Image Denoising Using Mean Curvature of Image Surface

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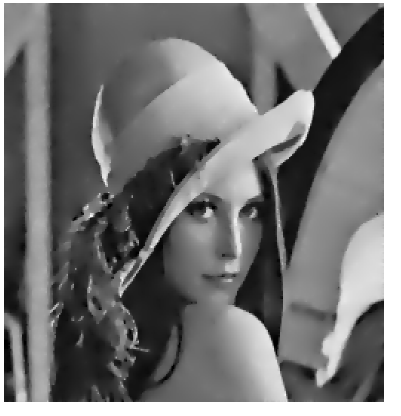
May 24, 2016

Outline

- Mean Curvature Based Image Denoising Model
- Fast Algorithm Using Augmented Lagrangian Method (ALM)
- Numerical Experiments
- New ALM Method
- New Numerical Experiments
- Summary and Future Work

Problem (Image Denoising)

- The presence of noise in images is unavoidable.
- Problem: **How to get clean images?**
  - Ideally, clean images should contain the most meaningful signals of given images and also include no noise.
  - Object boundaries are the most important signals depicted by images.
  - Object corners, part of object boundaries, are also important signals.
- Goal: Try to construct a model that is able to remove noise while keeping object boundaries, corners and image contrasts.



Typical Methods of Image Denoising

- Variational method, PDE-based method, statistical method and many other ones

- Variational method

$$\mathrm{f} = u + n$$

Given image **Desired clean image** **Noise**

$$\mathrm{f} : \Omega \rightarrow \mathrm{R}^1$$

How to decompose the given noisy image using appropriate regularizers?

Classical Variational Models

- Mumford-Shah (89)

$$E(u,K) = \int_\Omega (f-u)^2 + \lambda \int_{\Omega\setminus K} |\nabla u|^2 + \mu H^1(K)$$

goal true image　　goal boundary　　positive parameters

- Rudin-Osher-Fatemi (92)

$$E(u) = \lambda \int_\Omega |\nabla u| + \int_\Omega (f-u)^2, \quad \lambda > 0$$

  - Powerful & popular, excellent analytical properties
  - Preserve edges and sweep noise efficiently
  - Cannot preserve corner & image contrast
  - Suffers from the staircase effect

Related high-order models

- Ambrosio-Masnou-Morel's Euler's Elastica (03)

$$E(u) = \int_\Omega \left[a + b\left(\nabla \cdot \frac{\nabla u}{|\nabla u|} \right)^2 \right] |\nabla u| + \frac{1}{2}\int_\Omega (f-u)^2$$

  - Originally proposed for the disocclusion problem
  - Noise removal efficiently, no staircase effect
  - Solving a fourth-order differential equation

- Lysaker-Lundervold-Tai (LLT)(03)

$$L(u,\lambda) = \lambda \int_\Omega \sqrt{u_{xx}^2 + u_{xy}^2 + u_{yx}^2 + u_{yy}^2} + \frac{1}{2}\int_\Omega (f-u)^2$$

  - Excellent noise suppression, no staircase effect
  - Solving a fourth-order differential equation

Our Model

- Goal:
  - sweep noise while keeping object edges
  - preserve object corners and image contrasts
  - ameliorate the staircase effect

- Idea:
  - edges and corners are important concepts in differential geometry
  - geometry information of the given image function should be incorporated in the denoising process
  - use the mean curvature of the graph $(x, f(x))$ defined by the image function $f(x)$. (The idea of considering image graph is not new. Similar idea has been used in other works, such as Sochen et al.(98), Lysaker et al.(03))

Mean curvature of image surface

- Give an image :

$$f : \Omega \rightarrow R^1, \quad \Omega \subset R^2$$

- Consider the function :

$$\Phi(x,z) = z - f(x), \quad x \in \Omega$$

Its zero level set corresponds to the image surface $(x, f(x))$, whose mean curvature reads:

$$\frac{1}{2}\nabla_{(x,z)} \cdot \left(\frac{\nabla_{(x,z)}\Phi}{\left|\nabla_{(x,z)}\Phi\right|} \right) = \frac{1}{2}\nabla_{(x,z)} \cdot \left(\frac{(\nabla_x f, -1)}{\left|(\nabla_x f, -1)\right|} \right) = \frac{1}{2}\nabla_x \cdot \left(\frac{\nabla_x f}{\sqrt{1 + \left|\nabla_x f\right|^2}} \right) = H_f$$

Our Model (Zhu, Chan *SIIMS* 2012)

- Energy:

$$E(u) = \lambda \int_\Omega | H_u | + \int_\Omega (f-u)^2$$

$$= \frac{\lambda}{2} \int_\Omega \left| \nabla \cdot \left(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}} \right) \right| + \int_\Omega (f-u)^2$$

- Gradient Descent Equation:

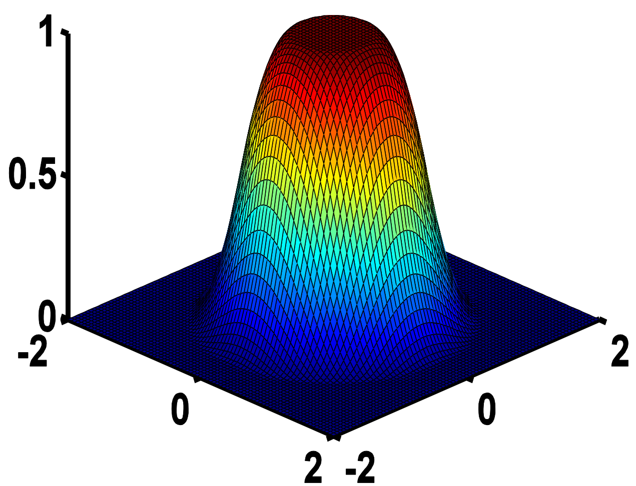
$$\frac{\partial u}{\partial t} = -\lambda \nabla \cdot \left[\frac{1}{\sqrt{1+|\nabla u|^2}} (I-P) \nabla(\Phi'(\kappa_u)) \right] + 2(f-u)$$

$$I(\vec{v}) = \vec{v}, \quad P(\vec{v}) = \left(\vec{v} \cdot \frac{\nabla u}{\sqrt{1+|\nabla u|^2}} \right) \frac{\nabla u}{\sqrt{1+|\nabla u|^2}}, \quad \Phi(x) = |x|$$

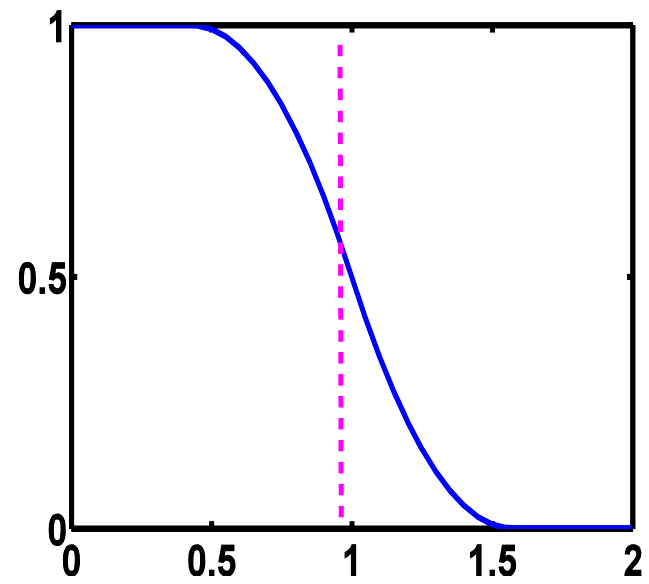
  - If $|\nabla u| << 1$, $\frac{\partial u}{\partial t} \approx -\lambda \Delta^2 u + 2(f-u)$, the bi-harmonic equation, explaining why small oscillation part can be removed effectively.

Why does our model preserve contrast?

- What is the value of the regularizer $\int_{R^2} |H_f|$ for $f = h\chi_E$, a multiplier of the characteristic function of a set E ?
  - When $E = B(0, R)$, $\int_{R^2} |H_f| = 2\pi R$.
    - choose an appropriate sequence of functions g_n that approximate f
    - calculate $\int_{R^2} |H_{g_n}|$, and define its limit as $\int_{R^2} |H_f|$.



an approximation to $f = \chi_{B(0,1)}$



generatrix function

Cont'd

- Using the same procedure, we can show that:

If E is an open set with C^2 boundary, then $\int_\Omega |H_f| = P(E,\Omega)$, the perimeter of set E inside the domain Ω

- These results suggest that the proposed model is able to preserve image contrasts, as the regularizer doesn't rely on the height of signal.

- Theorem (contrast preservation):

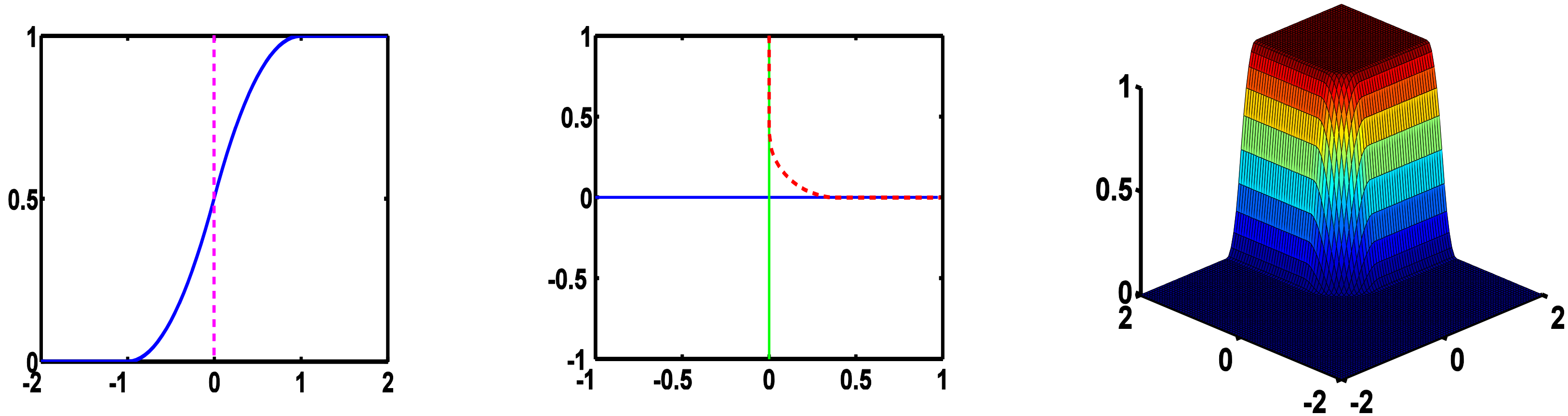
Let $f = h\chi_{B(0,R)}$ be an image defined on $\Omega = (-2R, 2R) \times (-2R, 2R)$. Define $S = \{u \in C^2(R^2) : u(x,y) = g(\sqrt{x^2 + y^2}), \text{g takes the same type of profile as shown.}\}$ then there exists a constant $C > 0$, such that if $\lambda < C$, then the following holds:

$$E(f) = \inf\{E(u) : u \in S\}$$

This property shows that the model attains a minimum at f if λ is small enough, i.e. the model restores f exactly and thus preserves contrast.

