



# **A Study on Iterative Schemes for Fully Coupled Reactive Transport and Flow in Variably Saturated Porous Media**

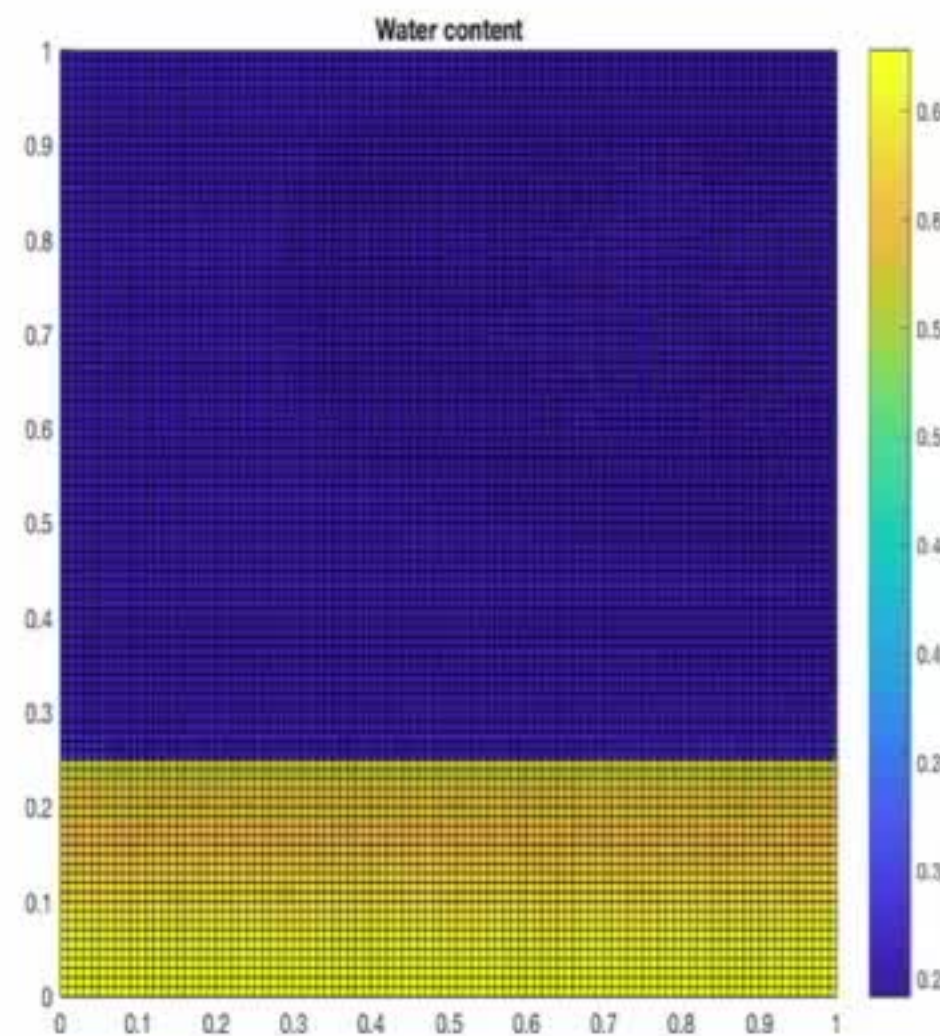
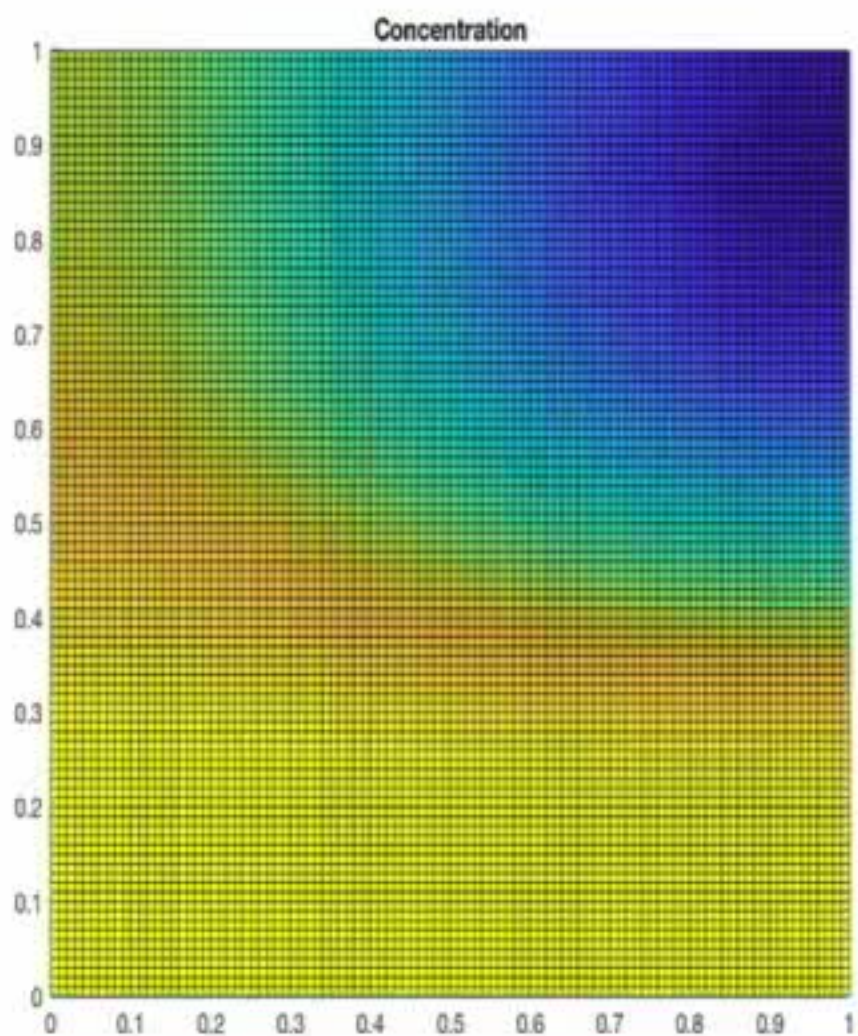
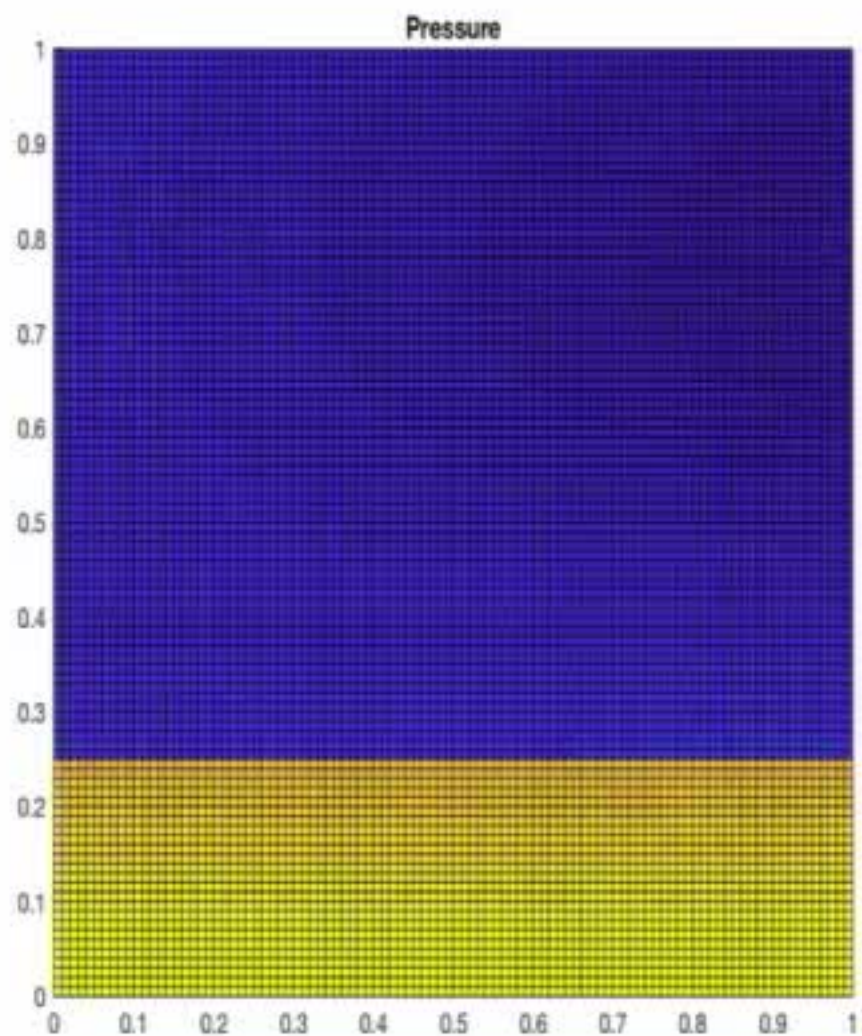
D. Illiano, F.A. Radu, I.S. Pop, K.-A. Lie

