Chimera and Chimera-Like States in Populations of Homogeneous and Heterogeneous Chemical Oscillators

Ken Showalter

SIAM Conference on Applications of Dynamical Systems
Snowbird, Utah, May 21 - 25, 2017

Co-Workers:
Simbarashe Nkomo
Jan Totz
Mark Tinsley
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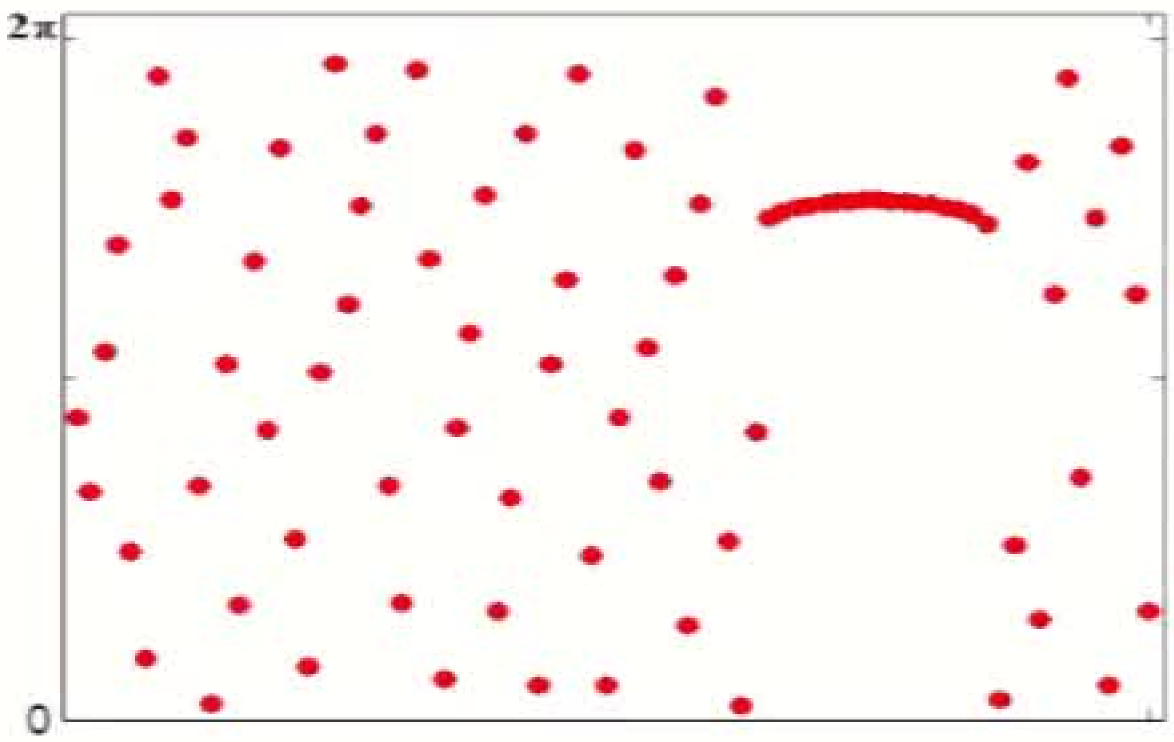

Funding:
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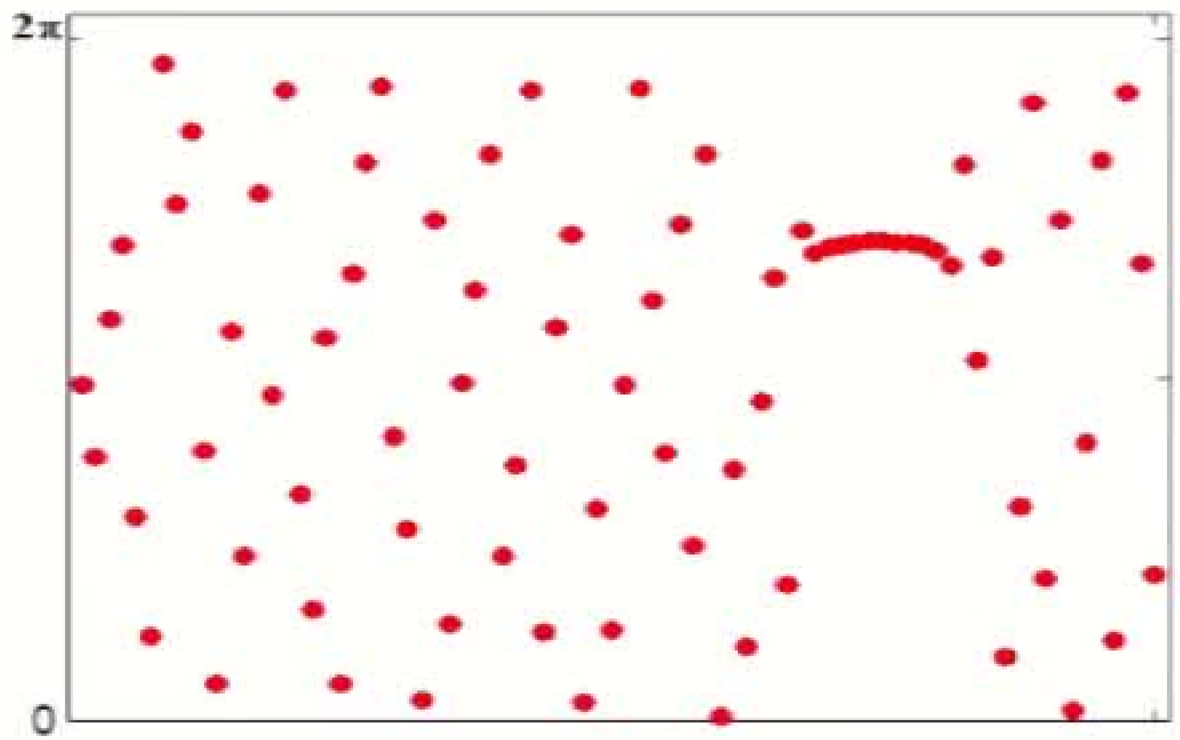

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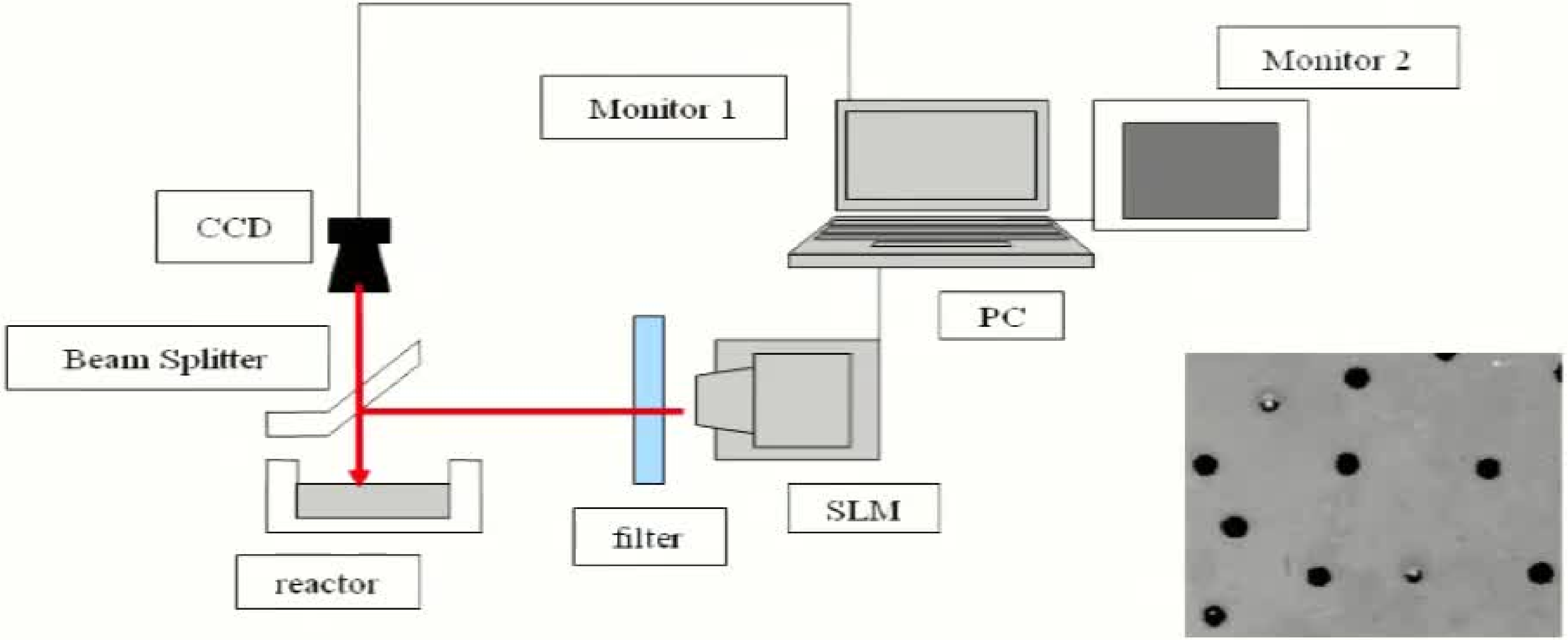
Photosensitive BZ Oscillator System: Experimental Setup

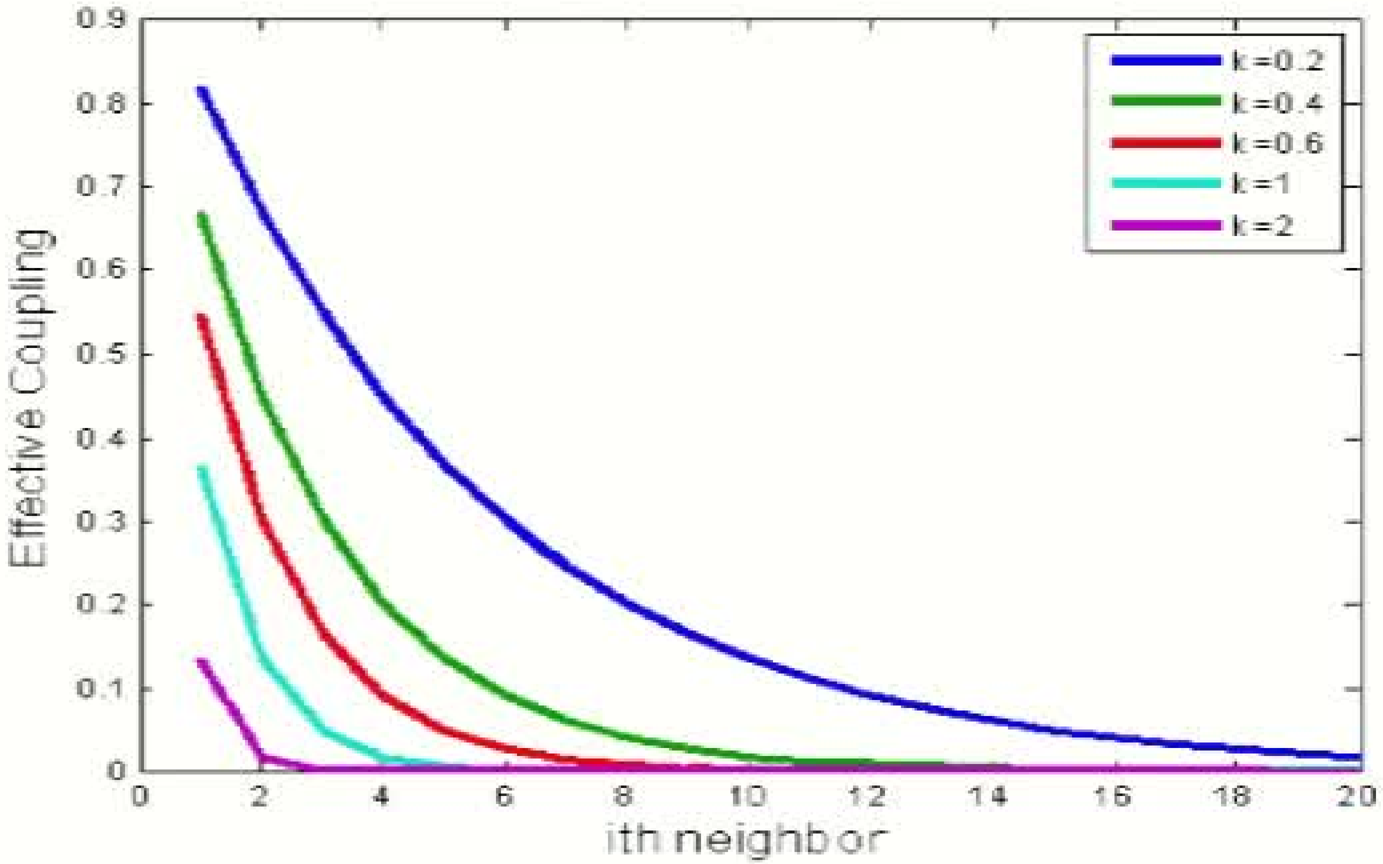


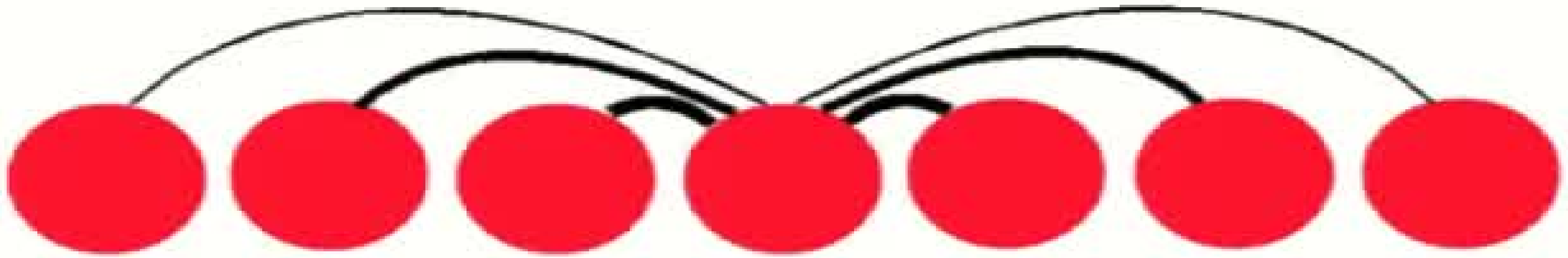
The projected light intensity on oscillatory particle j is:

$$\phi_j = \phi_0 + \sum_{\rho=j-n}^{j+n} K \left(I_\rho(t-\tau) - I_j(t) \right)$$

M. R. Tinsley, S. Nkomo, KS, *Nature Physics* **8**, 662 (2012).

Kuramoto Nonlocal Coupling: Chimera Dynamics





$$\phi_j = \phi_0 + \sum_{\rho=j-n}^{j+n} K\ (I_\rho(t-\tau) - I_j(t))$$

$$K = K' \exp(-\kappa|\rho - j|)$$

$$R(j,t) = \frac{1}{2m}\left|\sum^{j+m} \exp(i\theta(\rho,t))\right|$$

Nonlocal Coupling

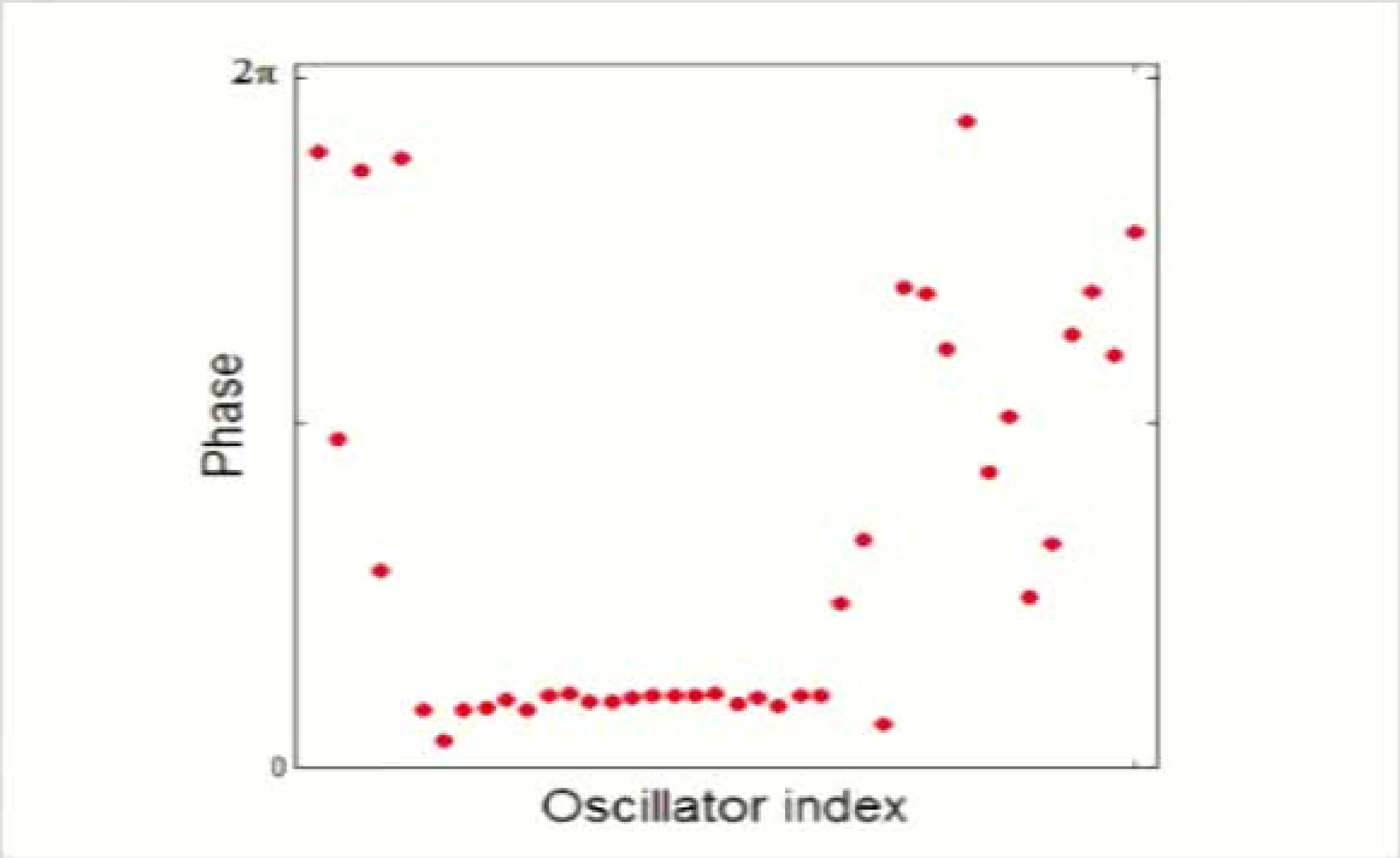
φ_j is the imposed light intensity on oscillator j, φ_0 is the background intensity, I_j and I_σ are the transmitted light intensities of oscillators j and σ, κ and K are the coupling constants, and τ is the time delay.

Local Order Parameter R

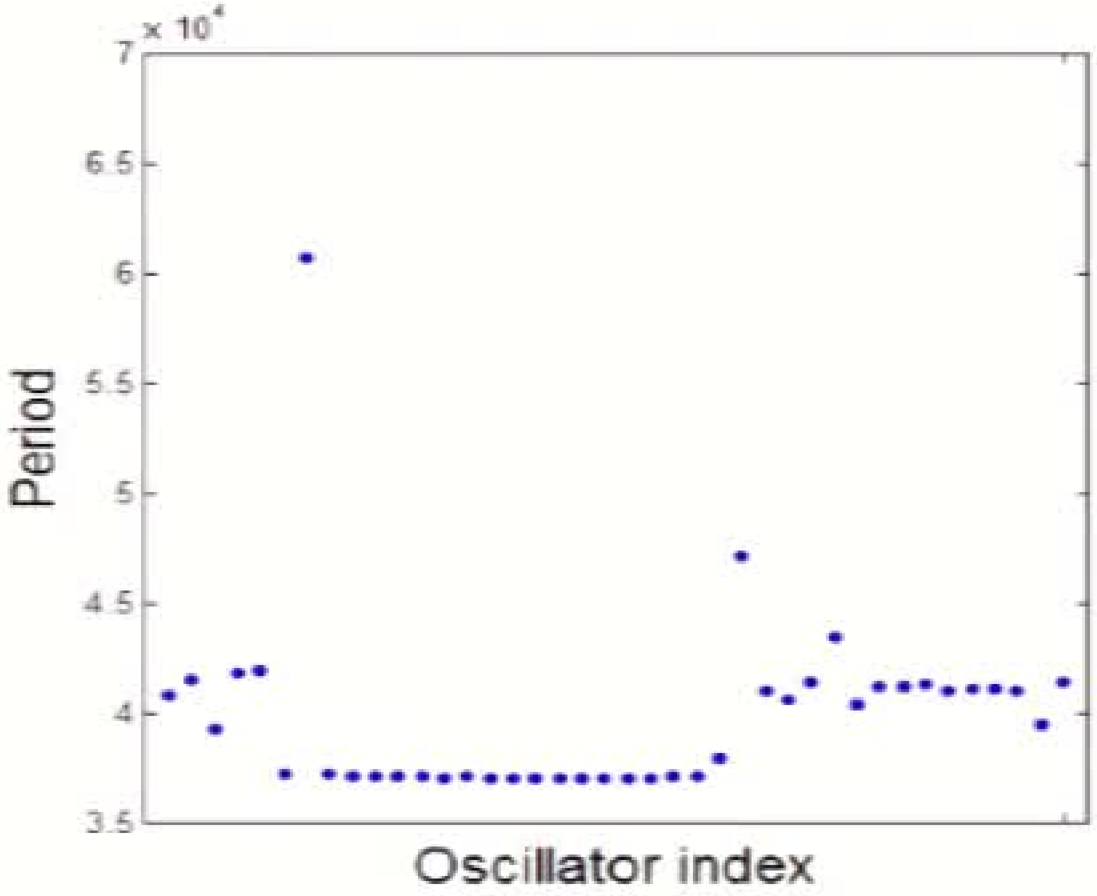
m is the sampling radius and θ is the phase.

S. Nkomo, M. R. Tinsley, KS, *Phys. Rev. Lett.* **110**, 244102 (2013).

Chimera Dynamics for Heterogeneous Oscillators



~50 oscillations



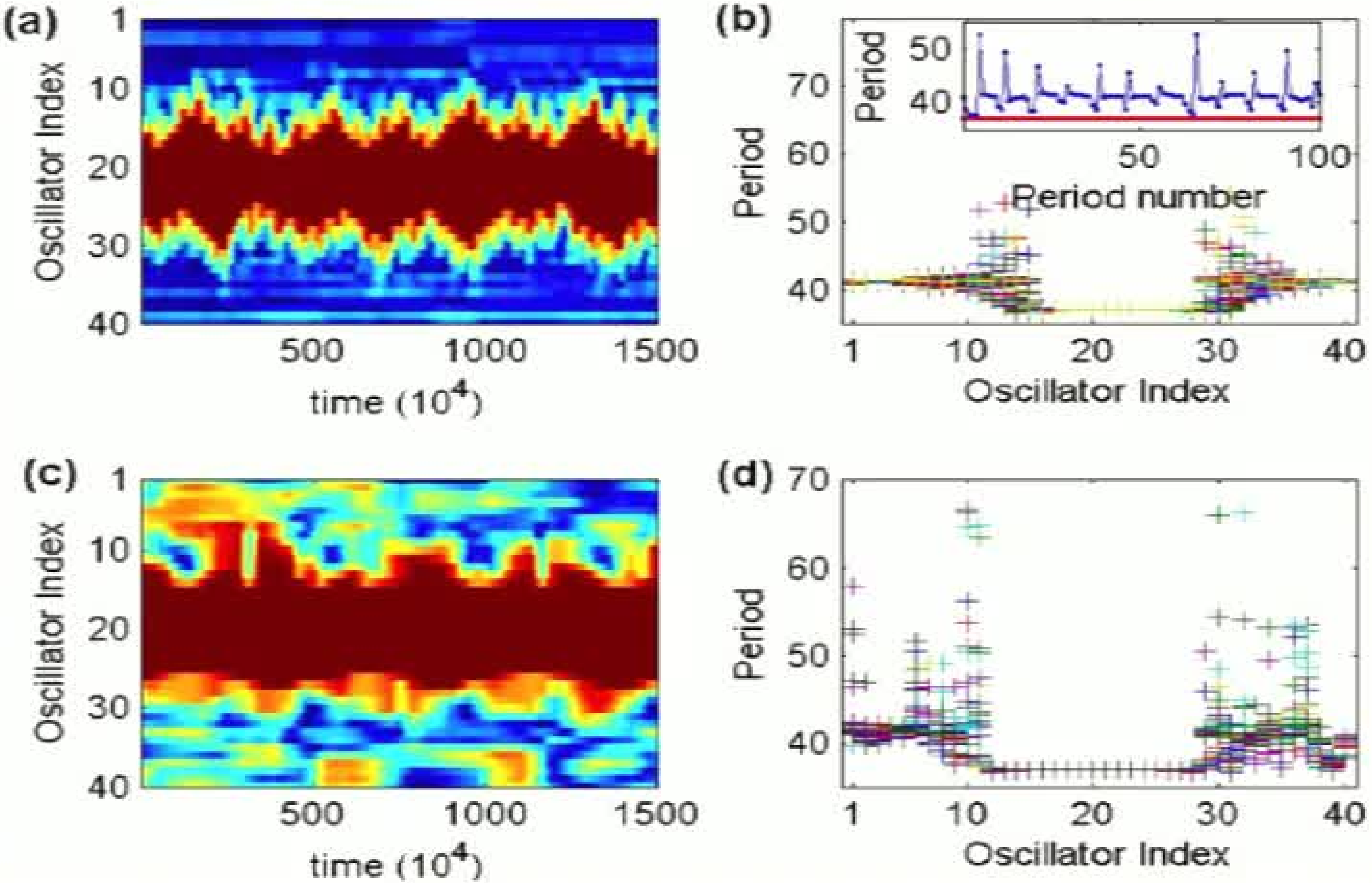
~250 oscillations

Random initial phase distribution $\kappa = 0.4$

40 oscillators

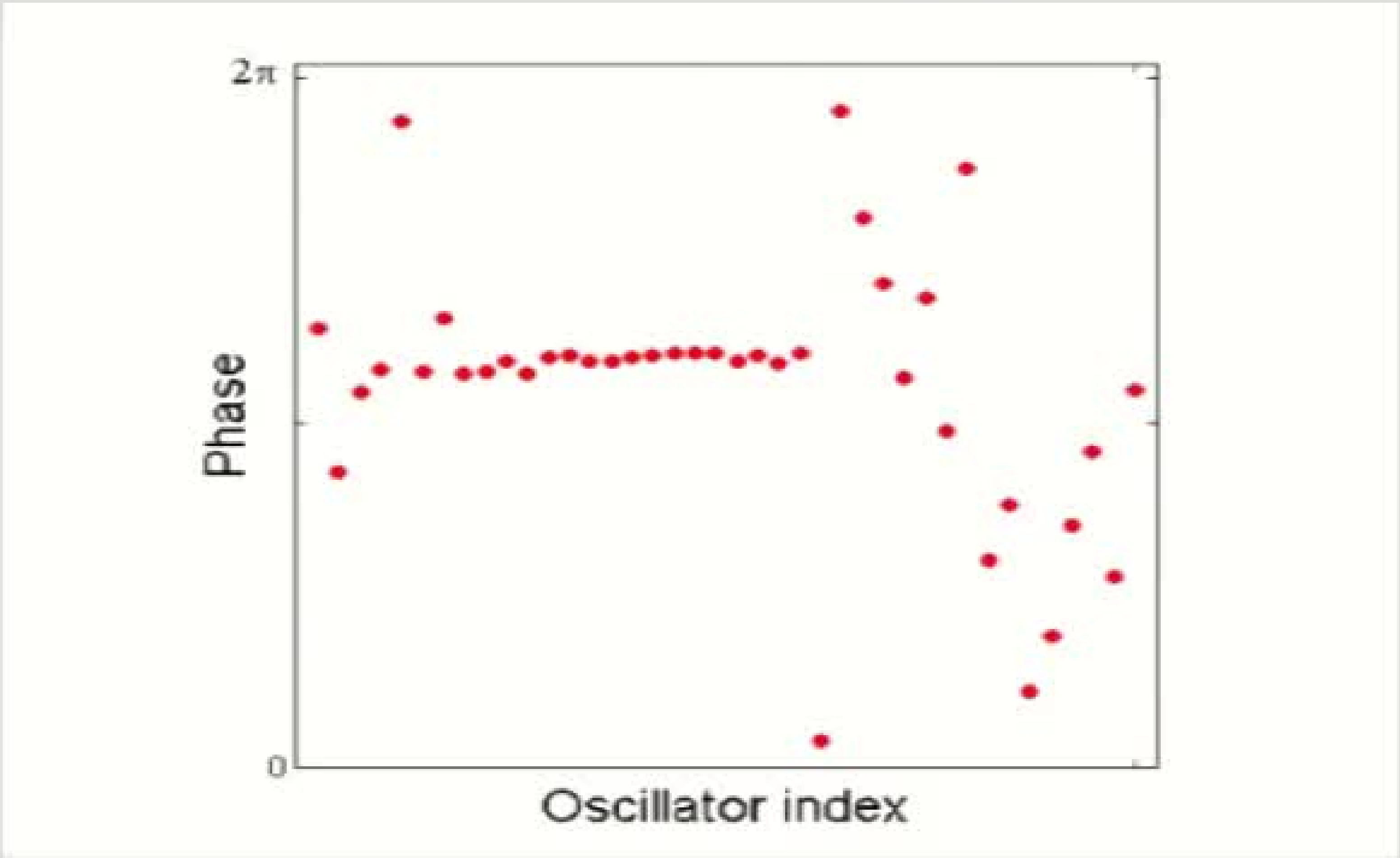
S. Nkomo, M. R. Tinsley, KS, *Chaos* **26**, 094826 (2016).

