Functional maps + products = $\heartsuit$

Michael Bronstein

USI Lugano / Imperial College / Intel

SIAM AM, Portland, 13 July 2018

>$10K

2005

$100

2010

$20

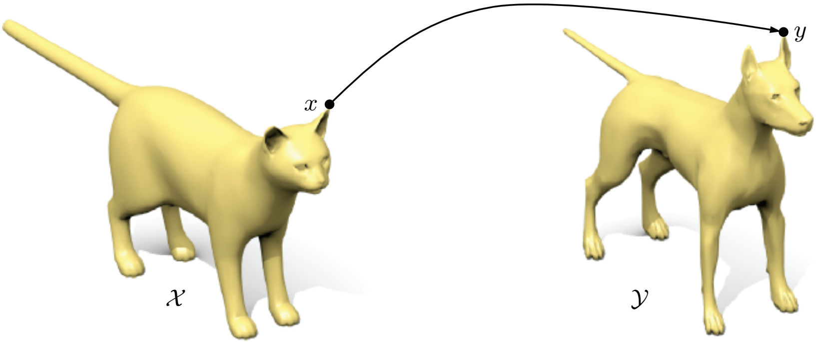
2014

$17

2017

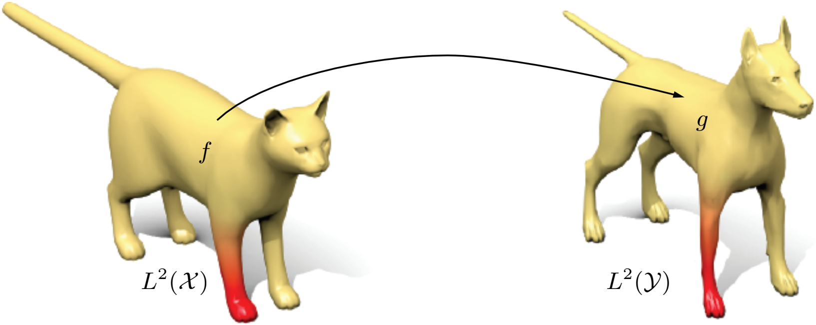
Faceshift (acquired by Apple in 2015)

Pointwise correspondence



Point-wise map $t\colon \mathcal{X} \to \mathcal{Y}$

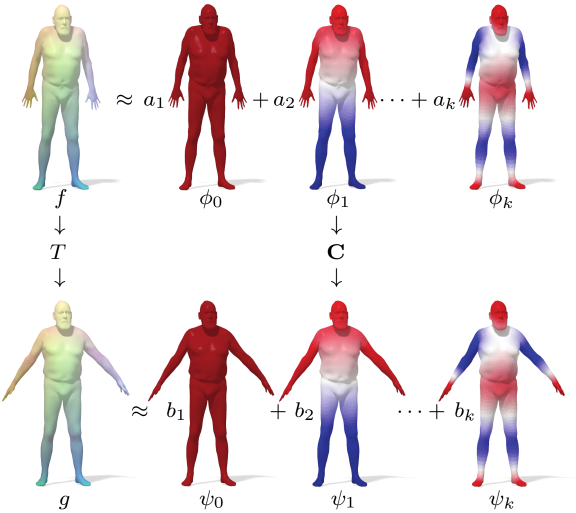
Functional correspondence



Functional map $T\colon L^2(\mathcal{X}) \to L^2(\mathcal{Y})$

Ovsjanikov, Ben-Chen, Solomon, Butscher, Guibas 2012

Functional maps in spectral domain



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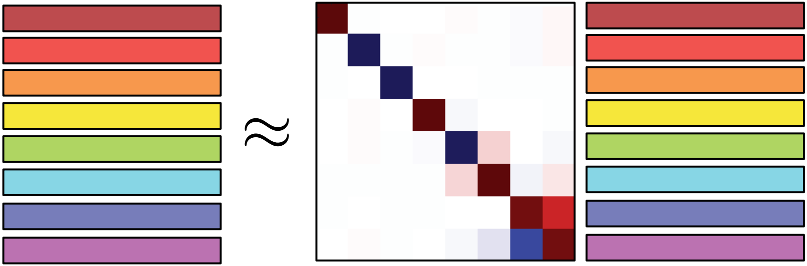


