Exceptional service in the national interest

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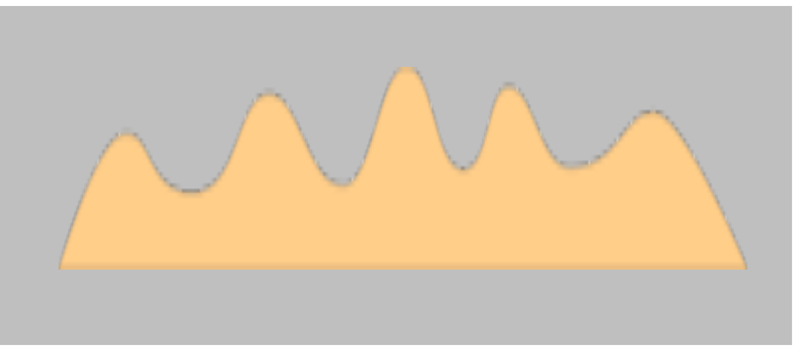
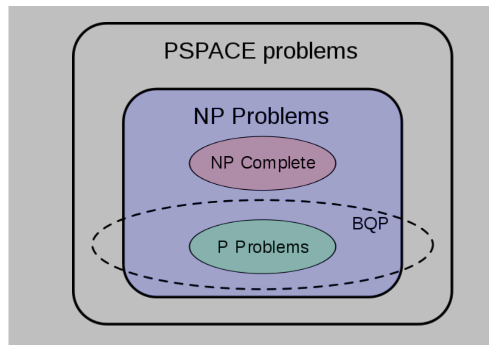
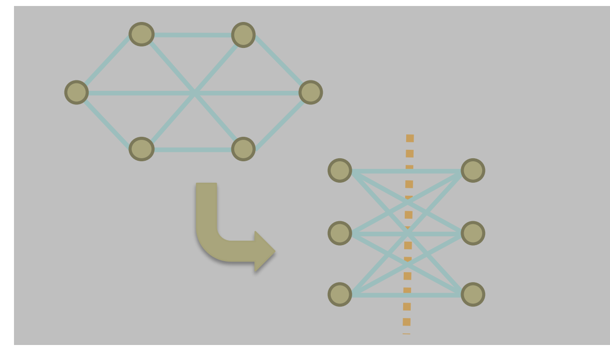






Quantum Approximation Algorithms

Ojas Parekh and Ciaran Ryan-Anderson

SIAM Annual Meeting, 2017

U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

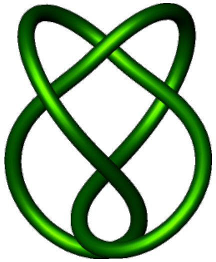
Why quantum algorithms?

- Potential power of quantum resources is too great to ignore
- Need quantum algorithms to guide quantum hardware investment and development
- Quantum perspective has inspired new classical algorithms!
- Desire for novel quantum applications and techniques

Limited bag of tricks for speedups

50+ algorithms: http://math.nist.gov/quantum/zoo

Phase Estimation (ca. 1994)



- Factoring
- Quantum chemistry
- Linear systems
- Topological invariants

Amplitude Amplification (ca. 1996)

- Unordered search
- Graph/network properties
- Data collision problems
- Matrix product verification

Hamiltonian Simulation (ca. 1996)

- Quantum chemistry
- Linear systems
- Maze solving

Quantum Walk (ca. 2002)

- Boolean formula evaluation
- Spatial search
- Quantum chemistry

New quantum algorithmic approaches are desperately needed!