SIAM 2019, Houston, Texas





# Numerical Example: Vadose zone

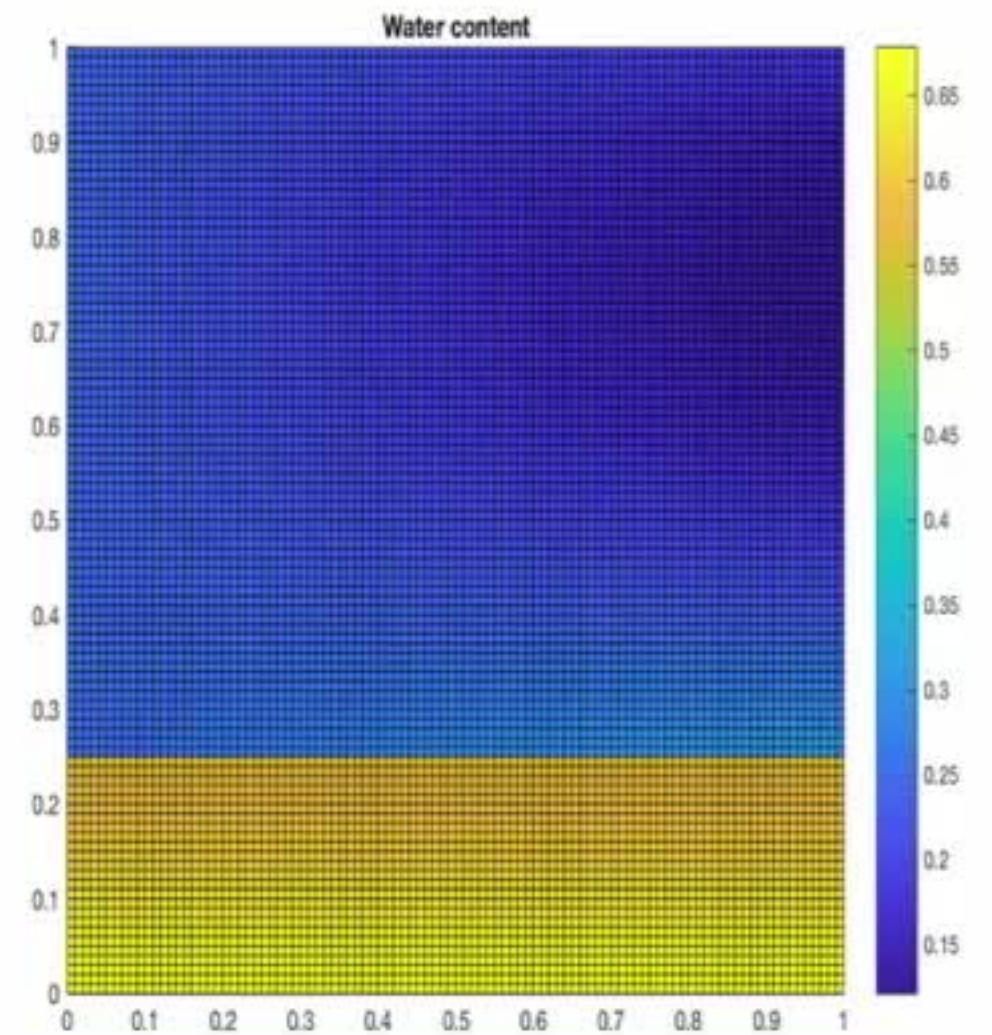
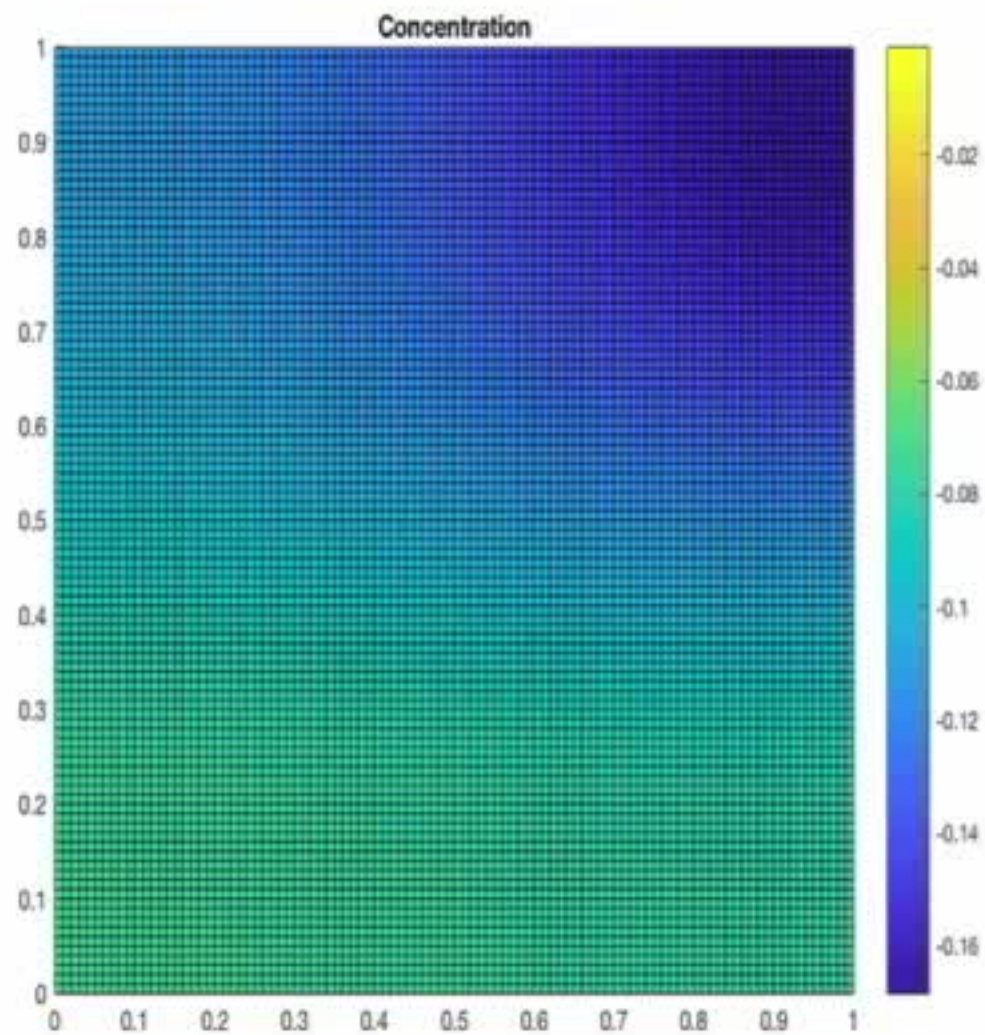
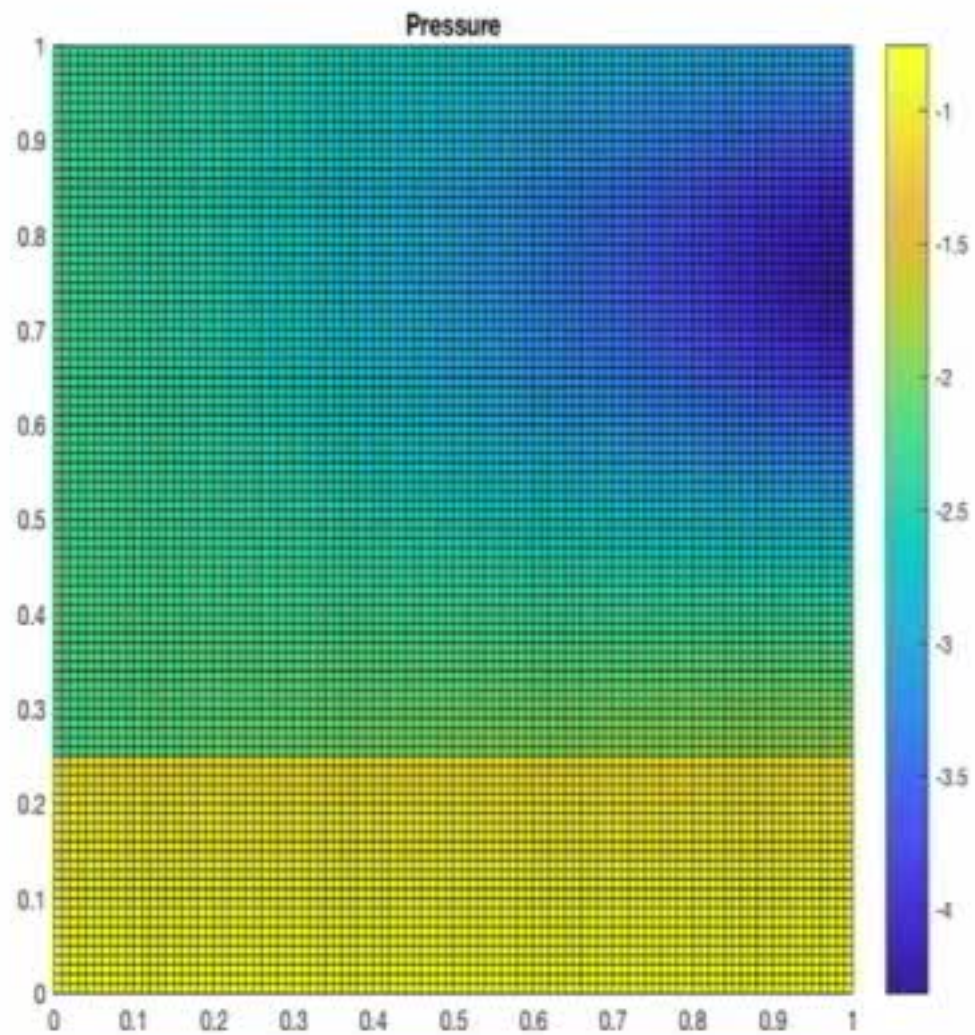