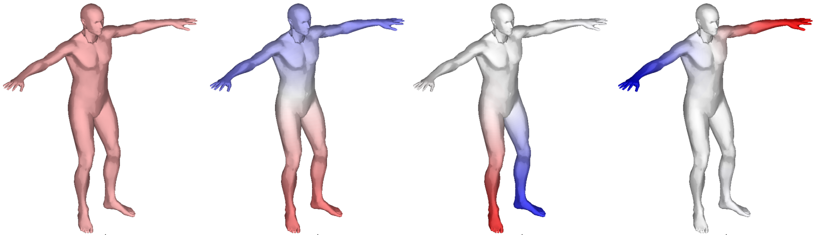
$\mathbf{B}_{k\times q} = (\langle \psi_i, g_j \rangle_{\mathcal{Y}})$ $\mathbf{C}_{k\times k}$ $\mathbf{A}_{k\times q} = (\langle \phi_i, f_j \rangle_{\mathcal{X}})$

where $\mathbf{A}$, $\mathbf{B}$ are Fourier coefficients of corresponding ‘probe’ functions

$$g_i \approx T f_i \qquad i = 1, \ldots, q \geq k$$

Ovsjanikov, Ben-Chen, Solomon, Butscher, Guibas 2012

Laplacian eigenbasis



ϕ_0 ϕ_1 ϕ_2 ϕ_3

For shapes with simple spectrum, Laplacian eigenfunctions are invariant (up to sign) to isometric deformations, $\psi_i = \pm T\phi_i$

ψ_0 ψ_1 ψ_2 ψ_3

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- **Is Laplacian eigenbasis the best way to represent functional maps?**

Pointwise maps = product preserving maps

Theorem Functional map $T : L^2(\mathcal{X}) \to L^2(\mathcal{Y})$ is a pointwise map iff

$$T(f \cdot h) = (Tf) \cdot (Th)$$

for all $f, h \in L^2(\mathcal{X})$

Kishor, Manhas 1993

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Kishor, Manhas 1993; Shtern, Kimmel 2013 (product-based pointwise descriptors)

Functional maps in product bases

Product basis

$$f \approx \sum_{i=0}^{k} a_i \phi_i$$

Nongeng, Melzi, Rodolà, Castellani, B, Ovsjanikov 2018

Product basis

$$f \approx \sum_{i=0}^{k} a_i \phi_i + \sum_{j=1}^{r} \tilde{a}_j \prod_{l=1}^{r_j} \phi_{i_{jl}} \qquad i_{jl} \in \{1, \ldots, k\}$$

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- Adds higher frequency information: for trigonometric bases 2nd order products max double the frequency since

$$\cos(nx) \cdot \cos(mx) = \tfrac{1}{2}[\cos((n+m)x)) + \cos((n-m)x))]$$

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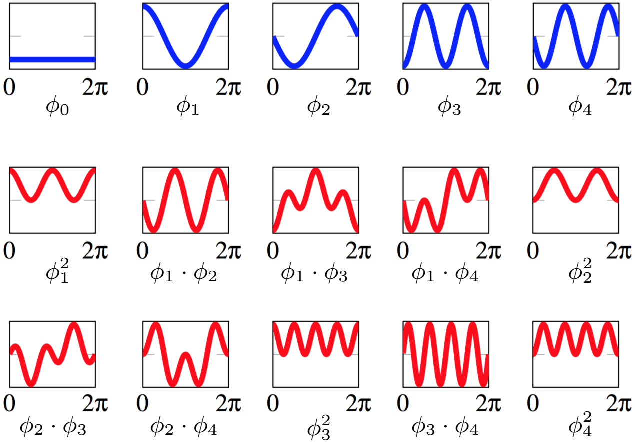
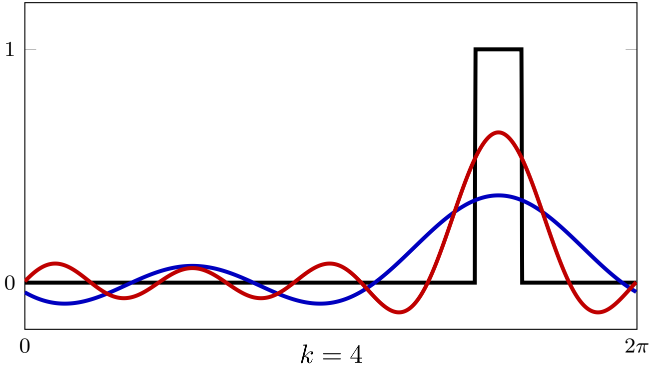
- Product basis is linearly dependent
- Orthogonality is lost
- Higher orders r become unstable
- Finding optimal approximation that minimizes the number of products used is NP-hard