State of quantum "speedups"

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- **Unproven** exponential speedup:
  Shor's quantum factorization algorithm

- Provable polynomial speedup:
  Grover's quantum search algorithm

- Provable exponential resource advantage (in specialized models of computation):
  Query and communication complexity

Quantum bits

State space

Classical bit:
(bit)

1 = Head

OR

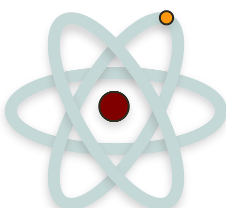
0 = Tail

{0, 1}

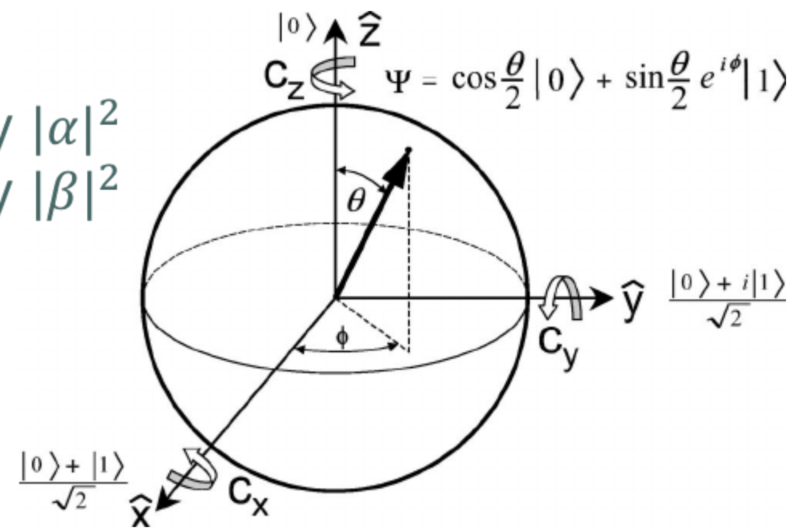
Prob. bit:
(p-bit)

0 with probability $1-p$
1 with probability p

Quantum bit:
(qubit)



$\alpha|0\rangle + \beta|1\rangle$
0 with probability $|\alpha|^2$
1 with probability $|\beta|^2$