Why does our model preserve corners?

- Consider the function $f = h\chi_E$ with $E=(0,R)\times(0,R)$ defined on $\Omega = (-R,R)\times(-R,R)$



a generatrix function — a rotating orbit — an approximation surface to $f = h\chi_E$

$$\int_\Omega |H_f| = 2R$$ the perimeter of E inside Ω

- Theorem (corner preservation)

  Let $f = h\chi_{(0,R)\times(0,R)}$ be an image defined on $\Omega = (-R,R)\times(-R,R)$. Define $Q = \{u : \text{the surface of } z = u(x,y) \text{ is obtained by rotating the generatrix along the orbit.}\}$, then there exists a constant $C > 0$, such that if $\lambda < C$, then the following holds

$$E(f) = \inf\{E(u) : u \in Q\}$$

Existing numerical methods for the model

- Multigrid Algorithm by C. Brito-Loeza and K. Chen, 2010
- Augmented Lagrangian method by W. Zhu, X.C. Tai, T. Chan, 2011
- Augmented Lagrangian method by M. Myllykoski, R. Glowinski, T. Karkkainen, 2015
- A new augmented Lagrangian method by W. Zhu

Augmented Lagrangian Method (ALM)

- Related functionals

$$E(u) = \lambda \int_\Omega |\nabla u| + \frac{1}{2}\int_\Omega (f-u)^2$$

- non-differentiable
- nonlinear

$$E(u) = \int_\Omega \left[a + b\left(\nabla \cdot \frac{\nabla u}{|\nabla u|} \right)^2 \right] |\nabla u| + \frac{1}{2}\int_\Omega (f-u)^2$$

- high order
- non-differentiable
- nonlinear

- ALM has been successfully applied to the minimization of these functionals by X.C. Tai et al. (*SIIMS 2010 & 2011*)
  - convert the original minimization problem to be a constrained optimization one;
  - search for saddle points of the resulting problem by solving several associated sub-problems **alternatingly and repeatedly**

- Key of ALM: whether the sub-problems can be solved efficiently

ALM for the Mean Curvature Denoising (*Zhu,Tai,Chan IPI 2013*)

- The functional of the mean curvature denoising model:

$$\mathrm{E(u)} = \lambda \int_\Omega \left| \nabla \cdot \left(\frac{\nabla \mathrm{u}}{\sqrt{1 + |\nabla \mathrm{u}|^2}} \right) \right| + \frac{1}{2} \int_\Omega (\mathrm{f} - \mathrm{u})^2$$

- Consider an equivalent constrained optimization problem

$$\min_{u,p,q,n} \lambda \int_\Omega |q| + \frac{1}{2} \int_\Omega (f - u)^2$$

$$\text{subject to} \quad p = \nabla u, \quad n = \nabla u / \sqrt{1 + |\nabla u|^2}, \; q = \nabla \cdot n$$

- How to handle the following constraint?

$$n = p / \sqrt{1 + |p|^2}$$

Cont'd

- Instead, introduce the following new variables

$$p = \langle \nabla u, 1 \rangle,\ n = \langle \nabla u, 1 \rangle / \left| \langle \nabla u, 1 \rangle \right|,\ q = \nabla \cdot n$$

NOT $p = \nabla u$

- Obtain a new constrained optimization problem

$$\min_{u,p,q,n} \lambda \int_\Omega |q| + \frac{1}{2} \int_\Omega (f-u)^2$$

subject to $p = \langle \nabla u, 1 \rangle,\ q = \nabla \cdot \langle n_1, n_2 \rangle,\ n = \langle n_1, n_2, n_3 \rangle = p / |p|$

- The way to treat the last constraint *(the idea borrowed from Tai et al. SIIMS 2011)*

$$\textit{If } m \neq 0, p \neq 0, \textit{ and } |m| \leq 1, \textit{ then}$$

$$m = p / |p| \Leftrightarrow p = m \bullet p$$

- *Consider a modified constrained problem*

$$\min_{u,p,q,n,m} \lambda \int_\Omega |q| + \frac{1}{2} \int_\Omega (f-u)^2$$

subject to $p = \langle \nabla u, 1 \rangle,\ q = \nabla \cdot \langle n_1, n_2 \rangle,\ n = \langle n_1, n_2, n_3 \rangle,\ n = m,\ m = p / |p|,\ |m| \leq 1$

Details of the proposed ALM

- The associated augmented Lagrangian functional

$$
\begin{aligned}
L(u,q,p,n,m;\lambda_1,\lambda_2,\lambda_3,\lambda_4) = {} & \lambda\int|q| + \frac{1}{2}\int(f-u)^2 \\
& + r_1\int(|p| - p\cdot m) + \int\lambda_1(|p| - p\cdot m) \\
& + \frac{r_2}{2}\int\left|p - \langle\nabla u,1\rangle\right|^2 + \int\lambda_2\cdot\left(p - \langle\nabla u,1\rangle\right) \\
& + \frac{r_3}{2}\int\left|q - \partial_x n_1 - \partial_y n_2\right|^2 + \int\lambda_3\left(q - \partial_x n_1 - \partial_y n_2\right) \\
& + \frac{r_4}{2}\int|n-m|^2 + \int\lambda_4\cdot(n-m) + \delta_R(m)
\end{aligned}
$$

L1 penalization for $m = p/|p|$

$$
\delta_R(m) = \begin{cases} 0 & m\in R \\ \infty & otherwise \end{cases}; \qquad R = \left\{m\in L^2(\Omega) : |m| \le 1\, a.e. in\, \Omega\right\}
$$

- The sub-problems

$$
\varepsilon_1(u) = \frac{1}{2}\int(f-u)^2 + \frac{r_2}{2}\int\left|p - \langle\nabla u,1\rangle\right|^2 + \int\lambda_2\cdot\left(p - \langle\nabla u,1\rangle\right),
$$

$$
\varepsilon_2(q) = \lambda\int|q| + \frac{r_3}{2}\int\left|q - \partial_x n_1 - \partial_y n_2\right|^2 + \int\lambda_3\left(q - \partial_x n_1 - \partial_y n_2\right),
$$

$$
\varepsilon_3(p) = r_1\int(|p| - p\cdot m) + \int\lambda_1(|p| - p\cdot m) + \frac{r_2}{2}\int\left|p - \langle\nabla u,1\rangle\right|^2 + \int\lambda_2\cdot\left(p - \langle\nabla u,1\rangle\right),
$$

$$
\varepsilon_4(n) = \frac{r_3}{2}\int\left|q - \partial_x n_1 - \partial_y n_2\right|^2 + \int\lambda_3\left(q - \partial_x n_1 - \partial_y n_2\right) + \frac{r_4}{2}\int|n-m|^2 + \int\lambda_4\cdot(n-m),
$$

$$
\varepsilon_5(m) = r_1\int(|p| - p\cdot m) + \int\lambda_1(|p| - p\cdot m) + \frac{r_4}{2}\int|n-m|^2 + \int\lambda_4\cdot(n-m) + \delta_R(m).
$$

Cont'd

- Minimizers of $\varepsilon_2(q), \varepsilon_3(p), \varepsilon_5(m)$ have closed-form solutions

$$\underset{q}{\text{Argmin}}\,\varepsilon_2(q) = \max\left\{0, 1 - \frac{\lambda}{r_3|\tilde{q}|}\right\}\tilde{q}, \quad \tilde{q} = \nabla\cdot\langle n_1, n_2\rangle,$$

$$\underset{p}{\text{Argmin}}\,\varepsilon_3(p) = \max\left\{0, 1 - \frac{r_1 + \lambda_1}{r_2|\tilde{p}|}\right\}\tilde{p}, \quad \tilde{p} = \langle\nabla u, 1\rangle + \frac{(r_1 + \lambda_1)m - \lambda_2}{r_2},$$

$$\underset{m}{\text{Argmin}}\,\varepsilon_5(m) = \begin{cases} \tilde{m} & |\tilde{m}| \le 1 \\ \tilde{m}/|\tilde{m}| & |\tilde{m}| > 1 \end{cases}, \quad \tilde{m} = n + \frac{(r_1 + \lambda_1)p + \lambda_4}{r_4}$$