Nongeng, Melzi, Rodolà, Castellani, B, Ovsjanikov 2018

Example: 1D product basis



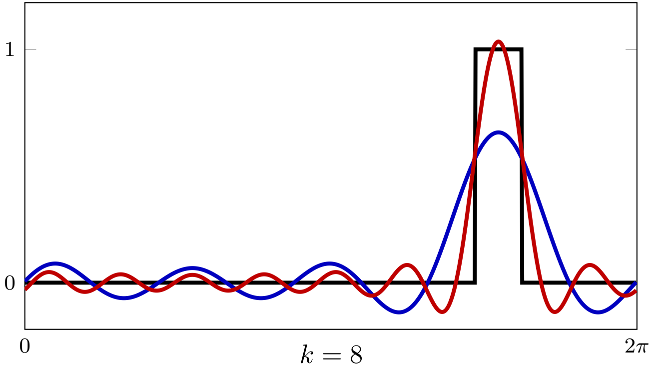
1D function approximation: standard vs product basis



Approximation of a step function (black) using standard (blue) and product (red) bases of order $r = 2$

Nongeng, Melzi, Rodolà, Castellani, B, Ovsjanikov 2018

1D function approximation: standard vs product basis



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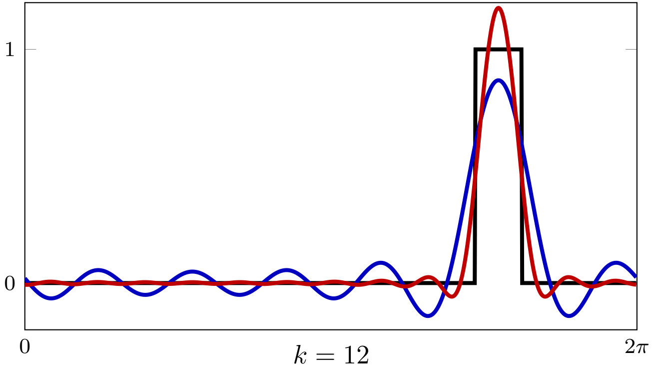

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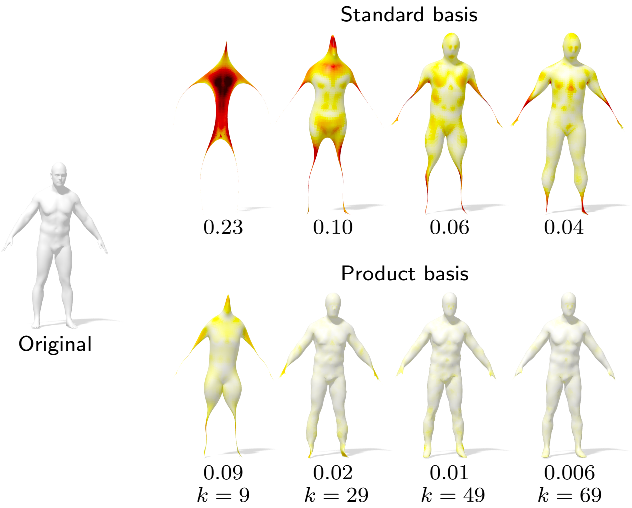
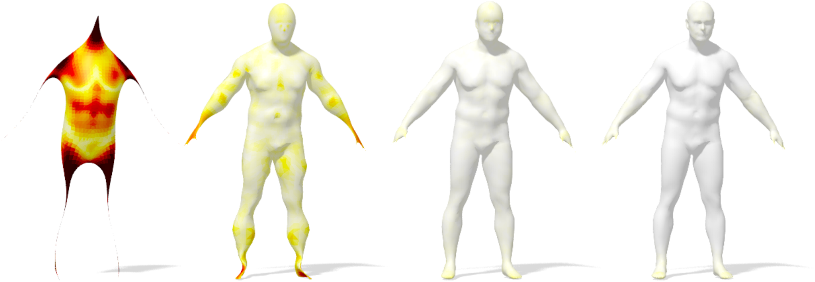
3D function approximation: standard vs product basis



