The Deffuant Model of Opinion Dynamics

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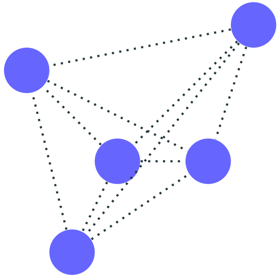
Opinion dynamics

- How does social influence affect opinion formation?
- Will the group form a consensus or become polarized?
- Model opinion formation at an individual level to try to describe behaviour on a global level.
- Many models of opinion dynamics
  - Continuous/Discrete opinions.
  - Large groups/round table discussion.

Deffuant model

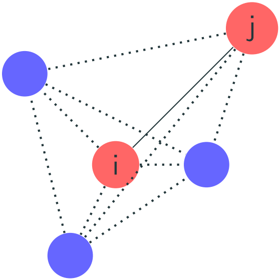
Network



Opinion Space

Deffuant model

Network

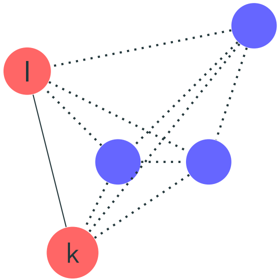


Opinion Space

Nodes i and j interact if $|x_i - x_j| < \epsilon = 1/2$

Deffuant model

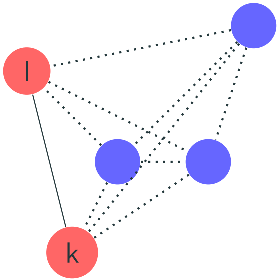
Network



Opinion Space

Deffuant model

Network



Opinion Space

$$x_k \to x_k + \mu \cdot (x_l - x_k)$$
$$\mu = \frac{1}{2}$$

Deffuant model

- We can simulate the evolution of the opinions on a network with N nodes.
- Each node i has an initial opinion x_i from a uniform distribution.
- At each time step choose two nodes i and j at random.
- If $|x_i - x_j| < \epsilon$ the two opinions get updated.

$$x_i \to x_i + \mu \cdot (x_j - x_i)$$
$$x_j \to x_j + \mu \cdot (x_i - x_j)$$

- Otherwise both opinions remain the same.

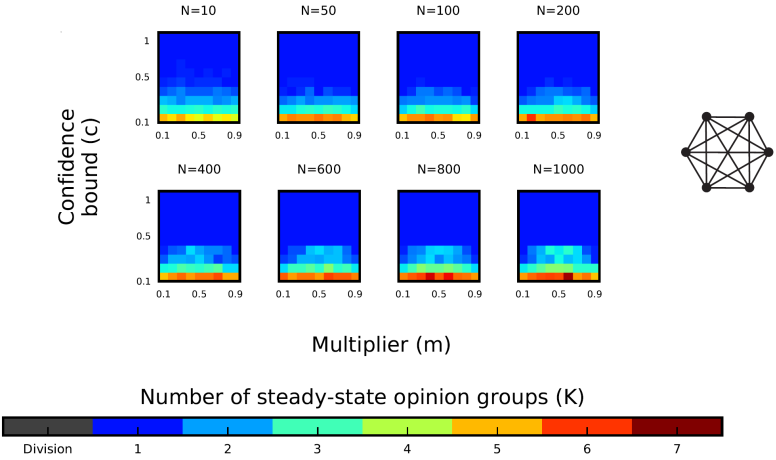
Deffuant model

Clusters of individuals with similar opinions will form.

We simulate the dynamics until

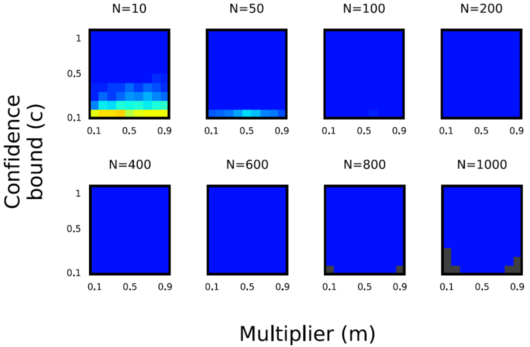
- Clusters are separated by distance ϵ.
- Opinion difference within clusters is < 0.02.

