An Ensemble Kalman Filter Method for Uncertainty Quantification in Full Waveform Inversion

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Goal of seismic tomography: infer physical subsurface parameters from seismic wavefield data

Full Waveform Inversion (Lailly, 1983; Tarantola, 1984) is routinely applied...

... but uncertainty estimation of FWI outputs weakly tackled in the literature

- crucial for the industry (risk managment, dry-well issues)
- major importance for geological and geodynamic interpretation of very deep targets (CMB for example)

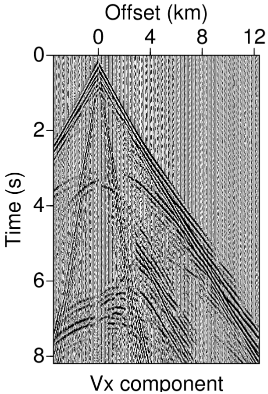
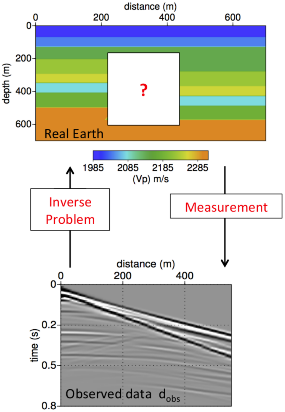


Figure 1: Exemple of recorded wavefield data.

Outline

- Full Waveform Inversion
- Data Assimilation with Ensemble
- Combining FWI and ETKF
- Applications
- Conclusion



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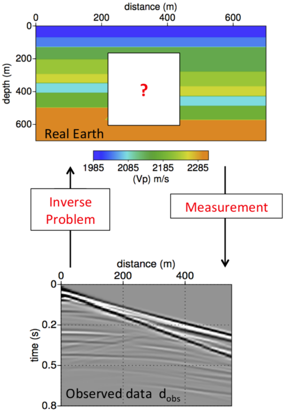
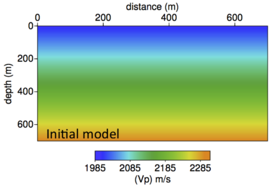
Full Waveform Inversion principle





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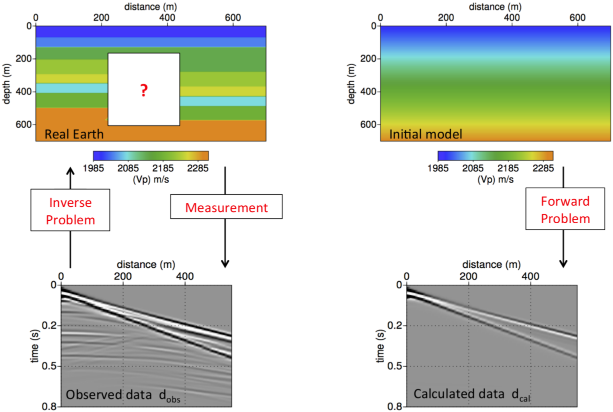

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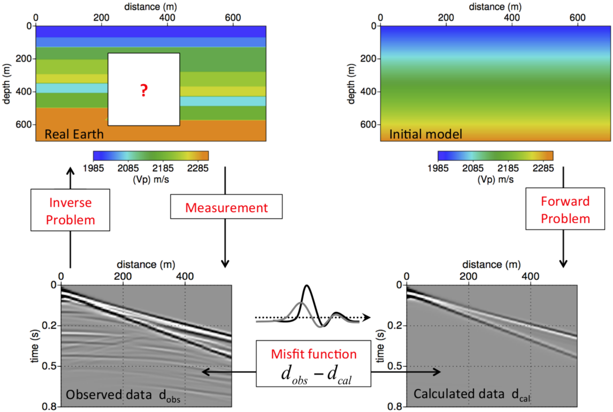

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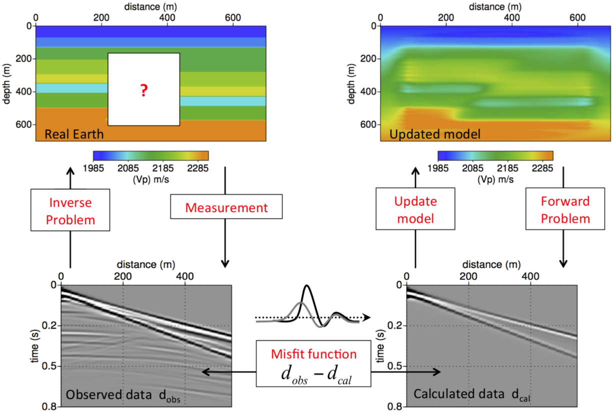

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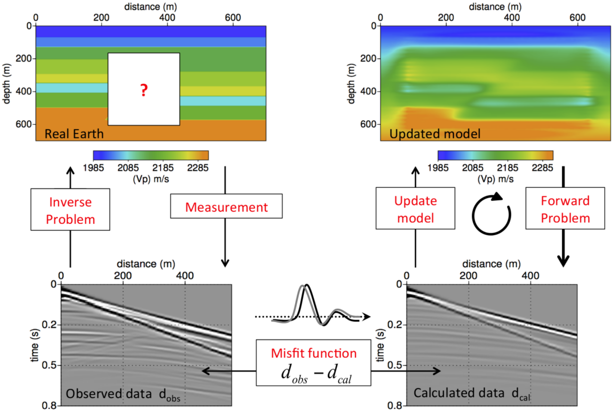
Full Waveform Inversion principle