Chimera for Homogeneous and Heterogeneous Oscillators



Homogeneous oscillators. (a) Local order parameter R as a function of time, with m = 3. (b) Scatter plot of 500 consecutive periods for each oscillator in (a); inset: length of consecutive periods of a synchronized oscillator (j = 20, red) and an unsynchronized oscillator (j = 12, blue). Heterogeneous oscillators. (c) R as a function of time. (d) Scatter plot of 500 consecutive periods for each oscillator in (c). Simulations with random initial phase distributions. Natural period: 41.0 and 41.0 ± 2.1

Chimera Dynamics for Heterogeneous Oscillators



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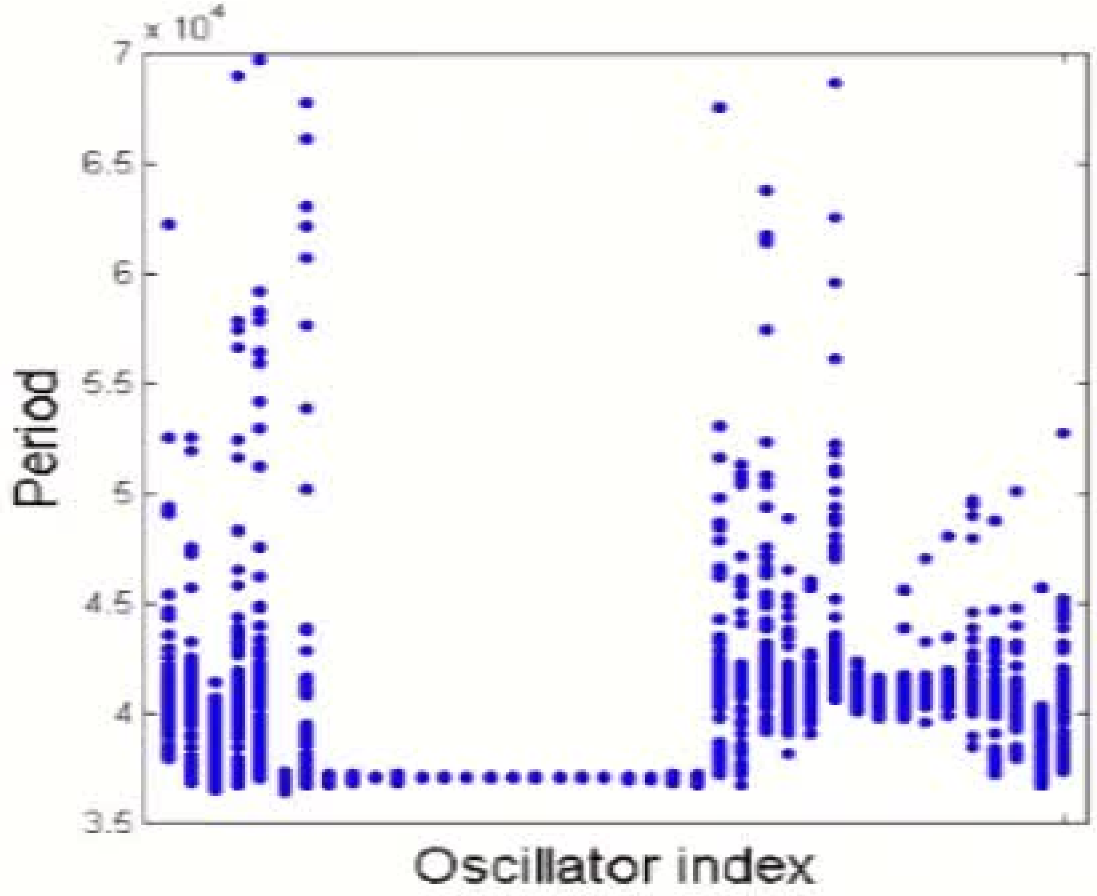

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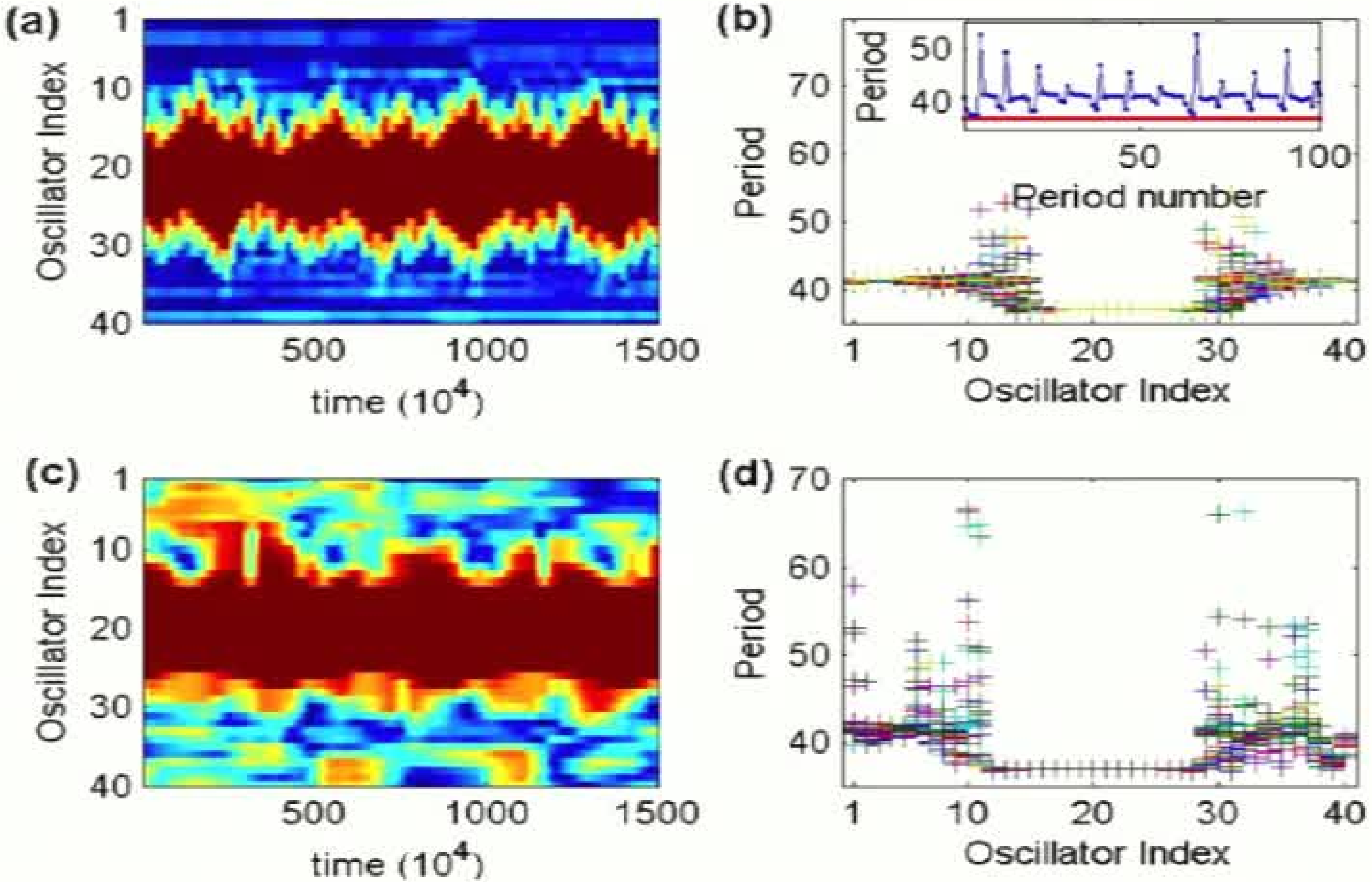

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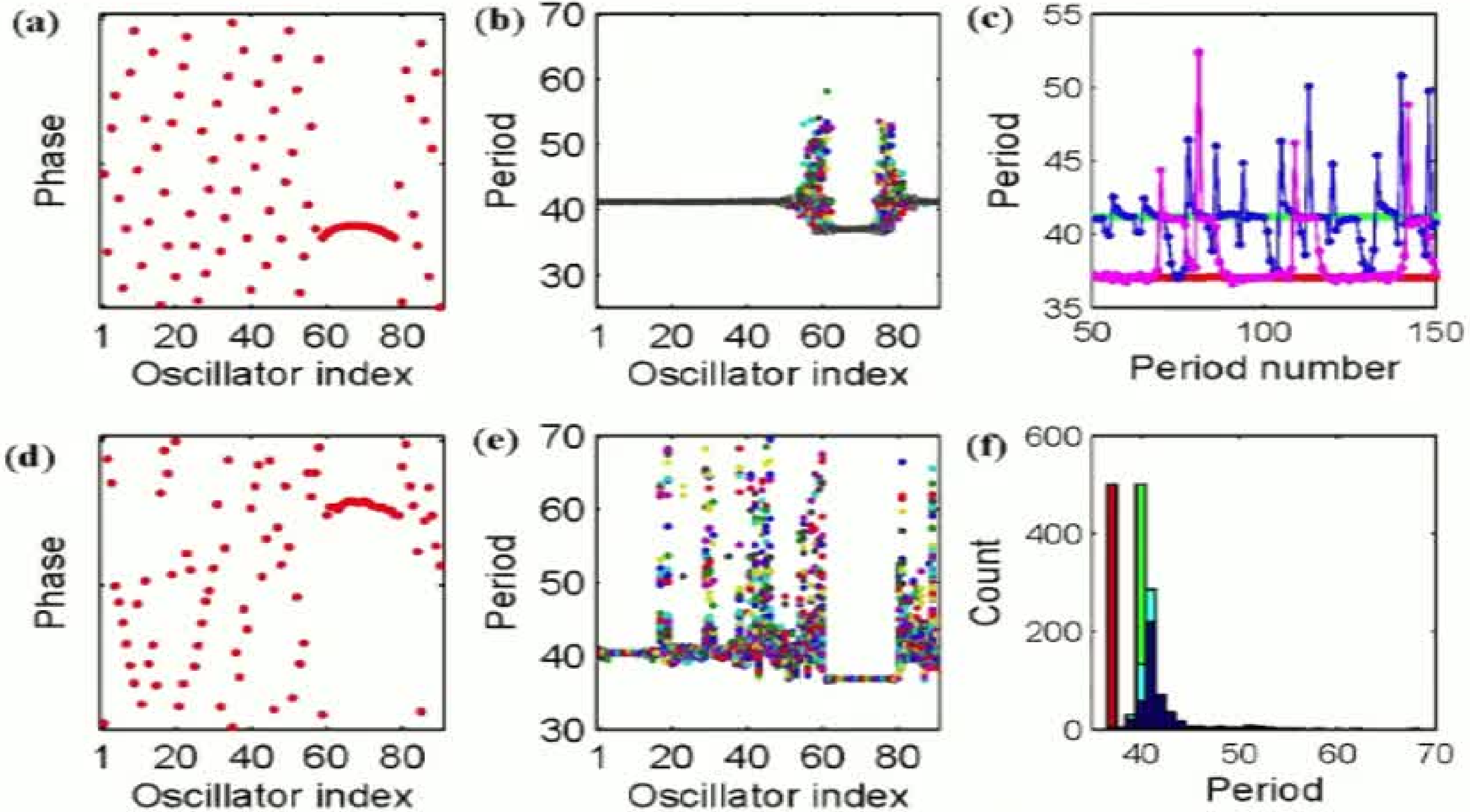
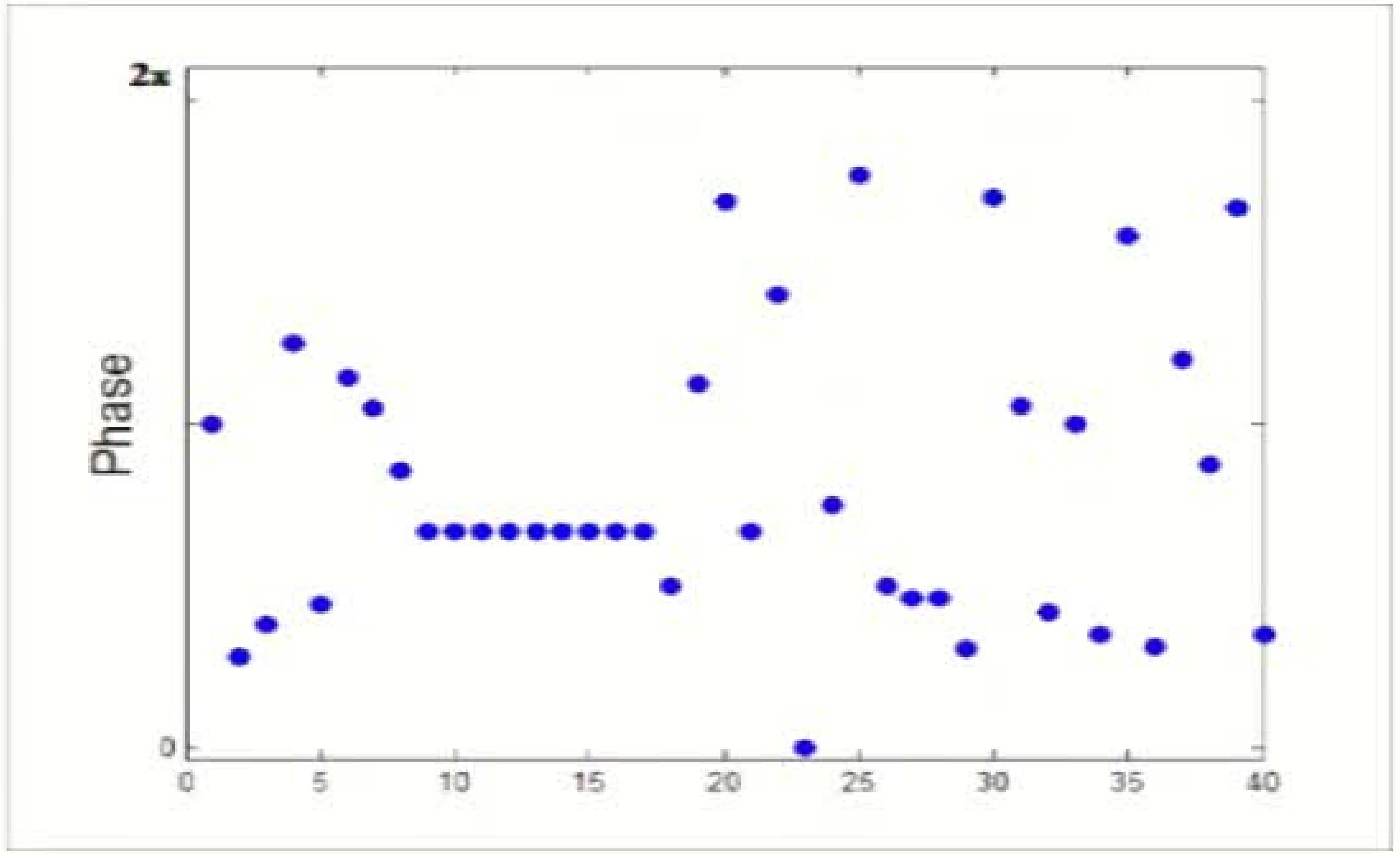
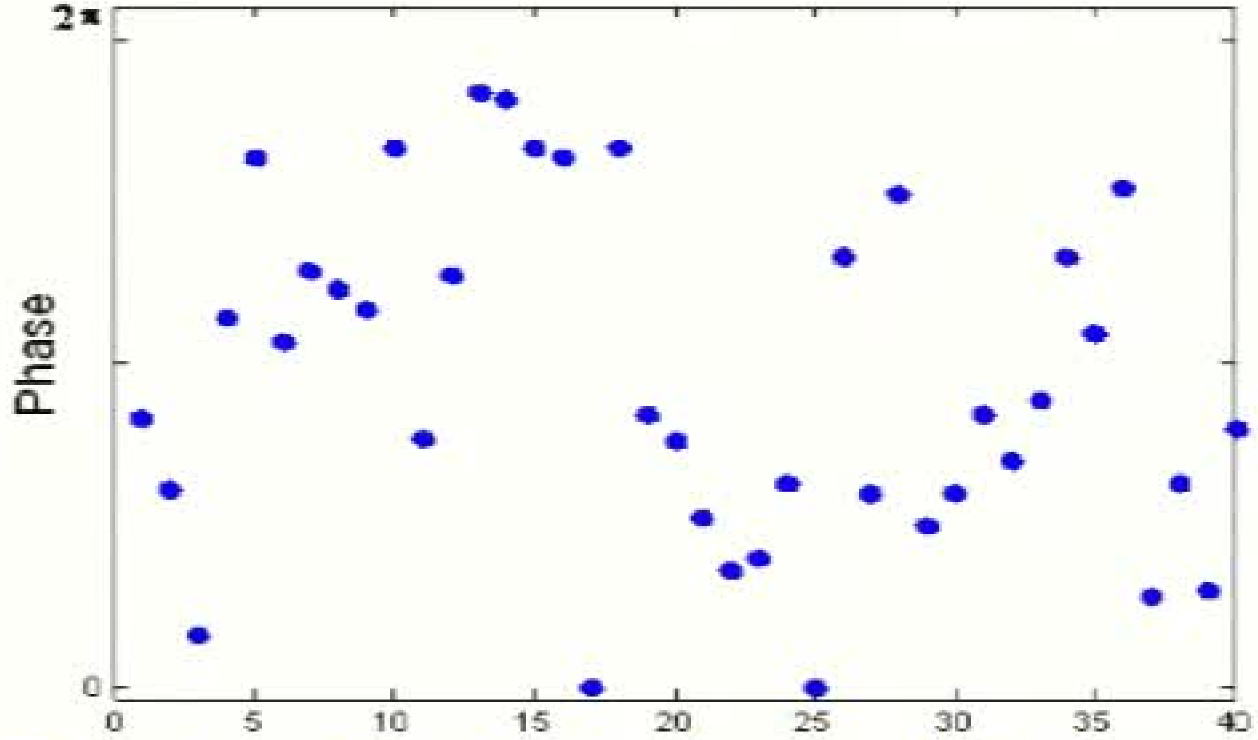
Chimera for Homogeneous and Heterogeneous Oscillators

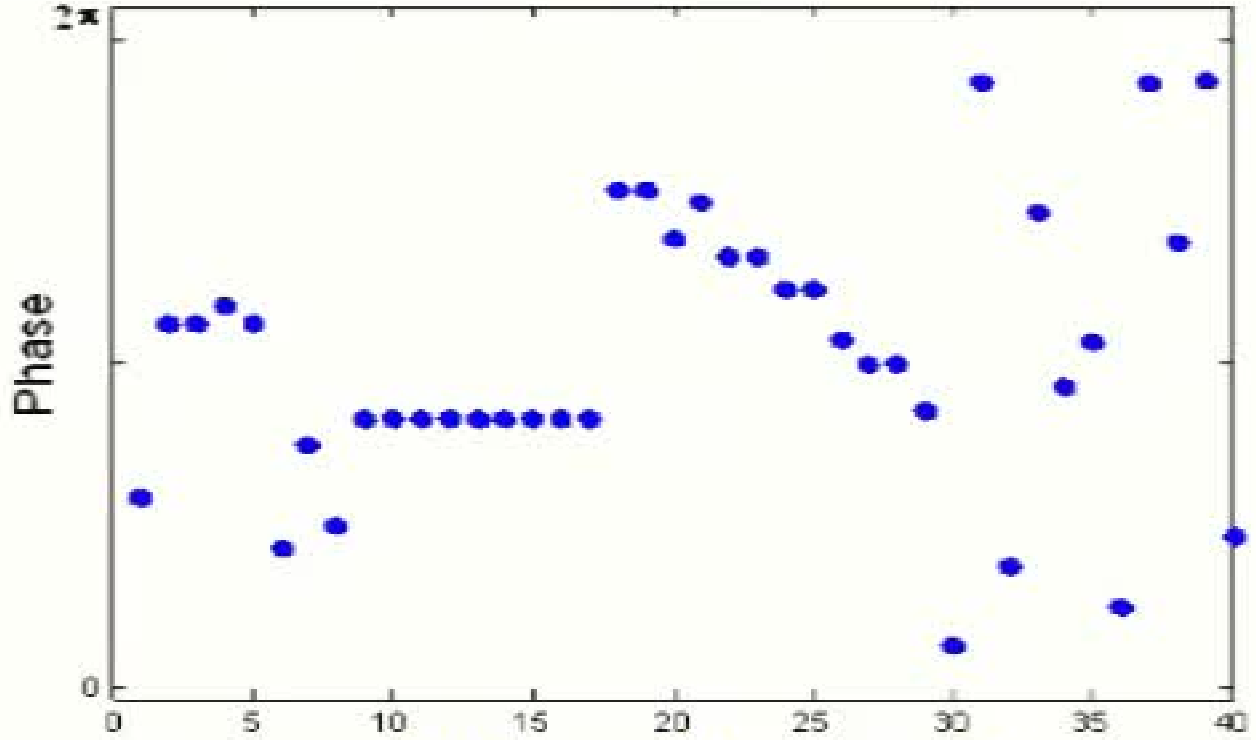


Homogeneous. (a) Snapshot of the phase for 90 oscillators. (b) Scatter plot of 500 periods. (c) Period vs period number for 100 periods: j = 40 (green), natural period; j = 58 (blue) and j = 60 (pink), aperiodic; j = 66 (red), synchronized period. Heterogeneous. (d) Phase for 90 oscillators. (e) Scatter plot of 500 periods. (e) Period histograms for synchronized (j = 75, red), phase wave (j = 10, green) , asynchronous (j = 58, dark blue) and phase dispersed (j = 50, light blue) oscillators.