Approximation of the shape 3D coordinates in standard and product bases

Nongeng, Melzi, Rodolà, Castellani, B, Ovsjanikov 2018

3D function approximation: standard vs product basis

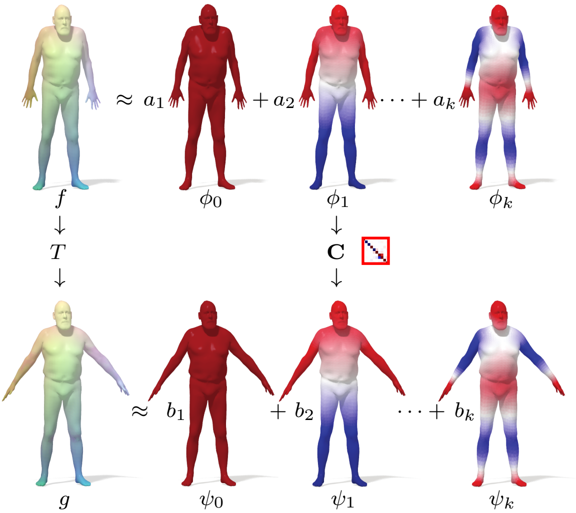


Standard basis 2^{nd} order 3^{rd} order 4^{th} order

Reconstruction of the shape 3D coordinates using standard and product bases of different order ($k = 20$)

Nongeng, Melzi, Rodolà, Castellani, B, Ovsjanikov 2018

Functional maps in product bases



Nongeng, Melzi, Rodolà, Castellani, B, Ovsjanikov 2018

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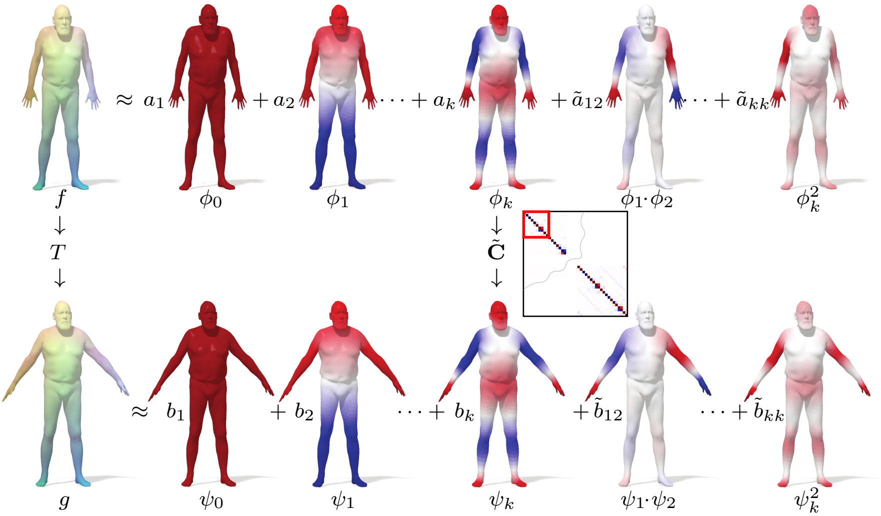

Nongeng, Melzi, Rodolà, Castellani, B, Ovsjanikov 2018

Functional maps in product bases

$$f \approx \sum_{i=0}^{k} a_i \phi_i + \sum_{i,j=1}^{k} \tilde{a}_{ij} \phi_i \cdot \phi_j$$

Nongeng, Melzi, Rodolà, Castellani, B, Ovsjanikov 2018

Functional maps in product bases

$$Tf \approx T\left(\sum_{i=0}^{k} a_i\phi_i + \sum_{i,j=1}^{k} \tilde{a}_{ij}\phi_i \cdot \phi_j\right)$$

Nongeng, Melzi, Rodolà, Castellani, B, Ovsjanikov 2018

Functional maps in product bases

$$Tf \approx \sum_{i=0}^{k} a_i T\phi_i + \sum_{i,j=1}^{k} \tilde{a}_{ij} T(\phi_i \cdot \phi_j)$$

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$$Tf \approx \sum_{i,j=0}^{k} a_i c_{ij} \psi_j + \sum_{i,j=1}^{k} \sum_{l,l'=0}^{k} \tilde{a}_{ij}\, c_{il} c_{il'}\, \psi_l \cdot \psi_{l'}$$

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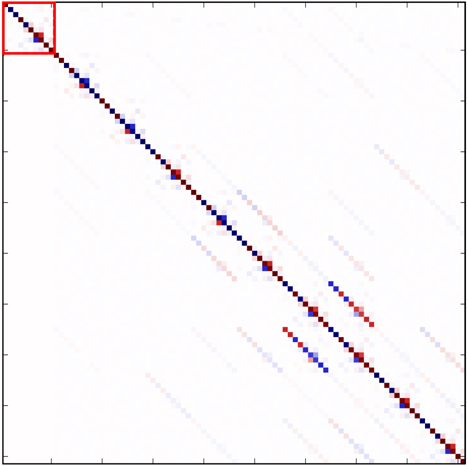
Structure of matrix $\tilde{\mathbf{C}}$