MRST – Matlab Reservoir Simulation Toolbox  
List, Radu, 2015, A study on iterative methods for solving Richards' equation





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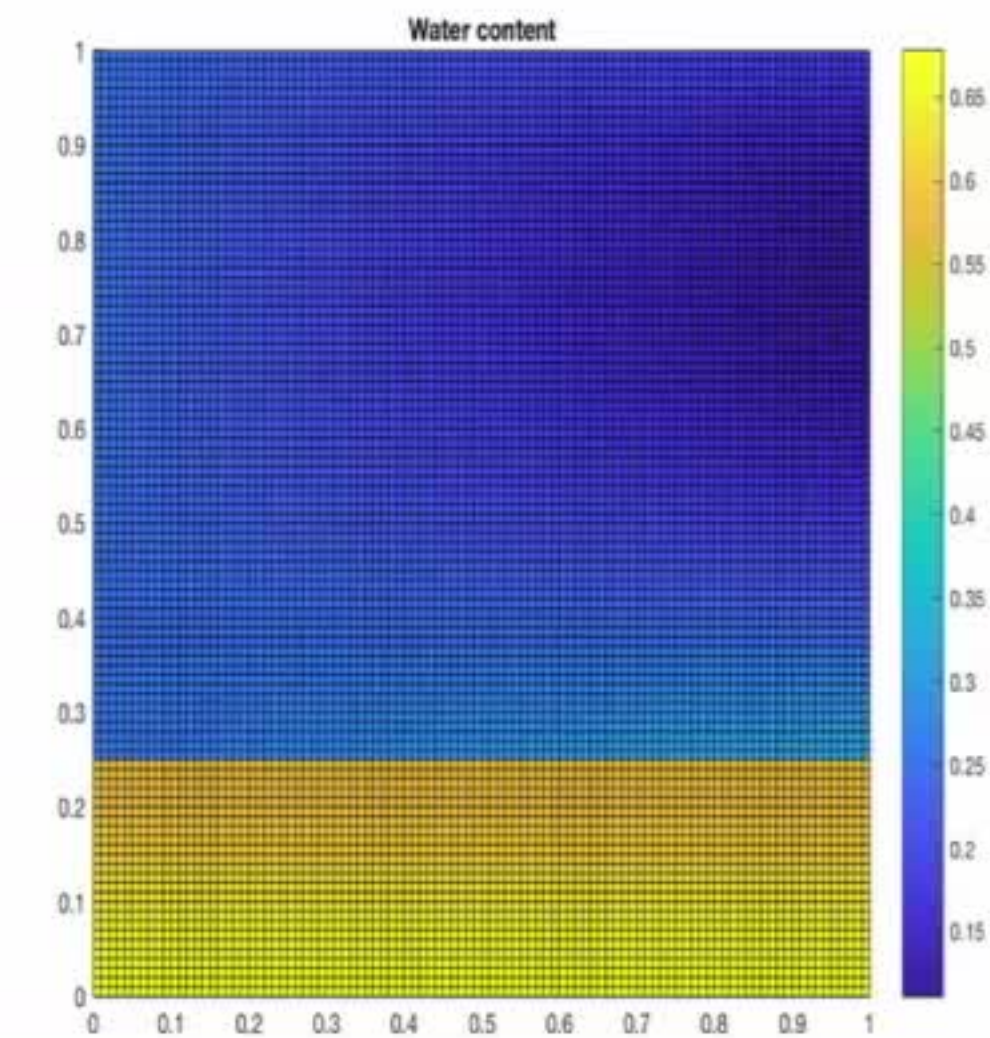
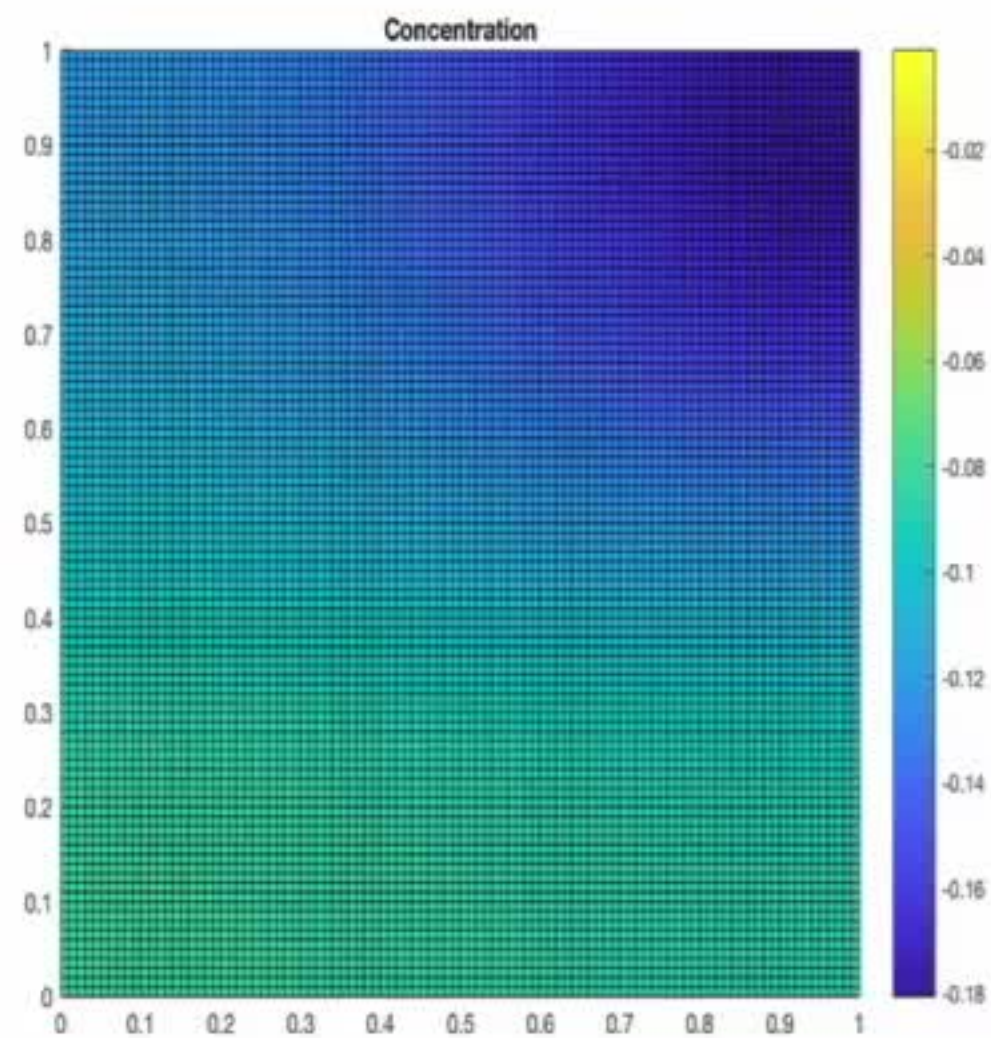
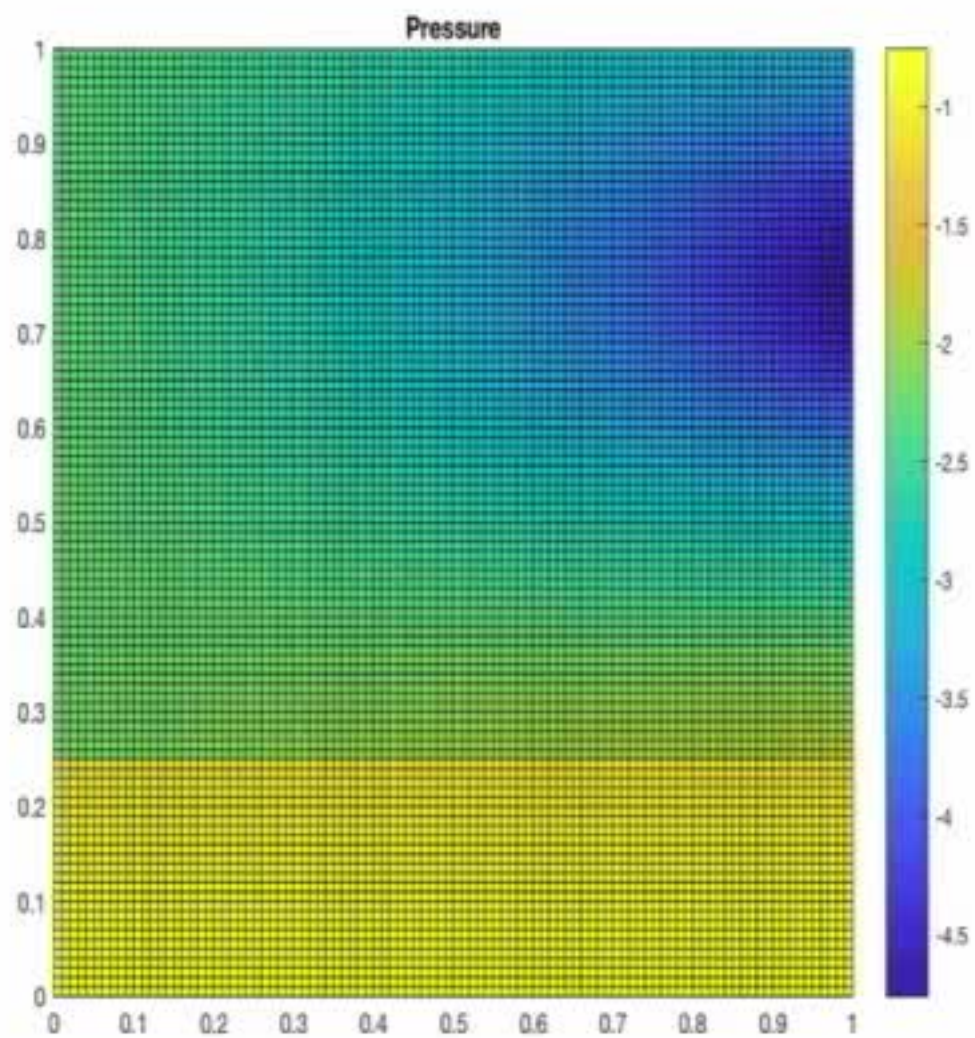