BZ Oscillator Experiment: Random Initial Phase Distribution

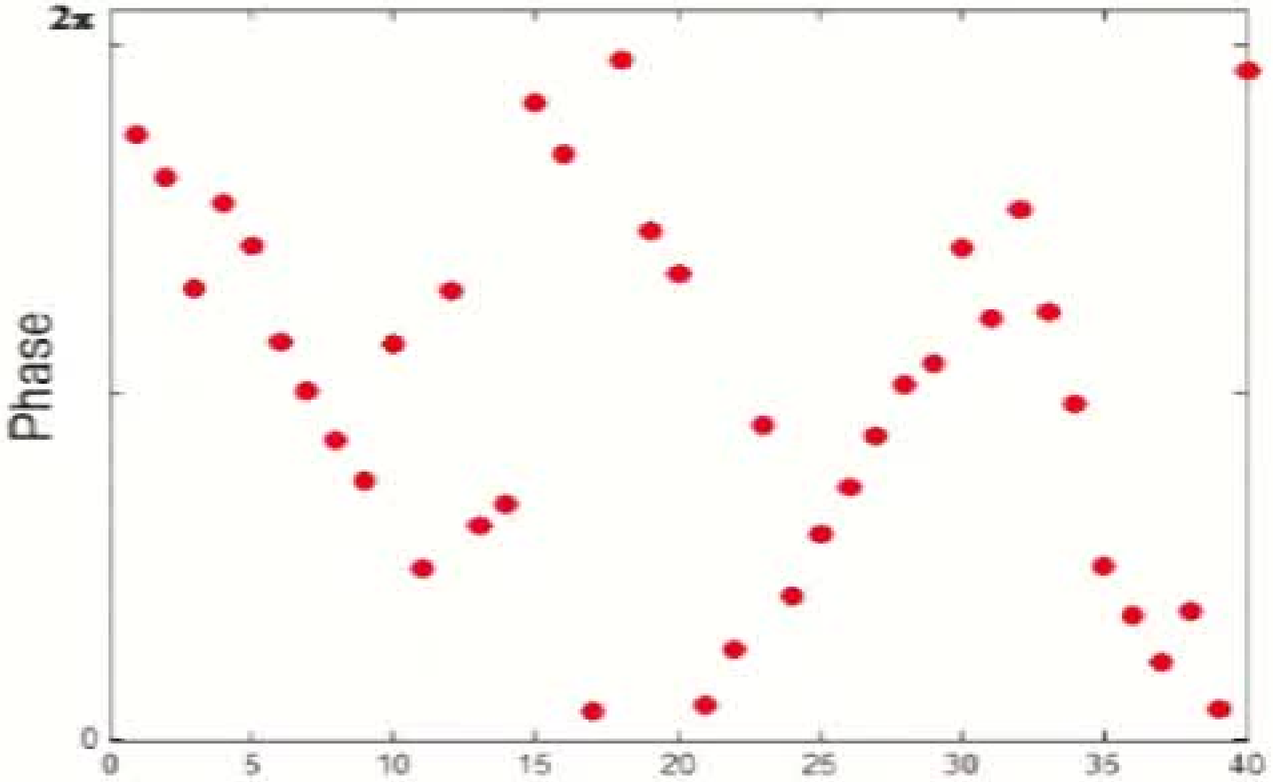


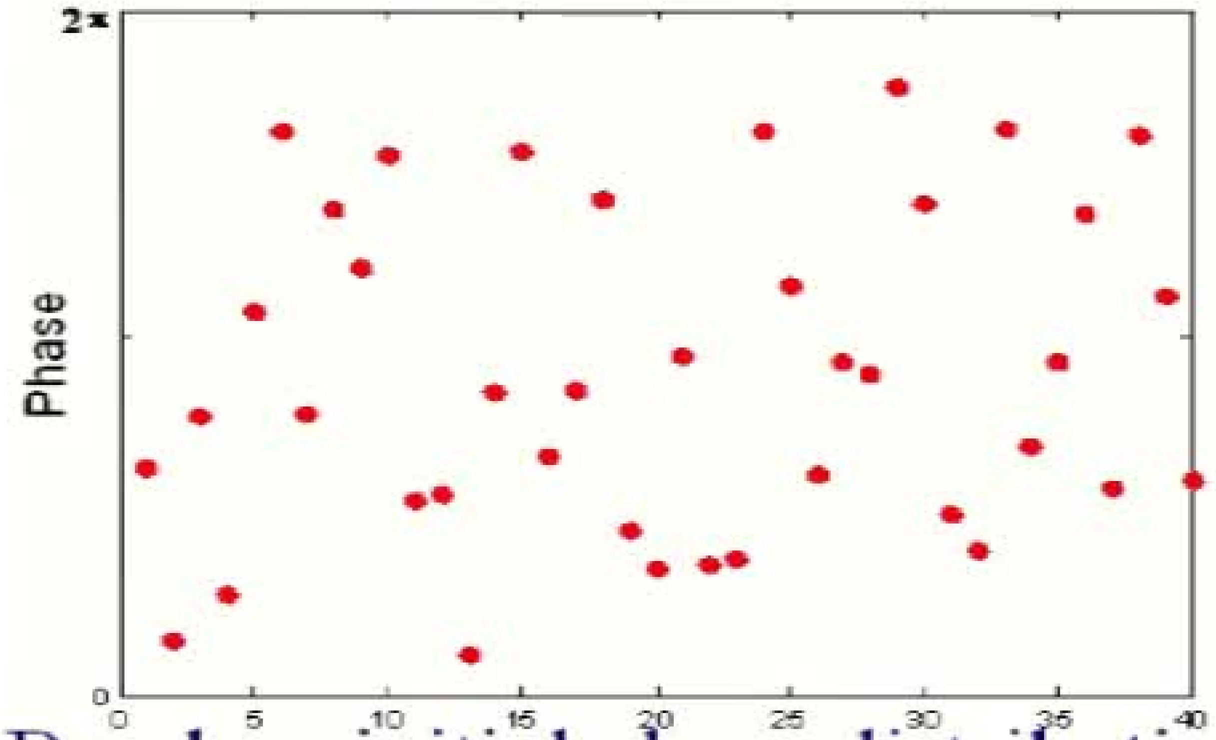


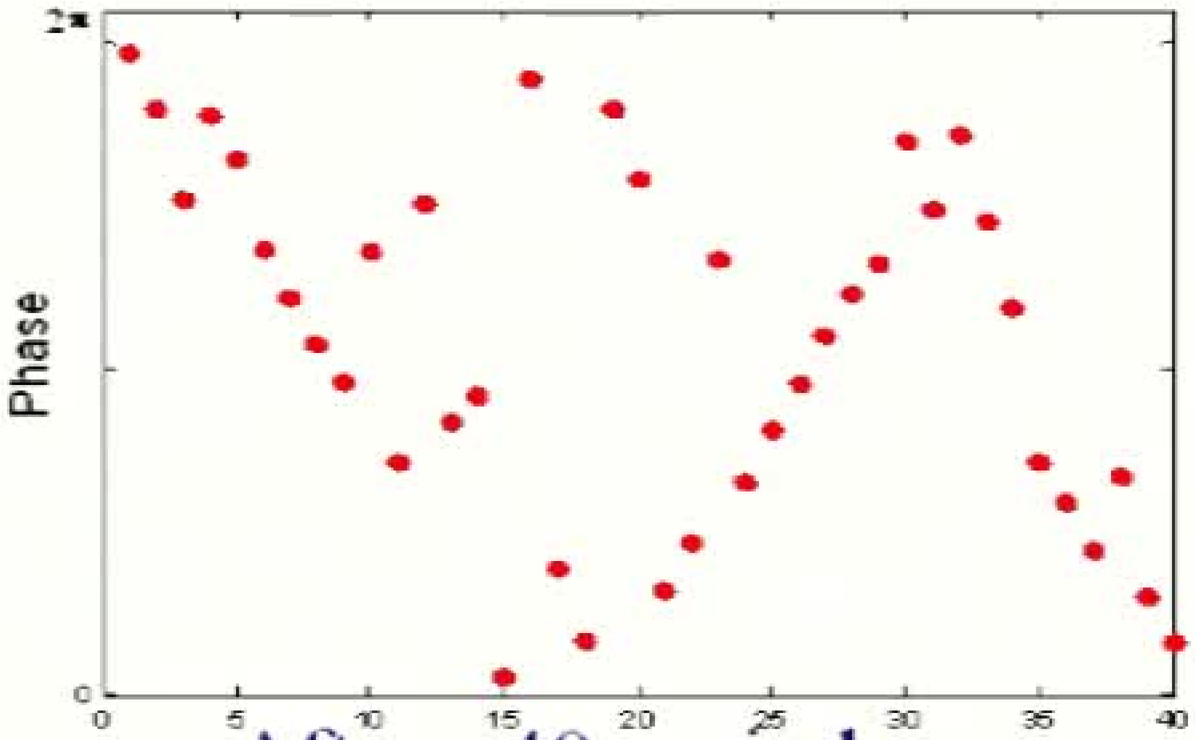


"Random" initial phase distribution　　$\kappa = 0.5$　　After ~40 periods

Heterogeneous Oscillators: Random Initial Phase Distribution





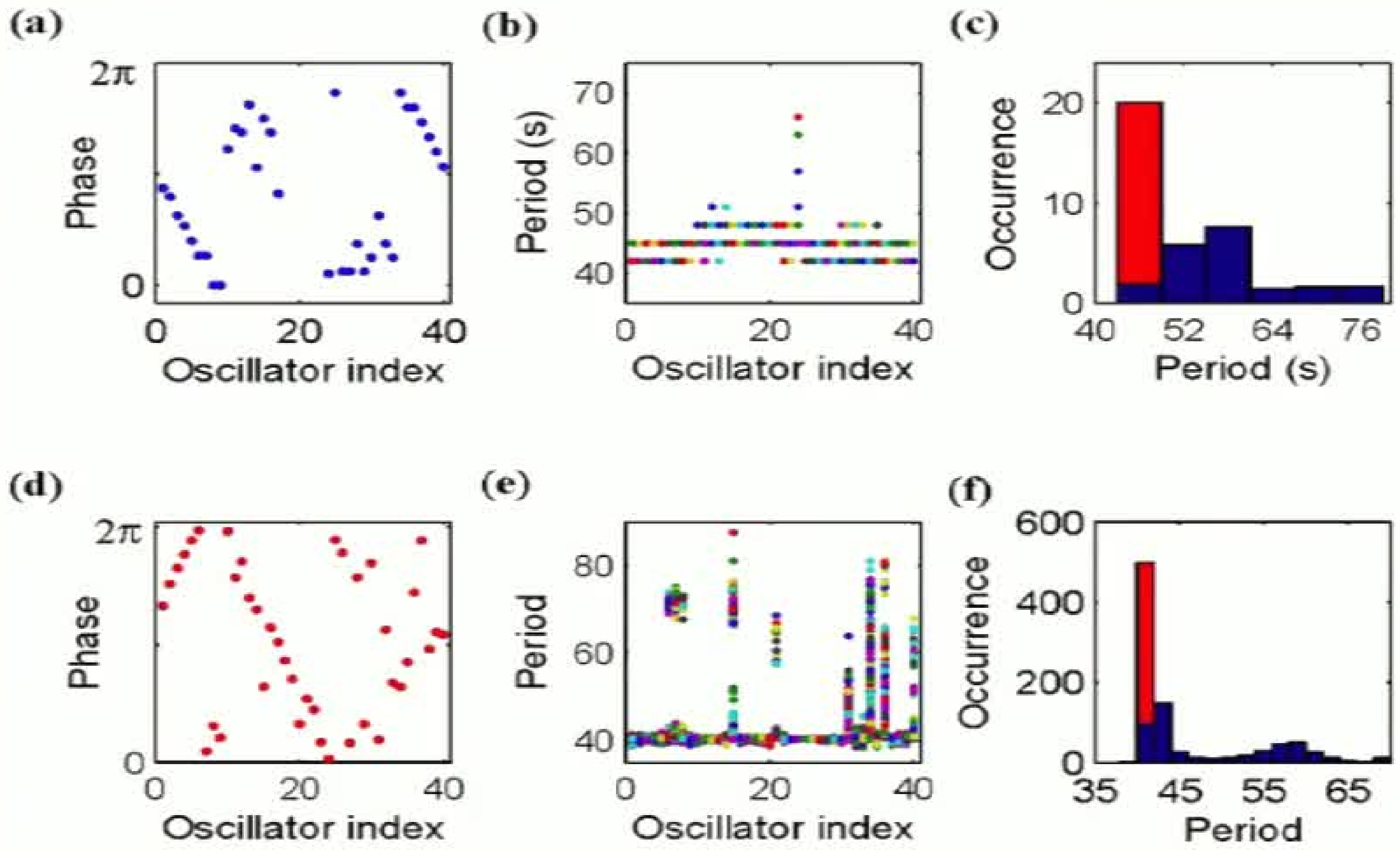


Random initial phase distribution $\kappa = 0.5$ After ~40 periods

Heterogeneous Oscillators: Chimera-Like States with Phase Waves

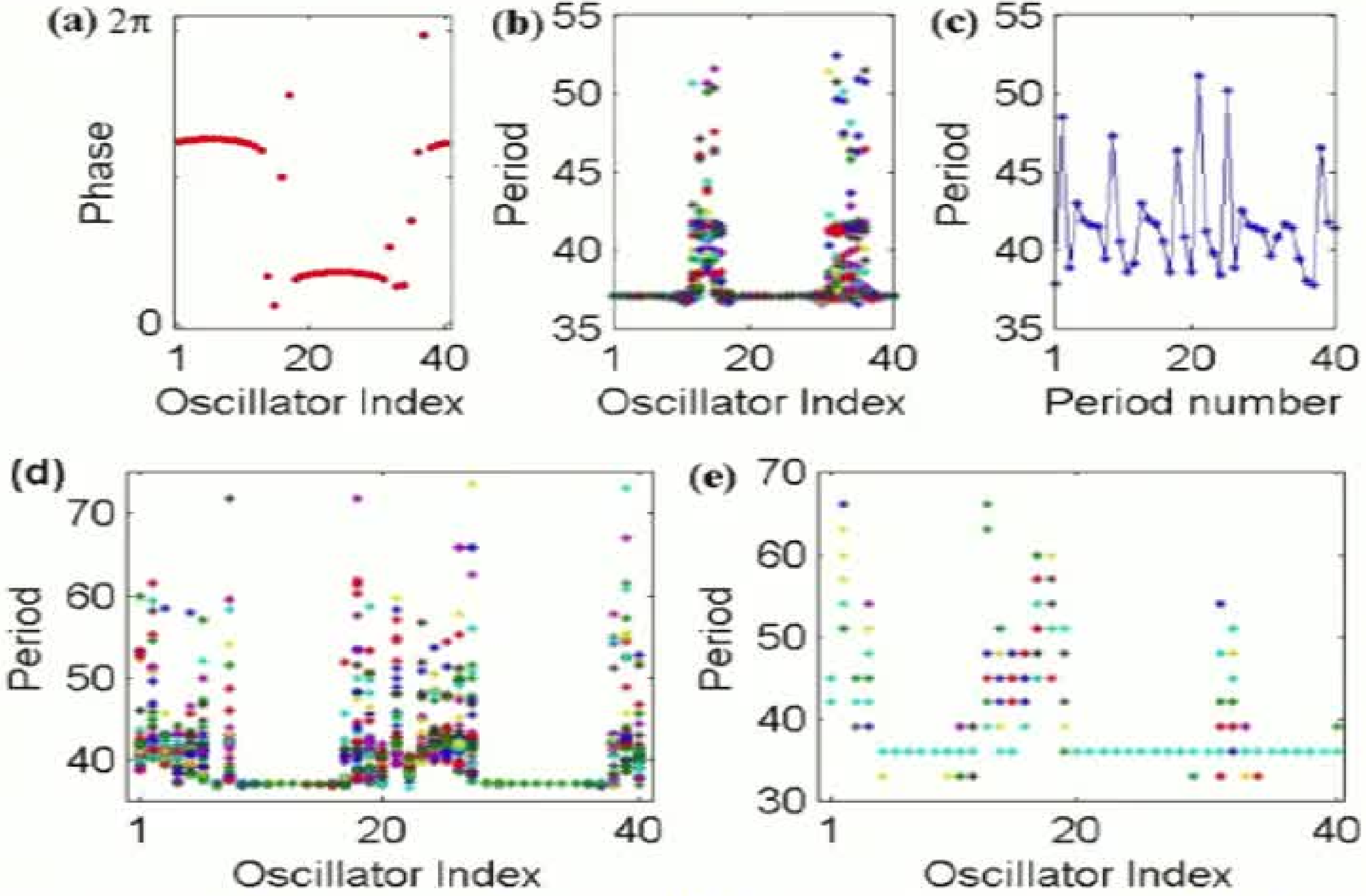
Experiment
40 oscillators

Simulation
40 oscillators



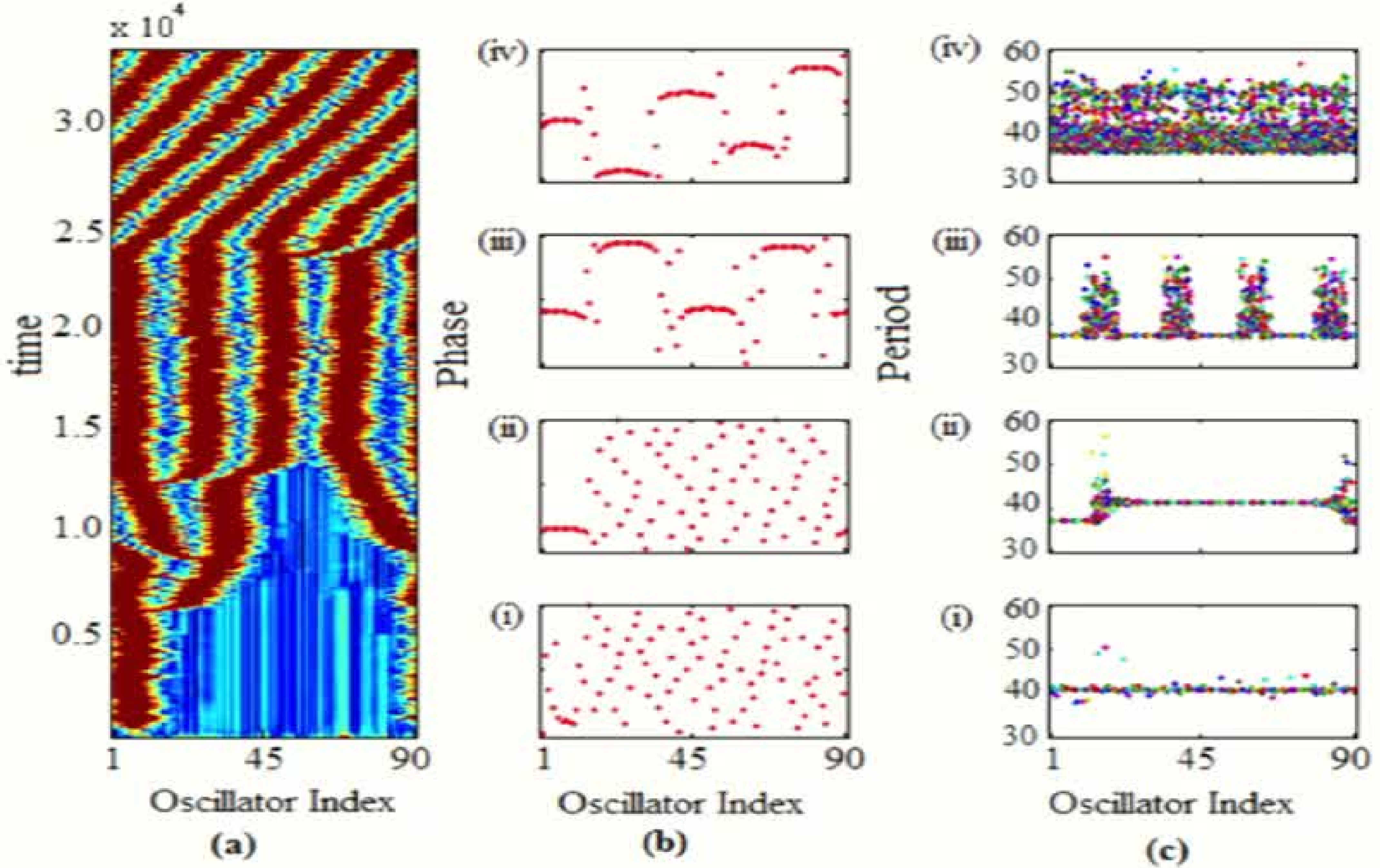
(a) Phase of each oscillator at t = 1200 s. (b) Period scatter plot over 20 periods. (c) The distribution of periods for an oscillator in the phase-wave group (j = 35, red) and for an oscillator in the incoherent group (j = 24, blue). (d) Phase snapshot at $t = 9.0 \times 10^4$. (e) Period scatter plot over 500 consecutive periods. (f) Distribution of periods for an oscillator in a phase-wave region (j = 27, red) and in an incoherent region (j = 36, blue).

Phase-Cluster Chimeras: Homogeneous & Heterogeneous Systems



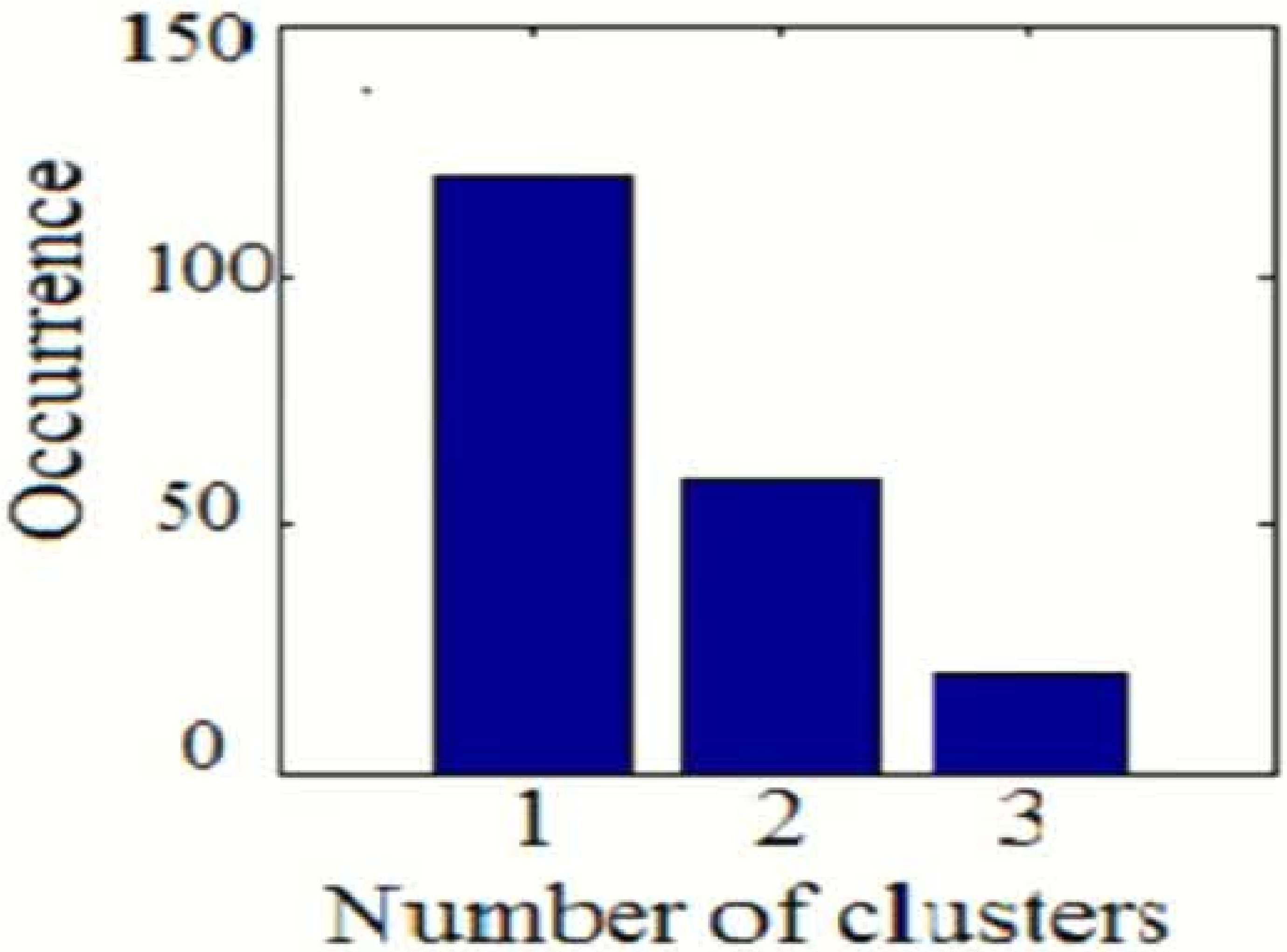
Phase-cluster chimera states in systems of homogeneous and heterogeneous oscillators. (a) Snapshot showing phase of each oscillator at $t = 1.8 \times 10^4$ in a system of 40 homogeneous oscillators. (b) Scatter plot of oscillator periods for 1000 consecutive cycles. (c) Period as a function of period number for oscillator $j = 33$. (d) Period scatter plot for 40 heterogeneous oscillators over 500 cycles. (e) Scatter plot of periods taken over 20 cycles in an experimental system of 40 BZ oscillators.

Homogeneous BZ Oscillators: Traveling Phase-Cluster Chimera



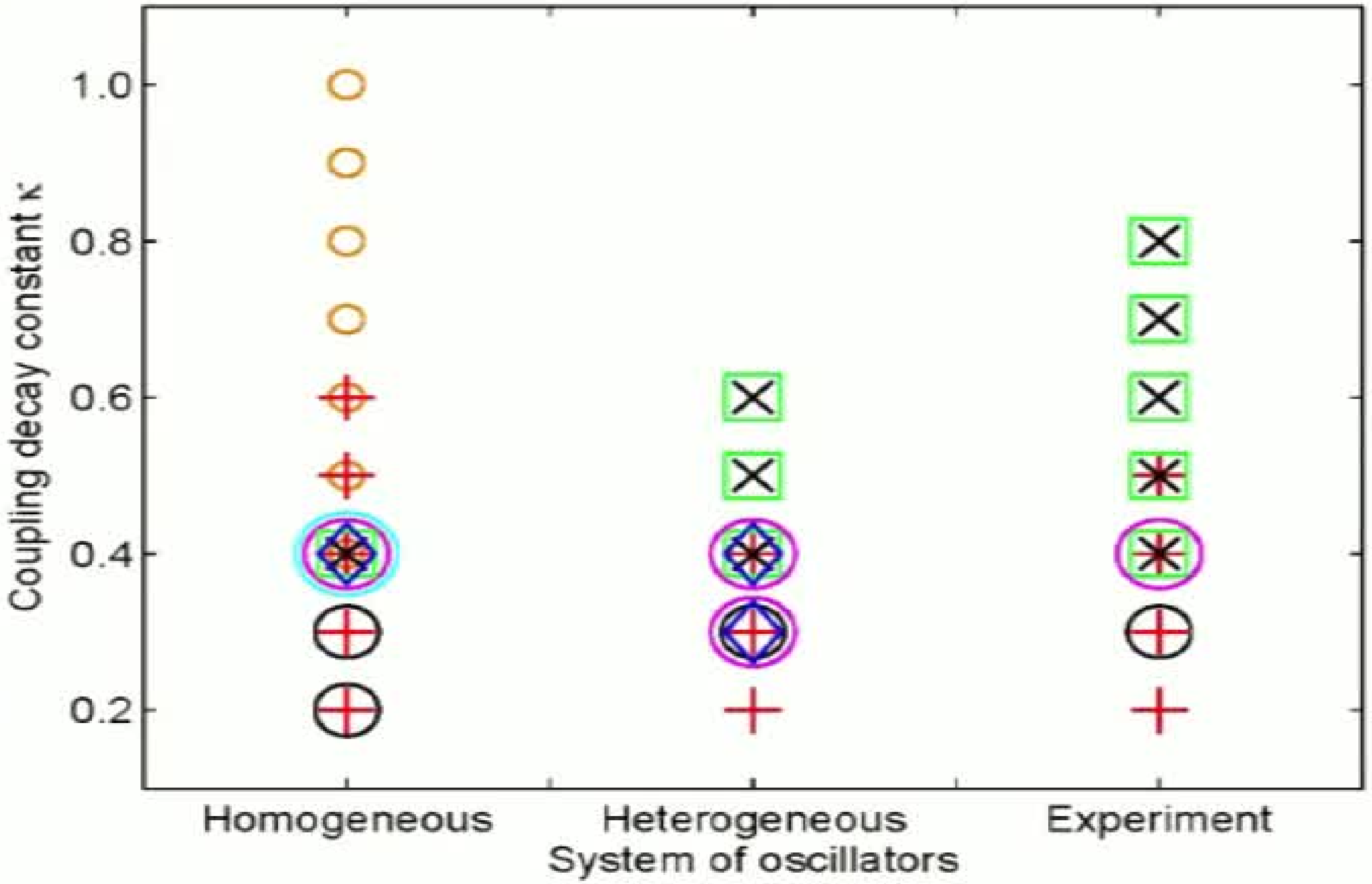
Simulations of 90 coupled identical BZ oscillators. (a) Local order parameter R as a function of time. (b) Snapshot of phase of each oscillator at times t = 3000, 4000, 20000, 32000. (c) Scatter plot of each oscillator for 500 periods. Random initial phase distribution. Simulations carried out for 10000 periods.

Homogeneous BZ Oscillators: Multistability



From 200 realizations with 40 homogeneous BZ oscillators:
121 simple chimera (1-phase-cluster), 58 2-phase-cluster chimeras, 20 traveling phase-cluster chimeras with 3 clusters, and 1 collapsed to a fully synchronized state. All parameters are the same except for the random initial seed for the unsynchronized oscillators. Simulations carried out for 12000 periods.

States for Different Values of κ



fully synchronized (+), synchronization clusters (O), splay state (O), simple chimera (×), phase-cluster chimera (□), traveling phase-cluster chimera (O), chimera-like states with phase waves (O), phase slip behavior (◇).

For $\kappa = 0.4$:

Homogeneous oscillator system:

fully synchronized, simple chimera, splay state, phase-cluster chimera, traveling phase-cluster chimera, and phase-slip behavior.

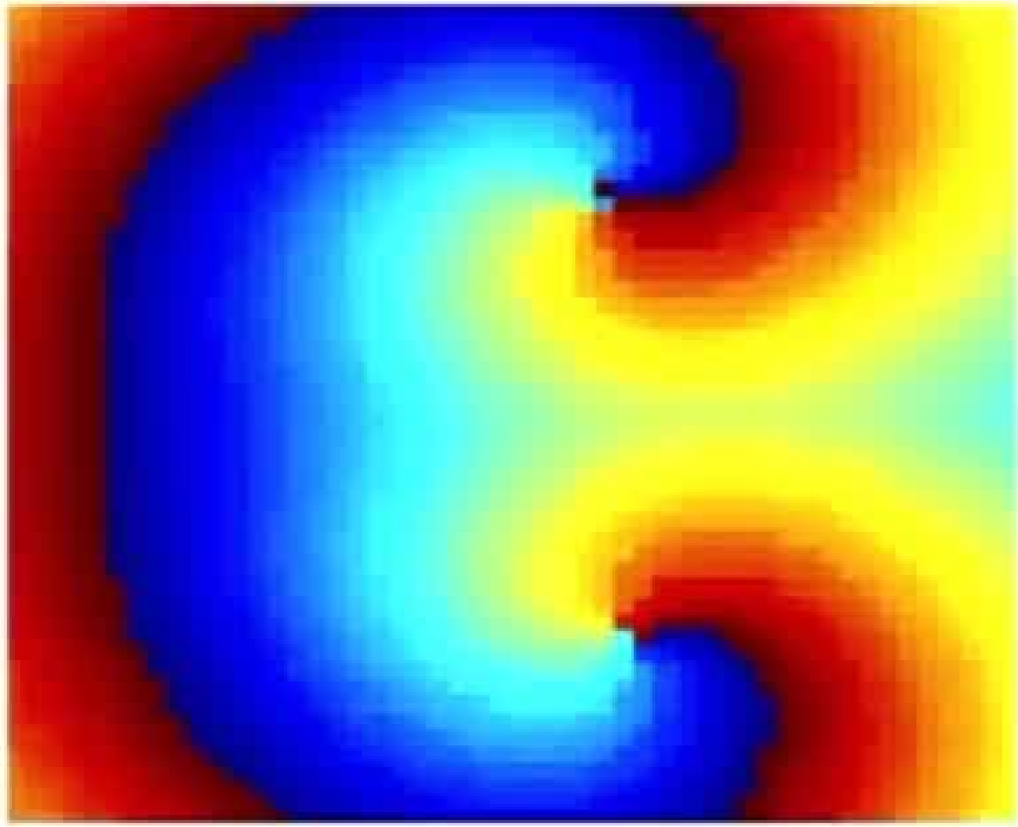
Heterogeneous oscillator system:

fully synchronized, simple chimera, phase-cluster chimera, phase-slip behavior, and phase-wave chimera behavior.