$\mathbf{C}$



$\tilde{\mathbf{C}}$

Nongeng, Melzi, Rodolà, Castellani, B, Ovsjanikov 2018

Structure of matrix $\tilde{\mathbf{C}}$

$$\tilde{\mathbf{C}}(\mathbf{C}) = \begin{bmatrix} \mathbf{C} & \phi_{00}\mathbf{c}_0^\top \otimes \mathbf{C}_{01} + \phi_{00} \begin{bmatrix} \mathbf{0} \\ \mathbf{C}_{11} \end{bmatrix} \otimes \mathbf{c}_0^\top \\ & \mathbf{C}_{11} \otimes \mathbf{C}_{11} \end{bmatrix}$$

is matrix of size $(k^2+k+1) \times (k^2+k+1)$ expressed in terms of $\mathbf{C}$, and

$$\mathbf{C}_{11} = \begin{bmatrix} c_{11} & \dots & c_{1k} \\ \vdots & \vdots & \vdots \\ c_{k1} & \dots & c_{kk} \end{bmatrix} \quad \mathbf{C}_{01} = \begin{bmatrix} c_{01} & \dots & c_{0k} \\ \vdots & \vdots & \vdots \\ c_{k1} & \dots & c_{kk} \end{bmatrix} \quad \mathbf{c}_0^\top = \begin{bmatrix} c_{00} & \dots & c_{0k} \end{bmatrix}$$

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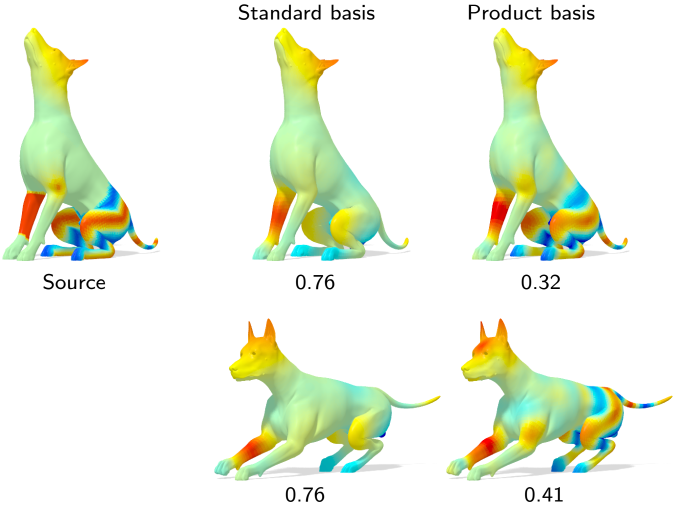
$\Rightarrow (k^2+k+1)^2$ **coefficients, but only** $(k+1)^2$ **degrees of freedom!**

Nongeng, Melzi, Rodolà, Castellani, B, Ovsjanikov 2018

Standard vs product bases



Function approximation and transfer error using standard and product bases

Nongeng, Melzi, Rodolà, Castellani, B, Ovsjanikov 2018

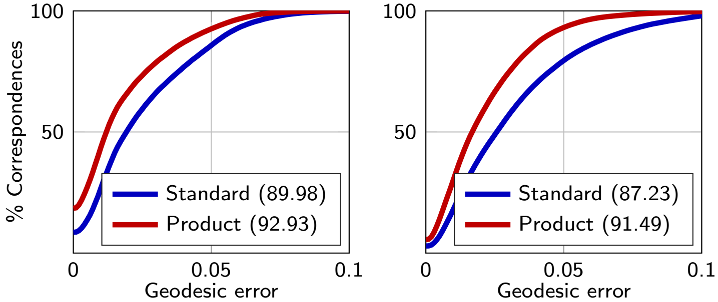
Example of correspondence on FAUST dataset

Source Standard basis Product basis

Correspondence (shown with matching colors) and correspondence error on SCAPE shapes using standard and product bases

Nongeng, Melzi, Rodolà, Castellani, B, Ovsjanikov 2018; data: Bogo et al. 2014

Correspondence quality



Quality of functional maps computed with standard and product bases on FAUST (left) and TOSCA (right) shapes

Nongeng, Melzi, Rodolà, Castellani, B, Ovsjanikov 2018

Future directions

- Instead of improving a given functional map, finding pointwise functional maps by solving the non-linear problem

$$\min_{\mathbf{C}\in\mathbb{R}^{k\times k}} \|\mathbf{B} - \tilde{\mathbf{C}}(\mathbf{C})\mathbf{A}\|_{\mathrm{F}}^2$$

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$$\min_{\mathbf{C}\in\mathbb{R}^{k\times k}} \|\mathbf{B} - \tilde{\mathbf{C}}(\mathbf{C})\mathbf{A}\|_{\mathrm{F}}^2$$