- Euler-Lagrange equations for $\varepsilon_1(u), \varepsilon_4(n)$

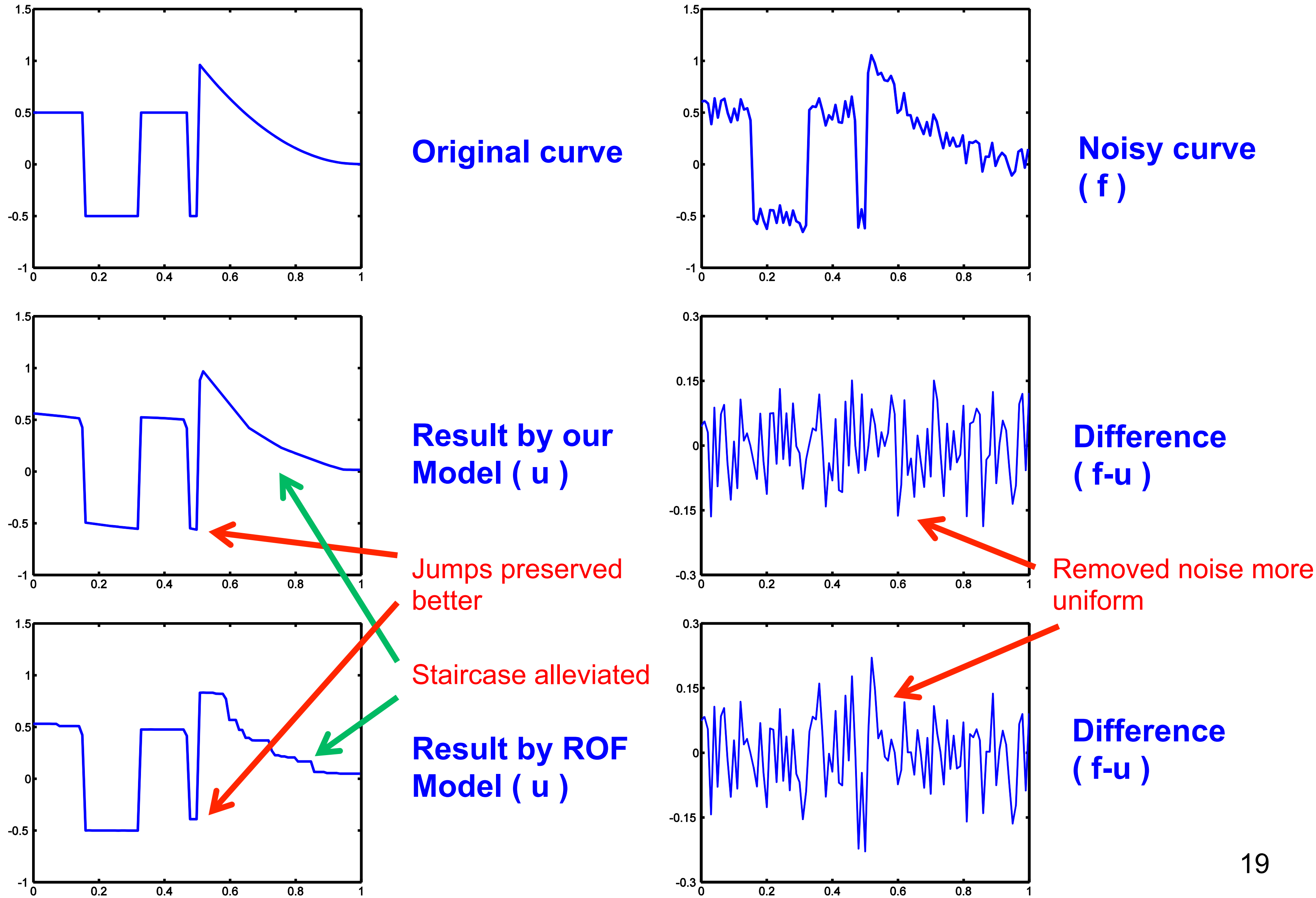
$$-r_2\Delta u + u = f - (r_2 p_1 + \lambda_{21})_x - (r_2 p_2 + \lambda_{22})_y,$$

$$\begin{cases} -r_3\left(\partial_x n_1 + \partial_y n_2\right)_x + r_4 n_1 = r_4 m_1 - \lambda_{41} - (r_3 q + \lambda_3)_x, \\ -r_3\left(\partial_x n_1 + \partial_y n_2\right)_y + r_4 n_2 = r_4 m_2 - \lambda_{42} - (r_3 q + \lambda_3)_y, \\ n_3 = m_3 - \lambda_{43}/r_4 \end{cases}$$

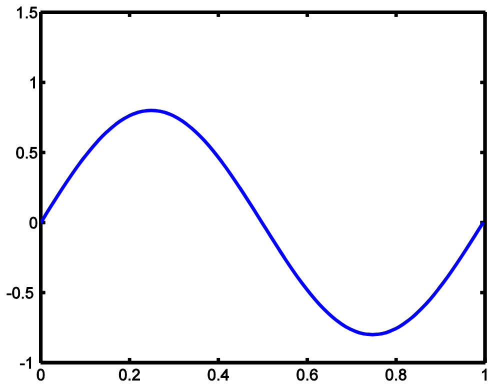
can be solved using FFT

All the sub-problems can be solved efficiently and accurately.

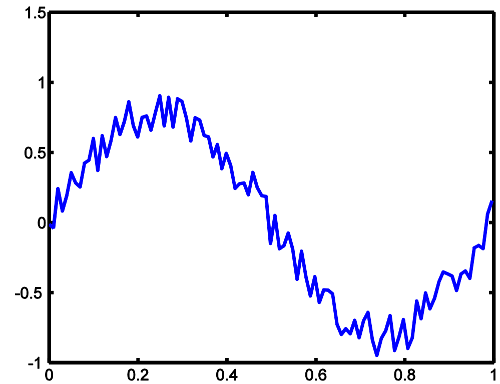
Experiments (1D)



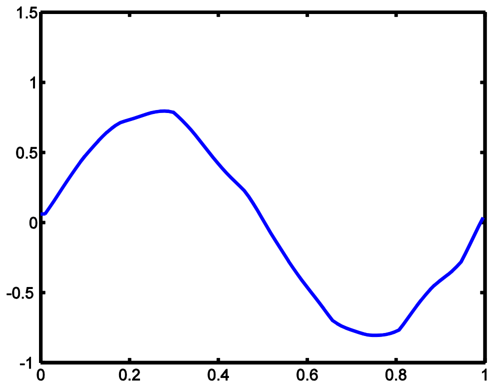
Experiments (1D)



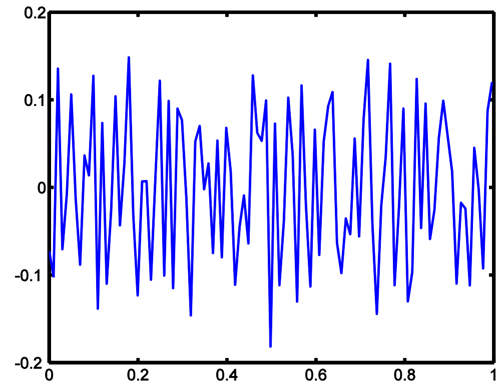
Original curve



Noisy curve (f)



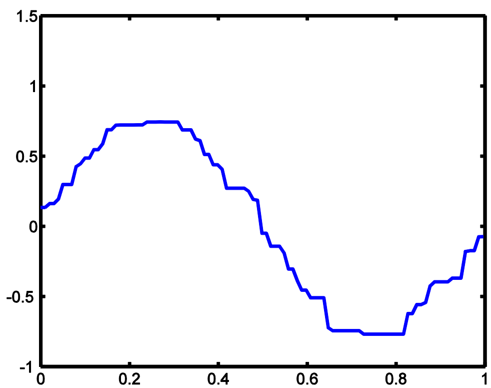
Result by our Model (u)



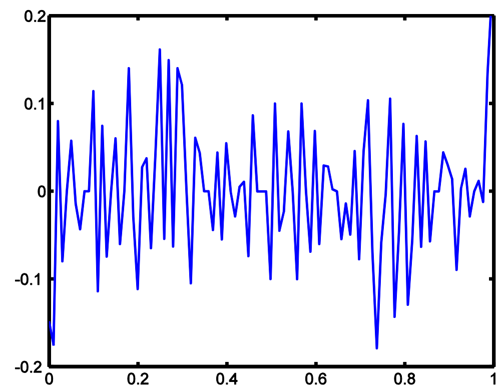
Difference (f-u)

Removed noise more uniform

Staircase alleviated

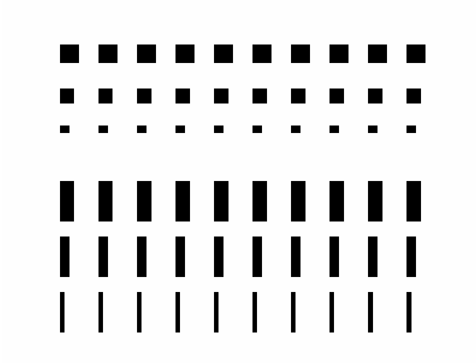


Result by ROF Model (u)

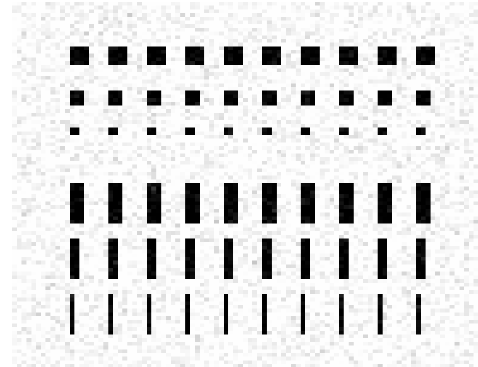


Difference (f-u)

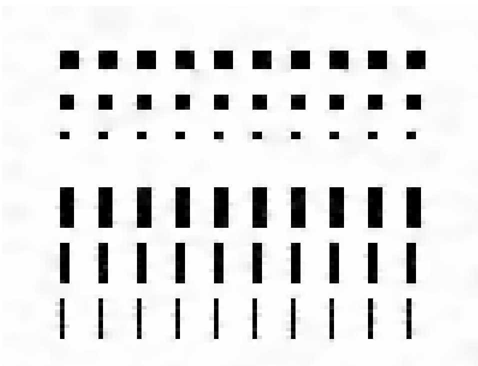
Experiments (2D)



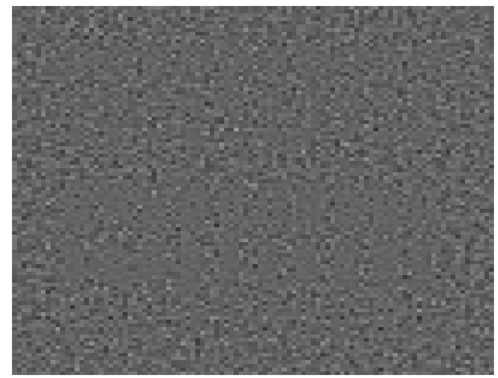
Original “Bars”



Noisy “Bars”
(f)

