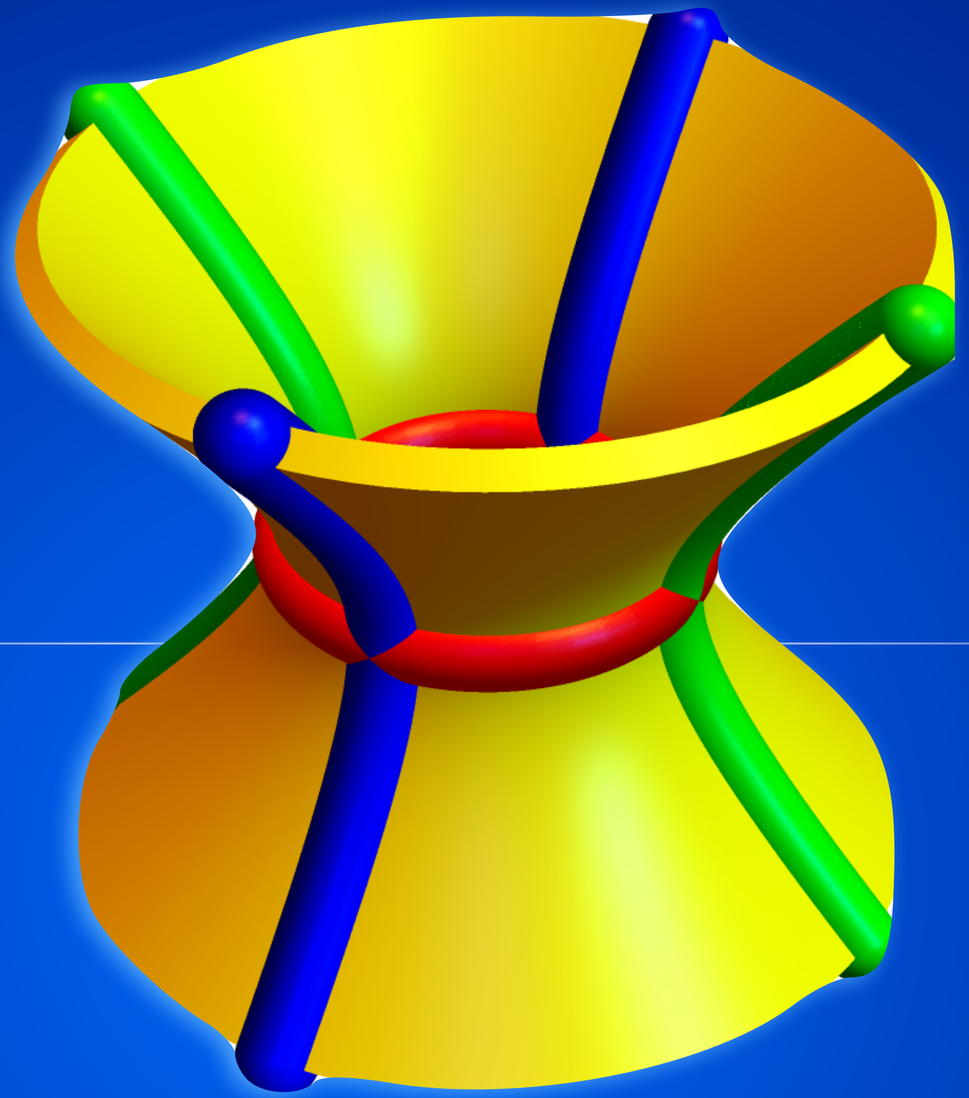
Surfaces



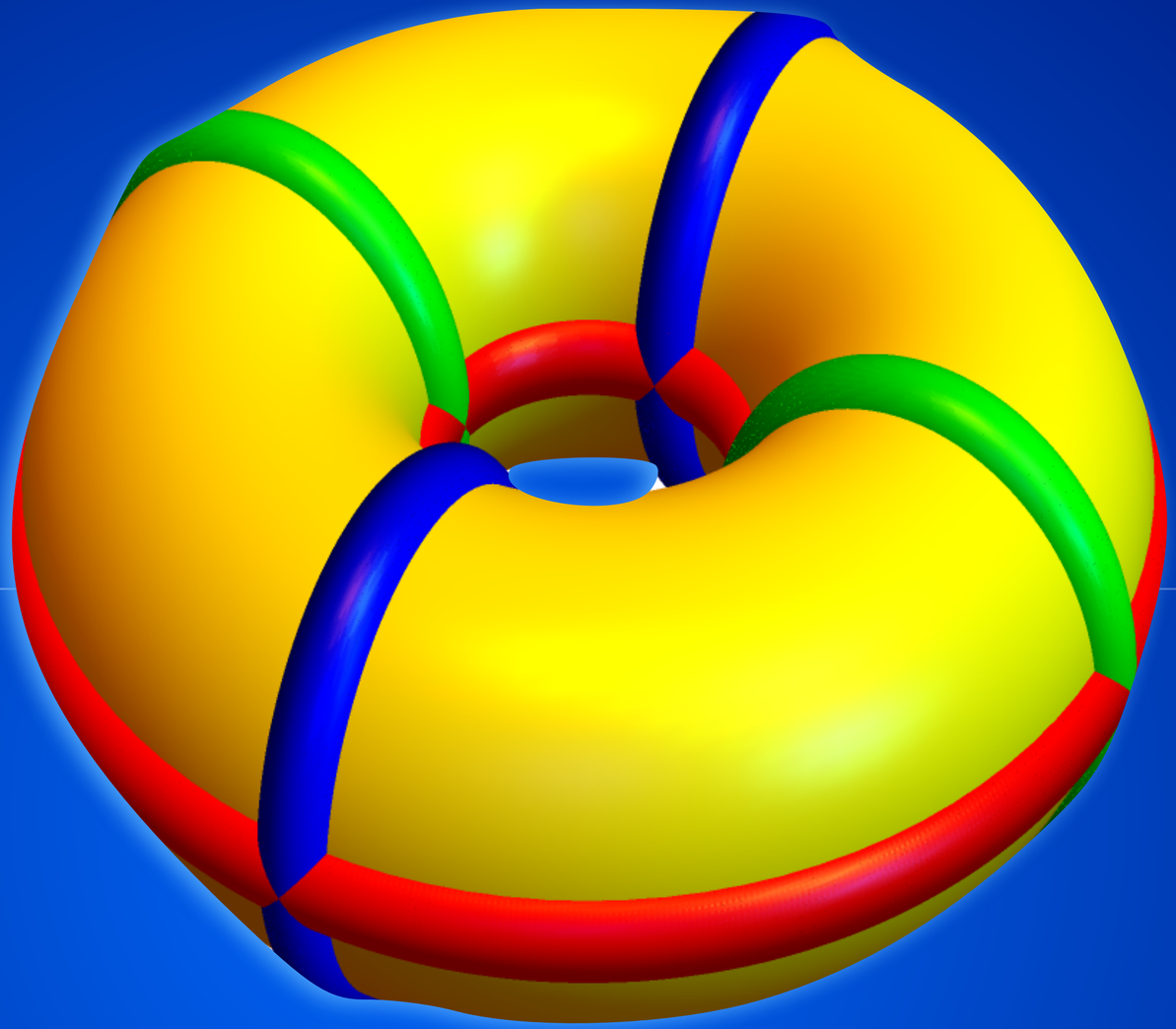
$$\vec{r}\,(\theta,\varphi) = \langle \cos(\theta)\sin(\varphi), \sin(\theta)\sin(\varphi), \cos(\varphi) \rangle$$



$$\vec{r}\,(x,y) = \langle x,y,f(x,y)\rangle$$



$$\vec{r}\,(\theta,z) = \langle g(z)\cos(\theta),\ g(z)\sin(\theta),\ z\rangle$$

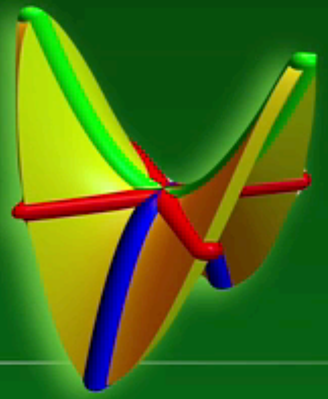
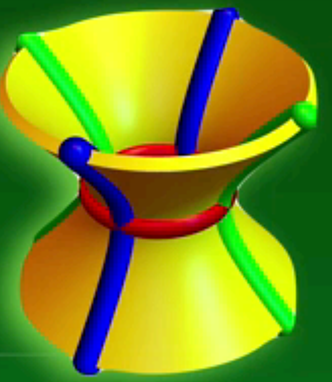