- More general definition of products (potentially combined with learning)

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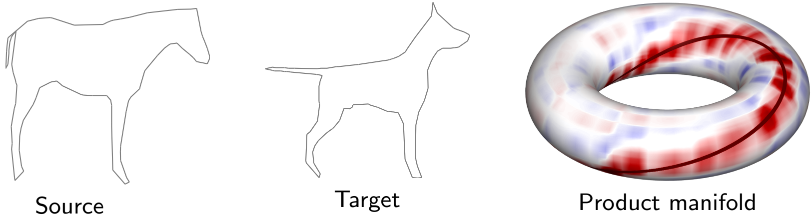
- **Is Laplacian eigenbasis the best way to represent functional maps?**

Functional maps in product spaces

Correspondence in the product space



Functional map $T_\mu : \mathcal{F}(\mathcal{M}) \to \mathcal{F}(\mathcal{N})$ associated with a density $\mu \in L^1(\mathcal{M} \times \mathcal{N})$ on the product manifold $(\mathcal{M} \times \mathcal{N}, g_\mathcal{M} \oplus g_\mathcal{N})$

$$T_\mu(g)(x) = \int_\mathcal{N} g(y)\mu(x, y)\mathrm{d}y$$

Rodolà, Lähner, BB, Solomon 2018

Laplacian eigenbasis on product manifold

Theorem Let $\mathcal{M} \times \mathcal{N}$ be a product manifold and let

$$\Delta_{\mathcal{M}\times\mathcal{N}}\xi = \gamma\xi$$

Then, there exist ϕ, ψ and α, β s.t. $\Delta_{\mathcal{M}}\phi = \alpha\phi$ and $\Delta_{\mathcal{N}}\psi = \beta\psi$ and

$$\gamma = \alpha + \beta \qquad \xi = \phi \wedge \psi$$

Berger, Gauduchon, Mazet 1971

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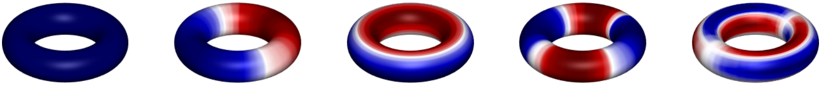

$\phi_0 \wedge \psi_0$ $\phi_1 \wedge \psi_0$ $\phi_0 \wedge \psi_1$ $\phi_2 \wedge \psi_0$ $\phi_3 \wedge \psi_0$

Berger, Gauduchon, Mazet 1971

Representation equivalence

Theorem Let $c_{ij} = \langle \phi_i, T_\mu(\psi_j) \rangle_{\mathcal{M}}$ be the representation of T_μ in orthogonal bases $\{\phi_i\}_{i \geq 1}$, $\{\psi_i\}_{i \geq 1}$ and let $p_{ij} = \langle \phi_i \wedge \psi_j, \mu \rangle_{\mathcal{M} \times \mathcal{N}}$ such that $\mu = \sum_{ij} (\phi_i \wedge \psi_j) p_{ij}$. Then $c_{ij} = p_{ij}$ for all i, j.

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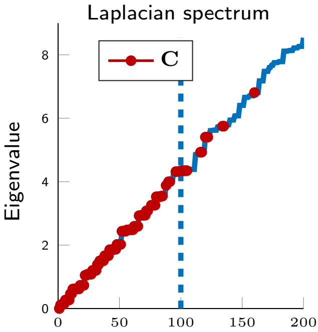
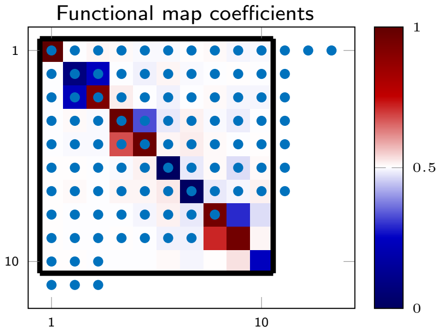
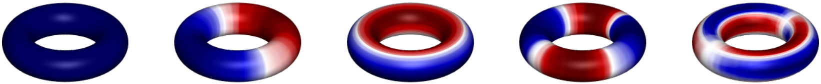




Rodolà, Lähner, BB, Solomon 2018

Representation efficiency

$$\mu = \sum_{\ell=0}^{k} p_\ell \xi_\ell$$



$\phi_0 \wedge \psi_0$ $\phi_1 \wedge \psi_0$ $\phi_0 \wedge \psi_1$ $\phi_2 \wedge \psi_0$ $\phi_3 \wedge \psi_0$