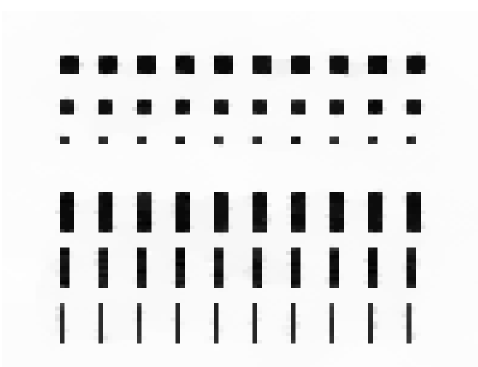


Result by our
Model (u)

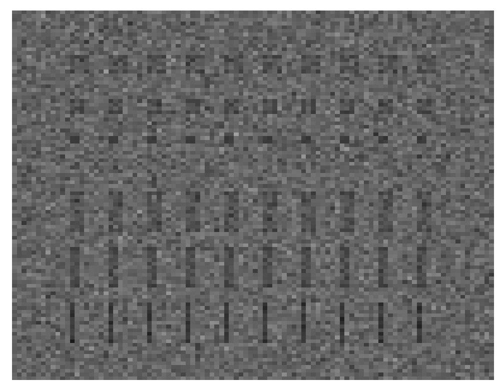


Difference
(f-u)

Contrast preserved
better

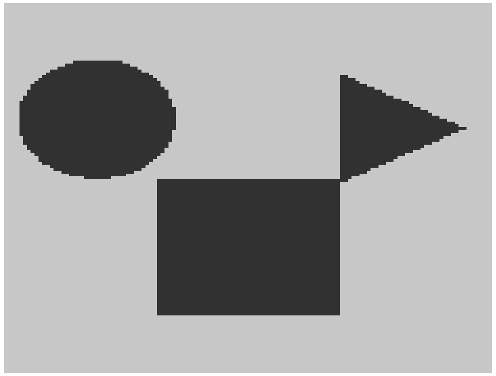


Result by ROF
Model (u)

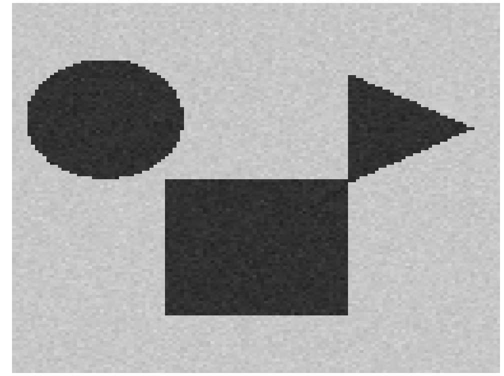


Difference
(f-u)

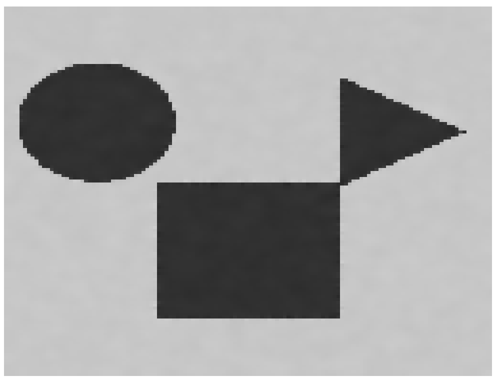
Experiments (2D)



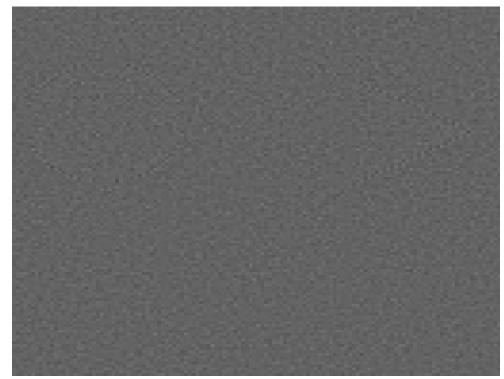
Original “Shapes”



Noisy “Shapes” (f)

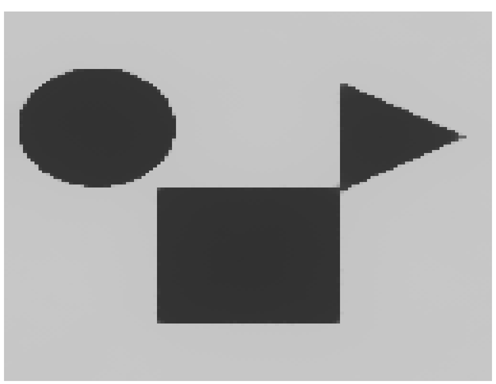


Result by our Model (u)

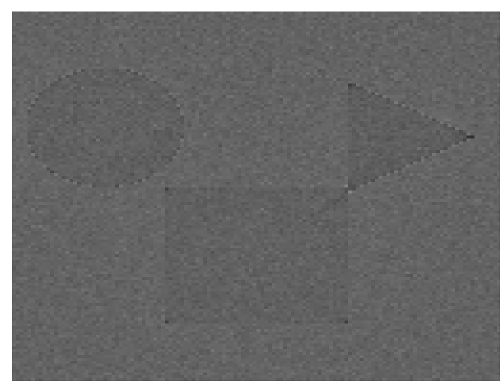


Difference (f-u)

As indicated in f-u, Contrast and corners Preserved better



Result by ROF Model (u)



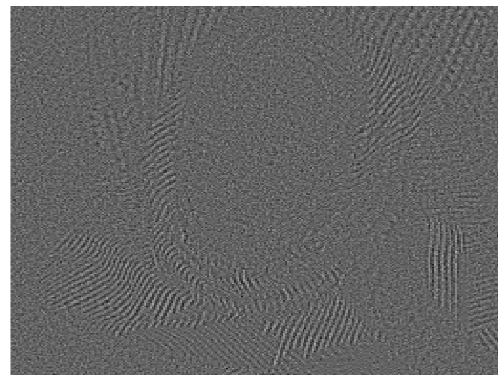
Difference (f-u)

Experiments (2D)

Original "Barbara"

Noisy "Barbara" (f)

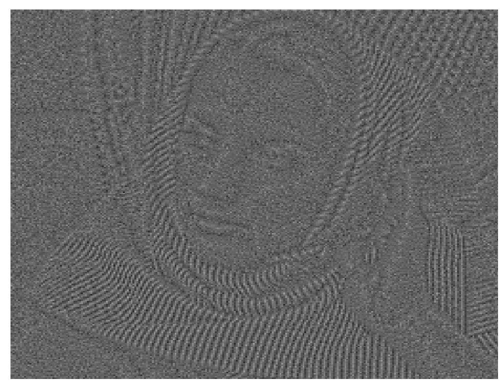
Result by our Model (u)



Difference (f-u)

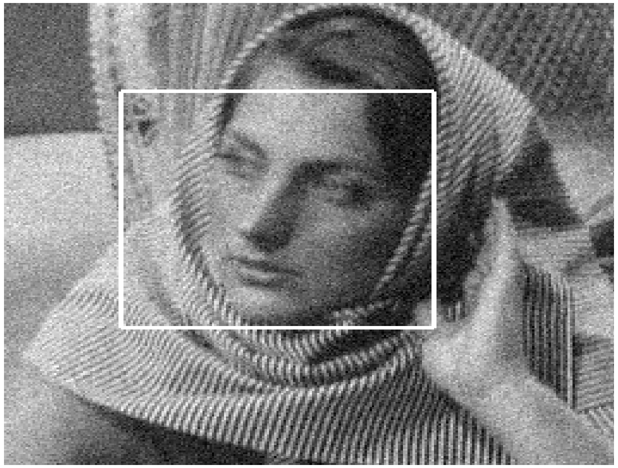
Large scale signal, such as face preserved better

Result by ROF Model (u)



Difference (f-u)

Experiments (2D)



Original “Barbara”

Local patch

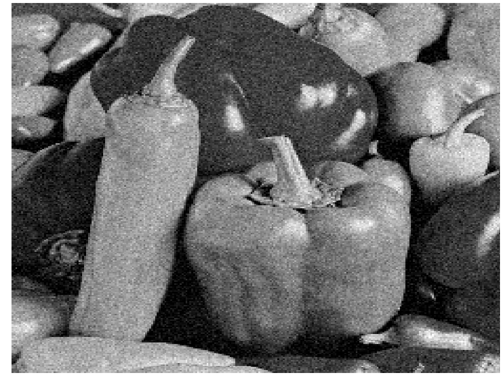
By our model

Staircase effect alleviated

By ROF model

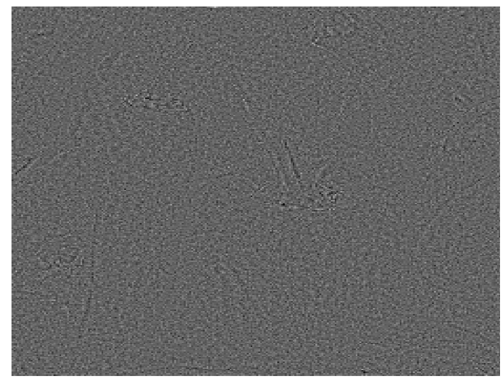
Experiments (2D)

Original "Peppers"



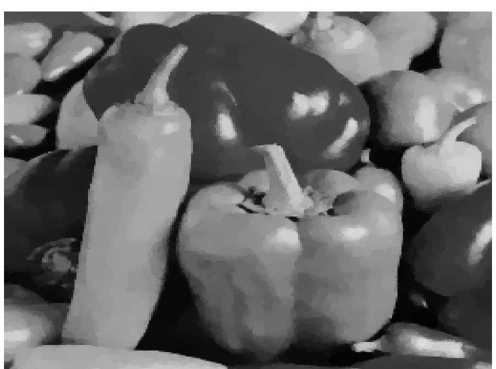
Noisy "Peppers" (f)

Result by our Model (u)

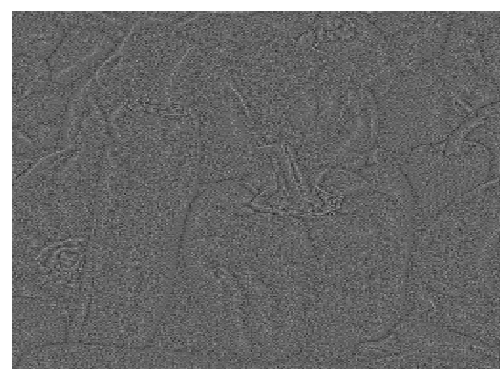


Difference (f-u)