Quantum gate

Can take the "square root" of ordinary logic gates

Conventional logic gate: NOT

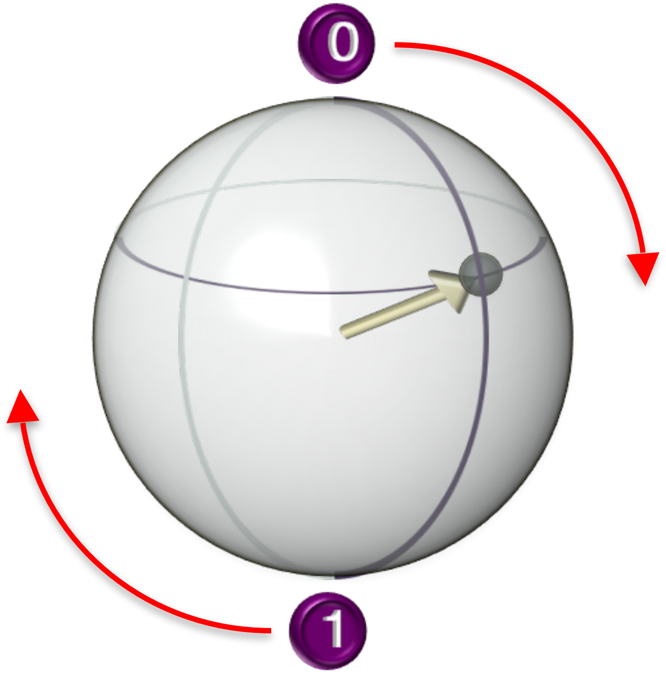
$$\text{yes} \rightarrow \text{no}$$

$$\text{no} \rightarrow \text{yes}$$

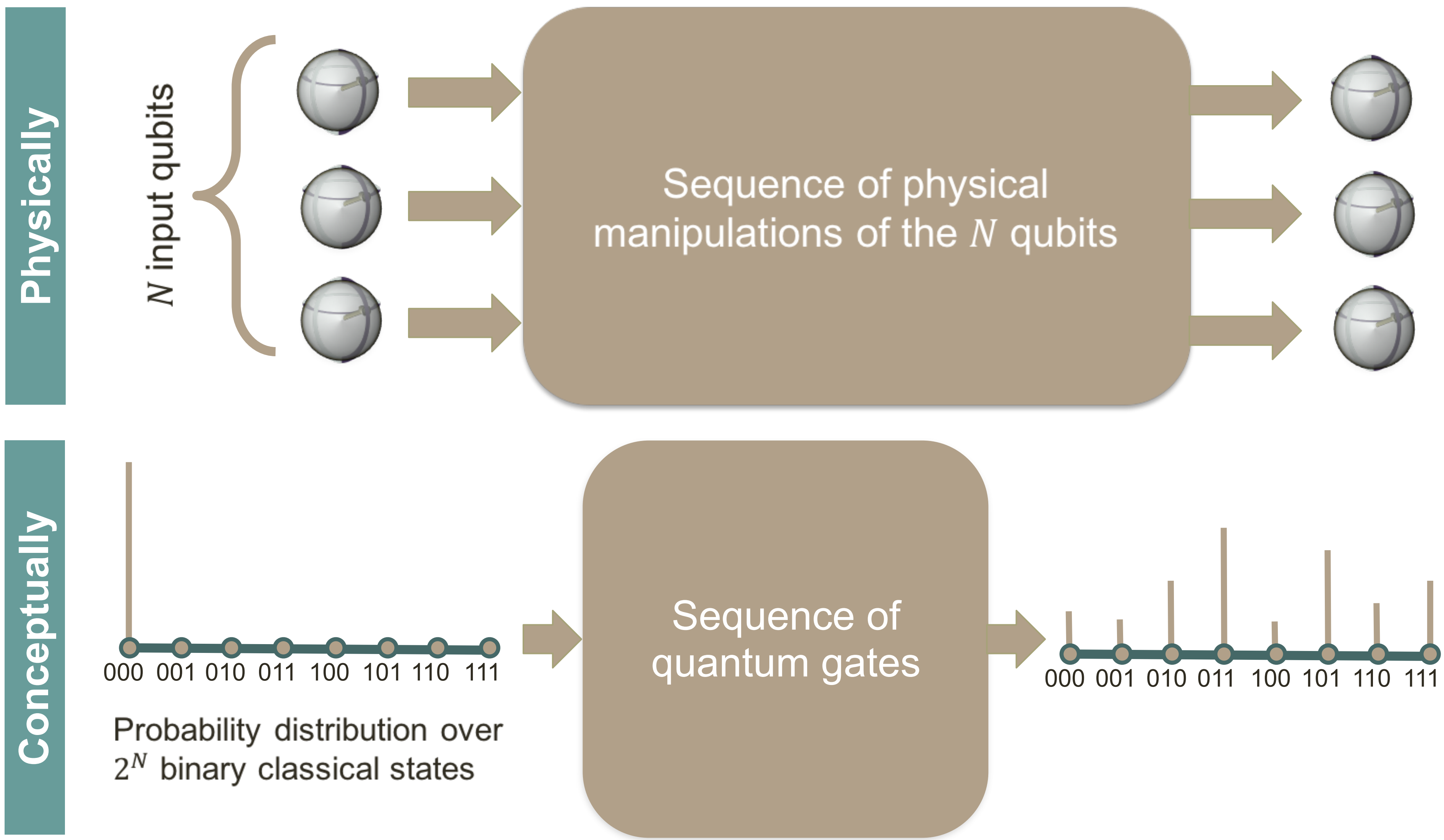
Quantum logic gate: $\sqrt{\text{NOT}}$

$$\text{yes} \rightarrow 50/50 \text{ chance of yes or no}$$

$$\text{no} \rightarrow 50/50 \text{ chance of yes or no}$$



Quantum algorithm



Entanglement by analogy

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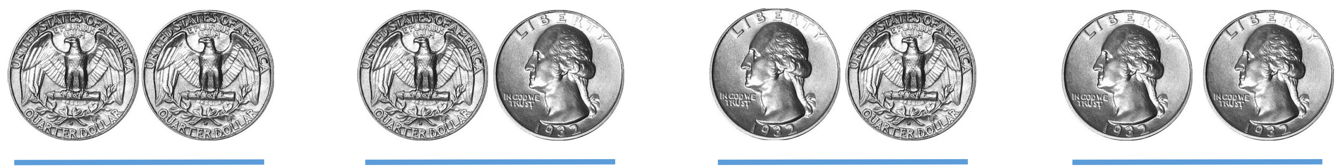
Physical world