Separable basis

Rodolà, Lähner, BB, Solomon 2018

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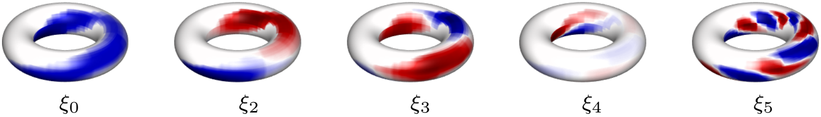
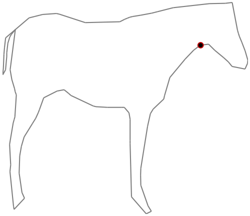


Non-separable localized basis

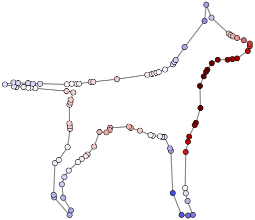
Eigenfunctions of the Hamiltonian operator $H = \Delta_{\mathcal{M}\times\mathcal{N}} + V$, where V is the localization potential

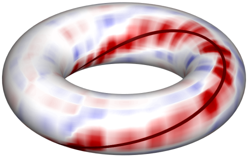
Rodolà, Lähner, BB, Solomon 2018; Choukroun et al. 2017; Melzi, Rodolà, Castellani, B 2017

Example: 1D correspondence



Source

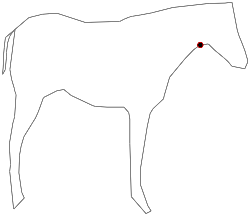




Delta function transfer using functional map on the product space computed in **separable basis**. Groundtruth correspondence shown in black.

Rodolà, Lähner, BB, Solomon 2018

Example: 1D correspondence



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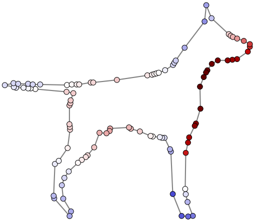
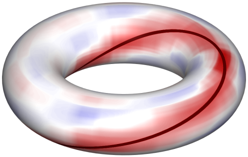
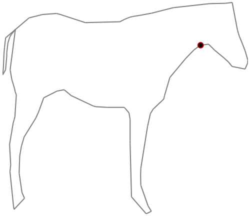




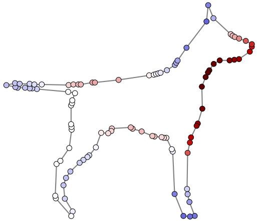
Delta function transfer using functional map on the product space computed in **localized basis (90% area)**. Groundtruth correspondence shown in black.

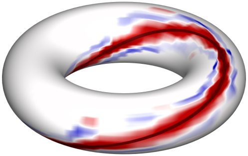
Rodolà, Lähner, BB, Solomon 2018

Example: 1D correspondence



Source

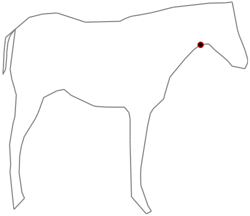




Delta function transfer using functional map on the product space computed in **localized basis (25% area)**. Groundtruth correspondence shown in black.

Rodolà, Lähner, BB, Solomon 2018

Example: 1D correspondence



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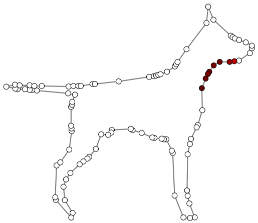
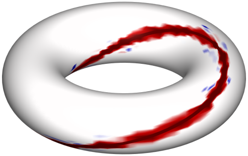
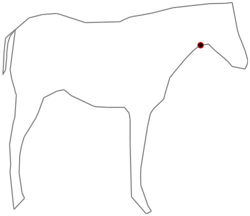


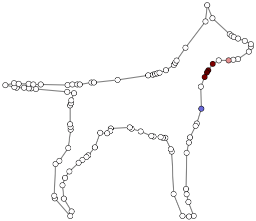


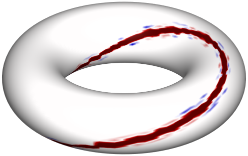
Delta function transfer using functional map on the product space computed in **localized basis (5% area)**. Groundtruth correspondence shown in black.