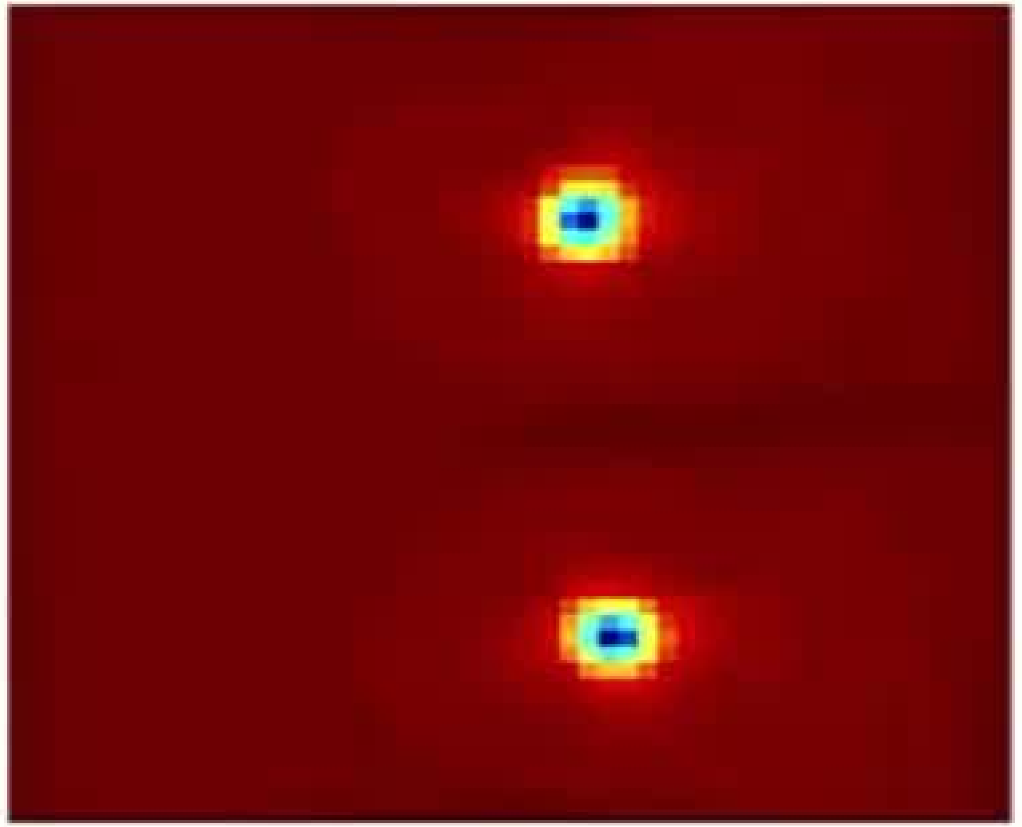
Experimental system:

same as heterogeneous system except no phase-slip behavior.

Spiral Chimeras: BZ Oscillator Simulation
Irregular Looping



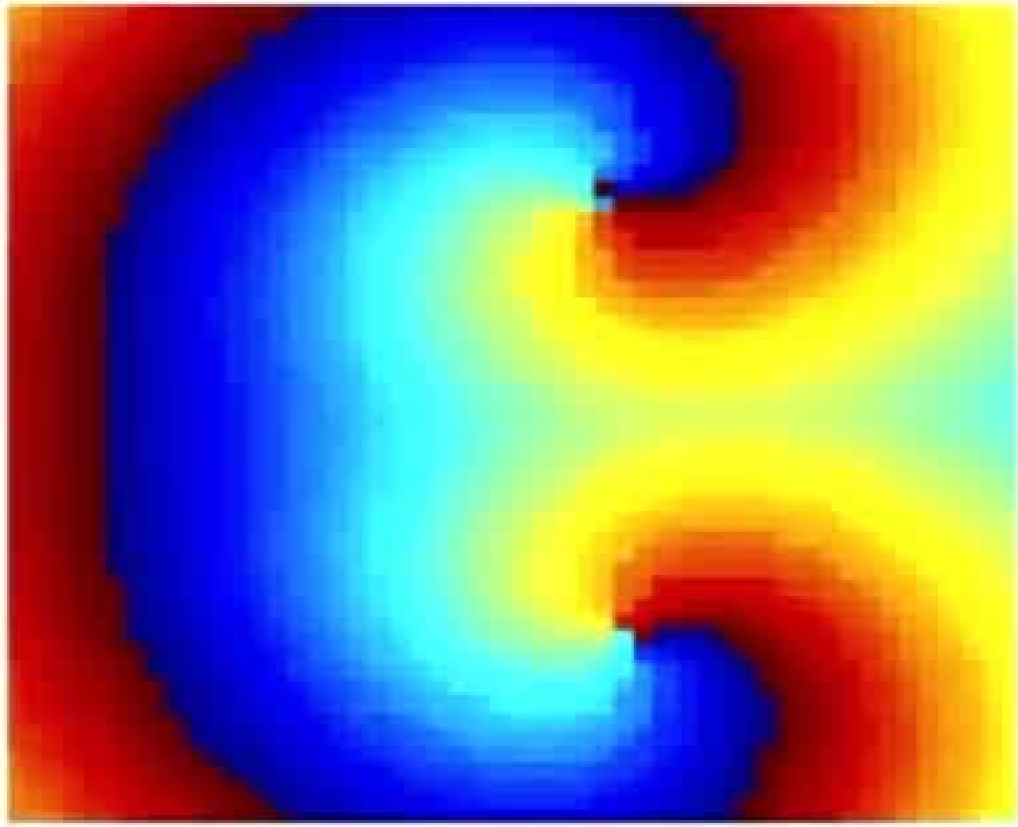
Phase: 2500
BZ oscillators



Local order parameter *R*.
Trajectory of asynchronous core.

50×50 BZ oscillators in a square lattice; coupling radius n = 4.
Simulation initiated with a pair of symmetric counter rotating spirals with $\tau = 0$.
Delay is switched on at t = 500 and simulation continued to t = 3500.
Delay $\tau = 3.4$.

Spiral Chimeras: BZ Oscillator Simulation
Irregular Looping



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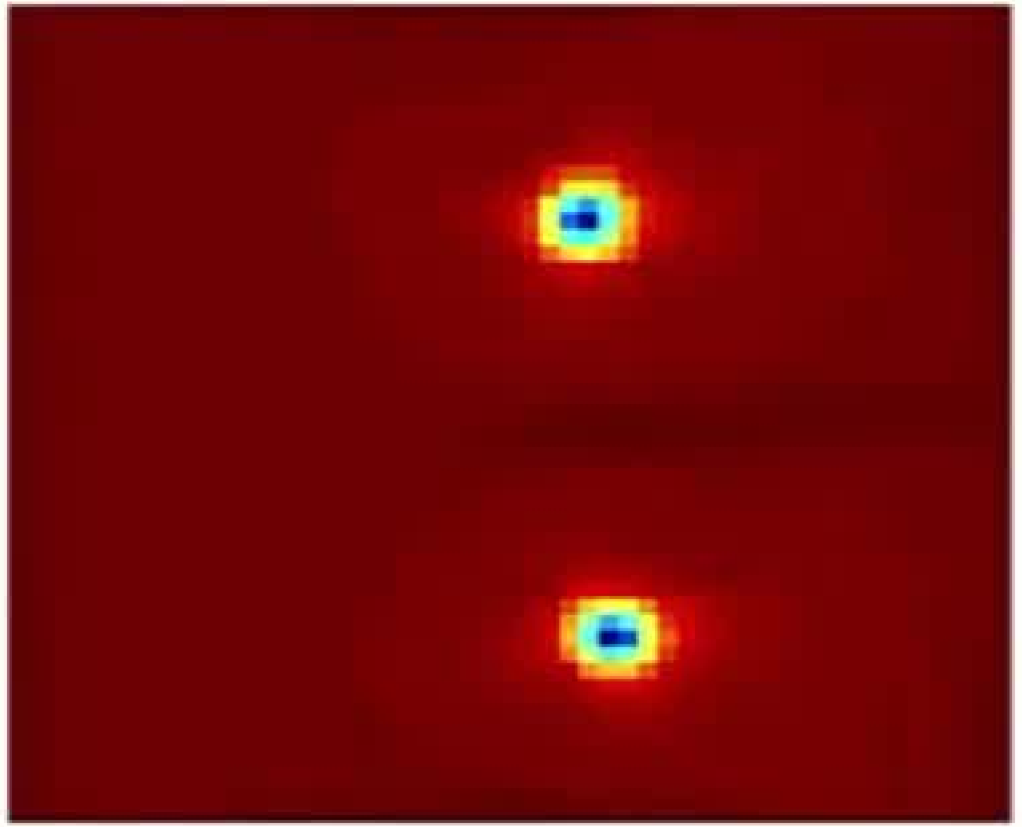

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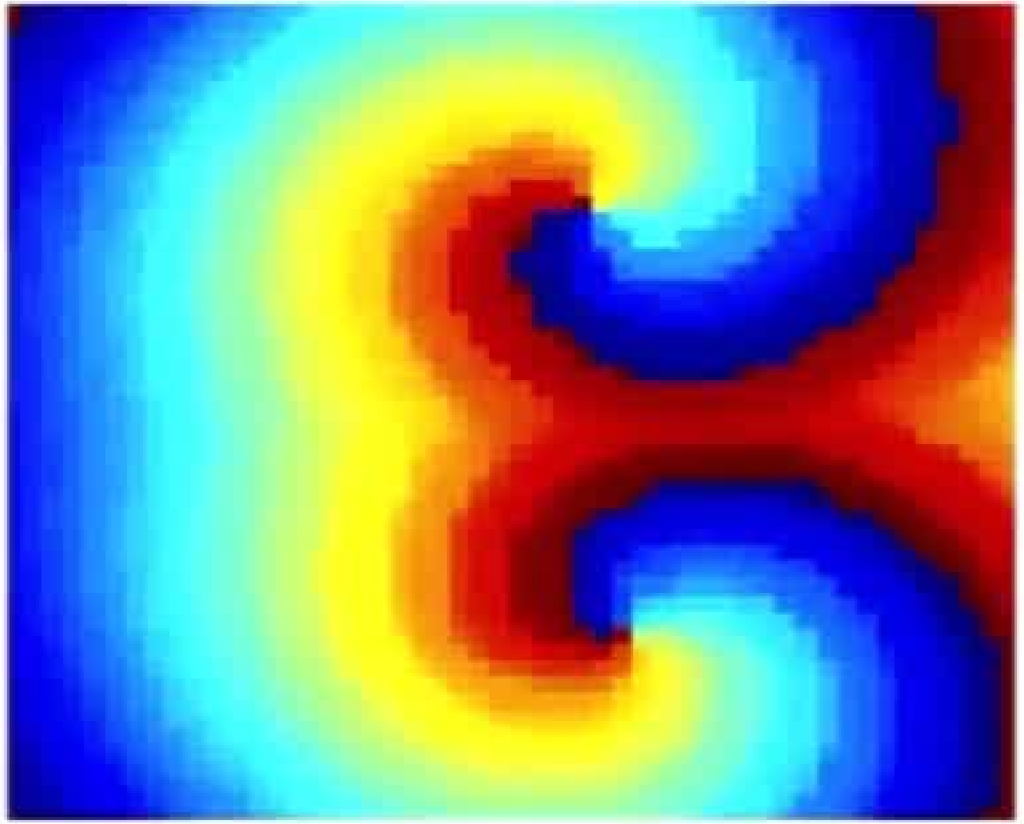

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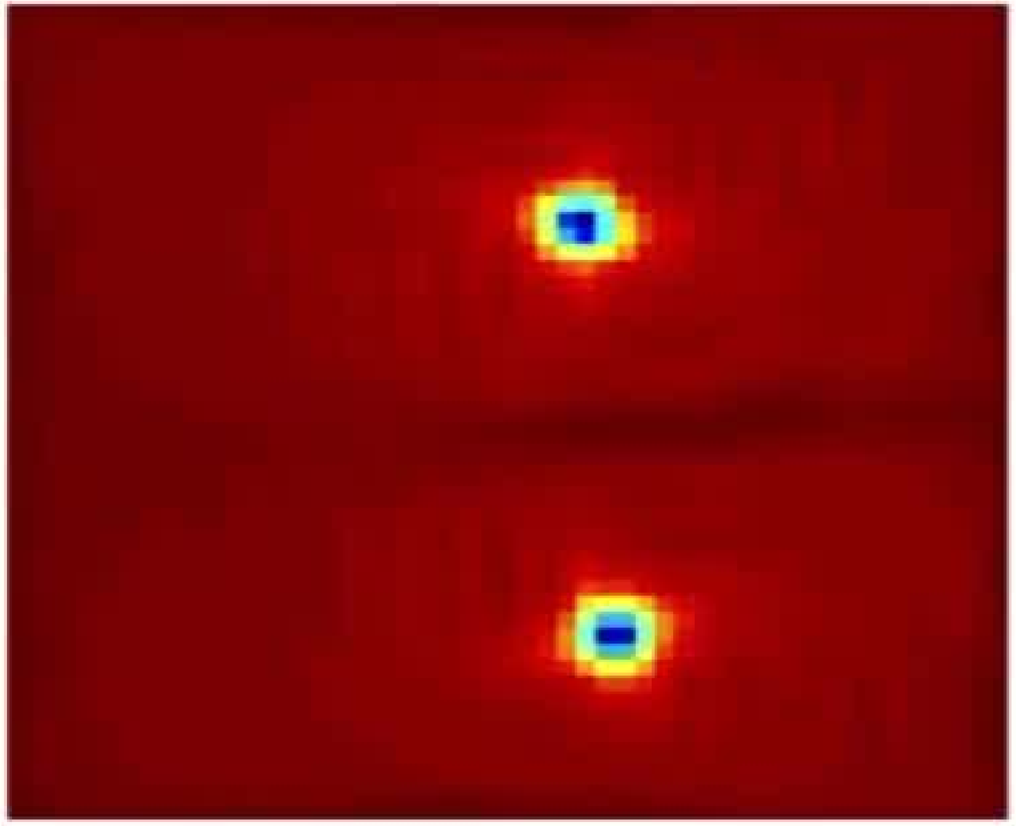
BZ oscillators



Local order parameter *R*.
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50 × 50 BZ oscillators in a square lattice; coupling radius n = 4.
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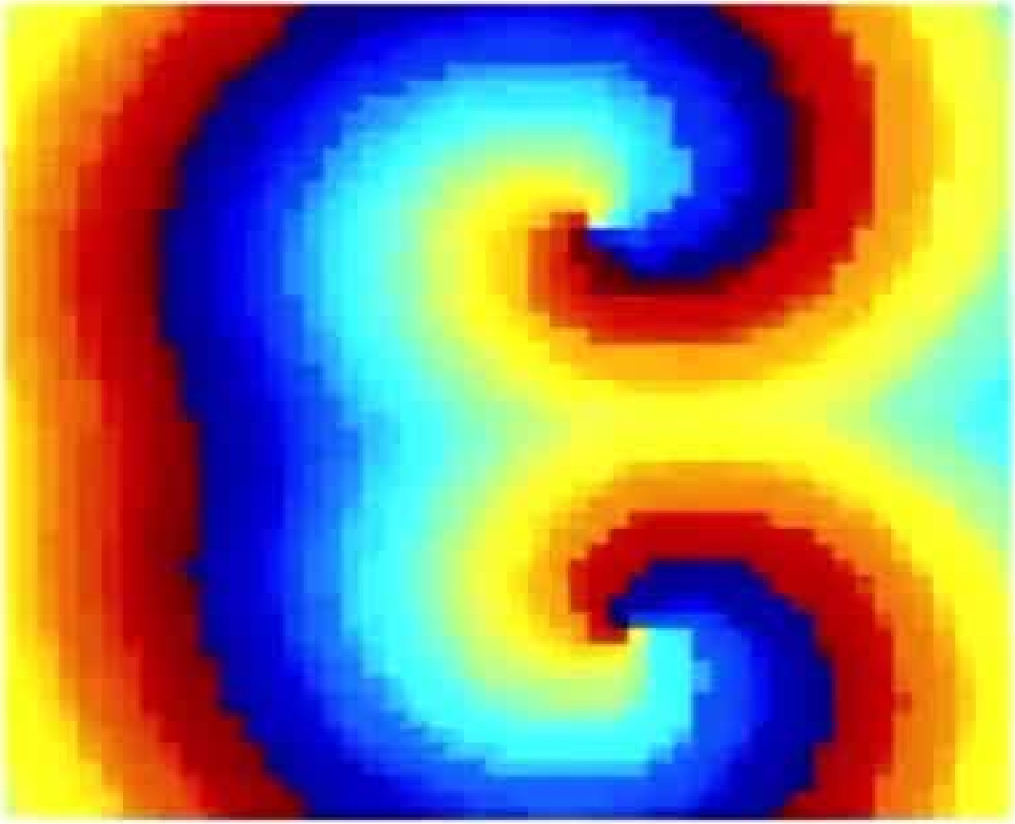

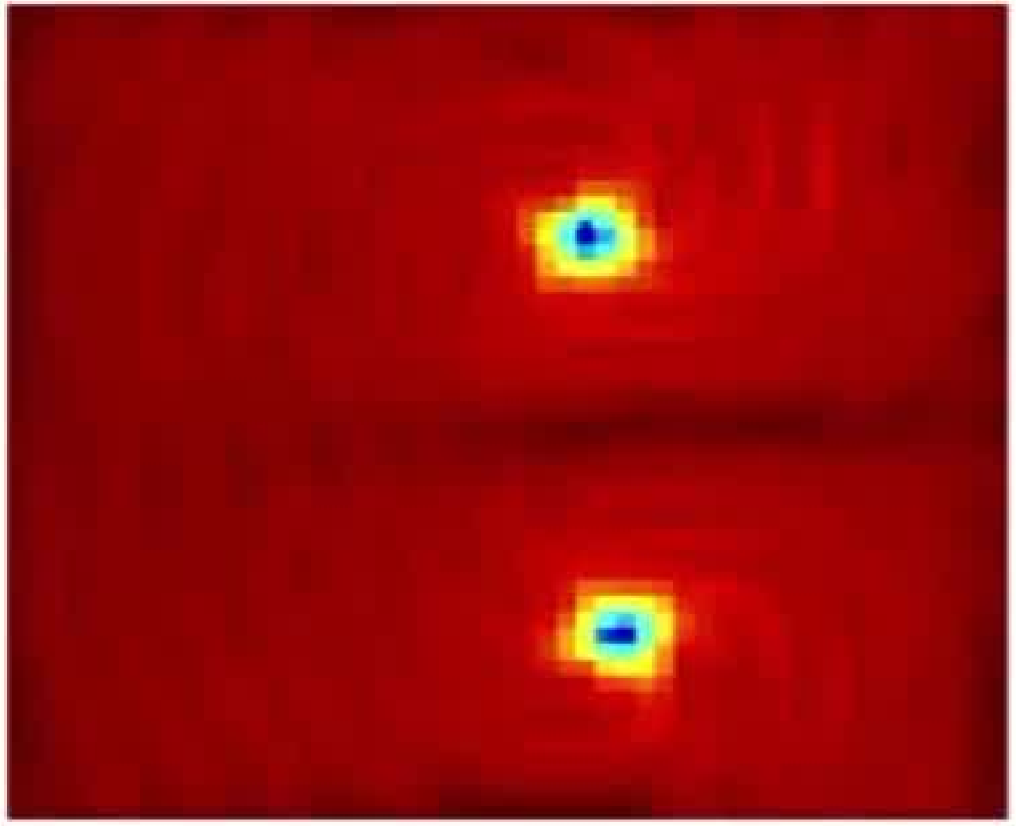

Phase: 2500 BZ oscillators

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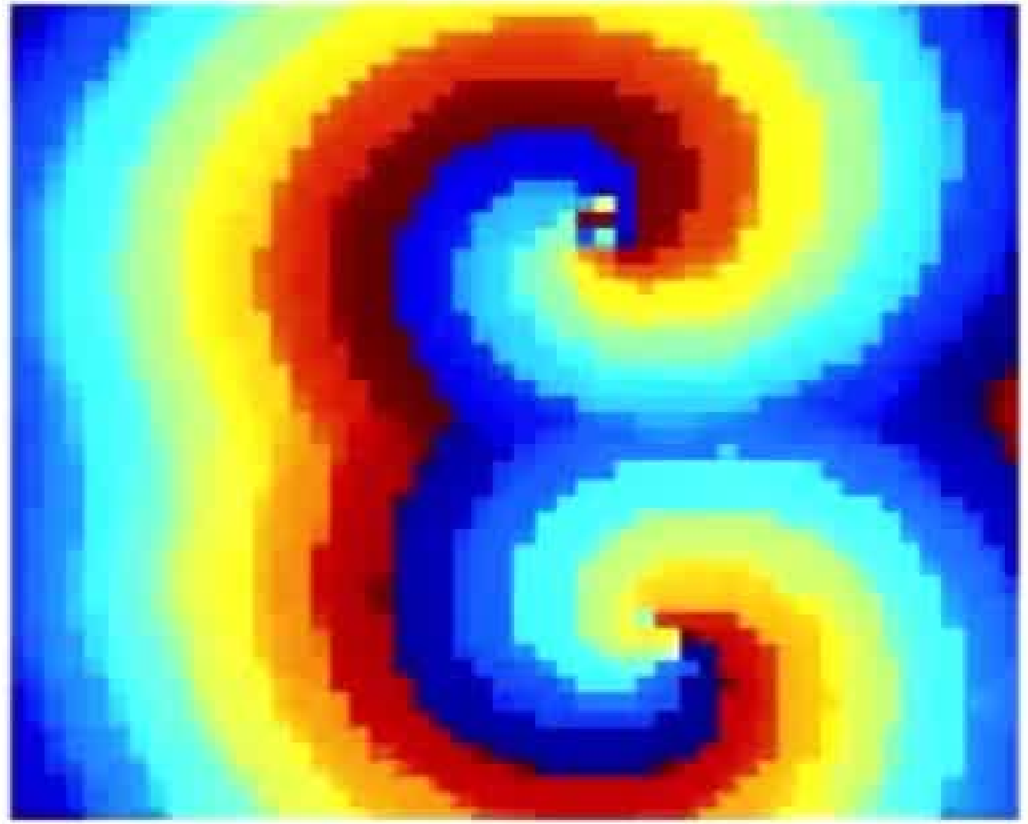
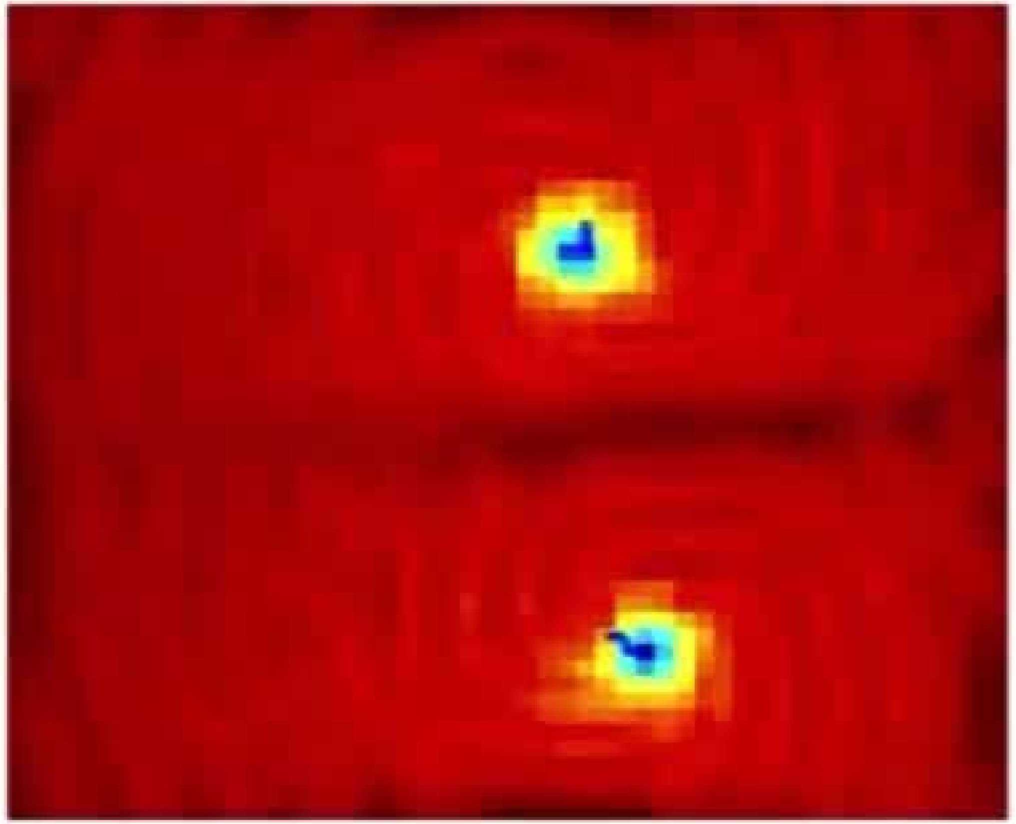
Irregular Looping



Phase: 2500 BZ oscillators



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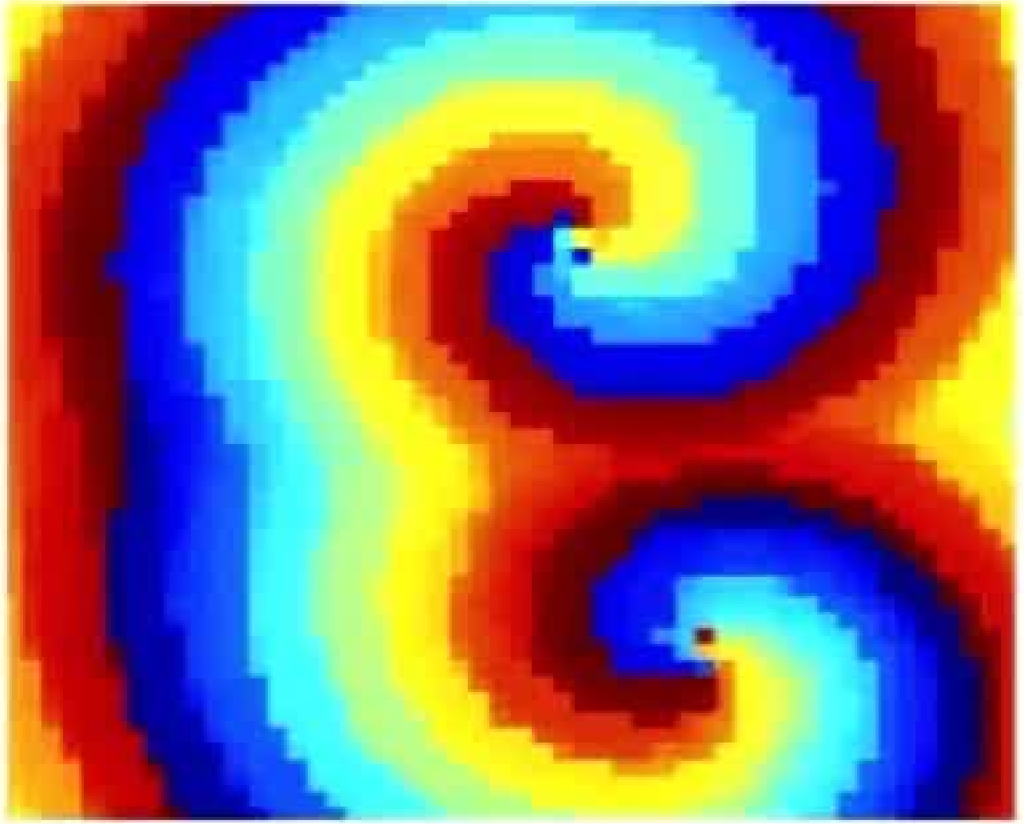