Superposition space
(possible measurement outcomes)

Probabilistic bits



prob. ¼ | prob. ¼ | prob. ¼ | prob. ¼

Entangled qubits

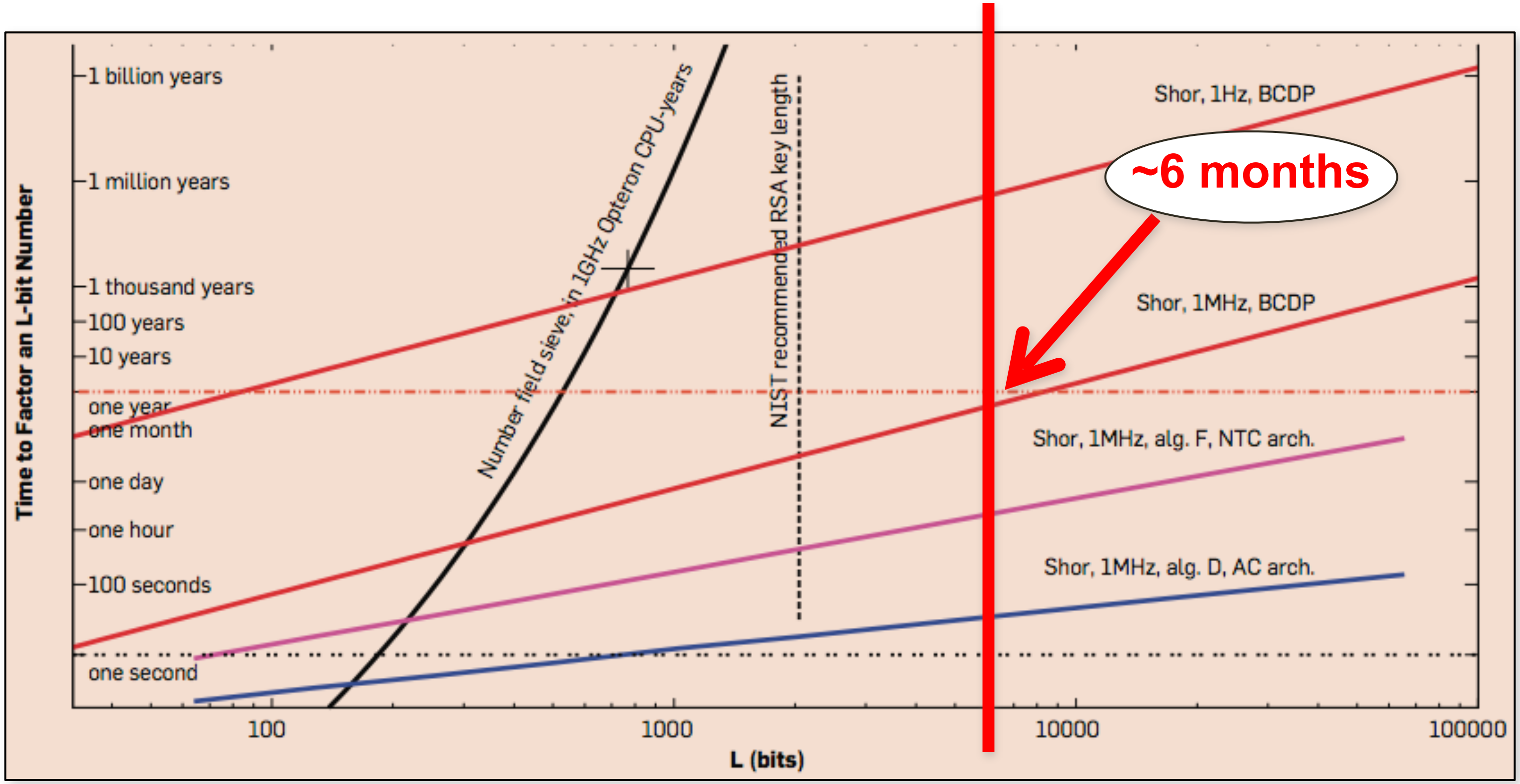
prob. ½ | prob. 0 | prob. 0 | prob. ½

The entangled qubits will always match, even if measured at different times and across space!

Quantum factoring

An exponential speedup

If all the silicon in the world's crust were converted to Pentium chips, it would take the age of the universe to factor a 5,000-bit number.



A blueprint for building a quantum computer, R. van Meter & C. Horsman, *Comm. ACM*, (2013) doi:10.1145/2494568

Adiabatic quantum computing

Not a 'universal' computer; may have no speedup

a.k.a. "quantum annealing"

$$\min_{x \in \{0,1\}} \left(\sum_{i,j=1}^{n} J_{ij} x_i x_j + \sum_{i=1}^{n} h_i x_i \right)$$

This problem is "NP-hard:" it is unlikely that even a quantum computer could solve it efficiently.

An adiabatic quantum computer ***could*** be made universal, if the technology were modified to allow the qubits to interact in more interesting ways.

Metrics status

Where we are, and where we might go

Metric	2016	2026