Large scale signal, such as surface of pepper, preserved better

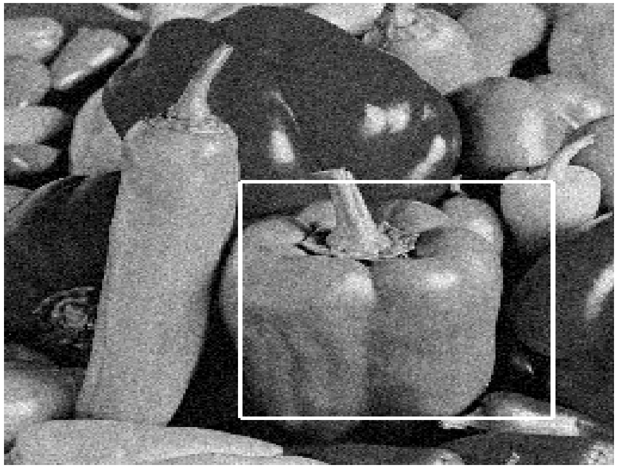


Result by ROF Model (u)

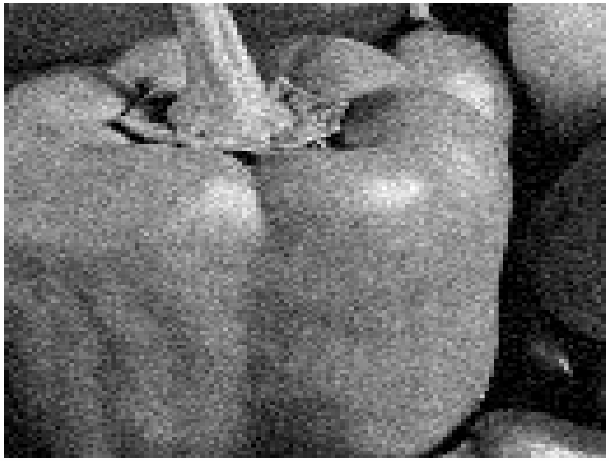


Difference (f-u)

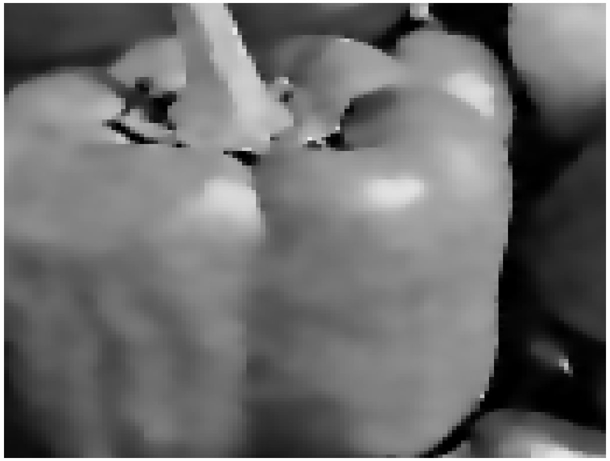
Experiments (2D)



Original “Peppers”



Local patch

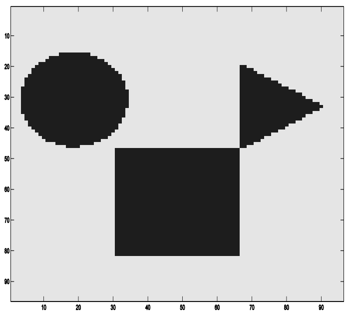


By our model

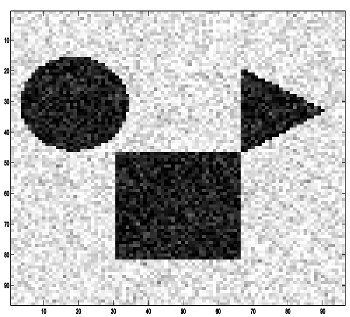
Staircase effect alleviated

By ROF model

Comparison with other high-order models

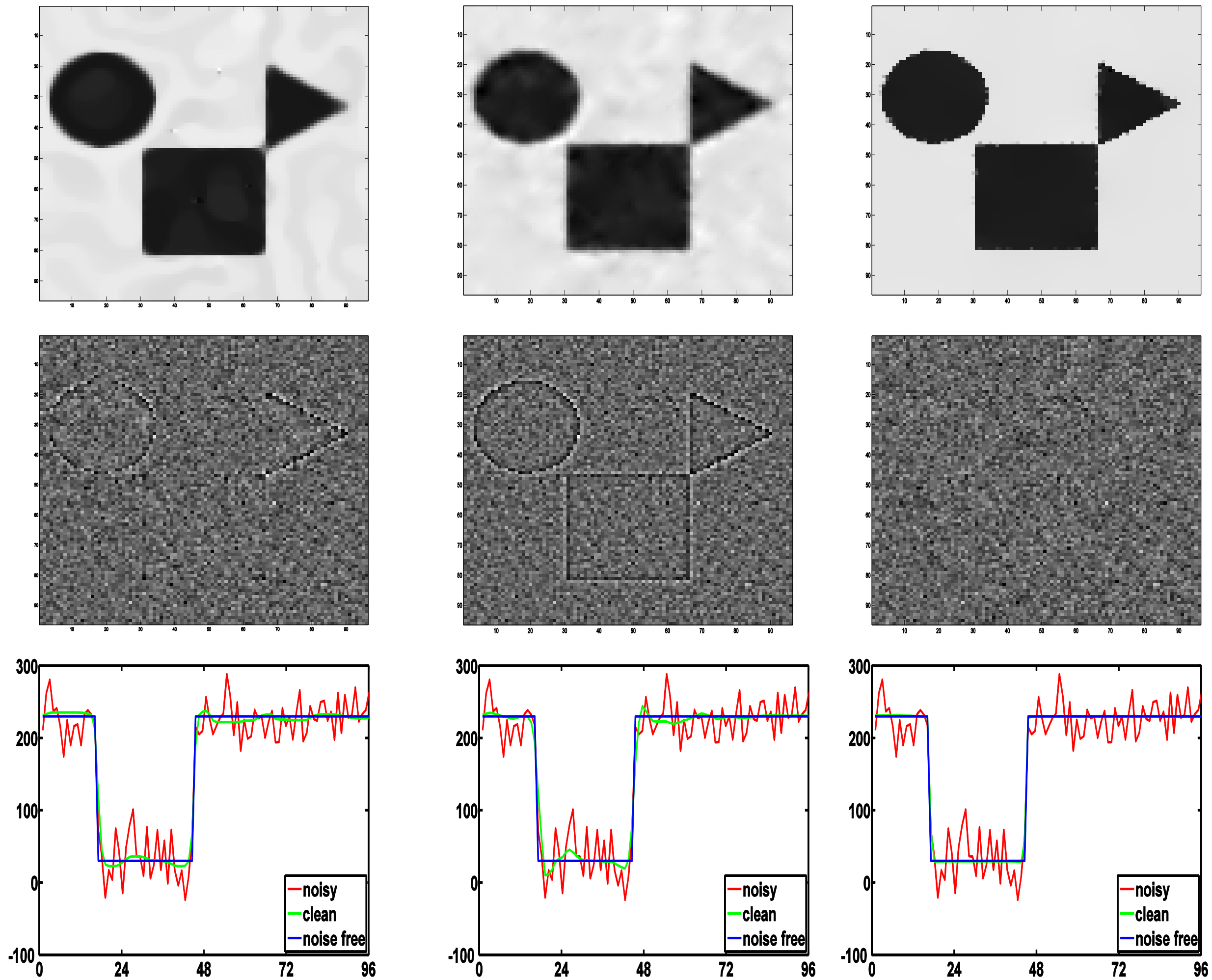


noise-free image



noisy image

A slice of the noise-free (B), noisy (R), and cleaned image (G)



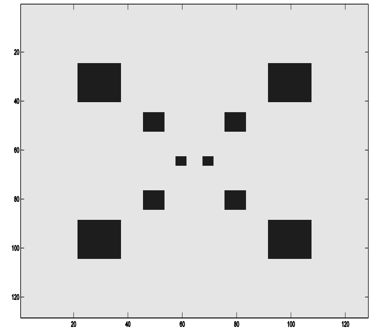
By Euler's elastica model

By the LLT model

By our model

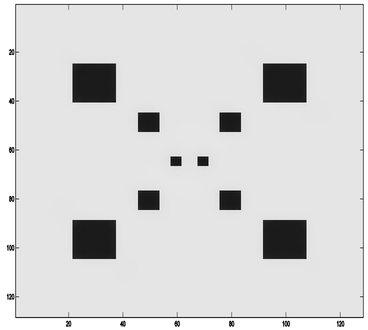
Contrast and corners preserved better than other models

Data-Driven Selection Property



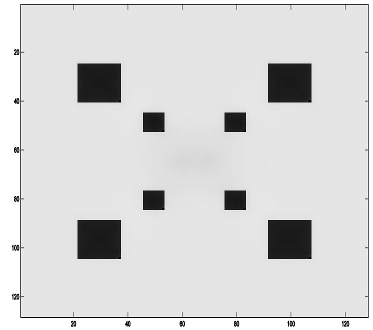
Original image f

When the regularization parameter increases, objects of small scales will be removed first and then the ones of relatively larger scales.