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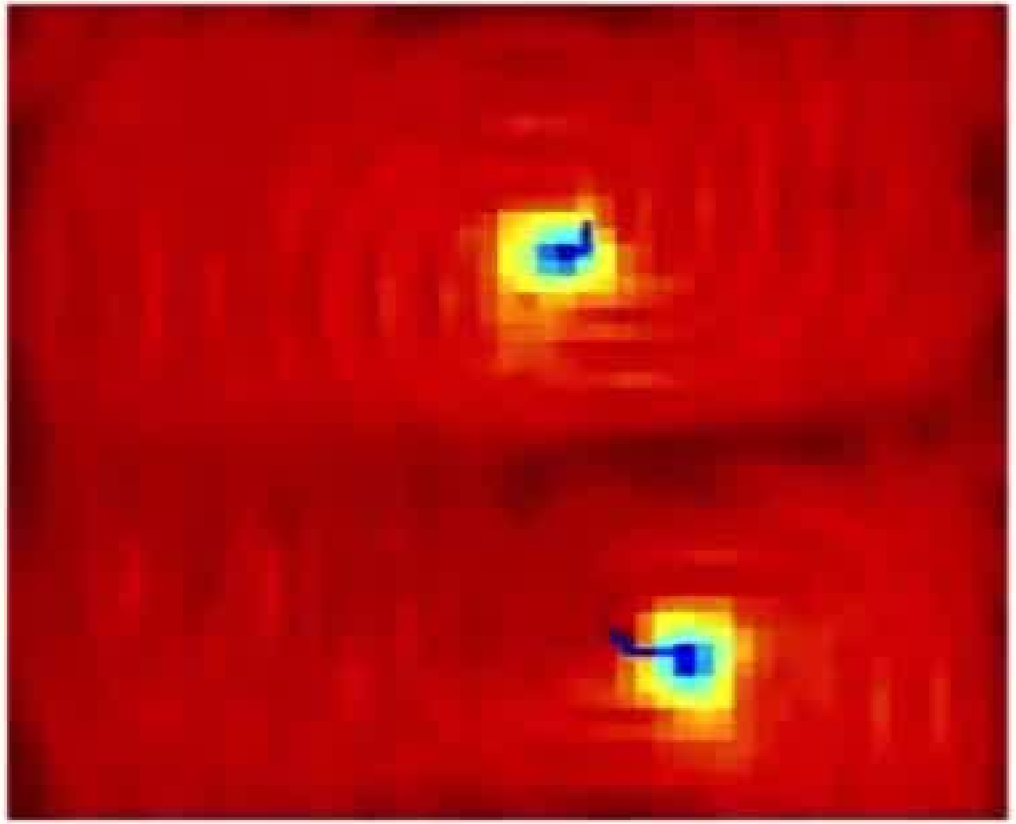

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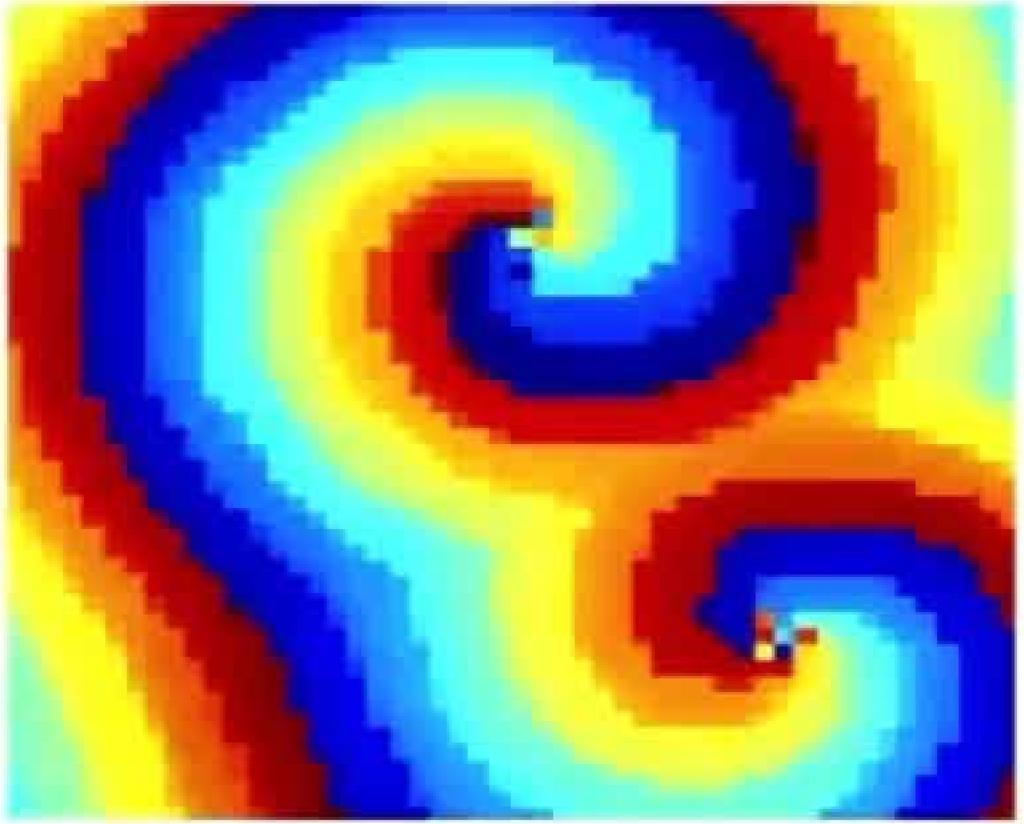
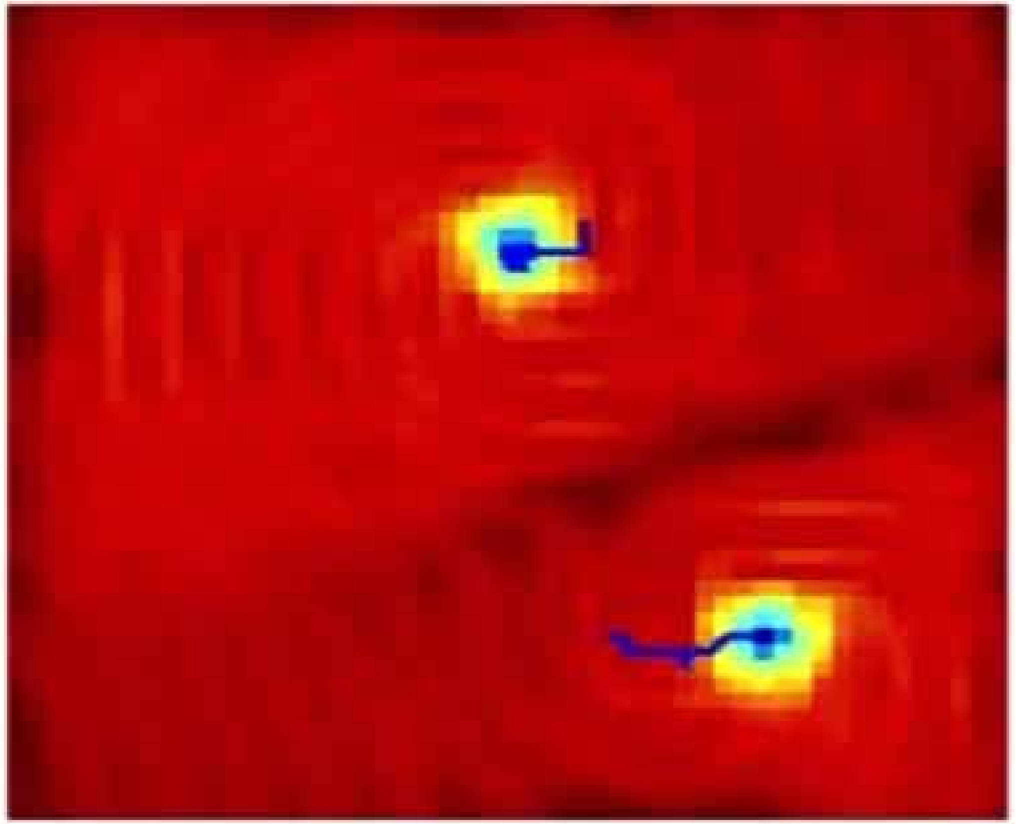


Phase: 2500
BZ oscillators



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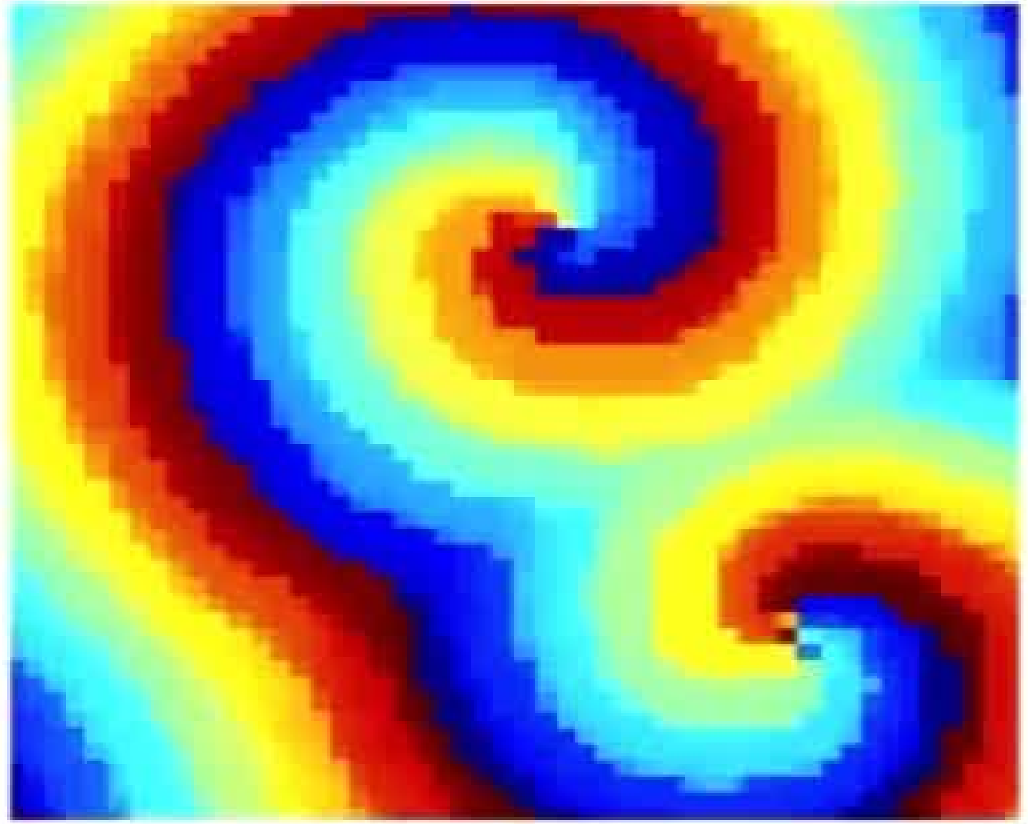

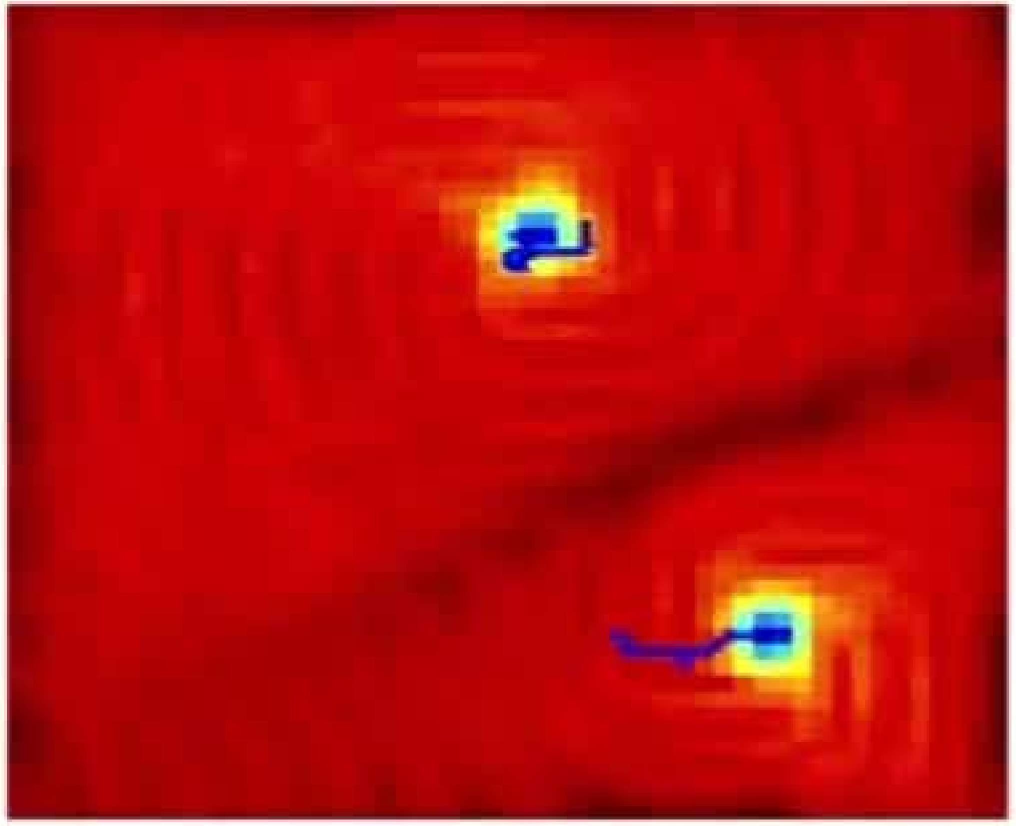

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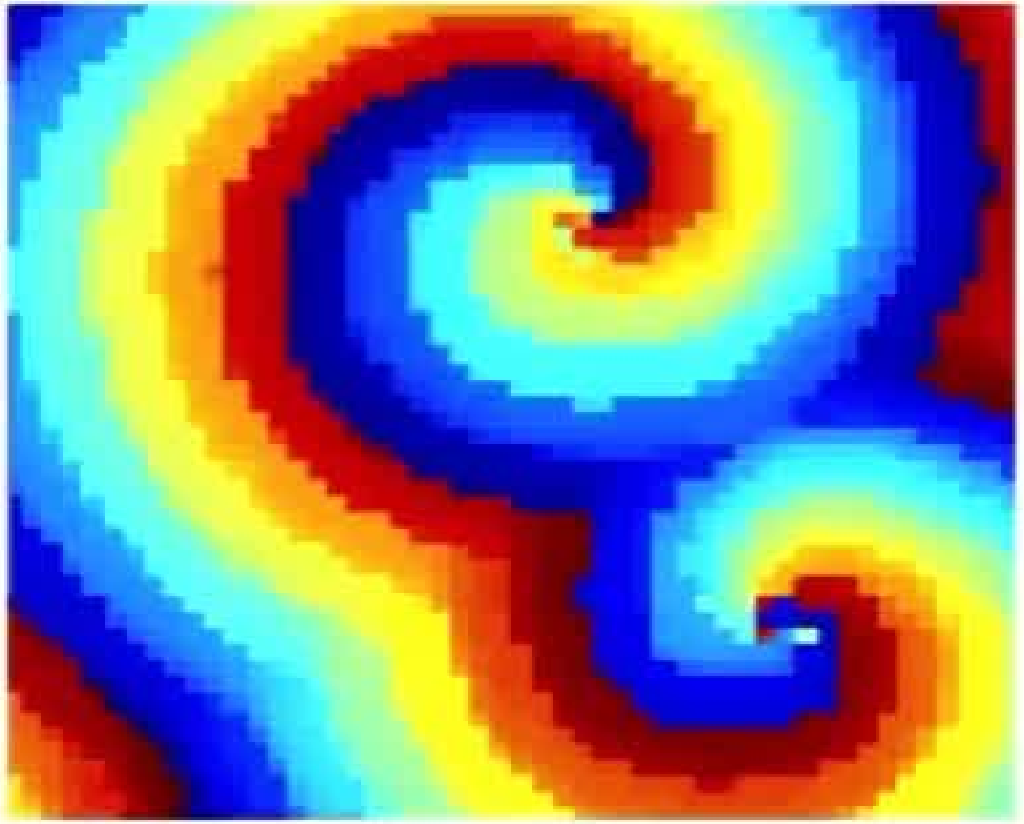
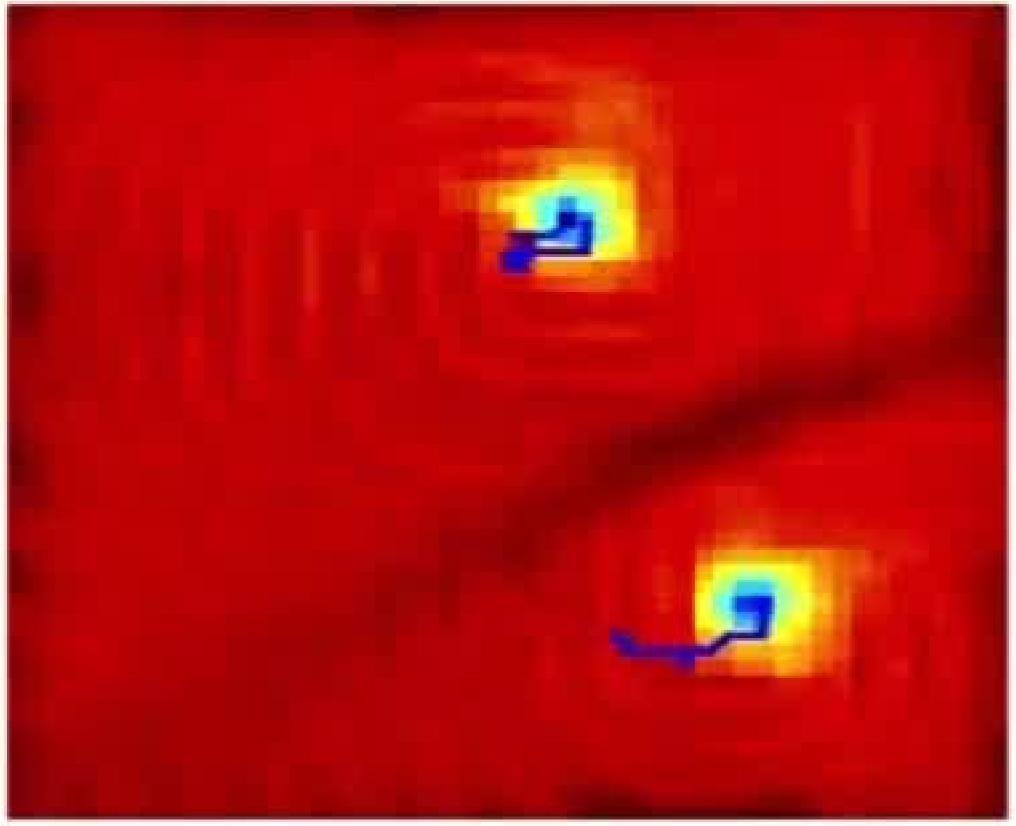


Phase: 2500
BZ oscillators



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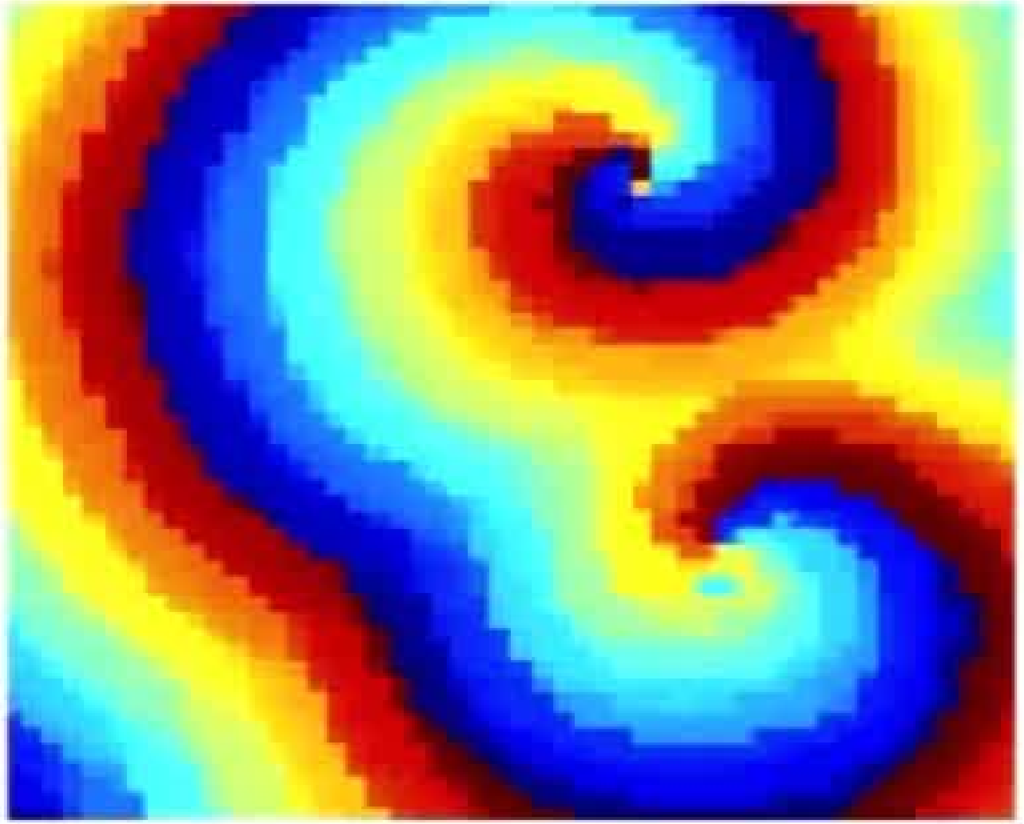

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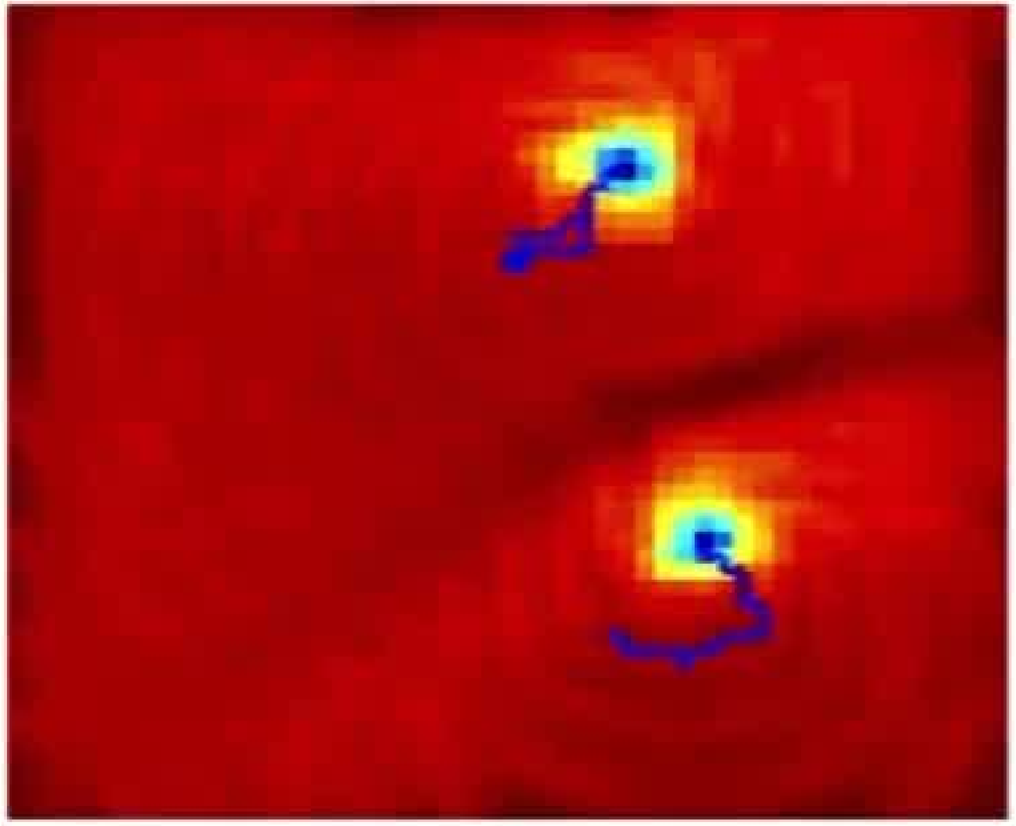

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Delay $\tau = 3.4$.

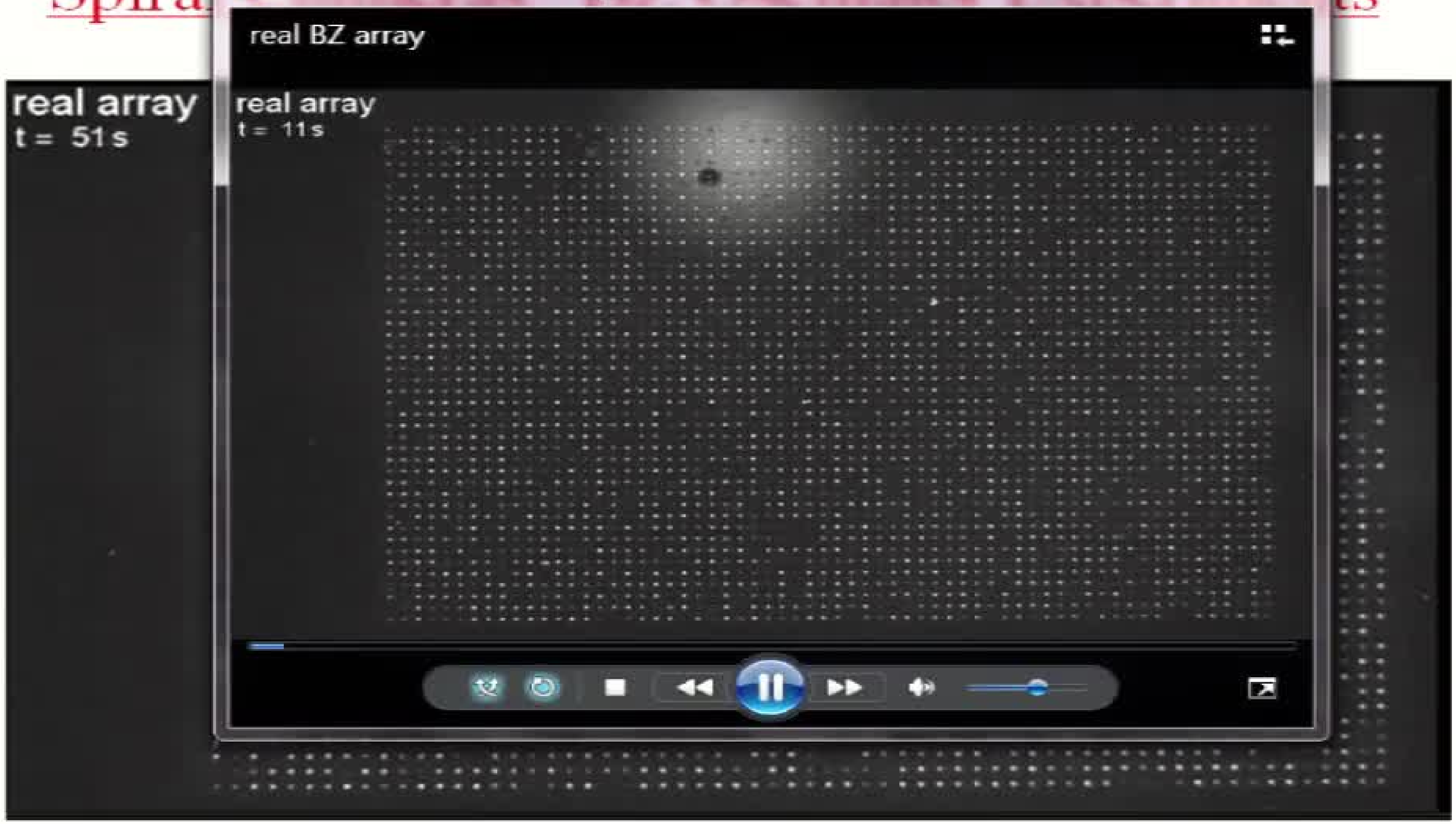
Spiral Chimeras BZ Oscillator ... ts

Movie

44 × 64 = 2816 BZ oscillators in real array. Ru(II) is excited by 440 nm illumination; phase of oscillator determined by fluorescence of Ru(II)*. Distance between oscillators is sufficient to prevent diffusive interaction. Oscillators are sorted for imperfections and to narrow frequency distribution.