Universal q. computer	~10 qubits, 100 ops	~1,000 qubits, 10,000 ops
1-qubit gates	~1 in 10,000 error rate	Scalable logical qubit
2-qubit gates	~1 in 100 error rate	Scalable logical qubit
Analog q. simulator	~1,000 qubits	~10,000 qubits
Quantum annealer	~1,000 qubits	~10,000 qubits

Benchmark: 50 qubits is beyond the simulation capabilities of today's best supercomputers.

Testbed QCs

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Google: 49-qubit goal by December 2017.

NSF: $3M/yr Ideas Lab: Practical Fully-Connected Quantum Computer Challenge (PFCQC), November 2017

DOE: $5M/yr Quantum Testbed User Facility (pending Congressional budget action)

IBM: Open-Access "Quantum Experience" online since 5/16: 40k users, 270k experiments, 15 published papers

limage: IBM

ETH Zurich: 45 *simulated* qubits on Cori II (#5) (4/17) [1]

[1] Haener and Steiger, arXiv:1704.01127 (2017).

Quantum Approximation Algorithms

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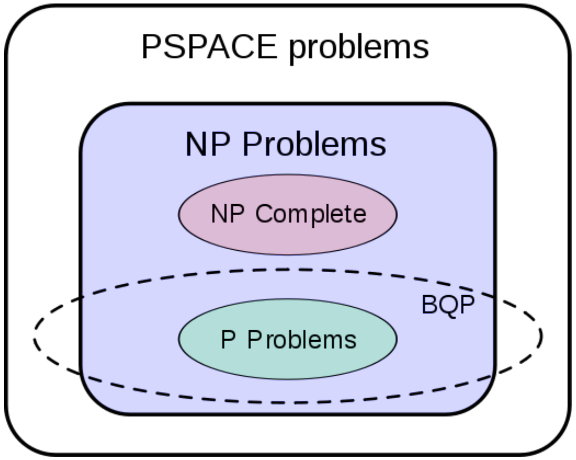
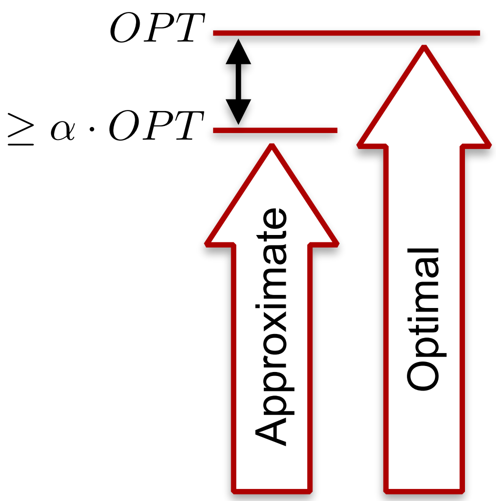
Motivation: hard to efficiently find optimal solutions for NP-complete optimization problems, **even for quantum computers**