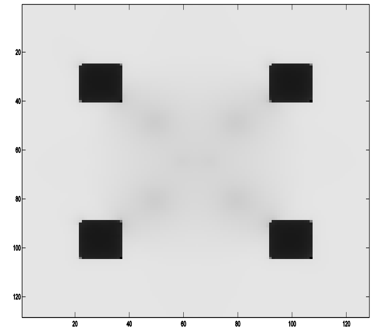
clean image u

$\lambda = 3.0 \times 10^3$



clean image u

$\lambda = 2.5 \times 10^4$



clean image u

$\lambda = 4.0 \times 10^4$

TV-L1 shares a similar property, but cannot preserve corners of objects

New ALM for the MC denoising (Z.,16)

- Goal: reduce the number of Lagrange multipliers
  - Ease the effort of choosing penalty parameters
  - With fewer Lagrange multipliers, the connections among variables become more tight so that curvature can be more faithfully captured

- Consider the following constrained problem

$$\min_{u,q,n} \lambda \int_\Omega |q| + \frac{1}{2}\int_\Omega (f-u)^2$$

$$\text{subject to } \quad n = \nabla u / \sqrt{1+|\nabla u|^2}, \; q = \nabla \cdot n$$

- and the following augmented Lagrangian functional

$$\begin{aligned} L(u,q,n;\lambda_1,\lambda_2) &= \lambda \int |q| + \frac{1}{2}\int (f-u)^2 \\ &+ \frac{r_1}{2}\int |q - \nabla \cdot n|^2 + \int \lambda_1 (q - \nabla \cdot n) \\ &+ \frac{r_2}{2}\int \left| \frac{\nabla u}{\sqrt{1+|\nabla u|^2}} - n \right|^2 + \int \lambda_2 \cdot \left(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}} - n \right) \end{aligned}$$

Only two Lagrange multipliers!

Cont'd

- Sub-problems

$$\varepsilon_1(u) = \frac{1}{2}\int (f-u)^2 + \frac{r_2}{2}\int \left| \frac{\nabla u}{\sqrt{1+|\nabla u|^2}} - n \right|^2 + \int \lambda_2 \cdot \left(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}} - n \right),$$

$$\varepsilon_2(q) = \lambda \int |q| + \frac{r_1}{2}\int |q - \nabla\cdot n|^2 + \int \lambda_1 (q - \nabla \cdot n),$$

$$\varepsilon_3(n) = \frac{r_1}{2}\int |q - \nabla\cdot n|^2 + \int \lambda_1 (q - \nabla \cdot n) + \frac{r_2}{2}\int \left| \frac{\nabla u}{\sqrt{1+|\nabla u|^2}} - n \right|^2 + \int \lambda_2 \cdot \left(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}} - n \right)$$

- Solving the sub-problems

$$Arg\min_q \varepsilon_2(q) = \max\left\{0, 1 - \frac{\lambda}{r_1|\tilde{q}|}\right\}\tilde{q}, \qquad \tilde{q} = \nabla\cdot n - \frac{\lambda_1}{r_1};$$

Closed-form solution

$$-r_1\nabla(\nabla\cdot n) + r_2 n = -\nabla(r_1 q + \lambda_1) + r_2 \frac{\nabla u}{\sqrt{1+|\nabla u|^2}} + \lambda_2$$

Can be solved using FFT

$$\nabla\cdot\left[\frac{A}{\sqrt{1+|\nabla u|^2}} - \left(\frac{A}{\sqrt{1+|\nabla u|^2}}\cdot\frac{\nabla u}{\sqrt{1+|\nabla u|^2}}\right)\frac{\nabla u}{\sqrt{1+|\nabla u|^2}}\right] + (f-u) = 0, \quad A = r_2\left(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}} - n\right) + \lambda_2$$

Compared with the previous ALM, this equation is much more complex;
We can use FFT to solve it by fixing the nonlinear terms

Quantities monitoring the convergence of iteration

- Residuals:

$$\left(R_1^k, R_2^k\right) = \frac{1}{|\Omega|}\left(\left|\widetilde{R}_1^k\right|_{L1}, \left|\widetilde{R}_2^k\right|_{L1}\right)$$

$$\widetilde{R}_1^k = q^k - \nabla \cdot n^k,$$

$$\widetilde{R}_2^k = \frac{\nabla u^k}{\sqrt{1 + \left|\nabla u^k\right|^2}} - n^k; \quad k = 1,2,\ldots$$

- Relative errors of Lagrange multipliers:

$$\left(L_1^k, L_2^k\right) = \left(\frac{\left|\lambda_1^k - \lambda_1^{k-1}\right|_{L1}}{\left|\lambda_1^{k-1}\right|_{L1}}, \frac{\left|\lambda_2^k - \lambda_2^{k-1}\right|_{L1}}{\left|\lambda_2^{k-1}\right|_{L1}}\right), \; k = 1,2,\ldots$$

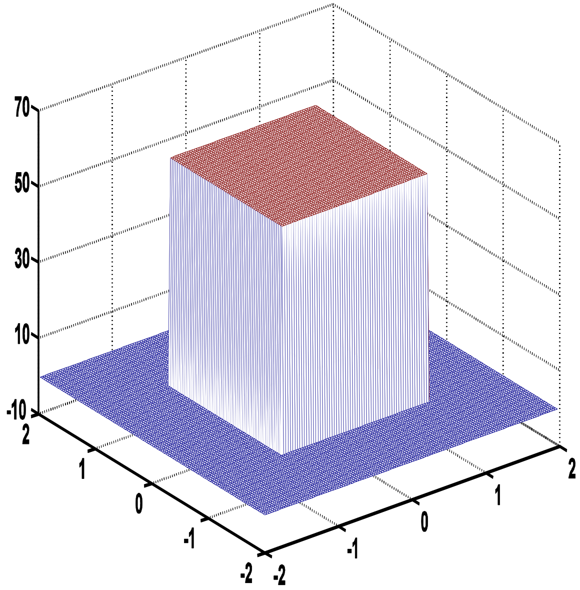
- Relative error of u^k:

$$\frac{\left|u^k - u^{k-1}\right|_{L1}}{\left|u^{k-1}\right|_{L1}}, \; k = 1,2,\ldots$$

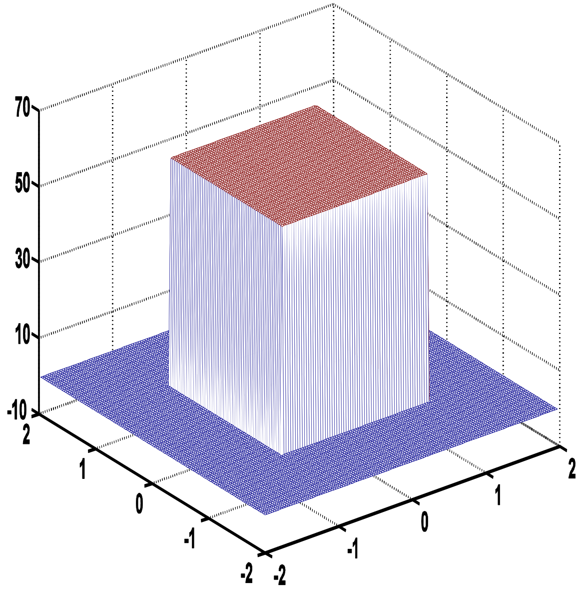
Preservation of contrasts and corners

- Consider an example

$$f(x)=\begin{cases}60 & x\in D\\ 0 & x\in\Omega\setminus D\end{cases}\qquad \Omega=[-2,2]\times[-2,2], D=[-1,1]\times[-1,1].$$



f

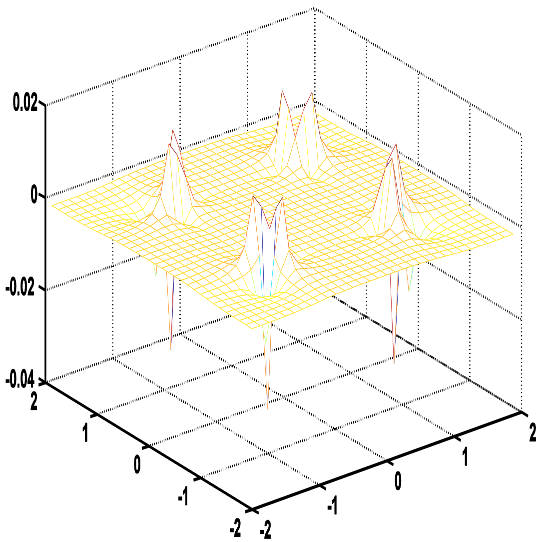


u

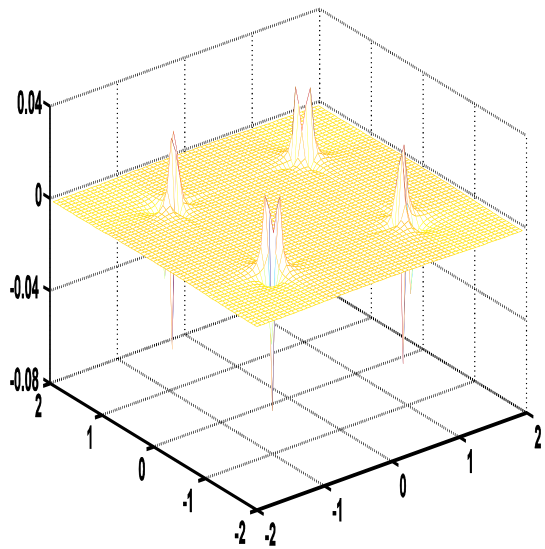
$$\lambda=1, N=128$$

Preservation of contrasts and corners

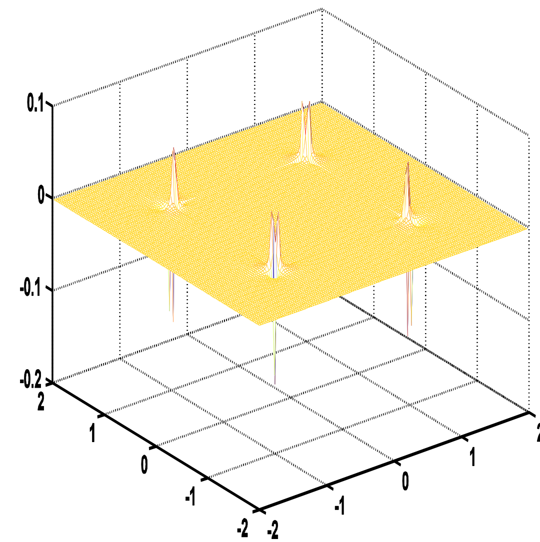
- The difference function “u-f”