Spira… BZ Oscillator …ts

Movie



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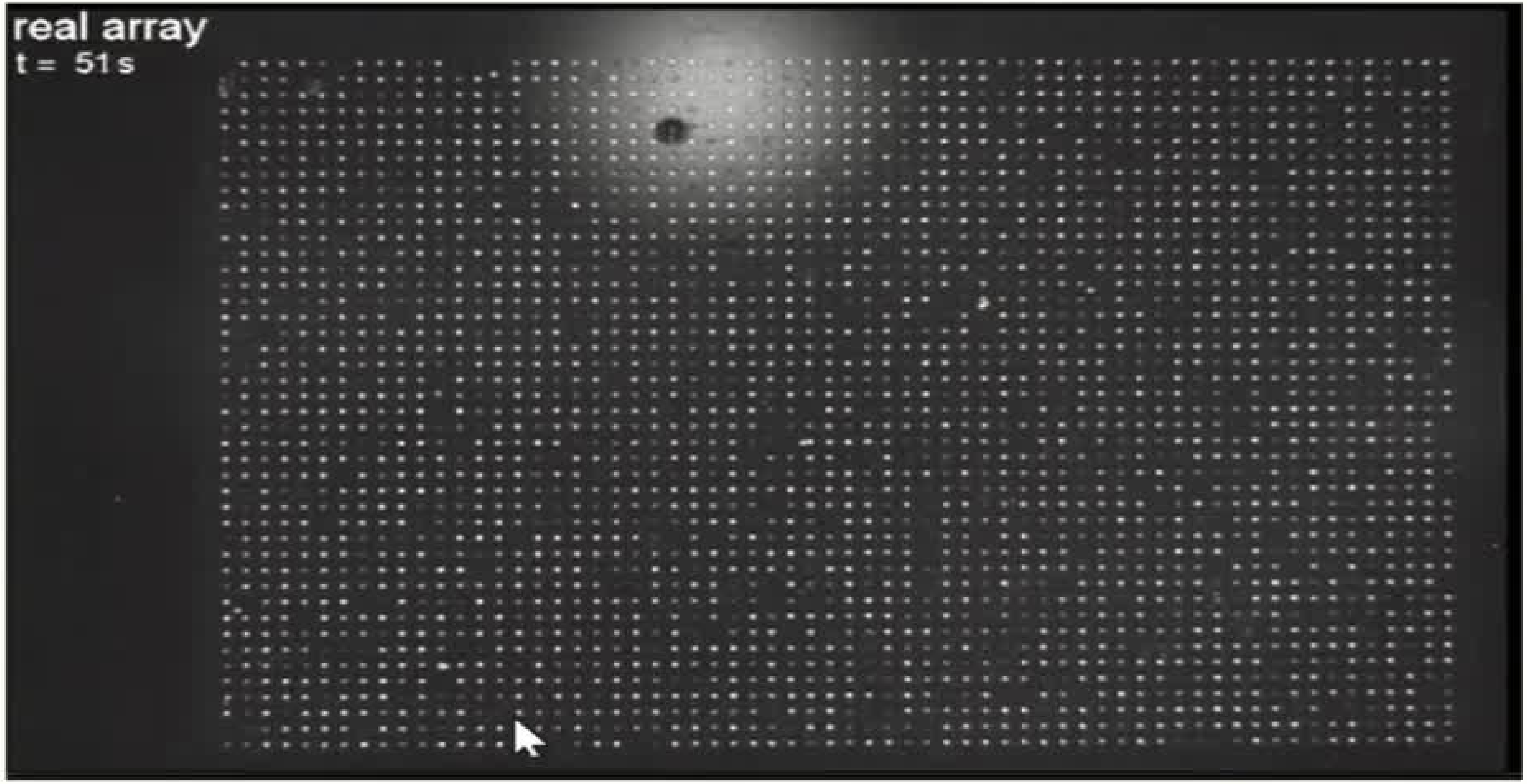
ray
real array
t = 217 s
00:07

Spiral Chimeras: BZ Oscillator Experiments

Movie



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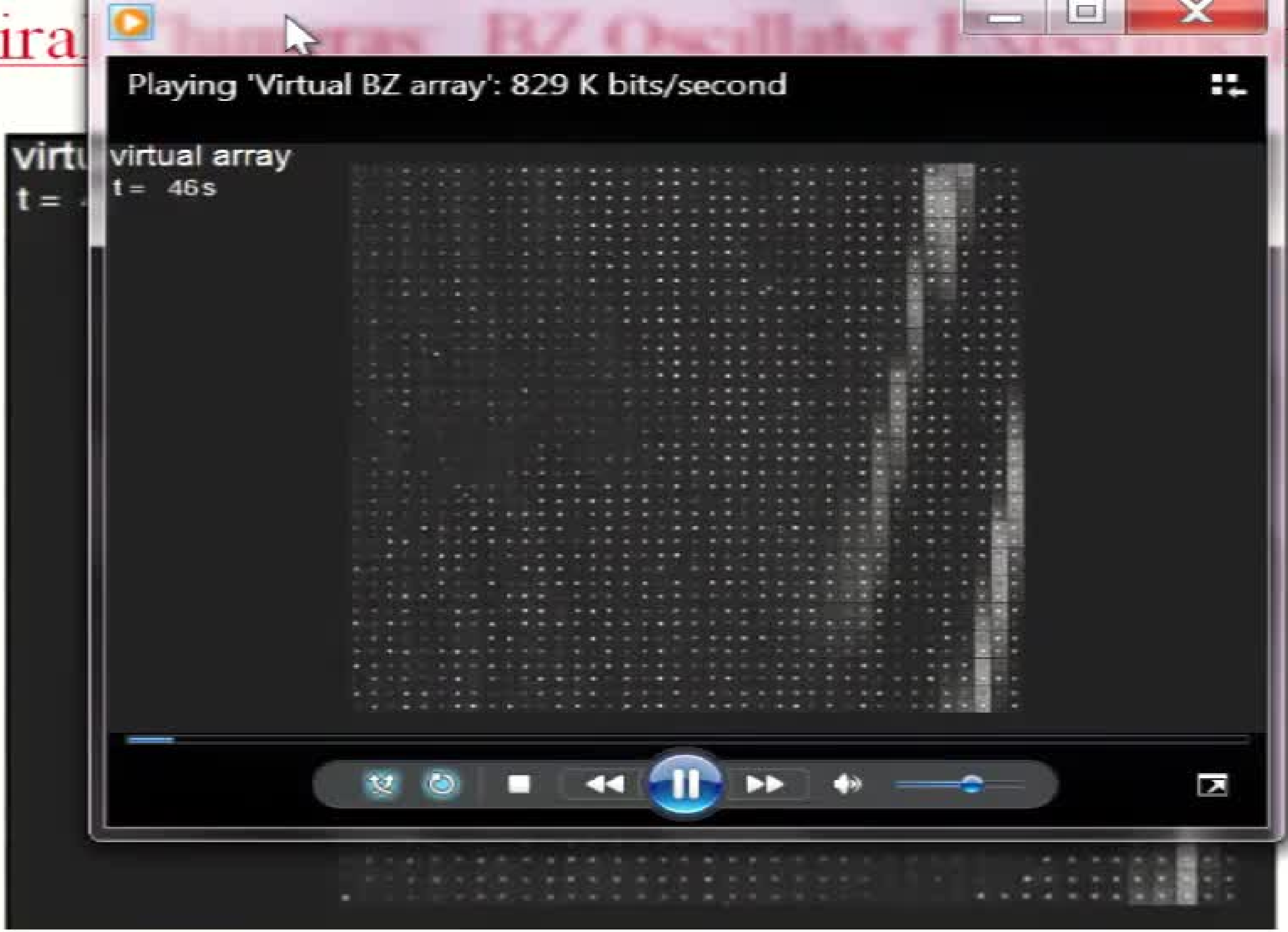
Spiral ... ts

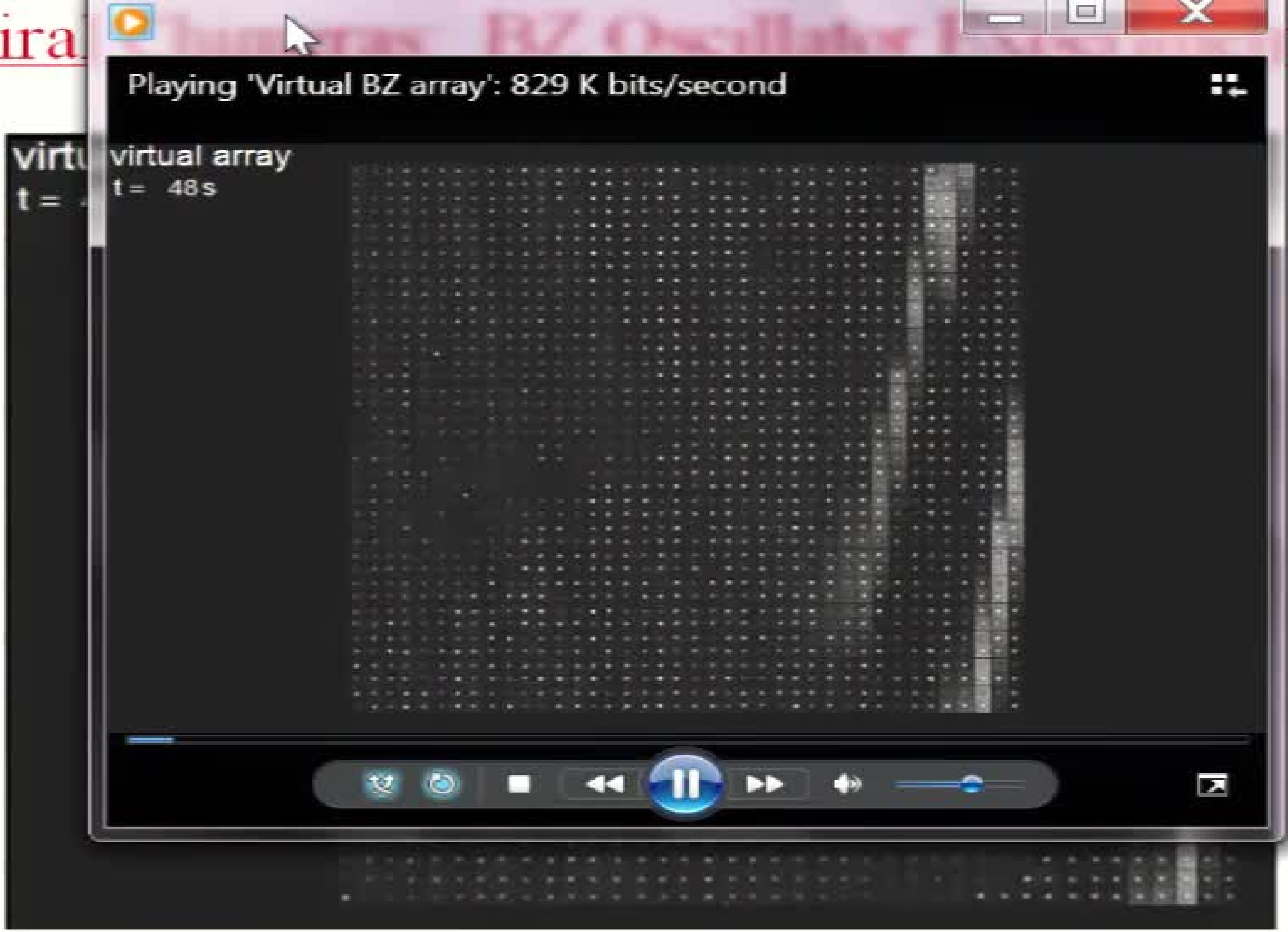
Movie



40 × 40 = 1600 BZ oscillators in virtual array.
Frequency distribution: T = 85.8 ± 7.0 s.
Periodic forcing initial conditions to give radially aligned phases. Delay τ = 2.0 s.

Spira… ts

Movie



$40 \times 40 = 1600$ BZ oscillators in virtual array.

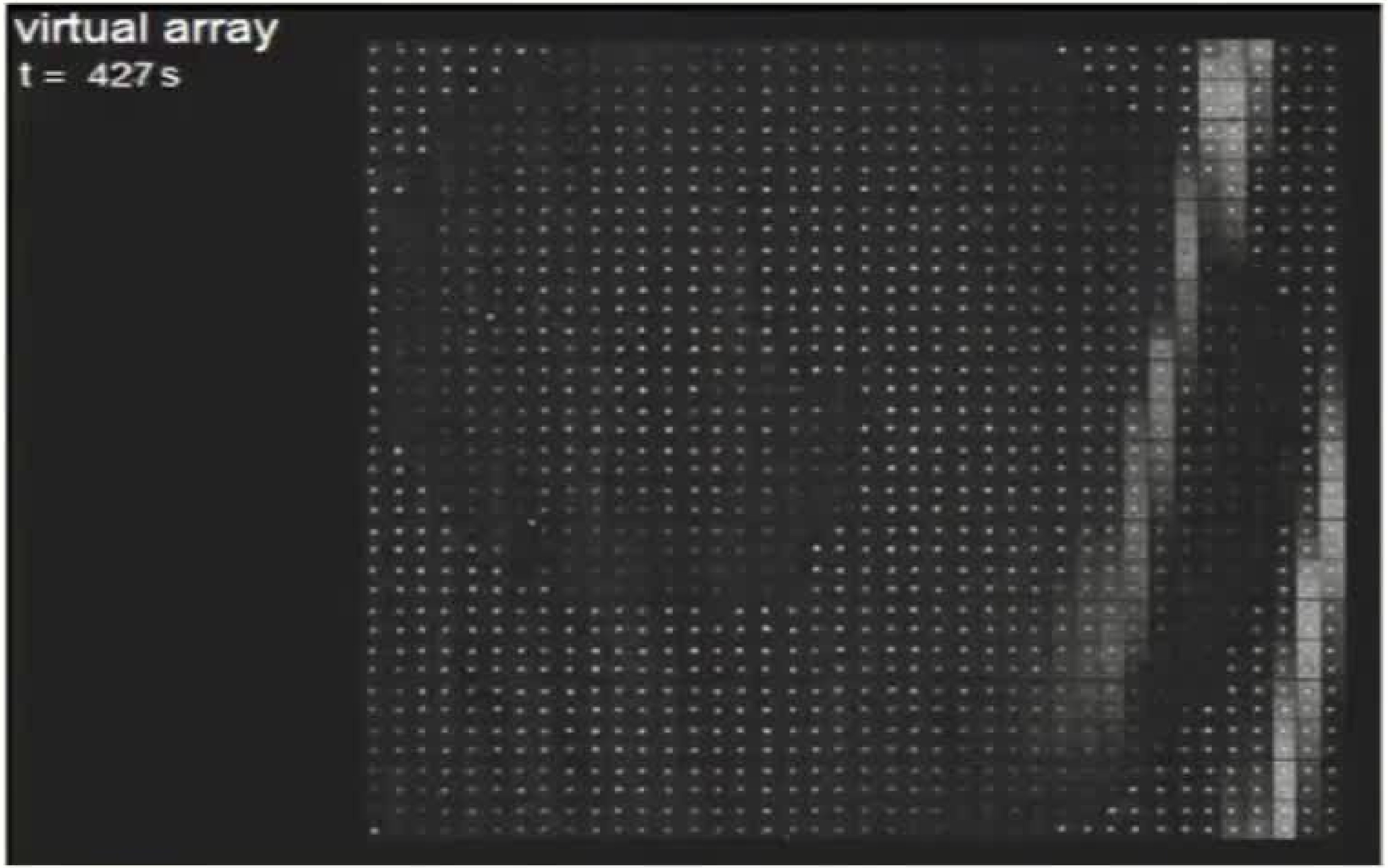
Frequency distribution: $T = 85.8 \pm 7.0$ s.

Periodic forcing initial conditions to give radially aligned phases. Delay $\tau = 2.0$ s.


array
virtual array
t = 71s
00:00

Spiral Chimeras: BZ Oscillator Experiments

Movie



40 × 40 = 1600 BZ oscillators in virtual array.
Frequency distribution: T = 85.8 ± 7.0 s.
Periodic forcing initial conditions to give radially aligned phases. Delay τ = 2.0 s.

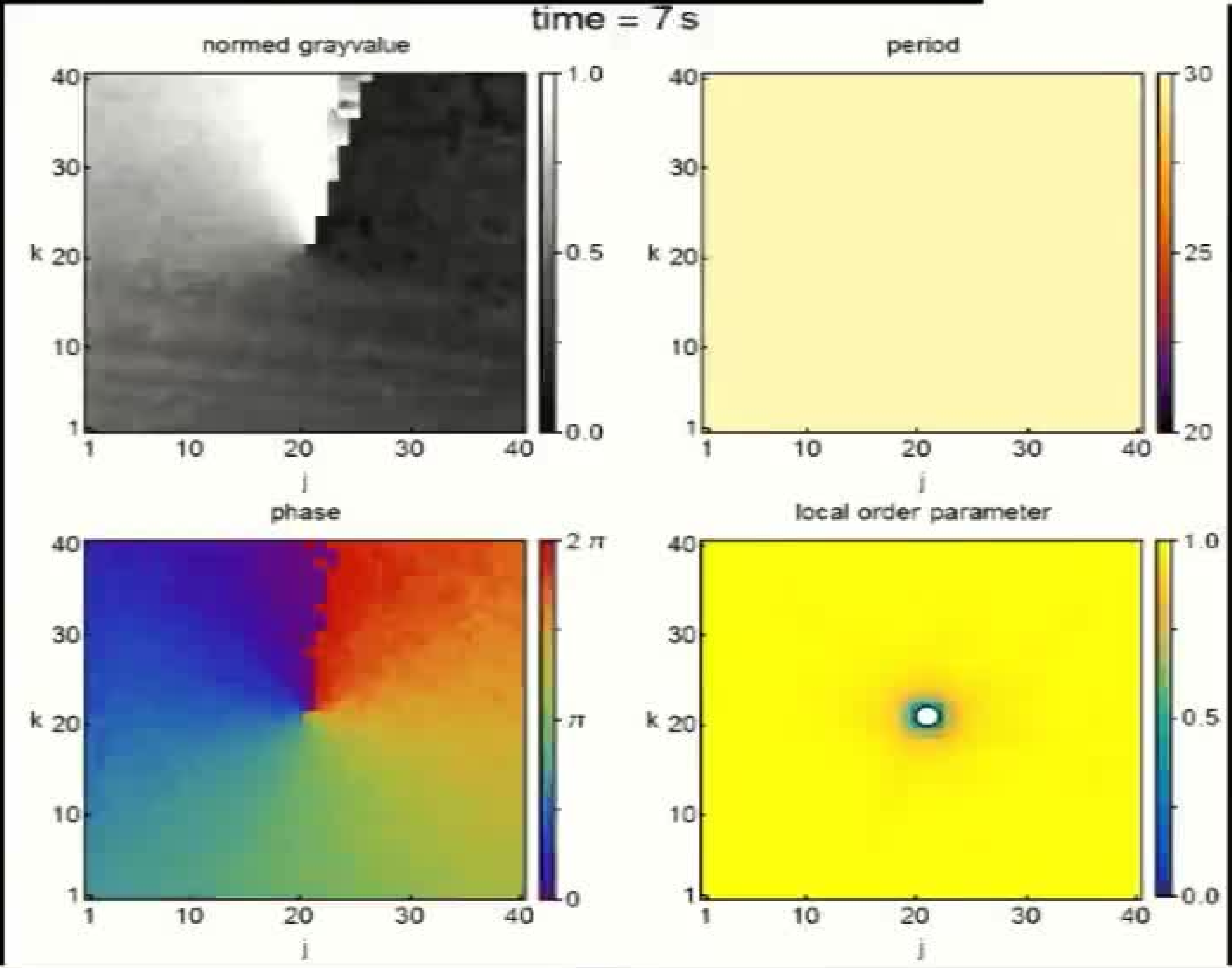
Spiral Chimeras – BZ Oscillator Experiments

Movie

40 × 40 = 1600 BZ oscillators in virtual array. Delay time $\tau = 2.0$ s.
Top row: panel 1 – raw data gray levels; panel 2 – instantaneous period.
Bottom row: panel 1 – oscillator phases; panel 2 – local order parameter, with core tracking.

Spir…

Movie



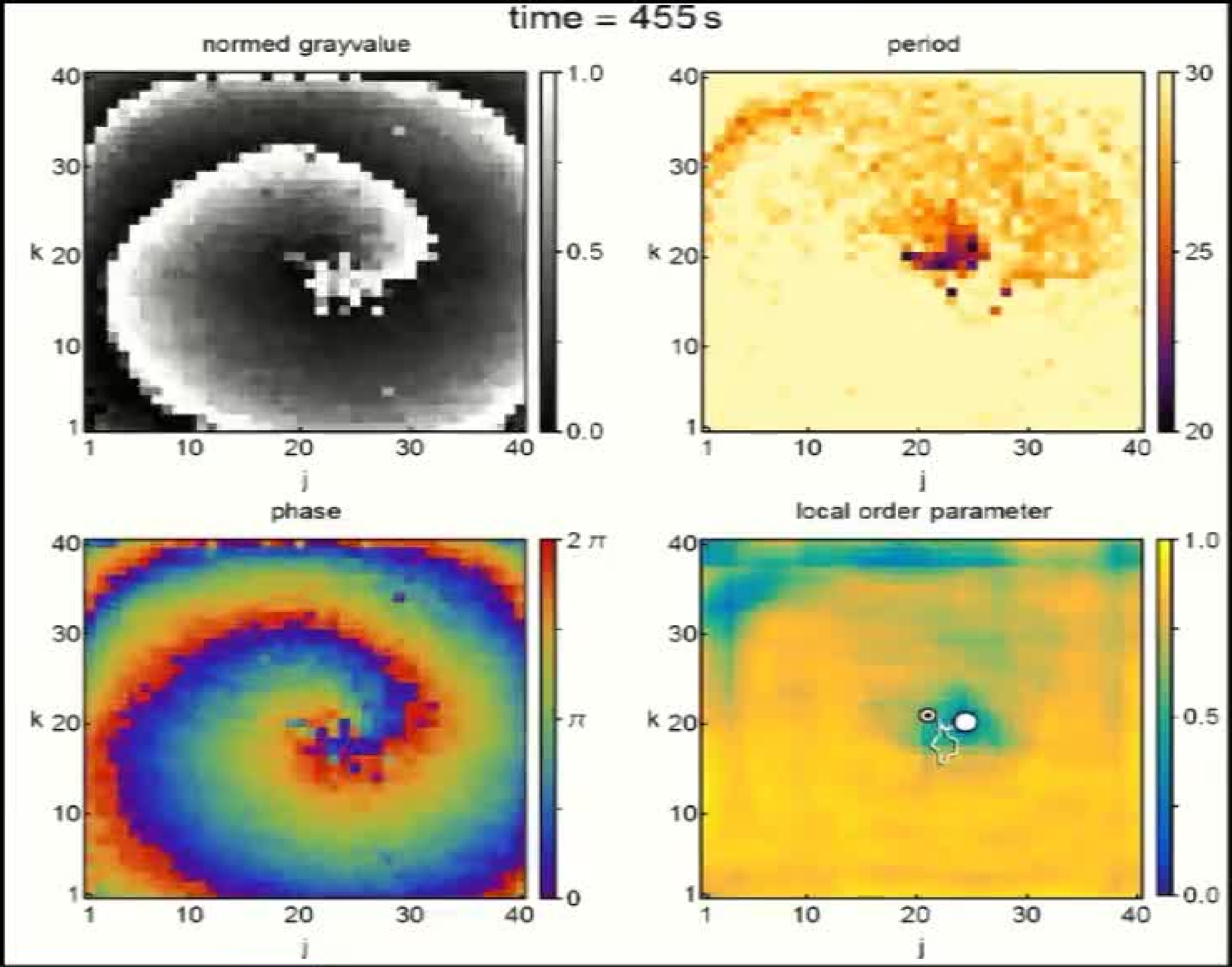
$40 \times 40 = 1600$ B…

Top row: panel 1 …

Bottom row: pane…

tracking.

core

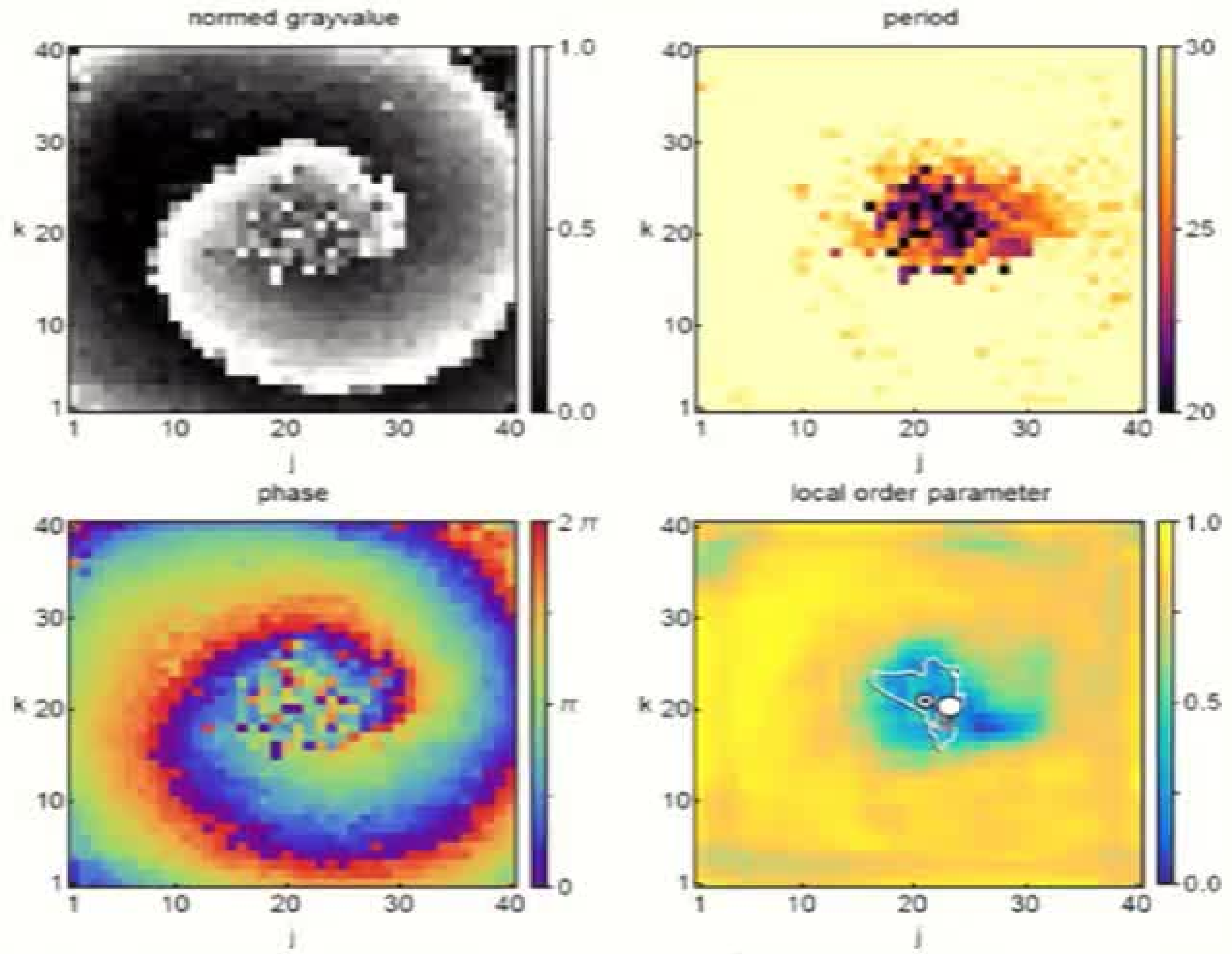


time = 455 s
normed grayvalue
period
phase
local order parameter
k
j
1.0
0.5
0.0
30
25
20
2 π
π
0

00:14

Spiral Chimeras: BZ Oscillator Experiments

Movie



$40 \times 40 = 1600$ BZ oscillators in virtual array. Delay time $\tau = 2.0$ s.