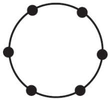
Simulations on networks - complete graph

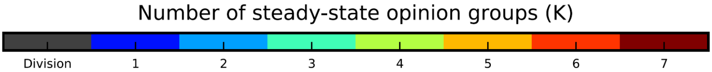


Meng et al. 2018

Simulations on networks - Cycle

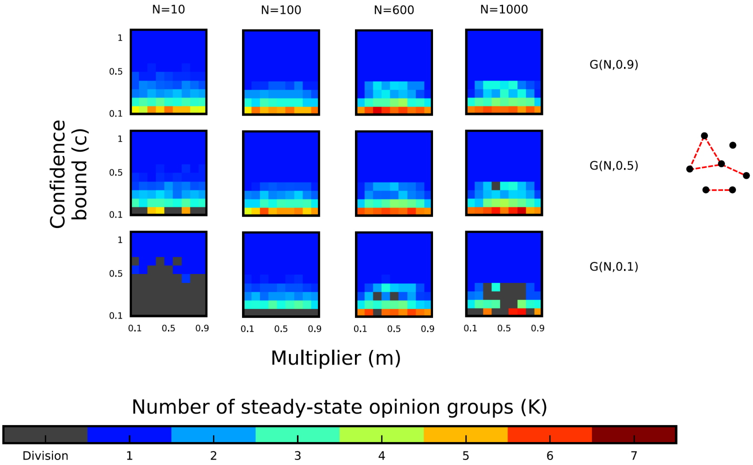






Meng et al. 2018

Simulations on networks - Erdös-Rényi graph



Meng et al. 2018

Mean field approximation

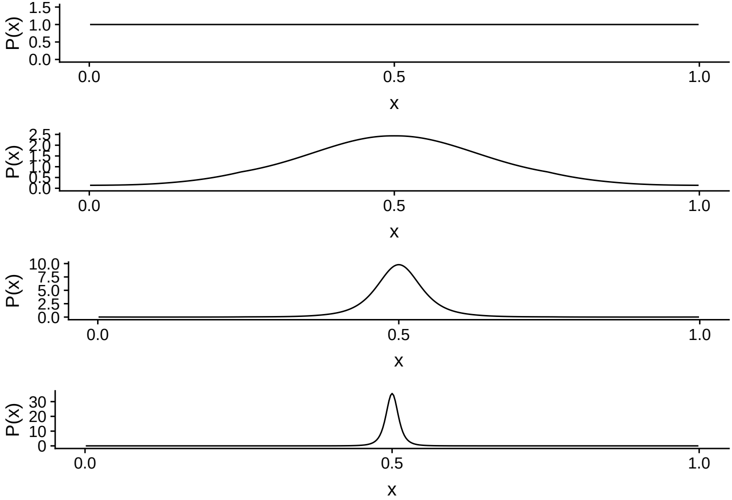
$$x_1 \to x_1 + \mu \cdot (x_2 - x_1)$$

- $P(x, t)$ = probability that an individual holds opinion x at time t.

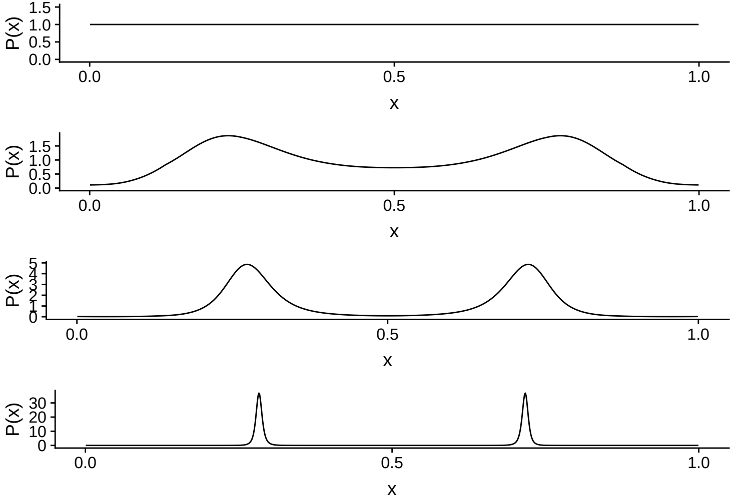
$$\frac{\partial}{\partial t} P(x,t) = \int \int_{|x_1 - x_2| < \epsilon} P(x_1, t) P(x_2, t) \delta\left(x_1 + \mu(x_2 - x_1) - x\right) dx_1 dx_2 - \int_{|x_1 - x| < \epsilon} P(x_1, t) P(x, t) dx_1$$