$N = 32$



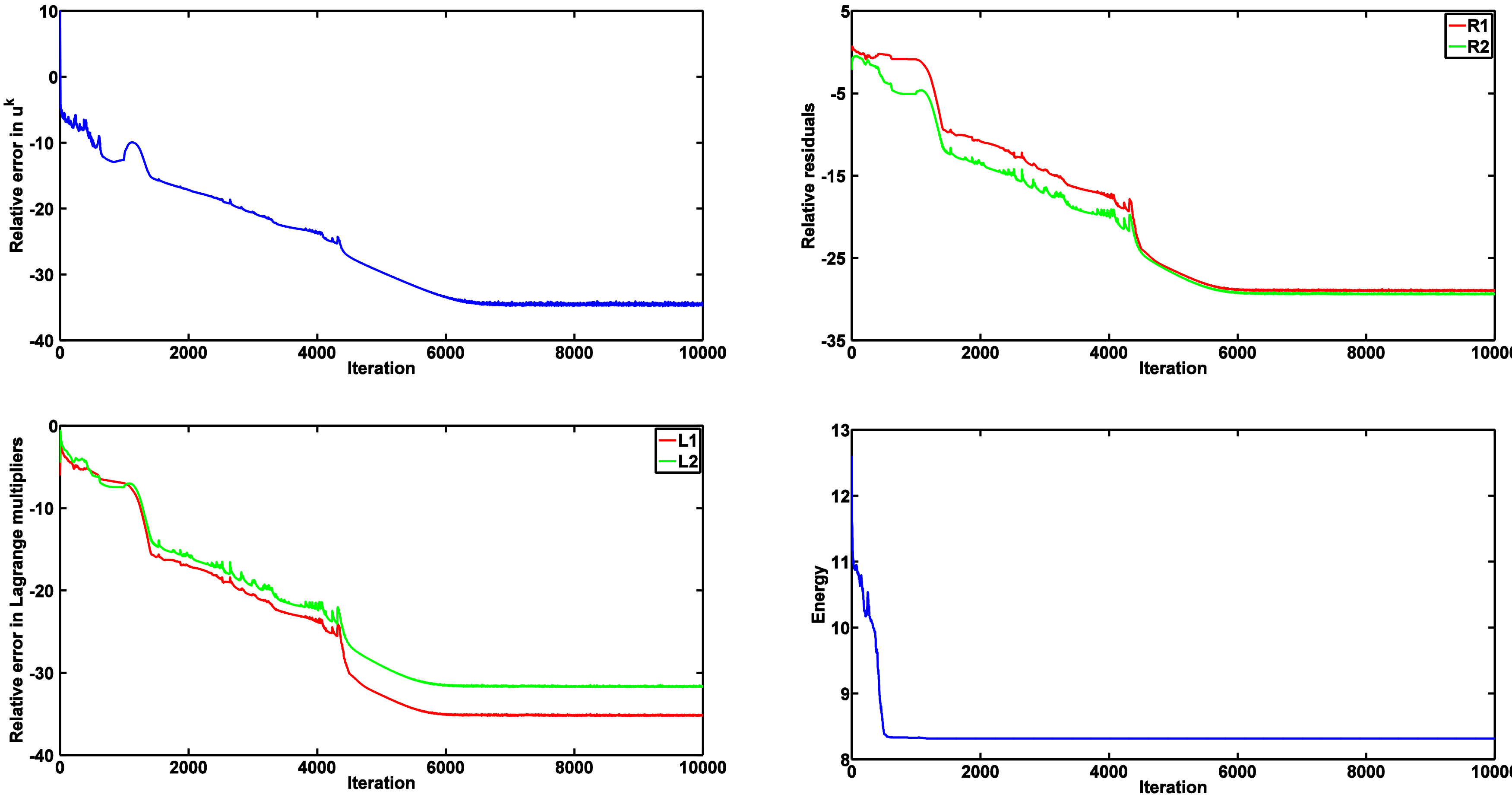
$N = 64$



$N = 128$

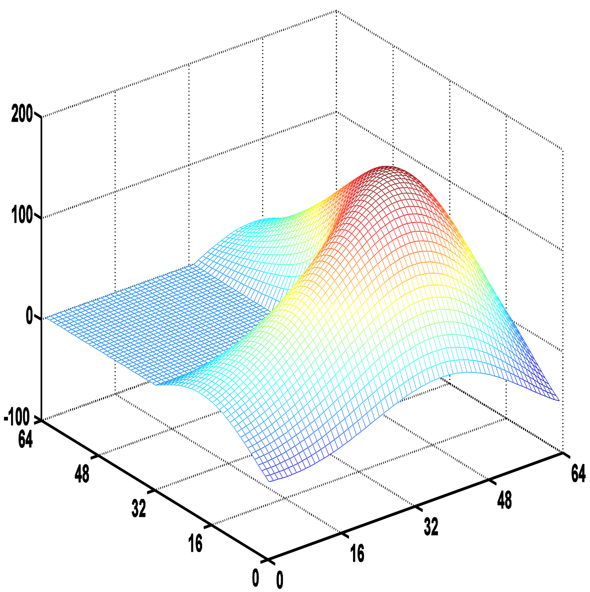
Image contrast and four corners can be well preserved

Convergence

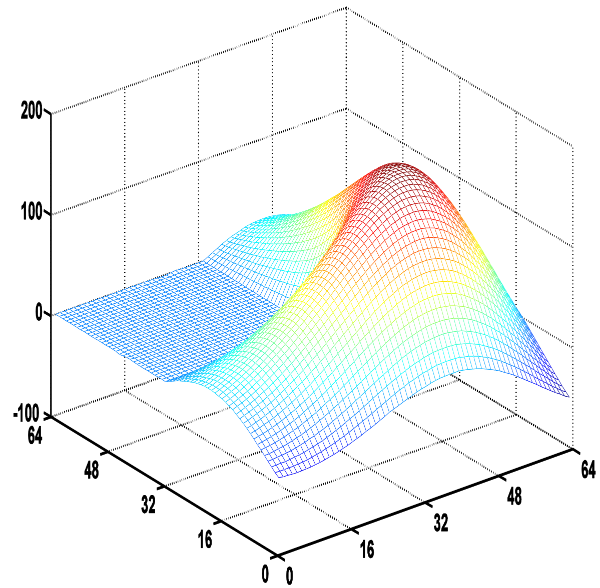


The relative errors around 1e-16, the machine precision of Matlab, indicating some minimizer is approached.

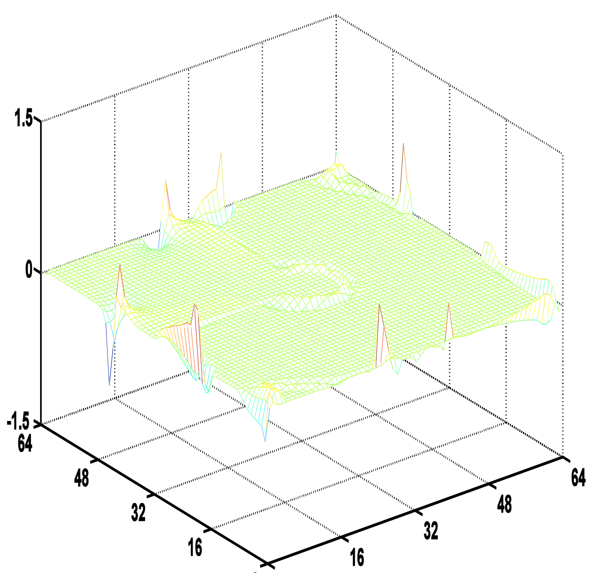
Suffering from staircase?