Approach: an ***approximation algorithm*** efficiently produces a near-optimal solution with a mathematically provable bound on quality

Benefit: ***quantum approximation algorithms (QAA)*** direct quantum resources towards **higher-quality solutions** instead of faster **running times,** sidestepping barriers to quantum speedups

The QAOA

The **Q**uantum **A**pproximate **O**ptimization **A**lgorithm was introduced by Farhi et al. in 2014

$$e^{i\sum_i \beta X_i} e^{i\gamma \sum_{ij\in E} Z_i Z_j} |+\rangle^{\otimes n}$$

Only known quantum approximation algorithm

Classical approximation algorithms have been studied since the 1960s

- Can be viewed as a discretization of adiabatic quantum computing
- Results in low-depth quantum circuits
- Generic framework for combinatorial optimization problems

QAOA for Max 3-XORSAT

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Goal of Max 3-XORSAT is to satisfy max number out of *m* given clauses:

$$(x_1 \oplus x_3 \oplus \neg x_4), (\neg x_1 \oplus x_2 \oplus x_3), \ldots$$

Restricted version: each variable appears in at most *d* clauses

Farhi et al. showed that QAOA beat the best known classical approx alg:

Authors	***Year***	***Result***	***Type***
Trevisan	***2000***	$\left(\frac{1}{2} + \frac{O(1)}{d}\right) m$	***Classical***
Farhi et al.	***2014***	$\left(\frac{1}{2} + \frac{O(1)}{d^{3/4}}\right) m$	***Quantum***
Barak et al.	***2015***	$\left(\frac{1}{2} + \frac{O(1)}{\sqrt{d}}\right) m$	***Classical***
Farhi et al.	***2015***	$\left(\frac{1}{2} + \frac{O(1)}{\log d\sqrt{d}}\right) m$	***Quantum***

Barak et al.'s result is best possible up to constants unless P=NP

Farhi et al., *A Quantum Approximate Optimization Algorithm…*, arXiv:1412.6062v2 (2015)

QAOA for Maximum Cut

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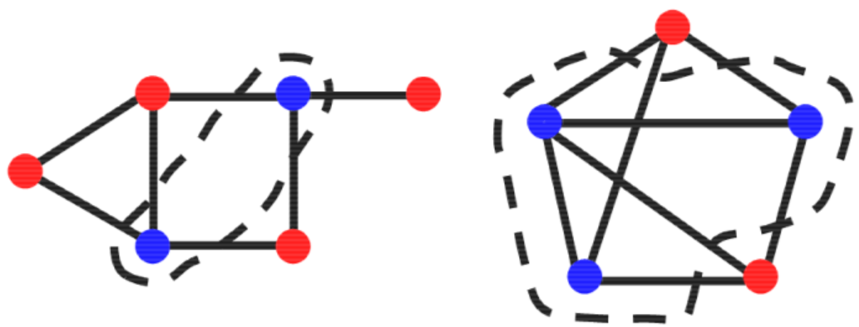
We show that QAOA outperforms best classical algorithm for the well-known Maximum Cut problem on *d*-regular triangle-free graphs with *m* edges