- Ben-Naim et al. 2003

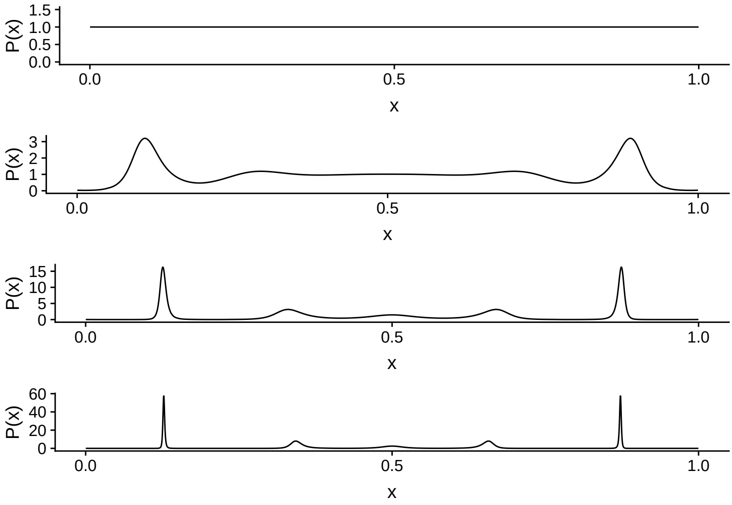
$\mu = 0.5, \epsilon = 0.5$



$\mu = 0.5, \epsilon = 0.25$

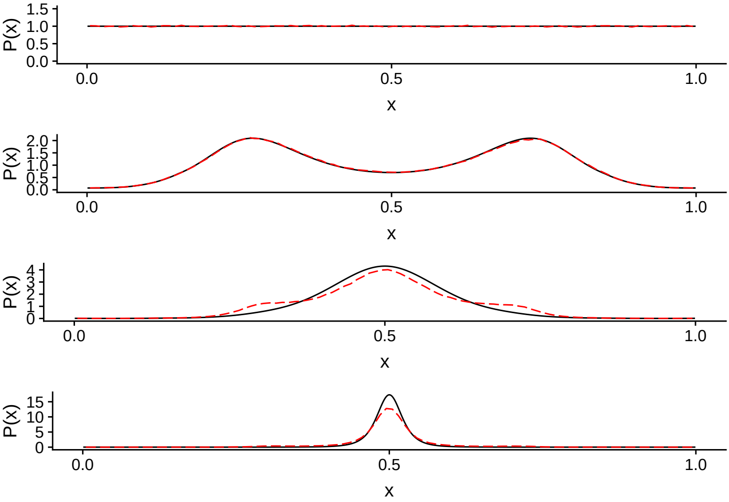


$\mu = 0.5, \epsilon = 0.1$



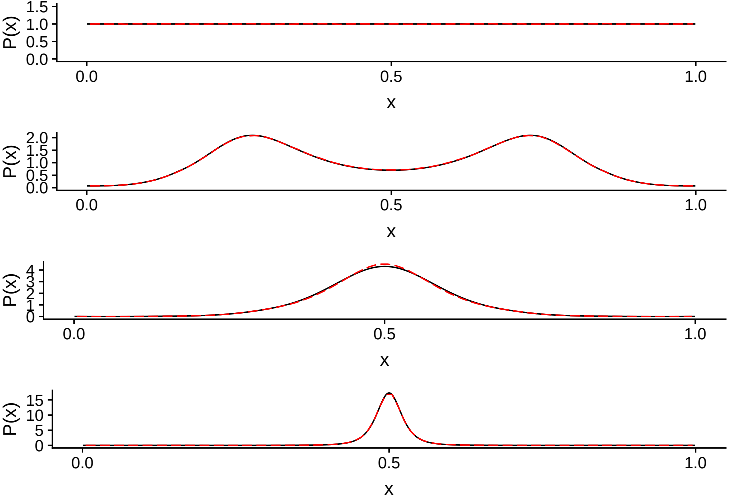
Comparison of mean field and simulations - complete graph