Rodolà, Lähner, BB, Solomon 2018

Example: 1D correspondence


Source

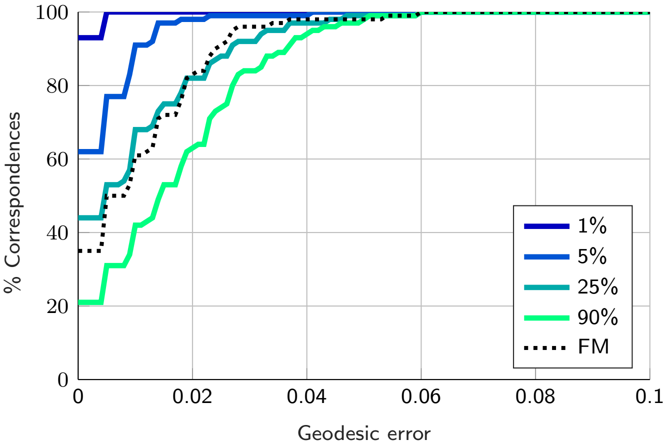




Delta function transfer using functional map on the product space computed in **localized basis (1% area)**. Groundtruth correspondence shown in black.

Rodolà, Lähner, BB, Solomon 2018

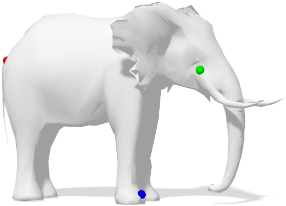
Example: 1D correspondence



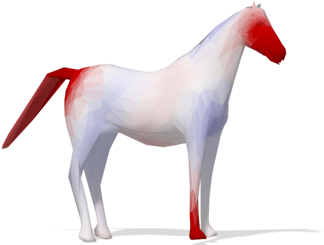
Quality of correspondence on product manifold using different basis localization

Rodolà, Lähner, BB, Solomon 2018

Example: 2D correspondence



Source

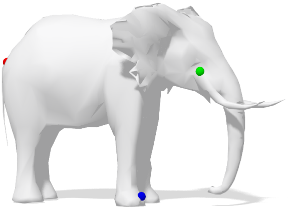


Target

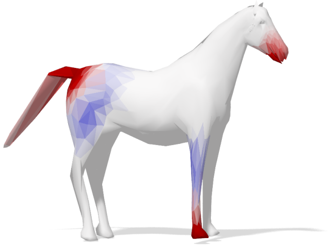
Delta function transfer using functional map on the product space computed in **separable basis**.

Rodolà, Lähner, BB, Solomon 2018

Example: 2D correspondence



Source

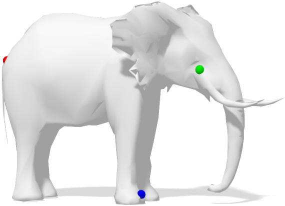


Target

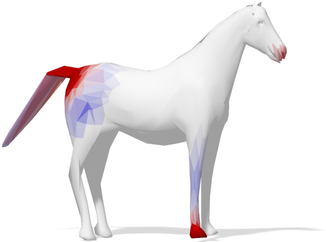
Delta function transfer using functional map on the product space computed in **localized basis (15% area)**.

Rodolà, Lähner, BB, Solomon 2018

Example: 2D correspondence



Source

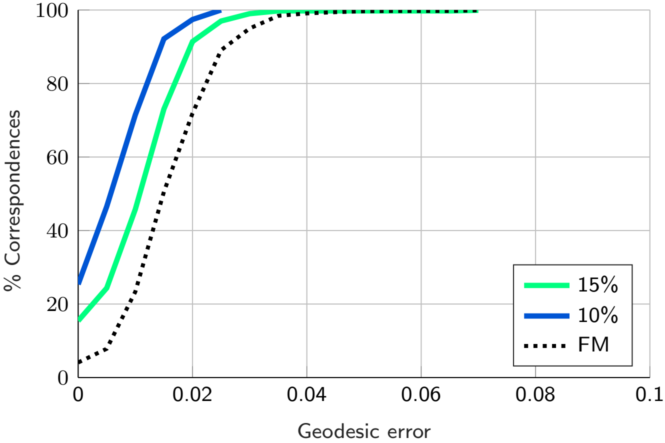


Target

Delta function transfer using functional map on the product space computed in **localized basis (10% area)**.

Rodolà, Lähner, BB, Solomon 2018

Example: 2D correspondence



Quality of correspondence on product manifold using different basis localization

Rodolà, Lähner, BB, Solomon 2018

Two completely different uses of products yield novel and interesting representations of functional maps and shed new light on old problems

Functional maps + products = ♡

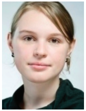

E. Rodolà

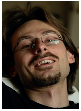

M. Ovsjanikov


D. Nogneng


U. Castellani


S. Melzi


Z. Lähner


A. Bronstein


J. Solomon

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