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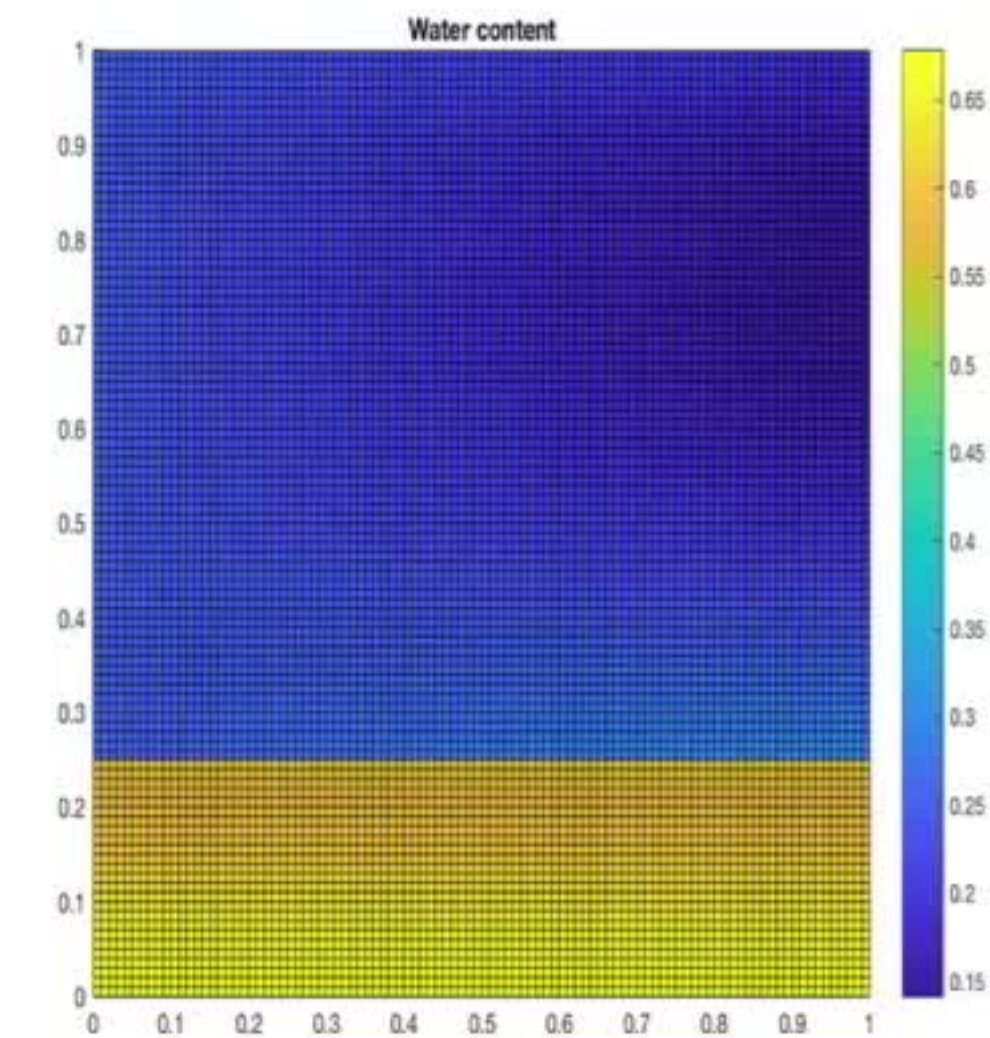
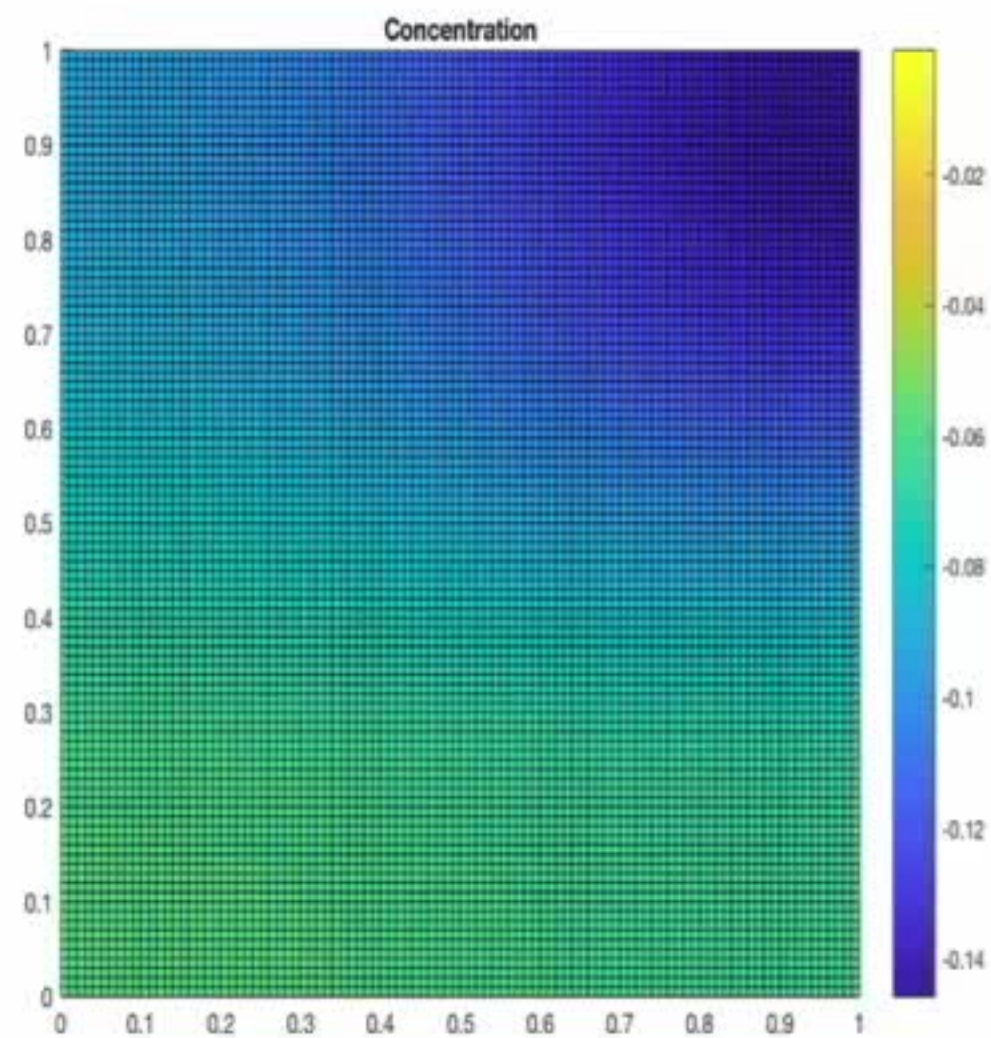
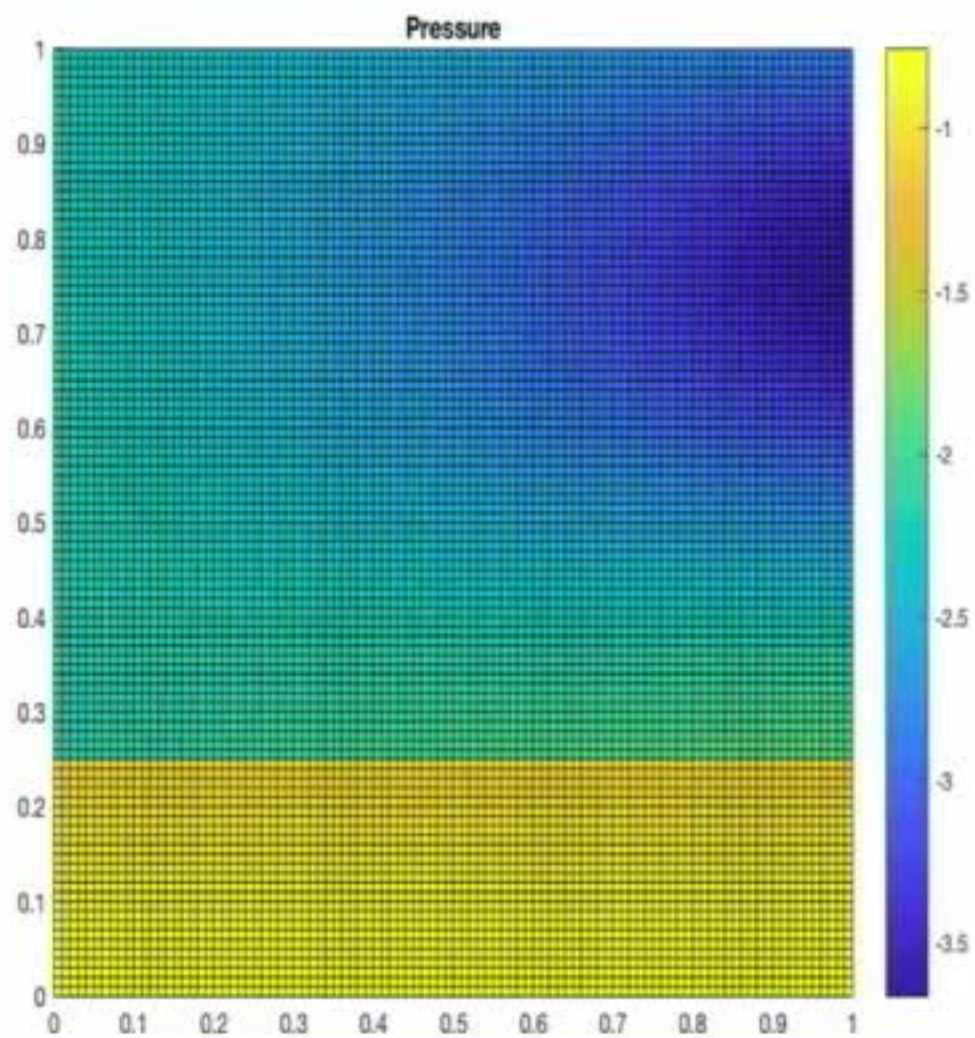