$\epsilon = 0.3, \ \mu = 0.5, \ N = 10^3$

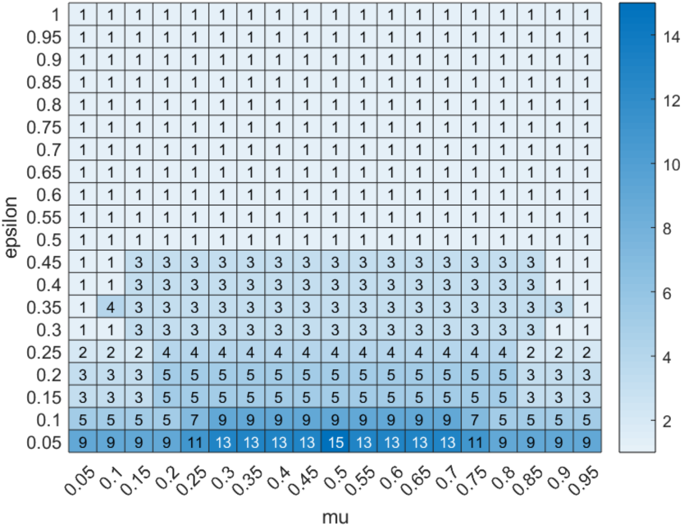


Comparison of mean field and simulations - complete graph

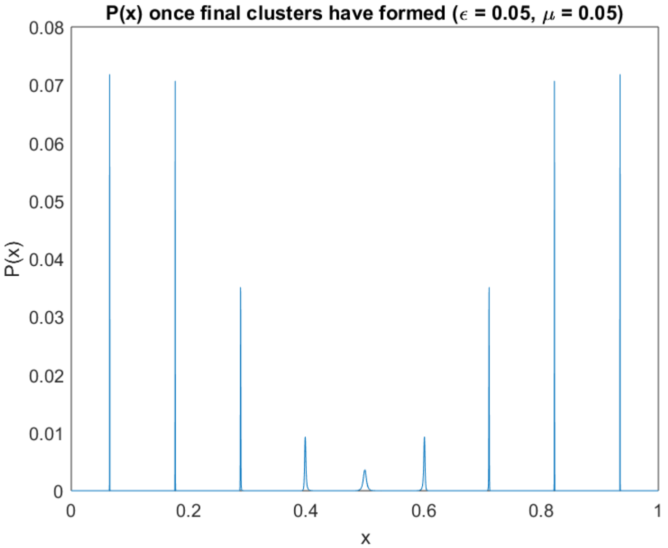
$\epsilon = 0.3,\ \mu = 0.5,\ N = 10^4$



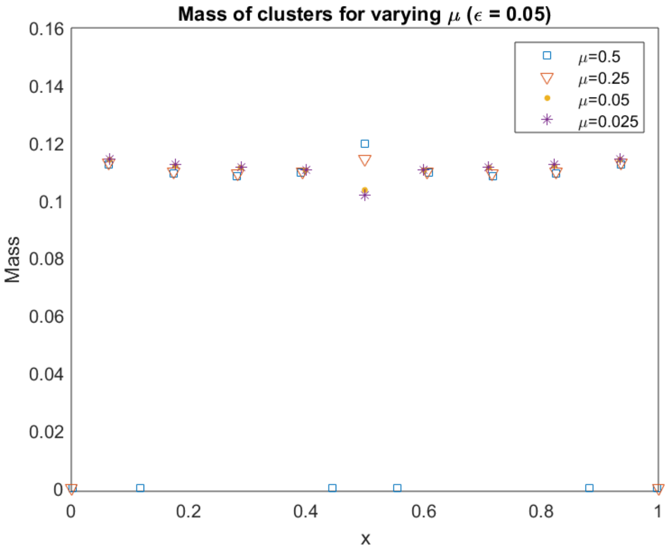
Number of clusters



Final distribution

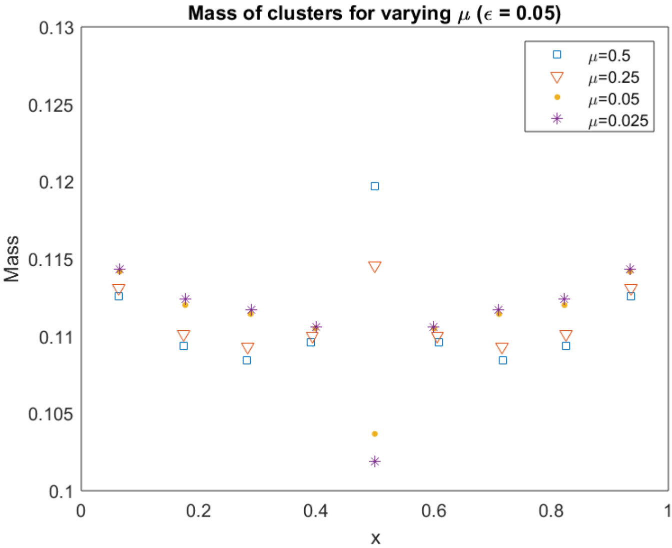


Effect of μ



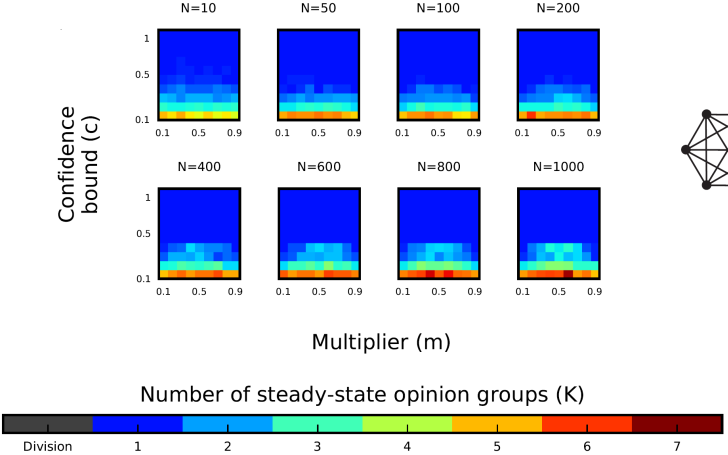
Mass of clusters for varying μ (ϵ = 0.05)
μ=0.5
μ=0.25
μ=0.05
μ=0.025
Mass
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
x
0
0.2
0.4
0.6
0.8
1

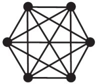
Effect of μ



Mass of clusters for varying μ (ϵ = 0.05)
μ=0.5
μ=0.25
μ=0.05
μ=0.025
Mass
0.13
0.125
0.12
0.115
0.11
0.105
0.1
0
0.2
0.4
0.6
0.8
1
x

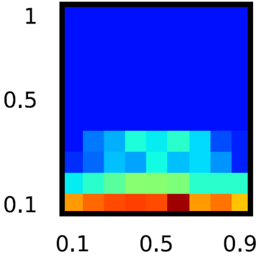
Comparison of mean field with simulations

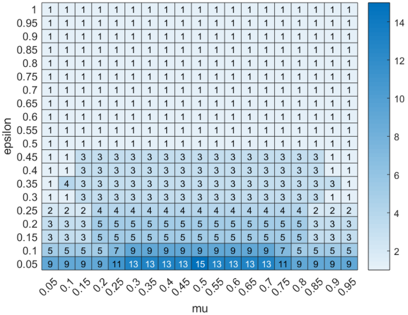




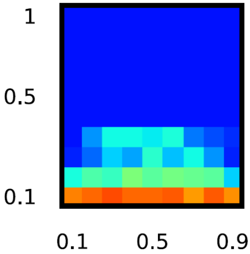
Meng et al. 2018

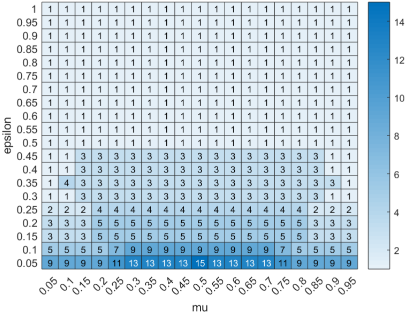
Complete network



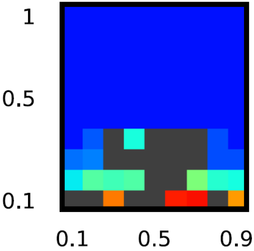


Erdös-Rényi N=1000, p=0.9





Erdös-Rényi N=1000, p=0.1



epsilon \ mu	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0.95	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0.9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0.85	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0.8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0.75	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0.7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0.65	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0.6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0.55	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0.5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
0.45	1	1	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	1	1
0.4	1	1	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	1	1
0.35	1	4	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	1
0.3	1	1	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	1	1
0.25	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	2	2	2
0.2	3	3	3	5	5	5	5	5	5	5	5	5	5	5	5	5	3	3	3
0.15	3	3	3	5	5	5	5	5	5	5	5	5	5	5	5	5	3	3	3
0.1	5	5	5	5	7	9	9	9	9	9	9	9	9	9	7	5	5	5	5
0.05	9	9	9	9	11	13	13	13	13	15	13	13	13	13	11	9	9	9	9

Future work

- Effect of degree distribution, modularity, initial conditions.
- Probability distributions $P_k(x, t)$ for nodes of degree k.
- What happens in $\mu \to 0$ limit?
- Does noise reduce the number of clusters that form?

References

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