f



u

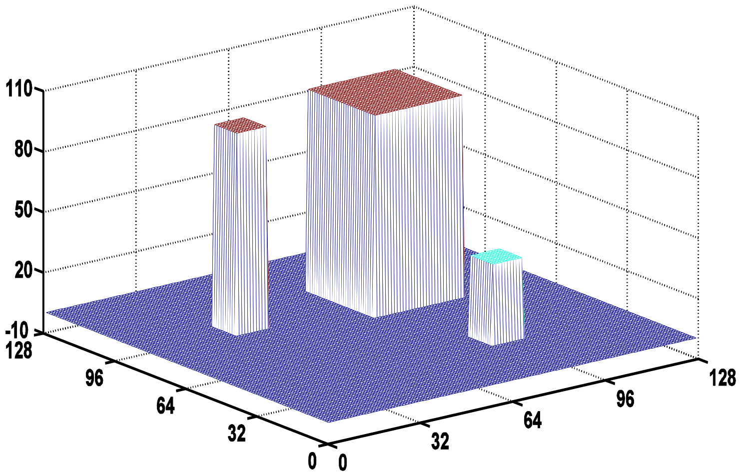


$u - f$

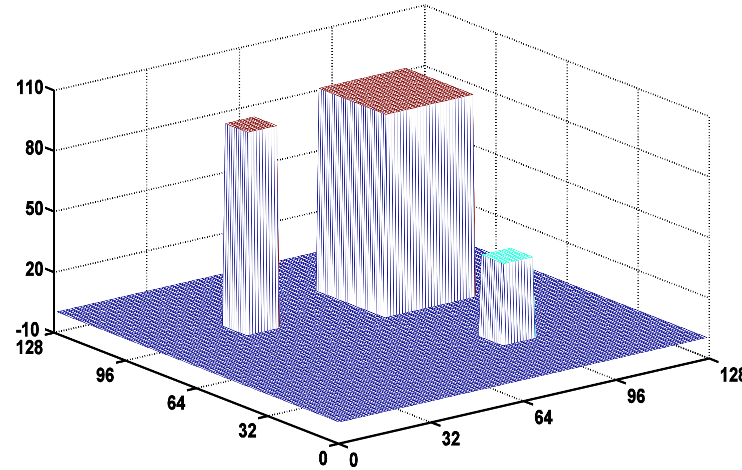
$$f = 160 * membrane\ (1,32)$$

The model is free from the staircase effect

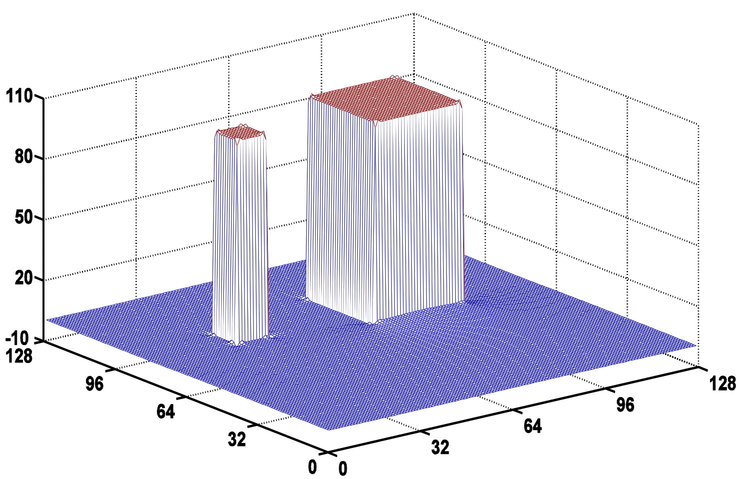
data selection



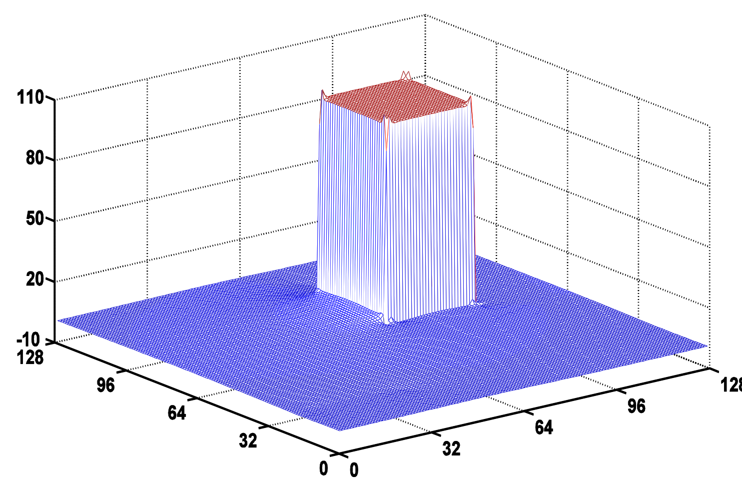
f



$u \ (\lambda = 10)$

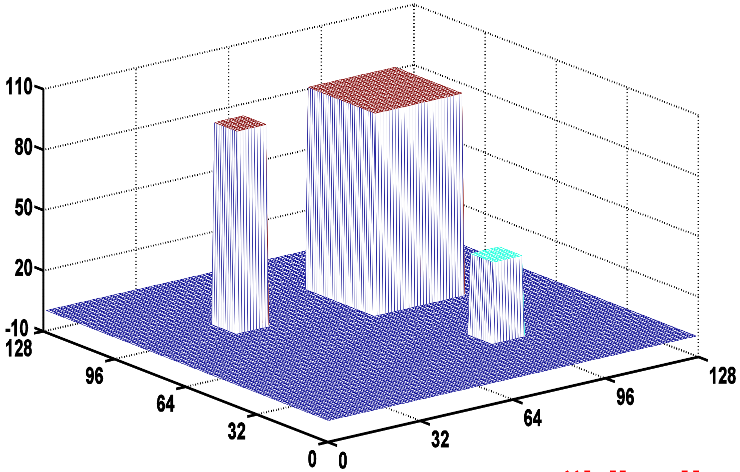


$u \ (\lambda = 1000)$



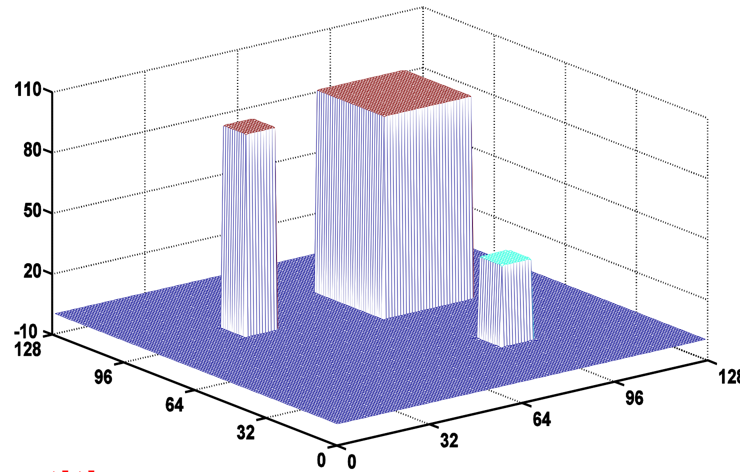
$u \ (\lambda = 2400)$

The role of spatial size h

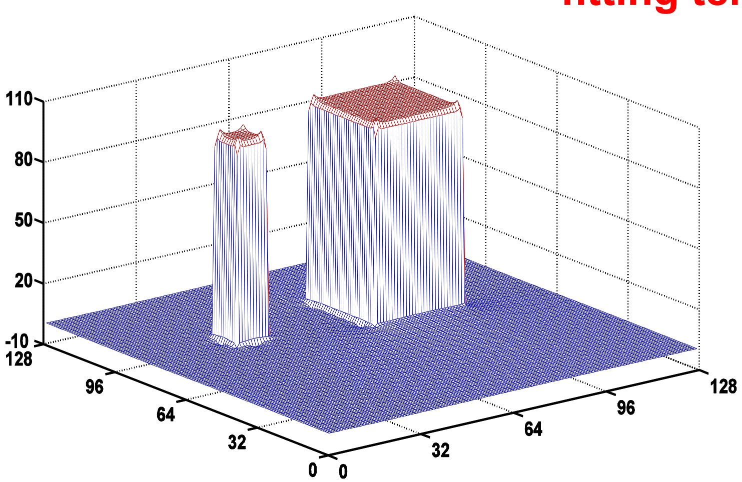


f

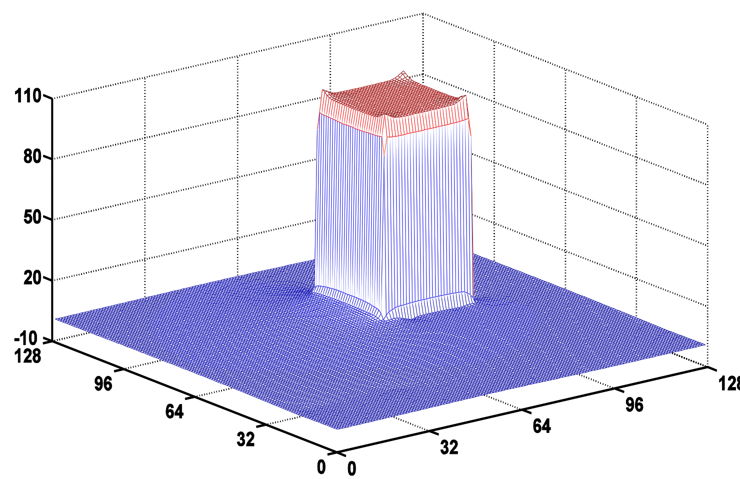
"h" adjusts the competition
Of the regularization and the
fitting terms



$u \ \ (\lambda = 100, h = 1)$



$u \ \ (\lambda = 100, h = 0.25)$



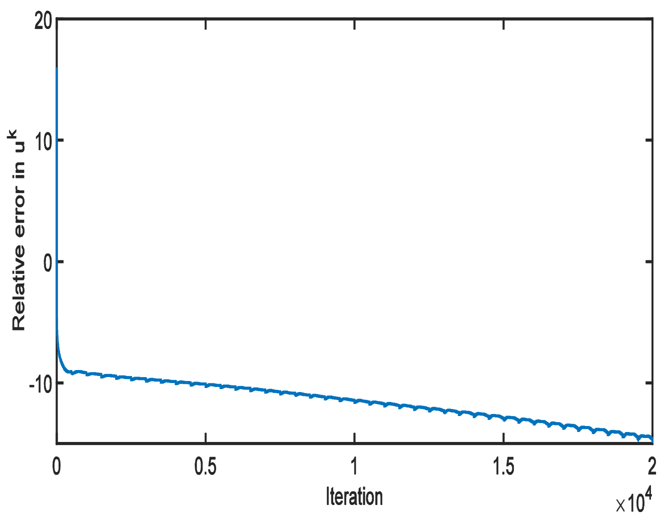
$u \ \ (\lambda = 100, h = 0.2)$

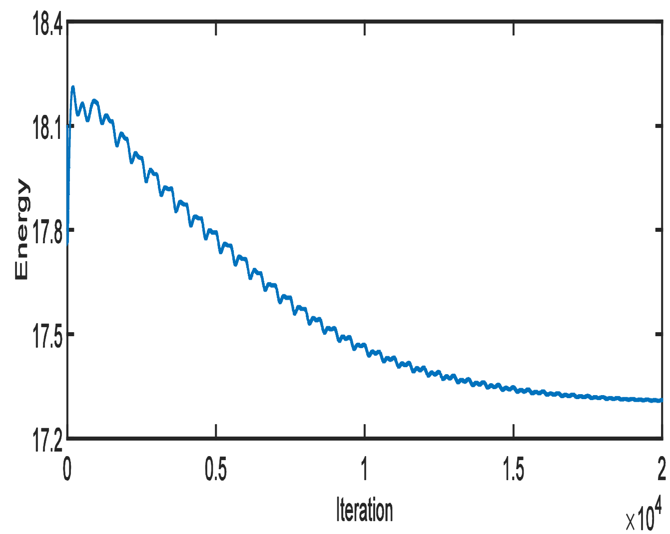
Real Image

f

u

h=10





How about L^{p}-mean curvature denoising?

f

$u\,(by\,L^1 - MC)$

$u\,(by\,L^2 - MC)$

This comparison suggests that using L^p -norm of mean curvature with $p \in [1,2]$ as a regularizer is also be a good choice.

Summary and future work

- Summary of the proposed model
  - sweep noise while keeping edges
  - preserve image contrast and corners
  - free of staircase effect
  - nonconvex
- Future work
  - Explore the features of L^p-norm of mean curvature based regularizers for $p \in [1,2]$ and apply them for other imaging problems
  - Construct more efficient numerical method for solving the u-subproblem of the new ALM

Our related papers

- *W. Zhu and T. Chan,*
  Image denoising using mean curvature of image surface, SIIMS 2012.
- *W. Zhu, X.C. Tai and T. Chan,*
  Augmented Lagrangian method for a mean curvature based image denoising model, Inverse Probl. Imag., 2013.
- *W. Zhu*
  A numerical study of a mean curvature based image denoising using augmented Lagrangian method, 2016.
- *W. Zhu*
  Image denoising using L^p-norm of mean curvature, 2016.

Thank you for your attention!!!