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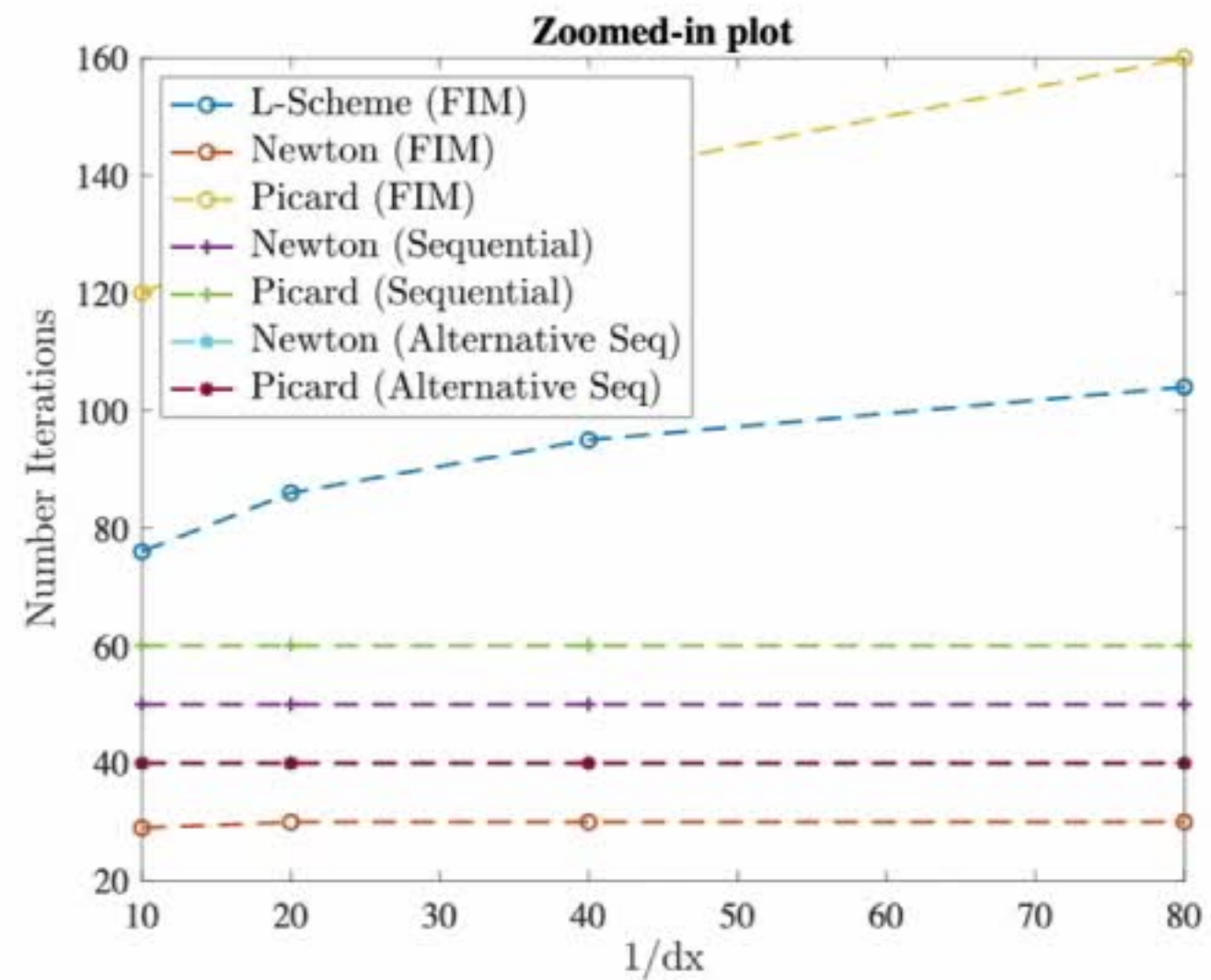
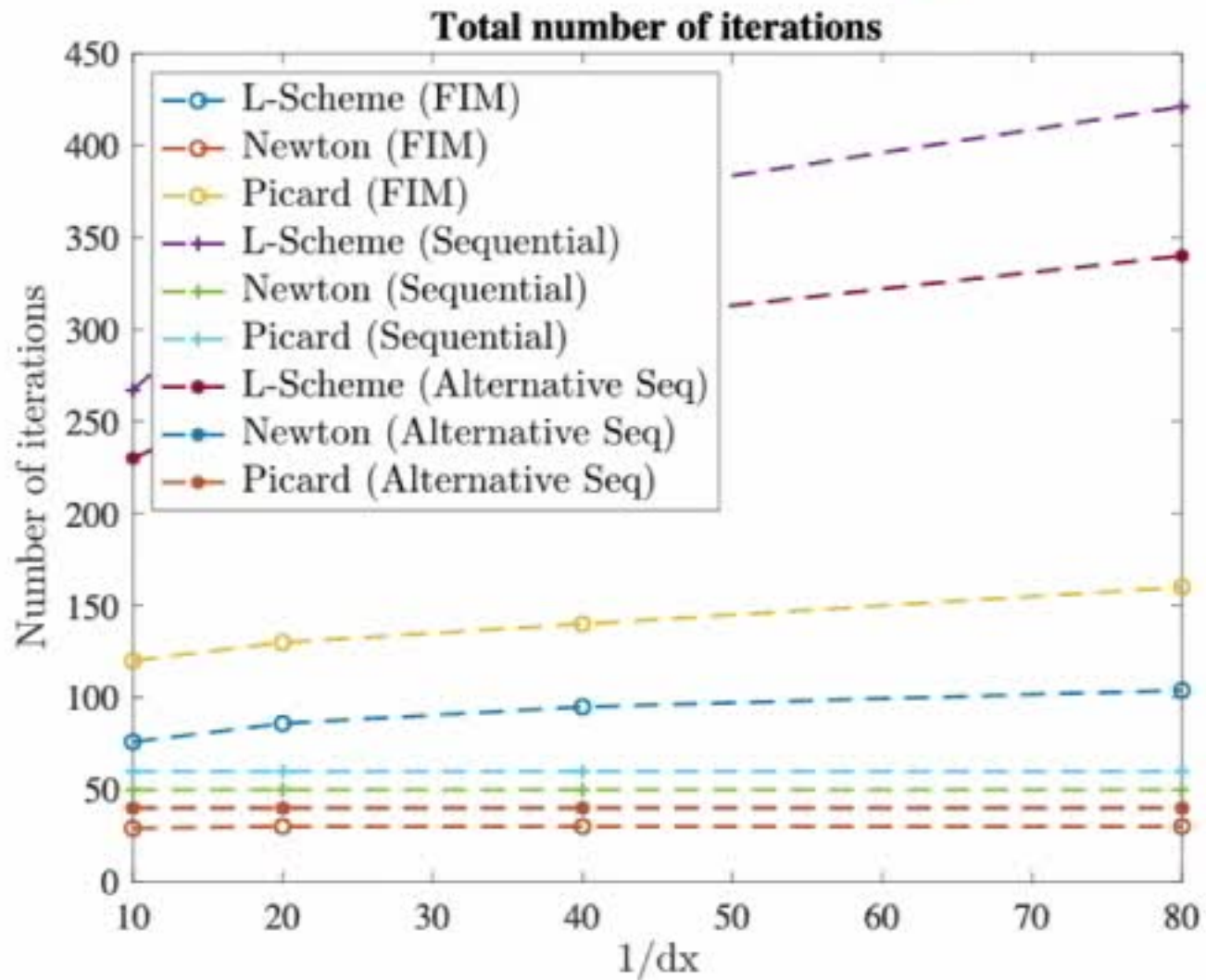