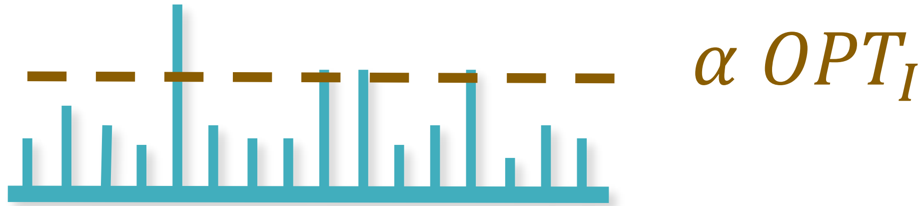
Authors	Year	Result	Type
Shearer	1992	$\left(\frac{1}{2}+\frac{0.177}{\sqrt{d}}\right)m$	Classical
Hirvonen et al.	2014	$\left(\frac{1}{2}+\frac{0.281}{\sqrt{d}}\right)m$	Classical
Parekh et al.	2017	$\left(\frac{1}{2}+\frac{0.303}{\sqrt{d}}\right)m$	Quantum



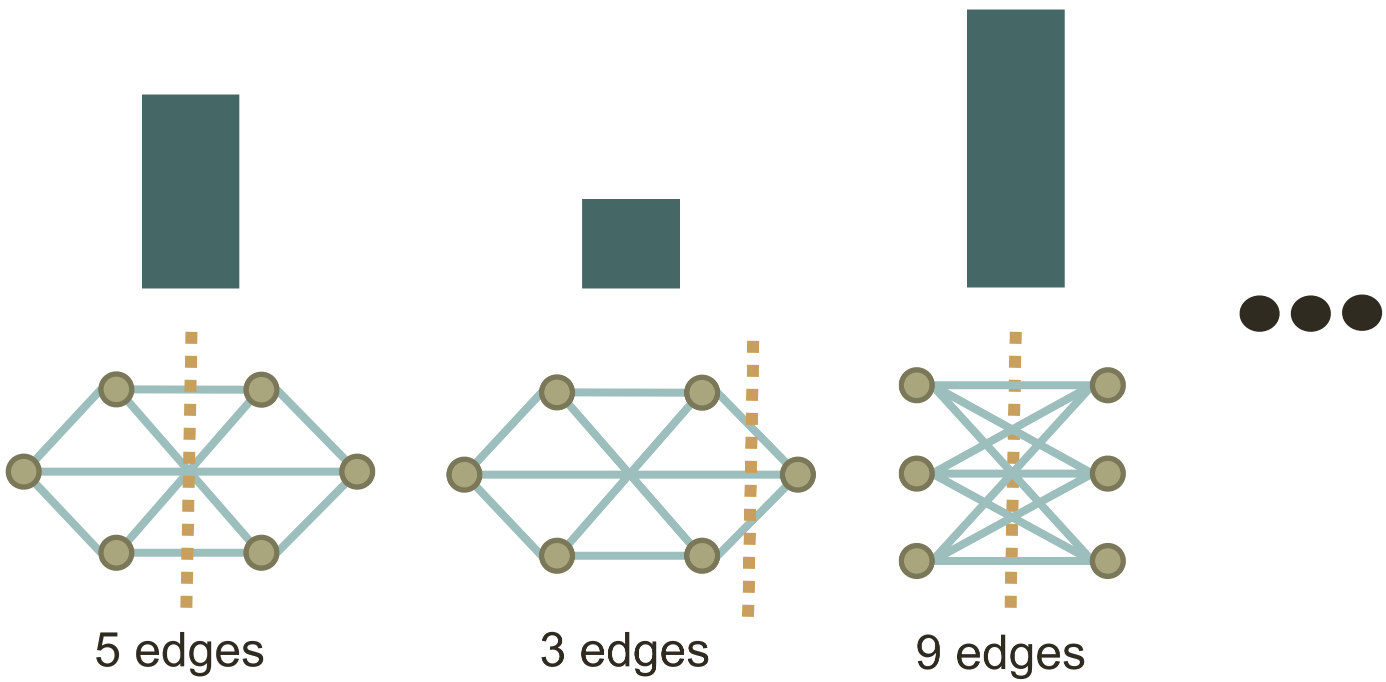
Only known quantum approximation algorithm outperforming the best-known classical algorithm