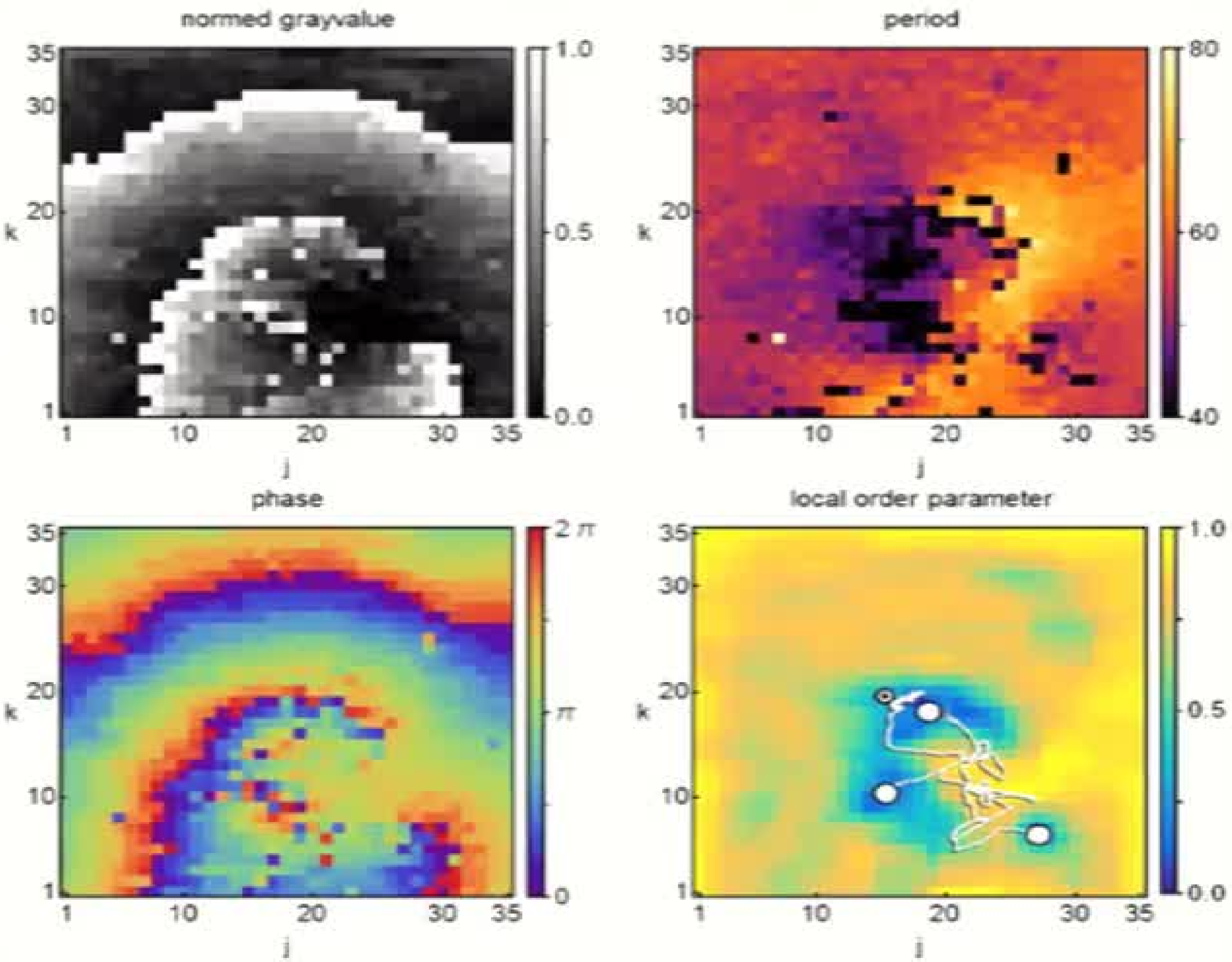
Top row: panel 1 – raw data gray levels; panel 2 – instantaneous period.

Bottom row: panel 1 – oscillator phases; panel 2 – local order parameter, with core tracking.

Spiral Chimeras: Core Splitting Experiments

Movie

Point\Presentations2015a\S4_



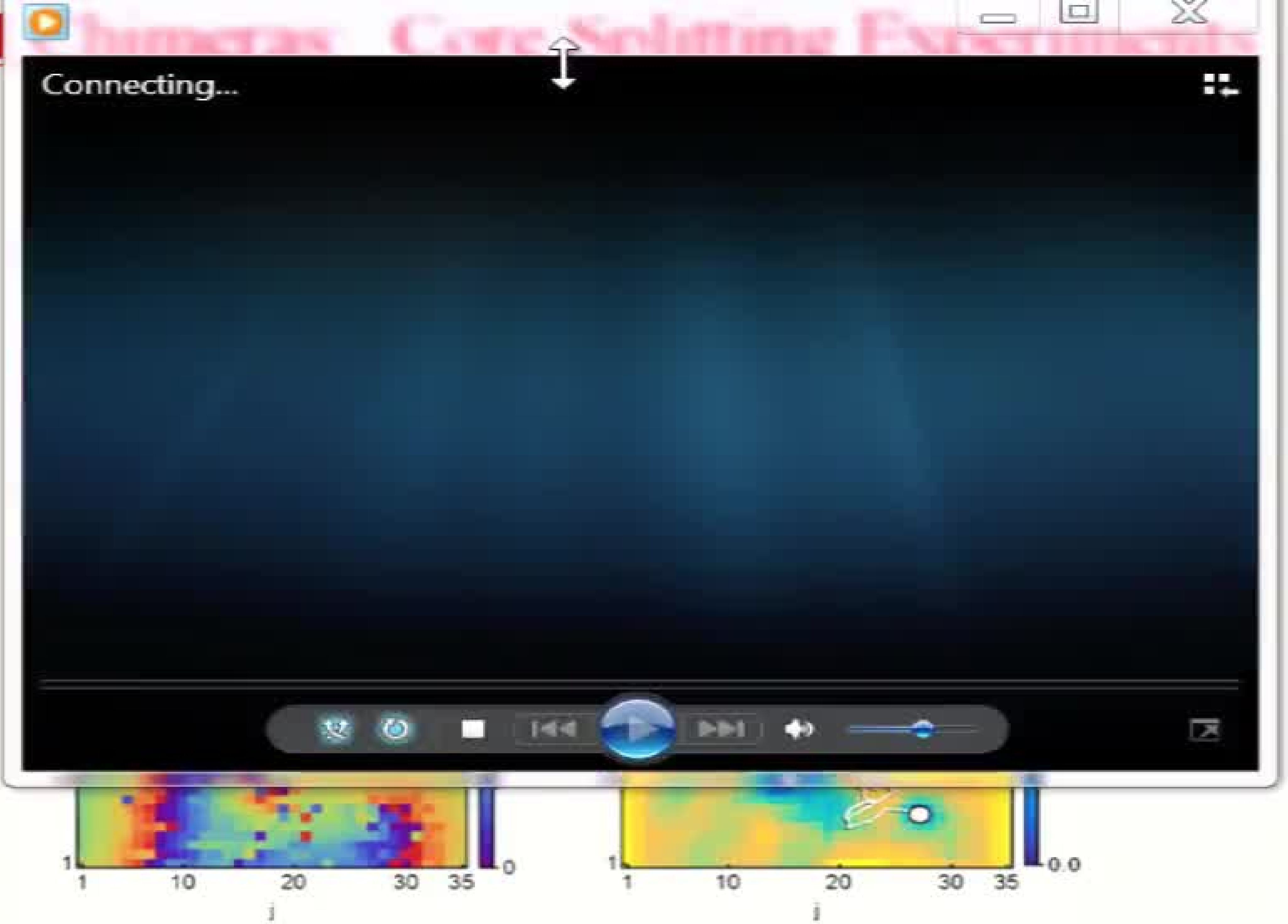
40 × 40 = 1600 BZ oscillators in virtual array. Delay time $\tau = 5.0$ s.

Top row: panel 1 – raw data gray levels; panel 2 – instantaneous period.

Bottom row: panel 1 – oscillator phases; panel 2 – local order parameter, with core tracking.

Spiral Chimeras – Core Splitting Experiments

Movie

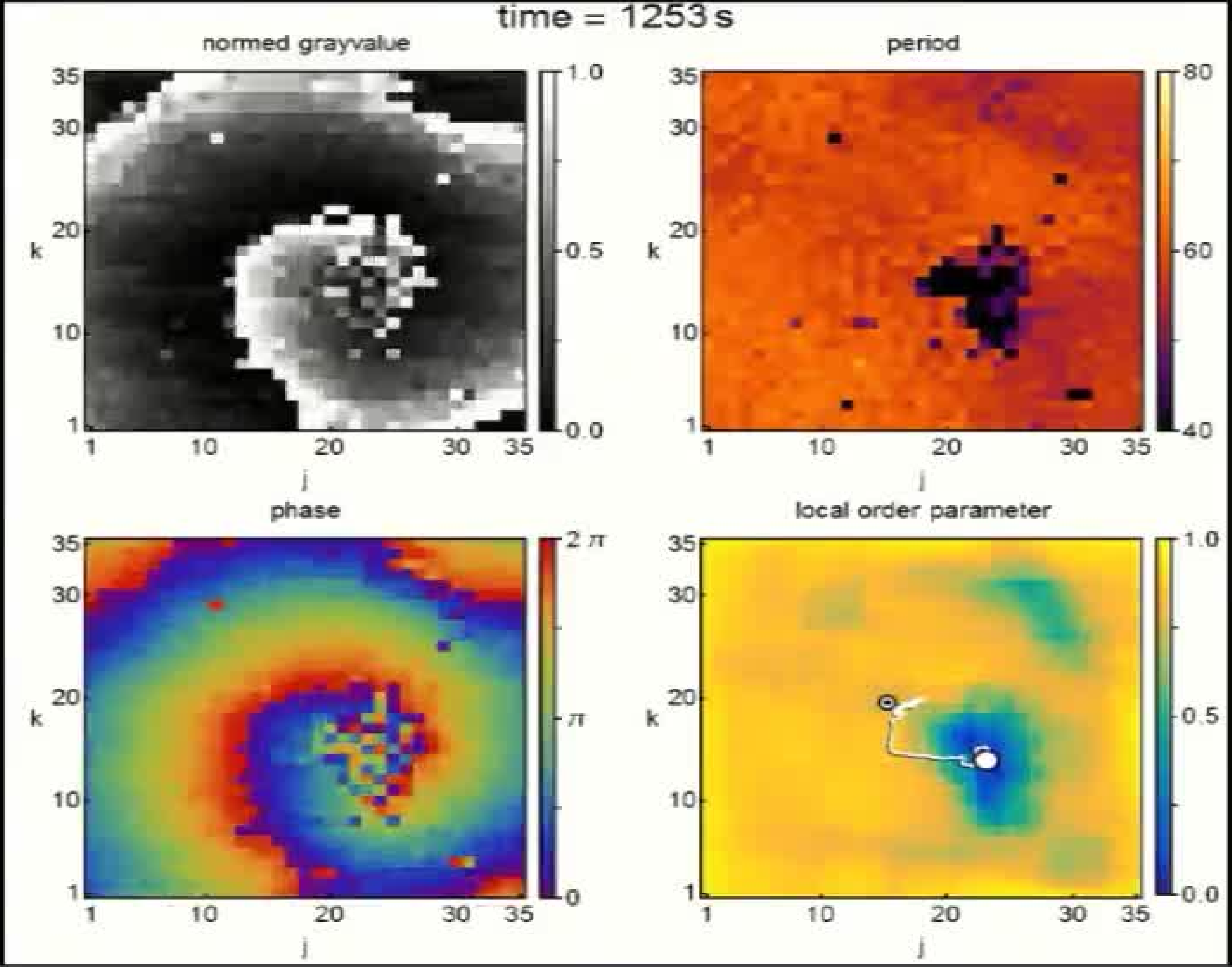


$40 \times 40 = 1600$ BZ oscillators in virtual array. Delay time $\tau = 5.0$ s.
Top row: panel 1 – raw data gray levels; panel 2 – instantaneous period.
Bottom row: panel 1 – oscillator phases; panel 2 – local order parameter, with core tracking.

Spiral

Movie

S4_split



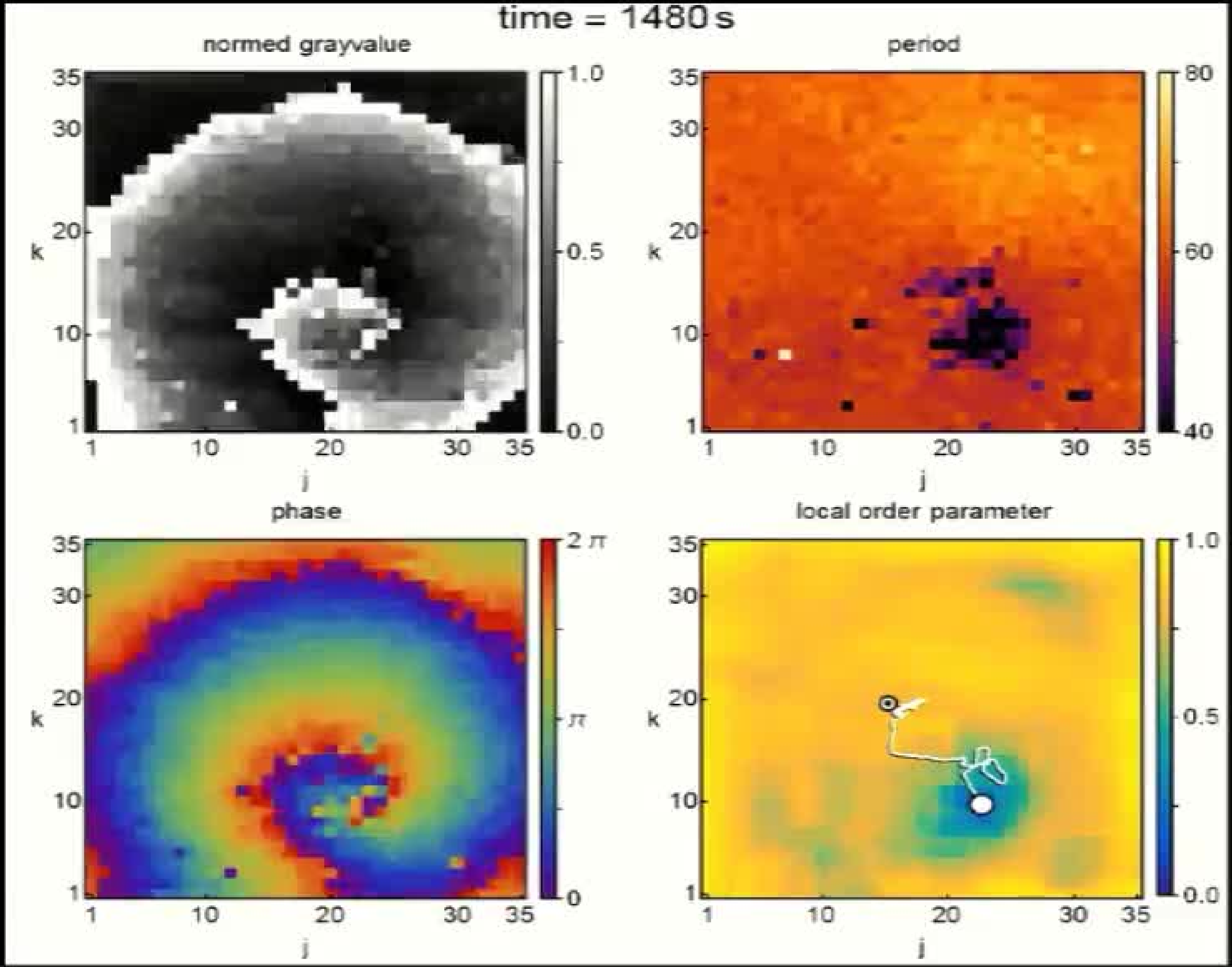
00:00

40 × 40 = 1600 B

Top row: panel 1

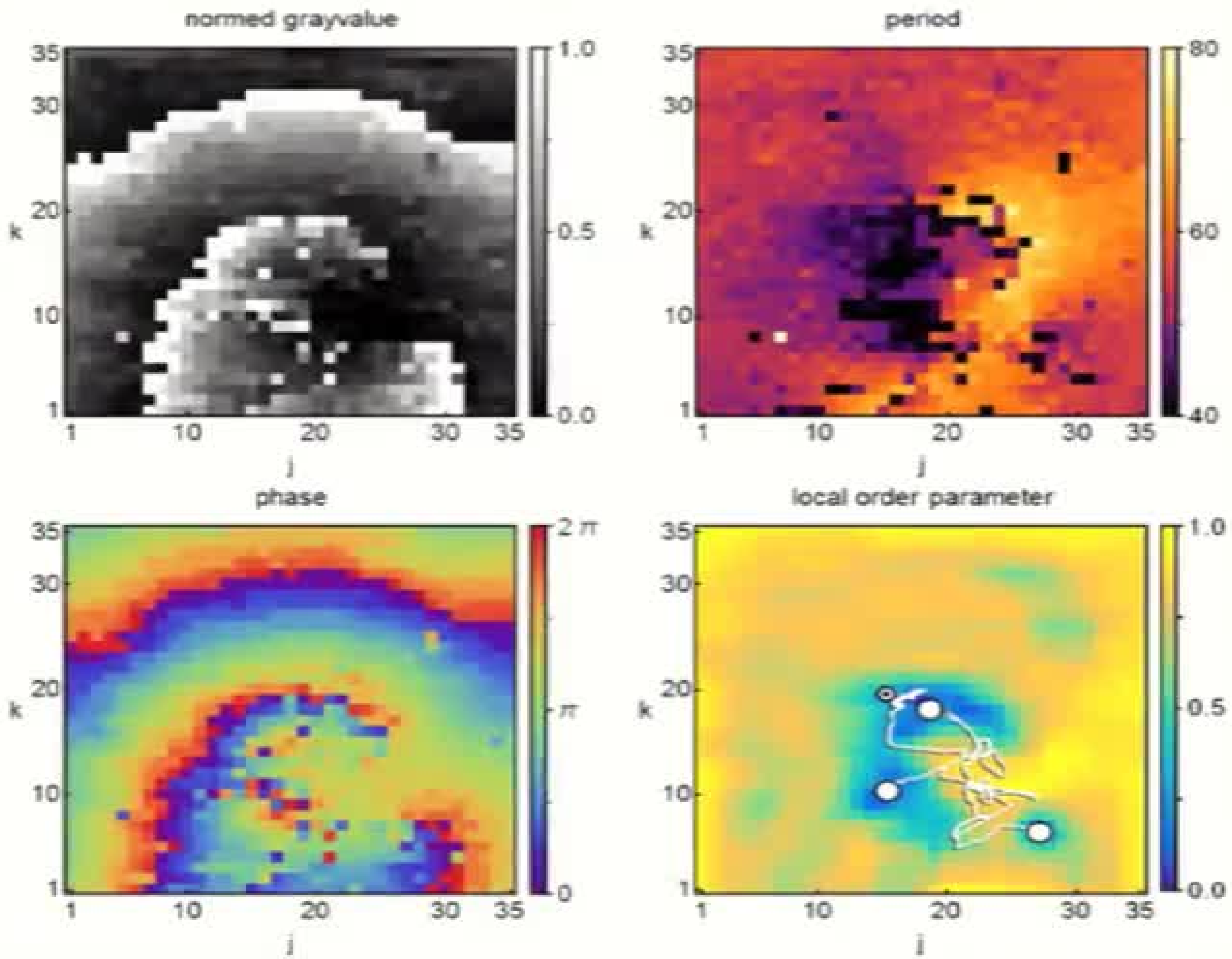
Bottom row: pane

tracking.


time = 1480 s
normed grayvalue
period
phase
local order parameter
k
j
1.0
0.5
0.0
80
60
40
2 π
π
0
00:04

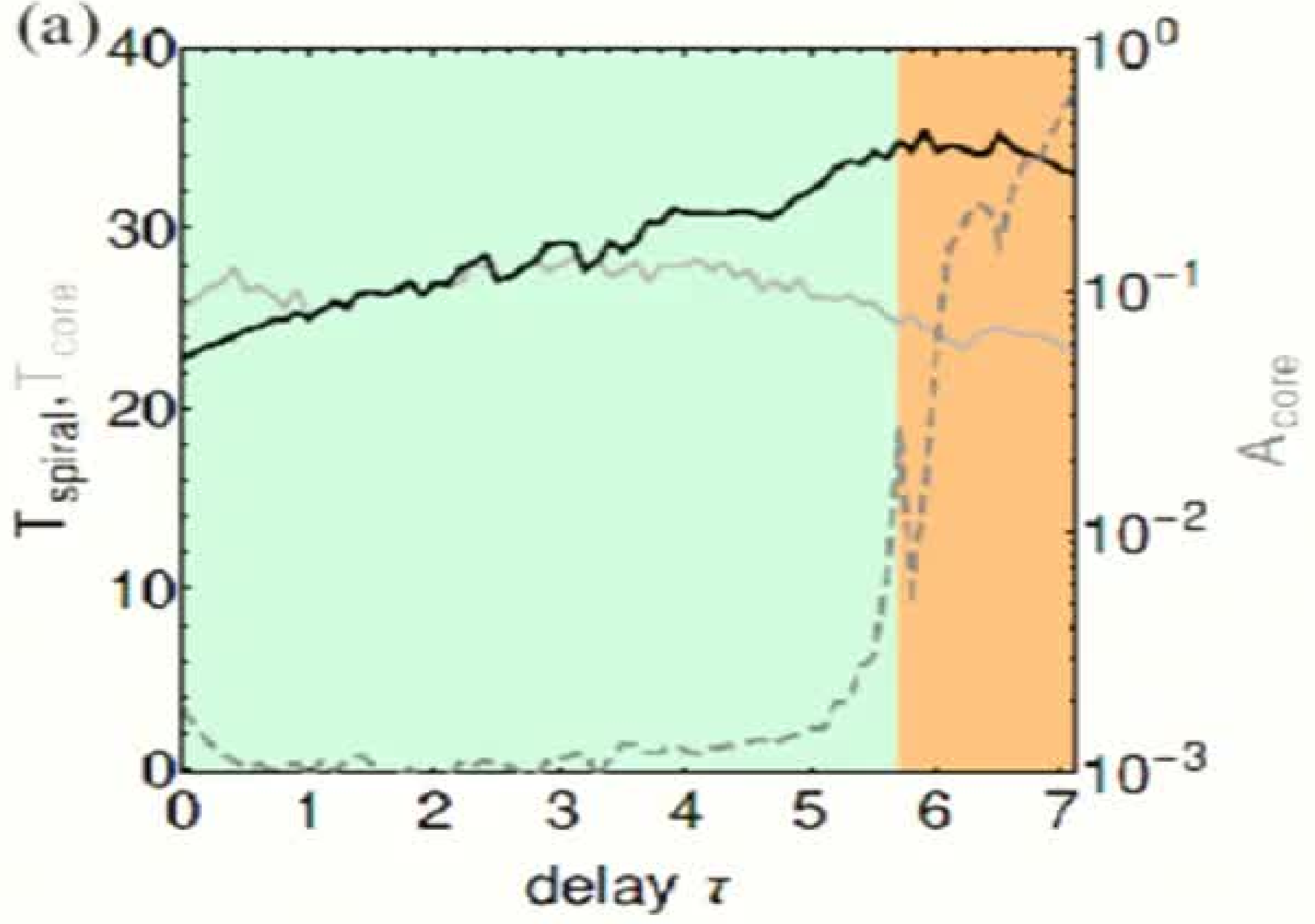
Spiral Chimeras: Core Splitting Experiments

Movie



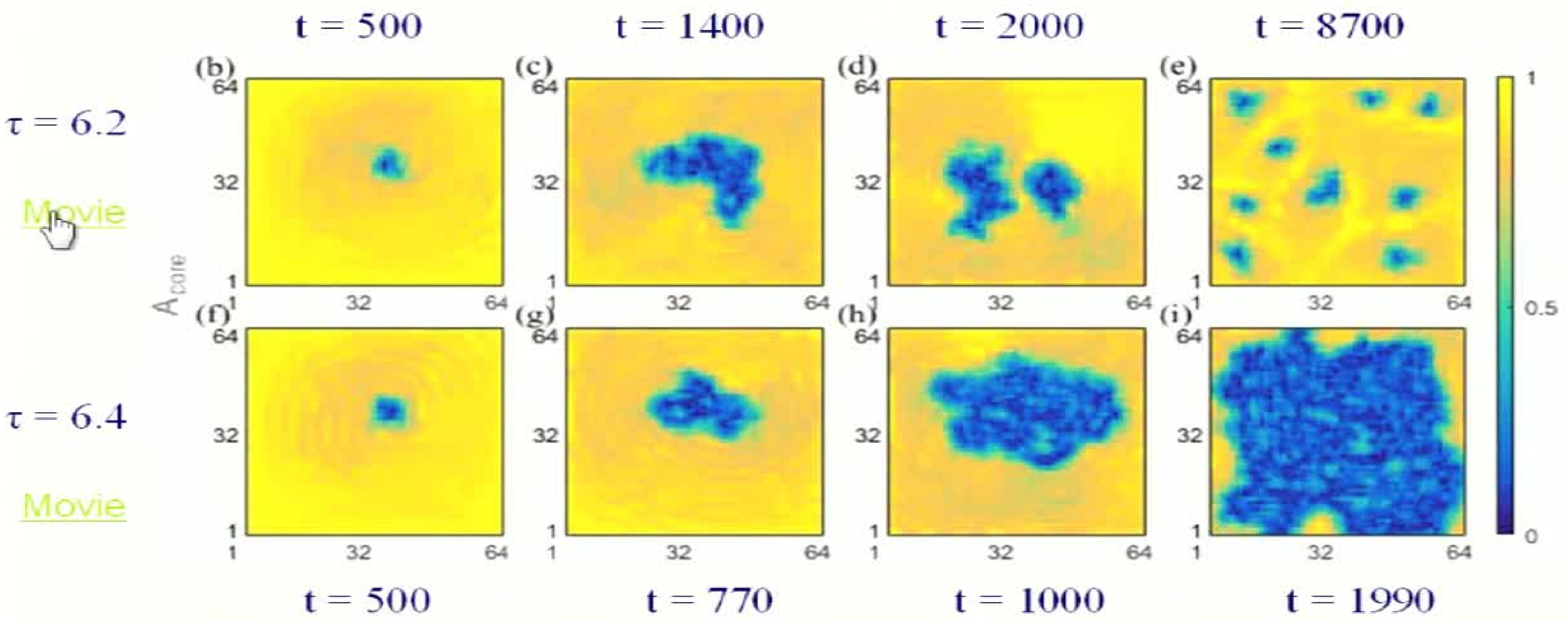
40 × 40 = 1600 BZ oscillators in virtual array. Delay time τ = 5.0 s.
Top row: panel 1 – raw data gray levels; panel 2 – instantaneous period.
Bottom row: panel 1 – oscillator phases; panel 2 – local order parameter, with core tracking.

Dependence on Delay Time



Rotational period of the spiral wave (solid black line)
Time-averaged period of the incoherent core oscillators (solid gray line)
Time-averaged fraction of the network made up of incoherent oscillators (dashed gray line).
A sharp increase in the fraction of the oscillators exhibiting asynchronous behavior can be seen in the salmon-colored region.

Transition from Spiral Wave Chimeras to Incoherence



Simulation of splitting and growth of spiral wave chimeras: 64 × 64 BZ oscillators.

(a) Asymptotic behavior is dynamic in nature with up to 9 small meandering cores.

(b) Initially small spiral wave chimera grows rapidly without splitting.

Behavior is highly dependent on initial conditions

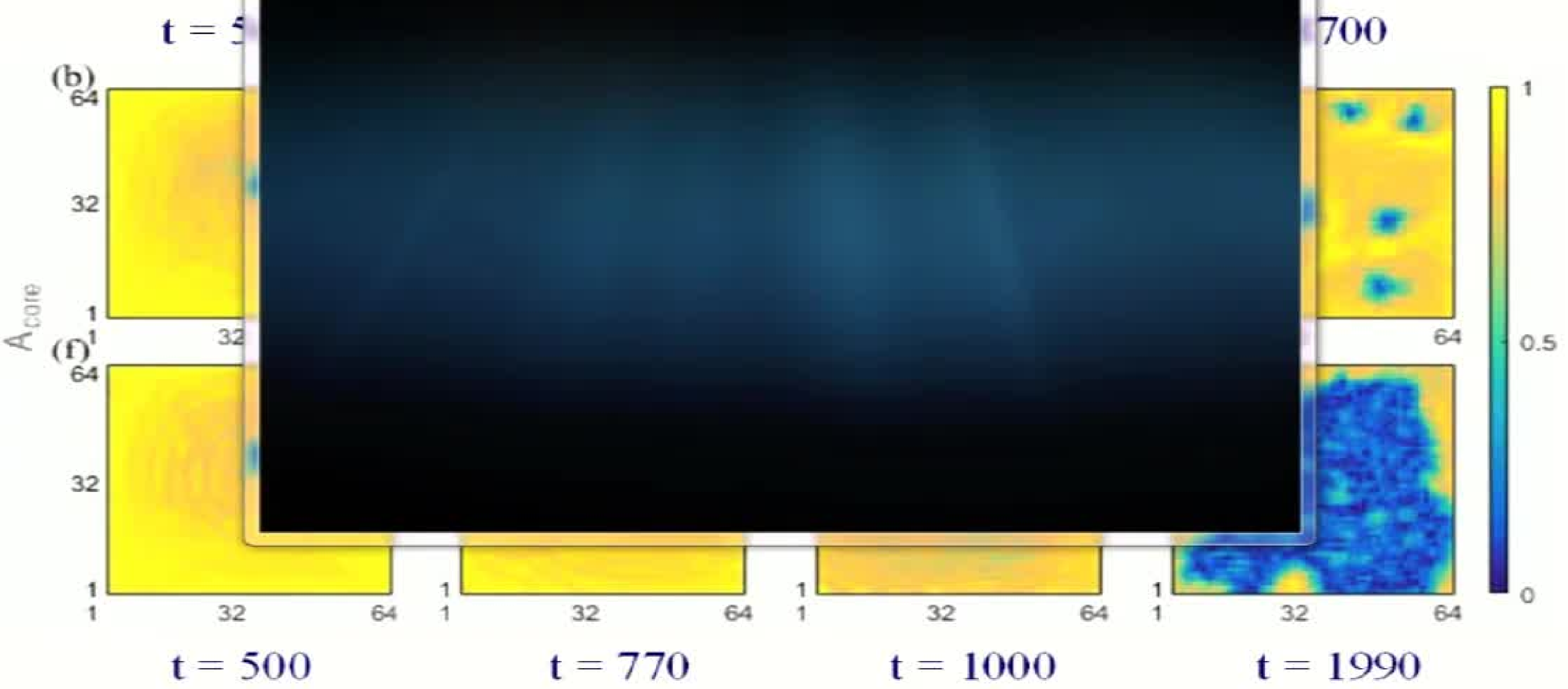
Transitio... ence

τ = 6.2

Movie

τ = 6.4

Movie



Simulation of splitting and growth of spiral wave chimeras: 64 × 64 BZ oscillators.