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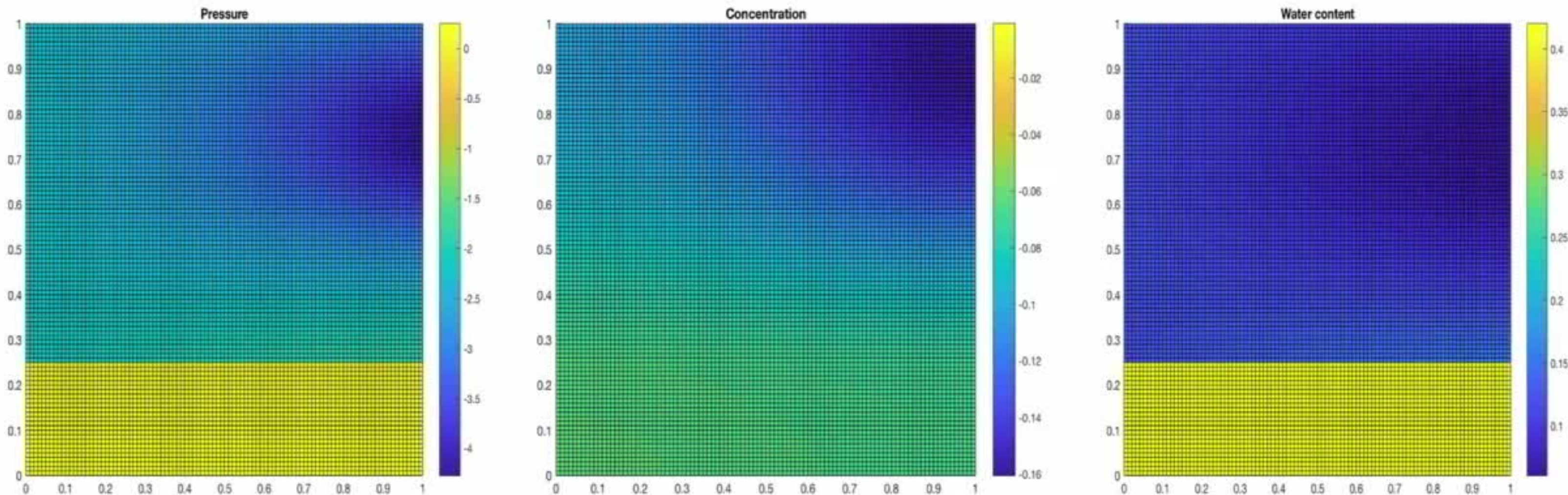


# Numerical Example: Vadose zone





# Numerical Example: variably saturated PM



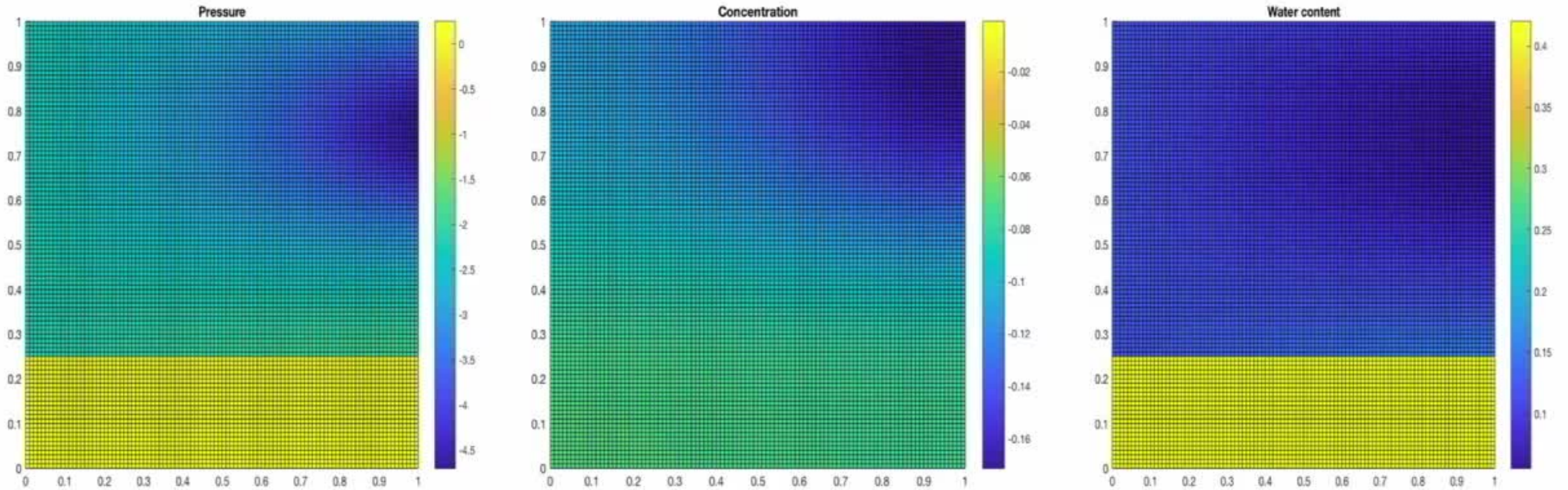
MRST – Matlab Reservoir Simulation Toolbox

List, Radu, A study on iterative methods for solving Richards' equation, *Comput. Geosci.*, 2016





# Numerical Example: variably saturated PM



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## Dynamic capillary pressure

$$\frac{\partial \theta}{\partial t} - \nabla \cdot (K(\theta) \nabla (\Psi + z)) = H_1$$

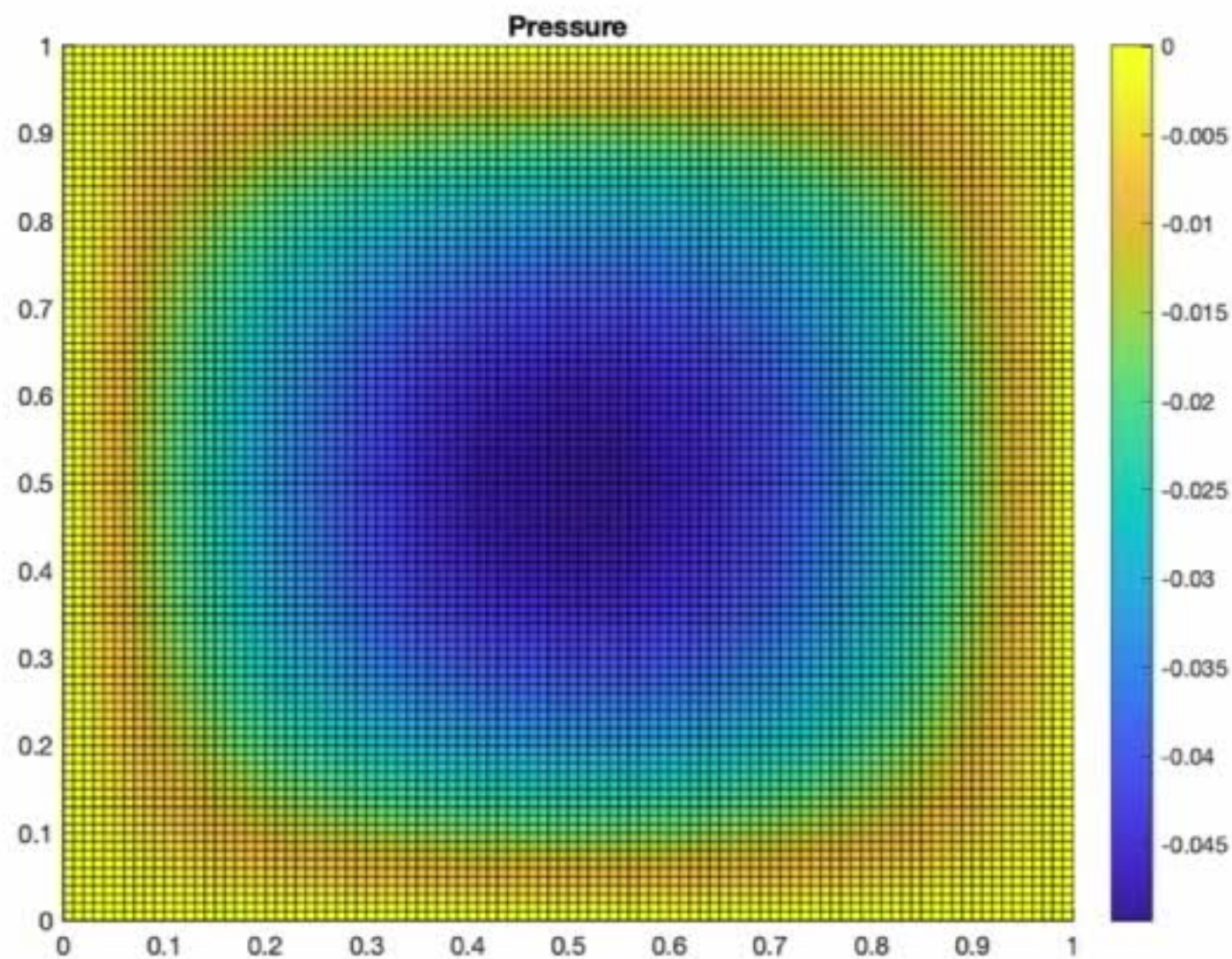
$$\frac{\partial c \theta}{\partial t} + \nabla \cdot (-D \nabla c + q_w c) + R(c) = H_2$$