$$\vec{r}\,(u,v) = \langle (2+\cos(v))\cos(u),\ (2+\cos(v))\sin(u),\ \sin(v) \rangle$$

Candy!





Math Candy

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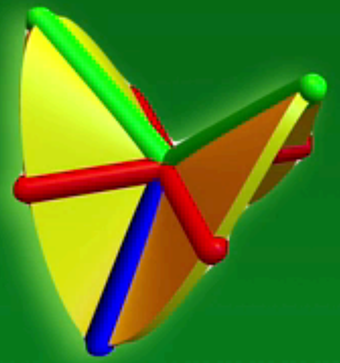
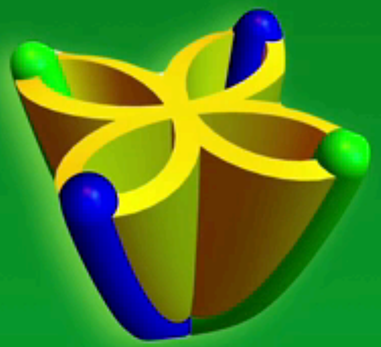


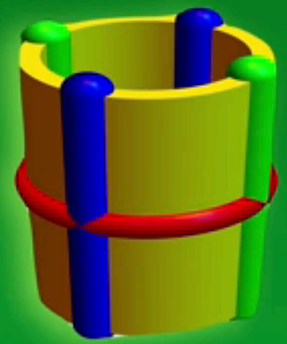



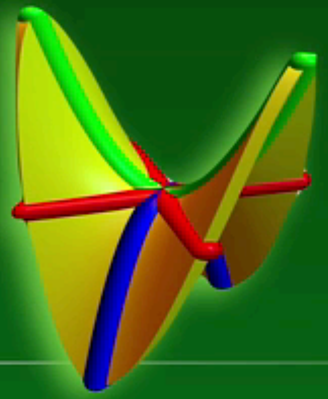
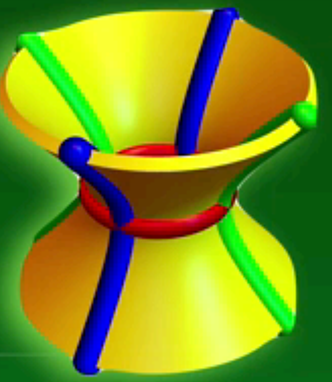

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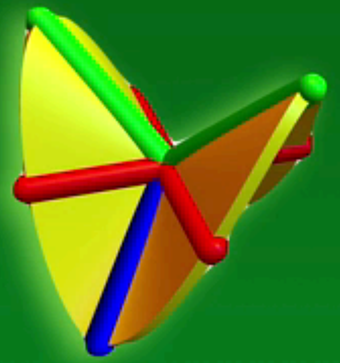
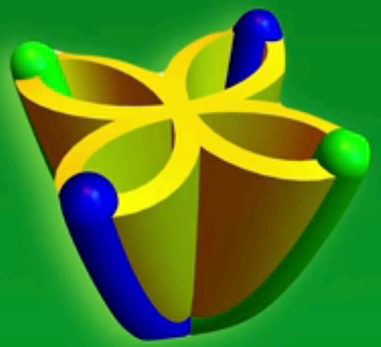


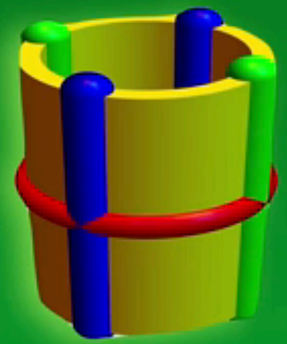


The end