(a) Asymptotic behavior is dynamic in nature with up to 9 small meandering cores.

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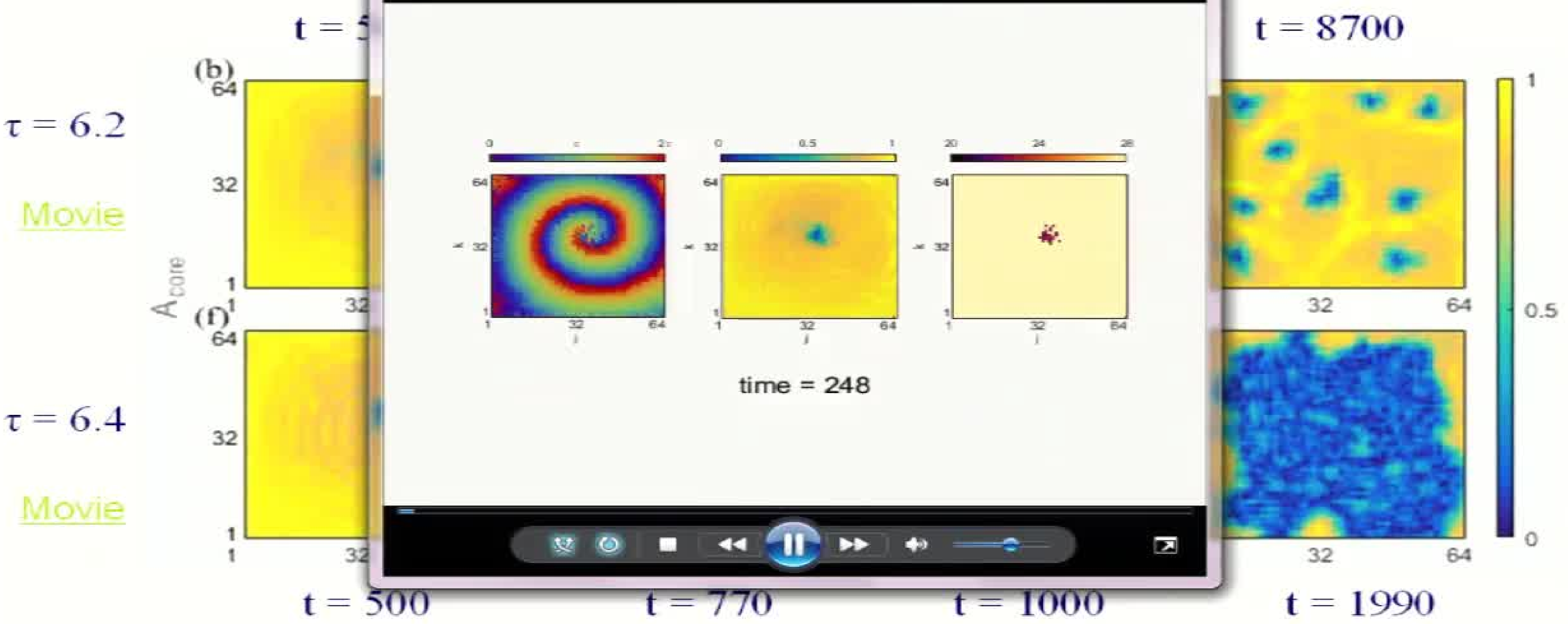
Transition ... coherence



Simulation of splitting and growth of spiral wave chimeras: 64 × 64 BZ oscillators.
(a) Asymptotic behavior is dynamic in nature with up to 9 small meandering cores.
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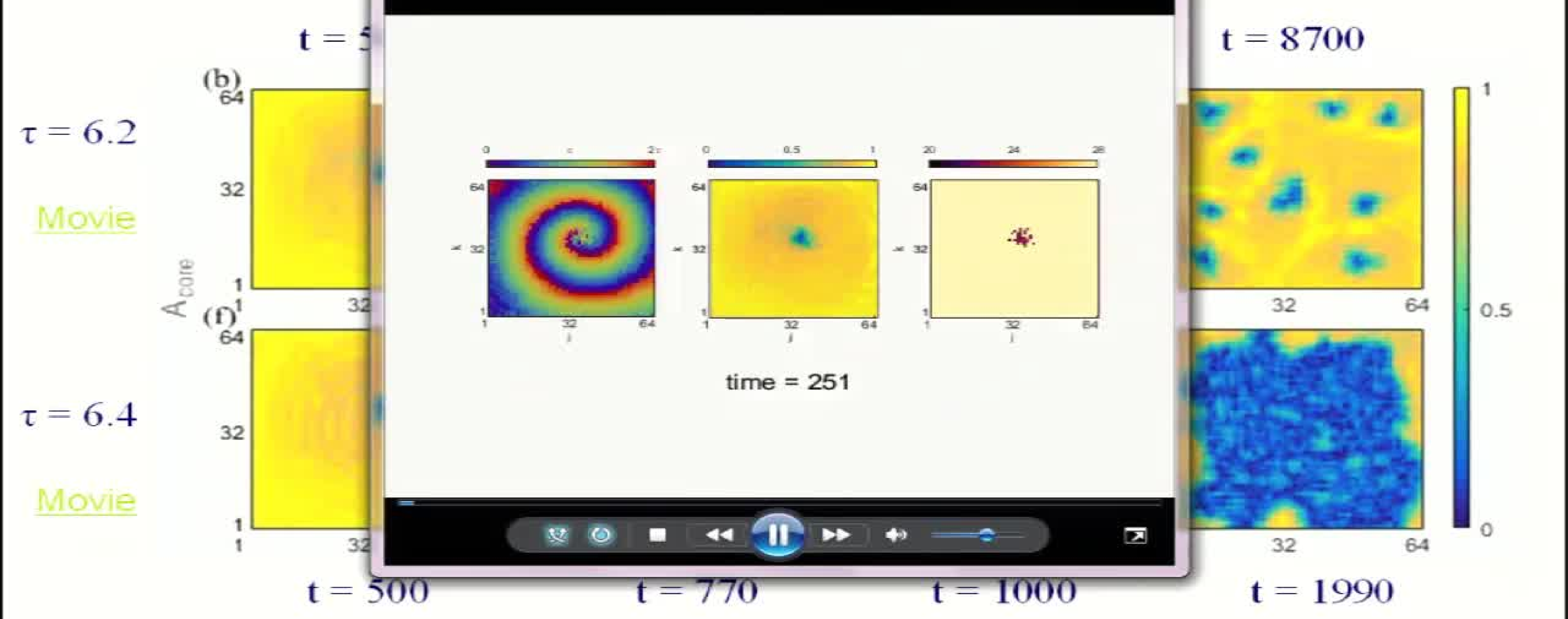
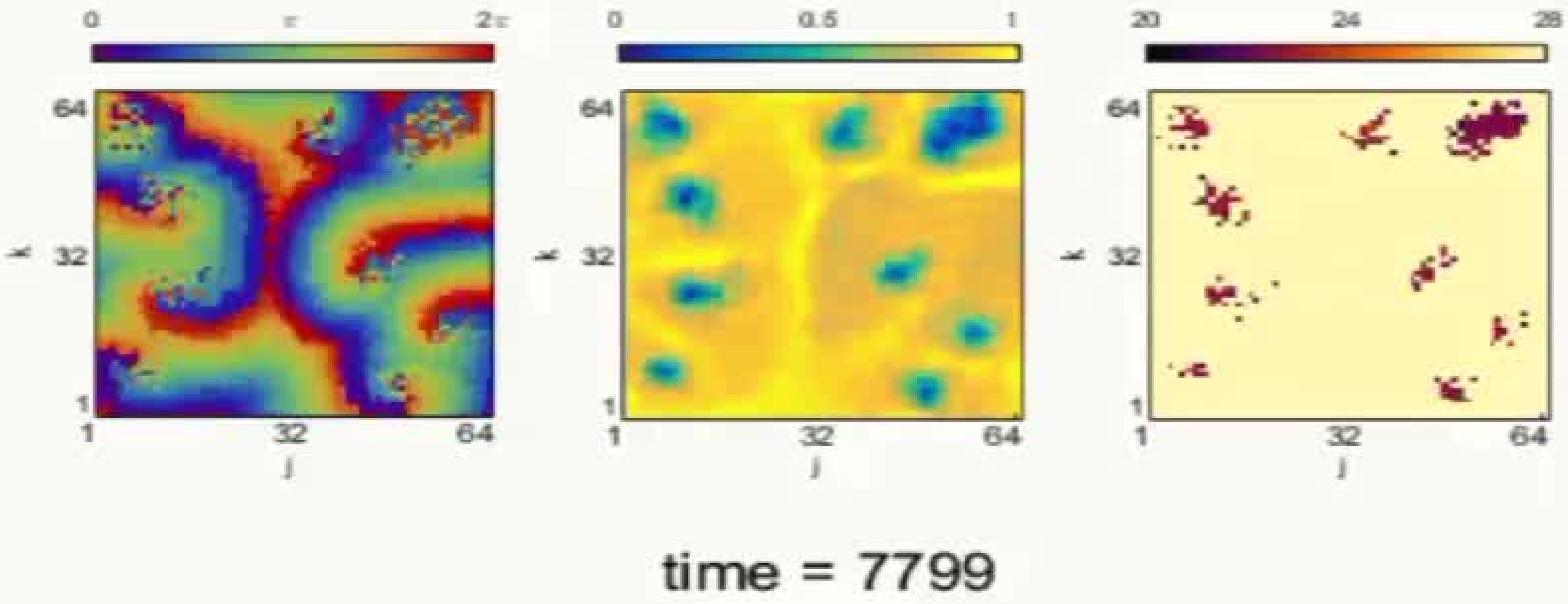
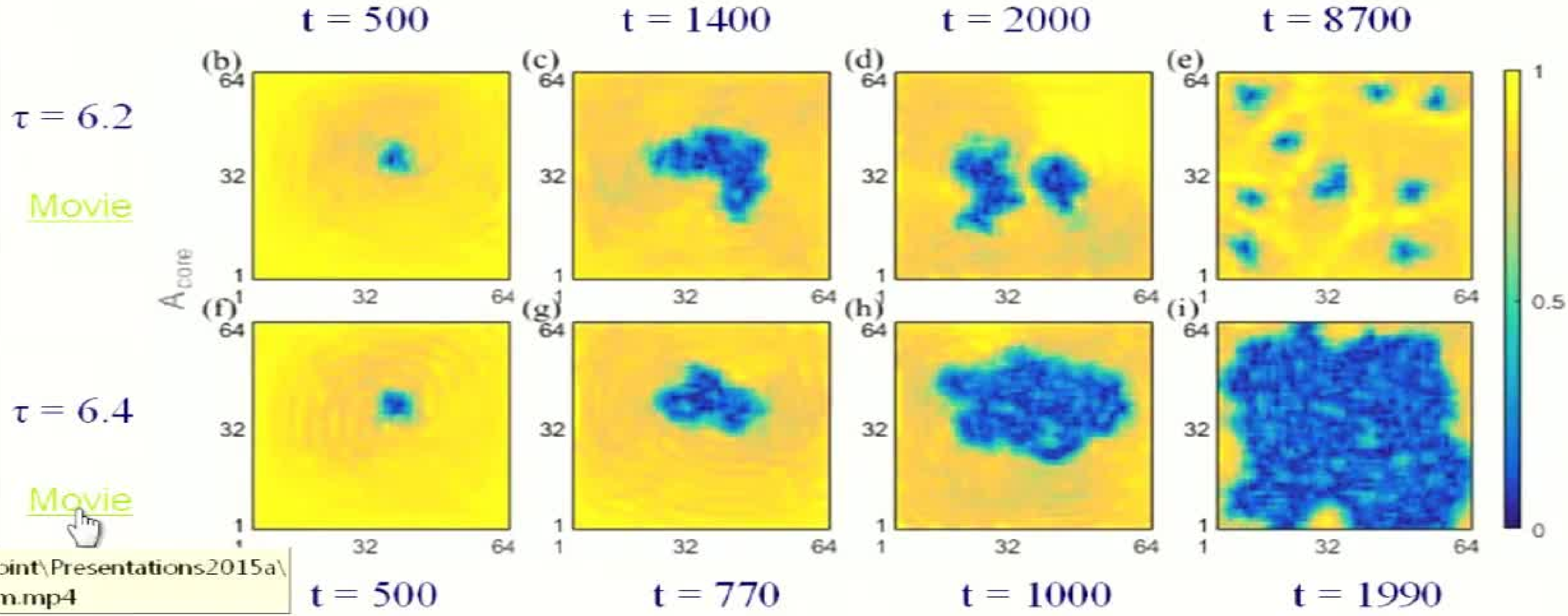
Transition ... coherence



Simulation of splitting and growth of spiral wave chimeras: 64 × 64 BZ oscillators.
(a) Asymptotic behavior is dynamic in nature with up to 9 small meandering cores.
(b) Initially small spiral wave chimera grows rapidly without splitting.


0
π
2π
0
0.5
1
20
24
28
64
32
1
k
j
time = 7799

Transition from Spiral Wave Chimeras to Incoherence



Simulation of splitting and growth of spiral wave chimeras: 64 × 64 BZ oscillators.

(a) Asymptotic behavior is dynamic in nature with up to 9 small meandering cores.

(b) Initially small spiral wave chimera grows rapidly without splitting.

Behavior is highly dependent on initial conditions.

Transitio[n] ... [...]ence

t = 5[...] ... [...]700

τ = 6.2

Movie

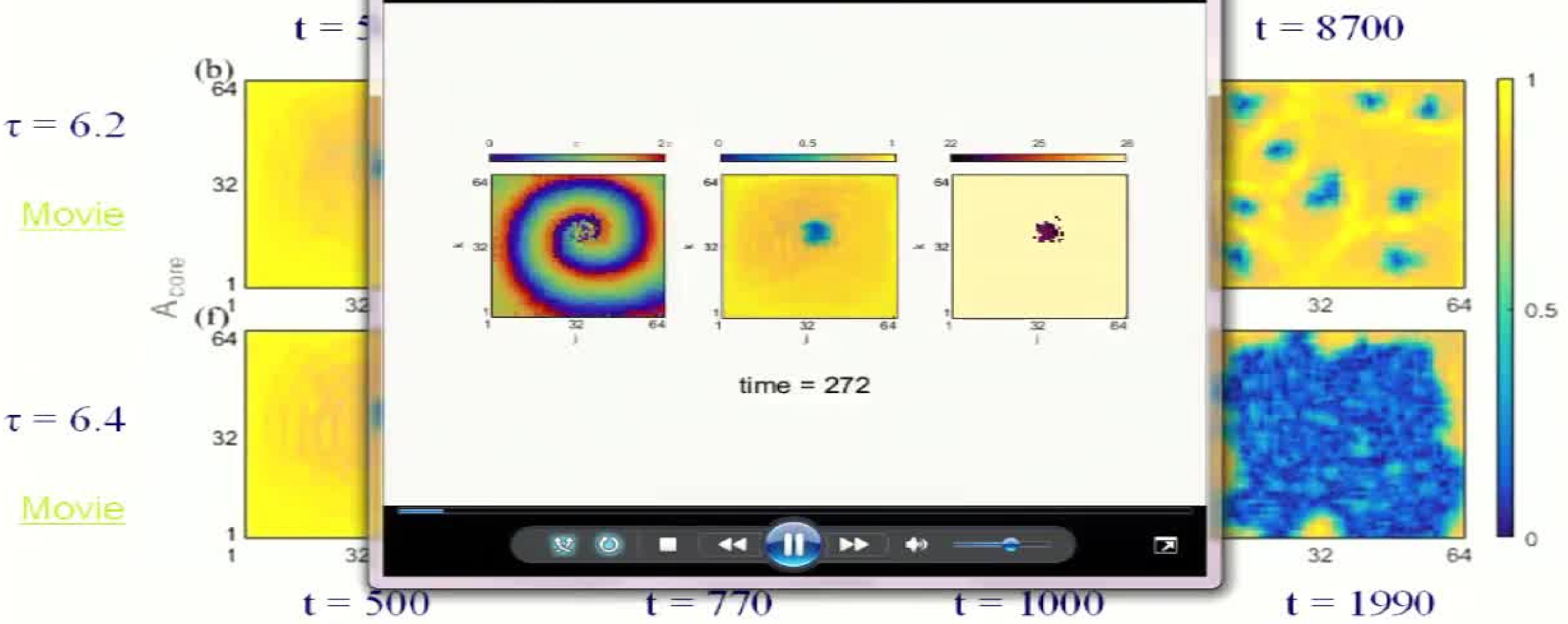
τ = 6.4

Movie

t = 500 t = 770 t = 1000 t = 1990

Simulation of splitting and growth of spiral wave chimeras: 64 × 64 BZ oscillators.
(a) Asymptotic behavior is dynamic in nature with up to 9 small meandering cores.
(b) Initially small spiral wave chimera grows rapidly without splitting.

Transition … coherence



Simulation of splitting and growth of spiral wave chimeras: 64 × 64 BZ oscillators.
(a) Asymptotic behavior is dynamic in nature with up to 9 small meandering cores.
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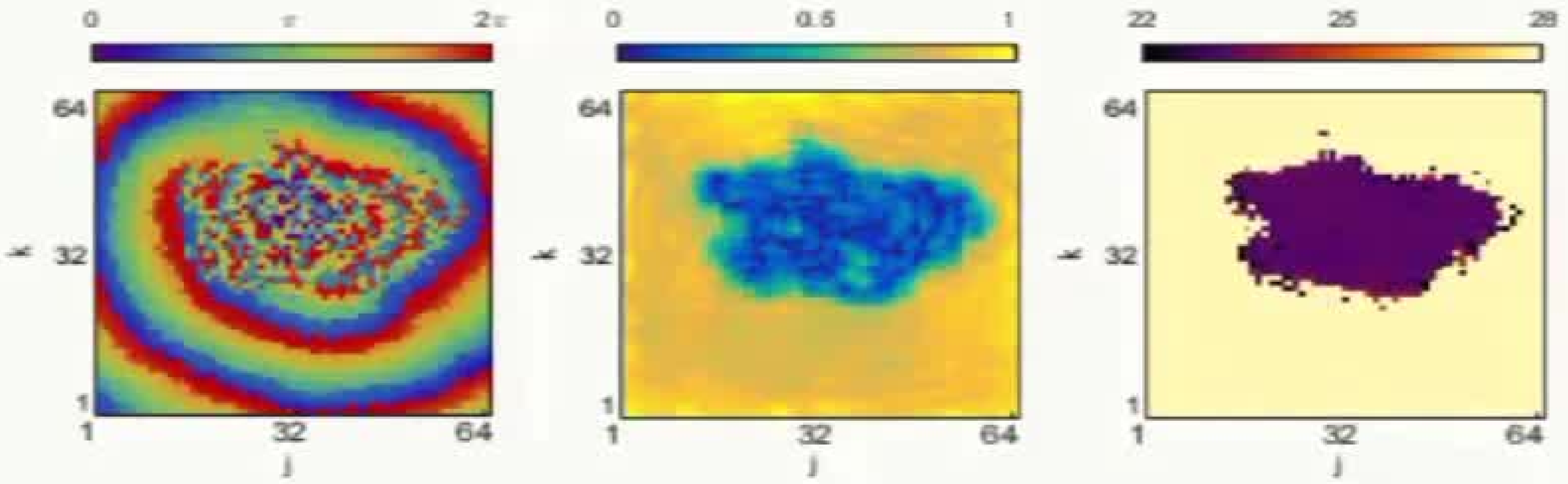
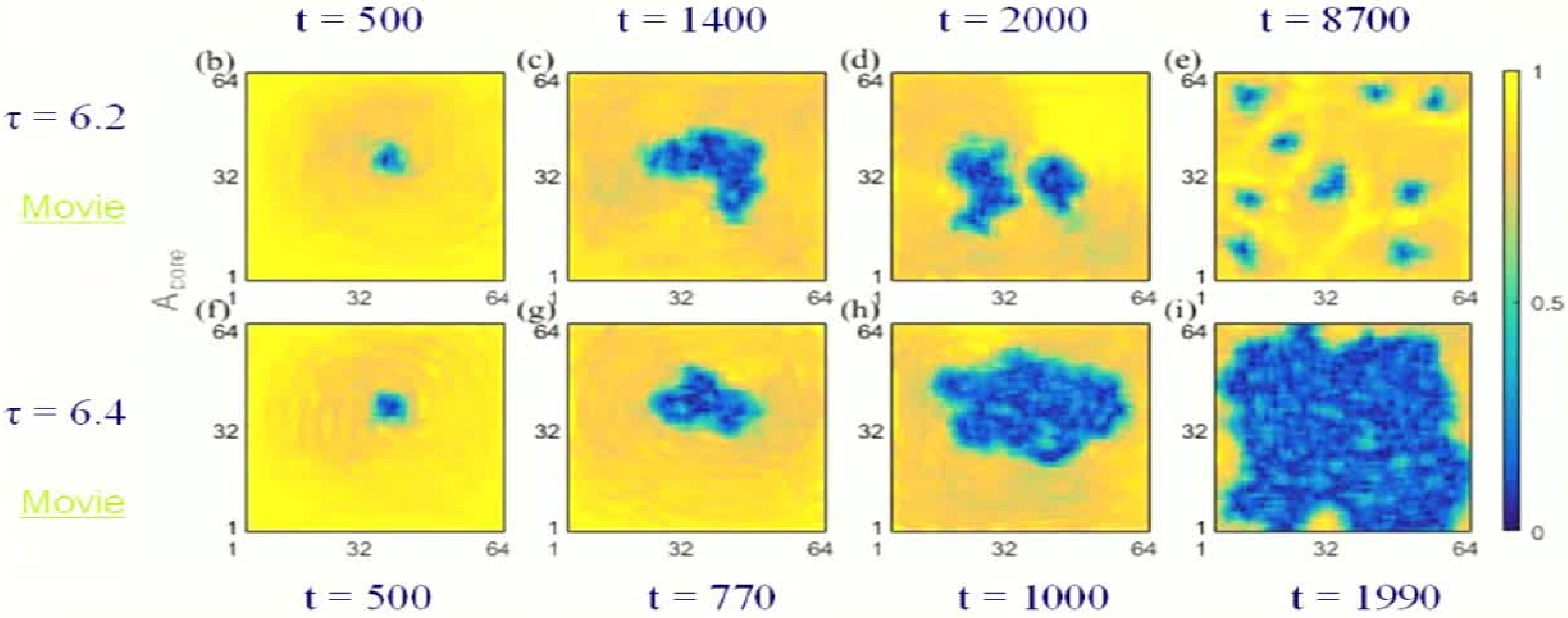


time = 731

Transition from Spiral Wave Chimeras to Incoherence



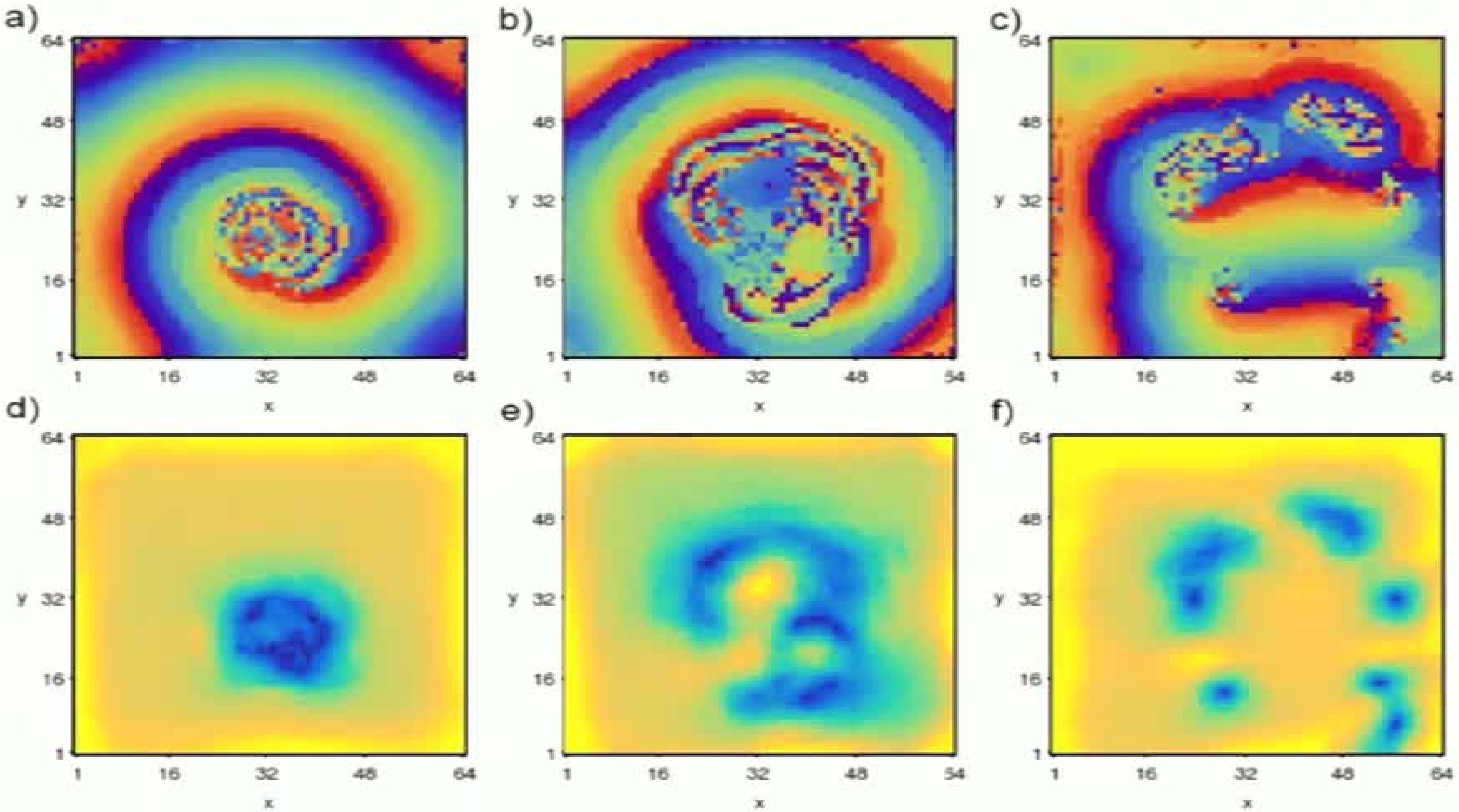
Simulation of splitting and growth of spiral wave chimeras: 64 × 64 BZ oscillators.

(a) Asymptotic behavior is dynamic in nature with up to 9 small meandering cores.

(b) Initially small spiral wave chimera grows rapidly without splitting.

Behavior is highly dependent on initial conditions.

Core Splitting in the FitzHugh-Nagumo Model



(a)-(c) The phase distribution as 64×64 FHN oscillators evolve in time. The spiral wave chimera exhibits core splitting, with $\tau = 0.51$.
(d)-(f) The corresponding local order parameter at times t = 67.8, 94.5, and 143.8.