$$\Psi + p_{cap}(\theta) - \tau \frac{\partial S_w}{\partial t} = 0$$





# Numerical example



Analytical solutions:

$$p_a = -t * x * (1 - x) * y * (1 - y)$$

$$c_a = -p_a$$

$$\theta_a = (1 - p_a^2)/2$$



VISTA



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BETWEEN THE NORWEGIAN ACADEMY OF  
SCIENCE AND LETTERS AND EQUINOR



## Conclusion

1. The comparison between the different linearization schemes shows that **the L-scheme is the only convergent method** in particularly complex configurations,
2. The **new approach here presented appears to be a valid alternative to the common formulation**, it gives equally accurate results, requiring fewer iterations,
3. The introduction of the dynamic effects removes the non-linearity of  $\theta$ ,
4. Best results were obtained with  $L_1 \approx 1/2 \max ||\partial \theta / \partial \Psi||$  and

$$L_2 \approx 1/2 \max ||\partial \theta / \partial c||.$$





# Dynamic capillary pressure

$$\frac{\partial \theta}{\partial t} - \nabla \cdot (K(\theta) \nabla (\Psi + z)) = H_1$$

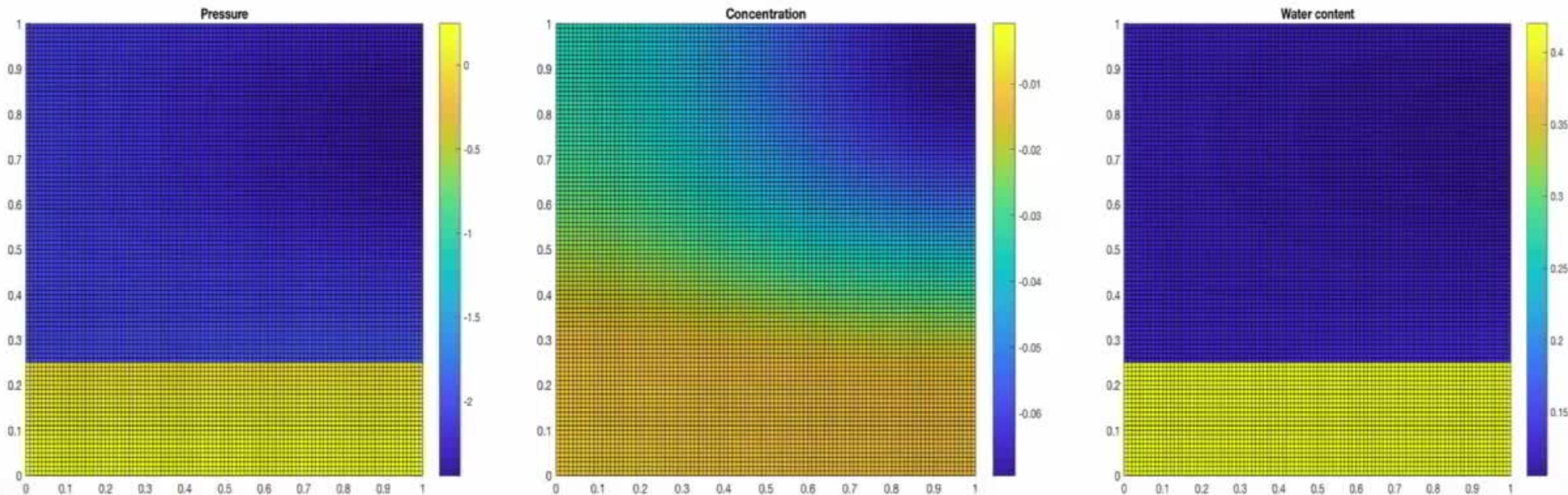
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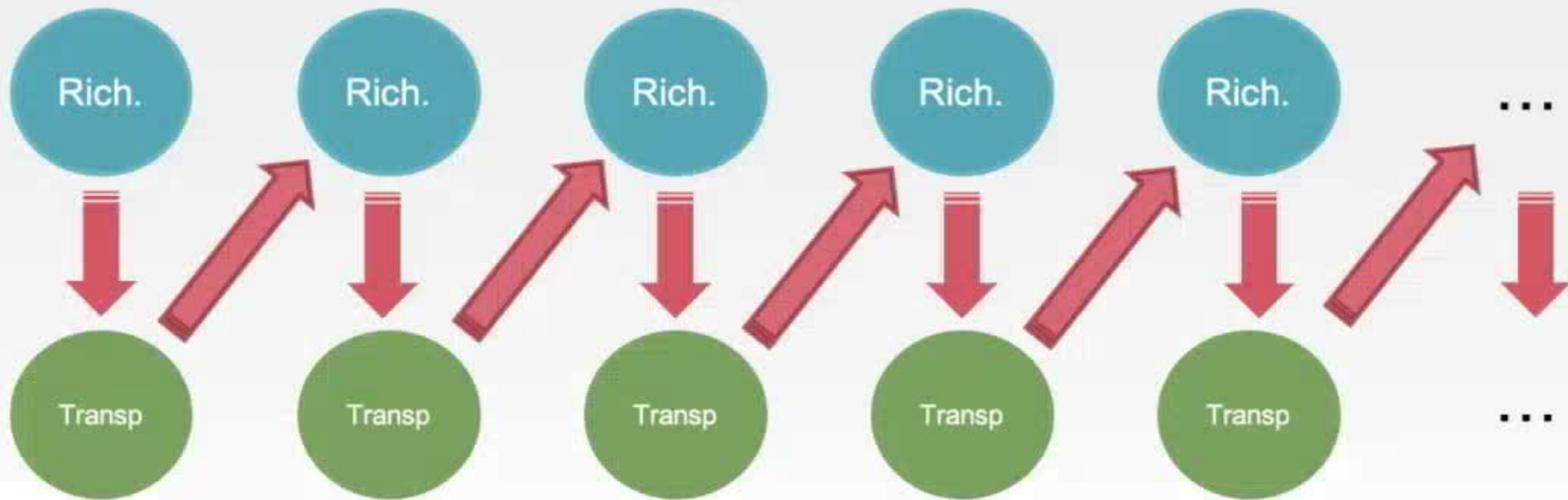
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# Alternative sequential approach



j loop