Image from https://cnx.org/contents/wEViGW7A@1/A-Combinatorial-Algorithm-for-

Sampling vs Optimization



Our quantum algorithm allows sampling from a probability distribution on cuts in a graph, likely to yield a cut with many edges

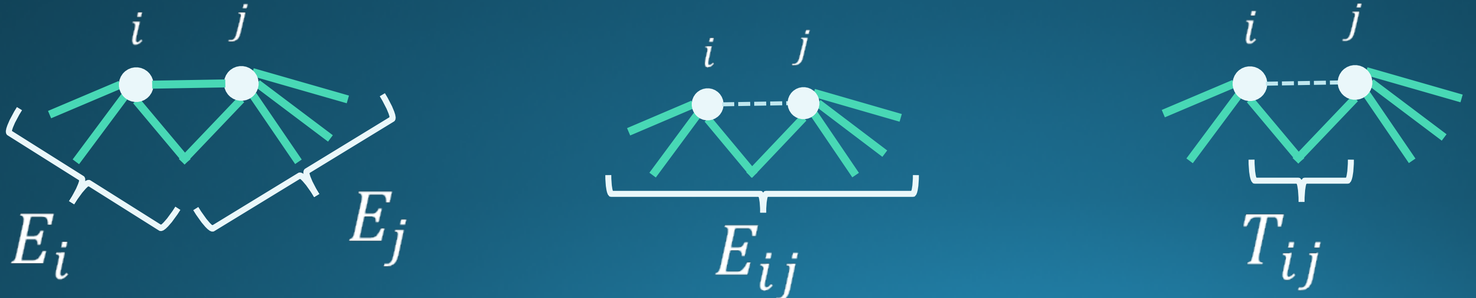


QAOA Analysis

$$\langle C \rangle = \sum_{\langle i,j \rangle \in E} \langle C_{\langle i,j \rangle} \rangle$$

$$\langle C_{\langle i,j \rangle} \rangle =$$

$$\frac{1}{2}\left[1 - \frac{1}{2}\sin(4\beta)\sin(2\gamma)\left\{\cos^{|E_i|-1}(2\gamma) + \cos^{|E_j|-1}(2\gamma)\right\} - \sin^2(2\beta)\cos^{|E_{ij}|-2|T_{ij}|}(2\gamma)\left\{\frac{1-\cos^{|T_{ij}|}(4\gamma)}{2}\right\}\right]$$

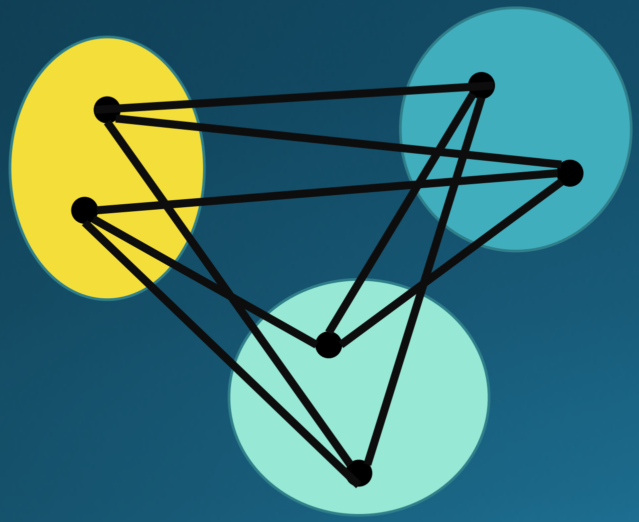


Classical Outperforms QAOA

- Linear time algorithm:

  W. Staton, ***Ars Combinatoria*** **10** (1980), 103-106.

Any 3-regular, connected graph (other than K_4):



Staton:

$$C(z) \geq \frac{7}{9} m = 0.\bar{7}\, m$$

QAOA-1:

$$\langle C \rangle \leq 0.692451\, m$$