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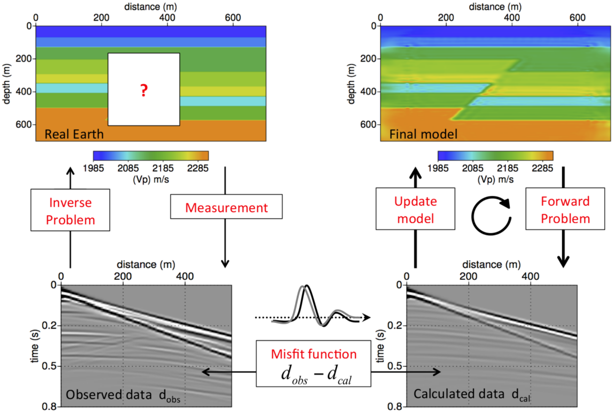
Full Waveform Inversion principle



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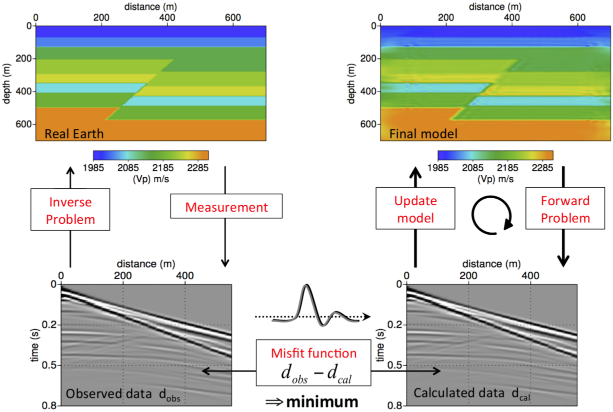

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The uncertainty problem

Following FWI properties

- ill-posed deterministic process
- high number of degrees-of-freedom
- computationaly expensive forward problem
- local optimization strategy (quasi-Newton)

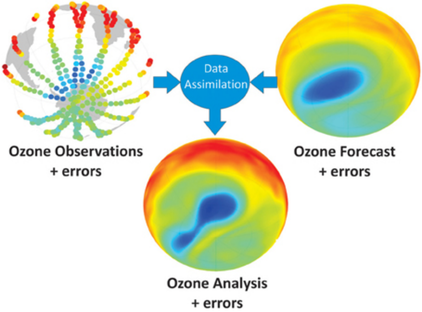
→ challenging uncertainty estimation

Only a few recent papers propose to tackle the uncertainy problem in FWI : still no systematic applications (Fichtner and van Leeuwen, 2015; Fang et al., 2018)

But Ensemble Data Assimilation offer an interesting framework to address those challenges!

Outline

- Generally used for large dynamic systems
- Combines sparse measurements with model to find the "best tradeoff"
- Yields state estimation of complex non-linear systems
- Uncertainty associated to state estimate



From *Data Assimilation: Making Sense of Observations*

The BLUE (Bayesian framework) $\rightarrow$ MAP estimator (considering all the statistics associated to models and observations)

- $m^b = m + \eta^b$, with $\eta^b \sim \mathcal{N}(0, \mathbf{P}^b)$
- $d_{obs} = \mathbf{H}m + \eta^0$, with $\eta^0 \sim \mathcal{N}(0, \mathbf{R})$

The BLUE solution m^a for the true state m is given by :

$$m^a = m^b + \mathbf{K}(d_{obs} - \mathbf{H}m^b) \quad ; \quad \mathbf{P}^a = (\mathbf{I} - \mathbf{KH})\mathbf{P}^b \tag{1}$$

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with the gain matrix being given by

$$\mathbf{K} = \mathbf{P}^b\mathbf{H}^T(\mathbf{H}\mathbf{P}^b\mathbf{H}^T + \mathbf{R})^{-1} \tag{2}$$

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This solution is equivalent to solving the following linear optimization problem :

$$\mathcal{C}(m) = \frac{1}{2}(m - m^b)^T(\mathbf{P}^b)^{-1}(m - m^b) + \frac{1}{2}(d_{obs} - \mathbf{H}m)^T\mathbf{R}^{-1}(d_{obs} - \mathbf{H}m) \tag{3}$$

Kalman Filter (Kalman, 1960) generalises BLUE to linear dynamic problems

Forecast step

$m_k^f = \mathbf{F} m_{k-1}^a + \eta^q$
$\mathbf{P}_k^f = \mathbf{F}\mathbf{P}_{k-1}^a \mathbf{F}^T + \mathbf{Q}_{k-1}$

- projects $\{m, P\}$ along k axis

Analysis step

$m_k^a = m_k^f + \mathbf{K}_k (d_{obs,k} - \mathbf{H} m_k^f)$
$\mathbf{P}_k^a = (\mathbf{I} - \mathbf{K}_k \mathbf{H})\mathbf{P}_k^f$

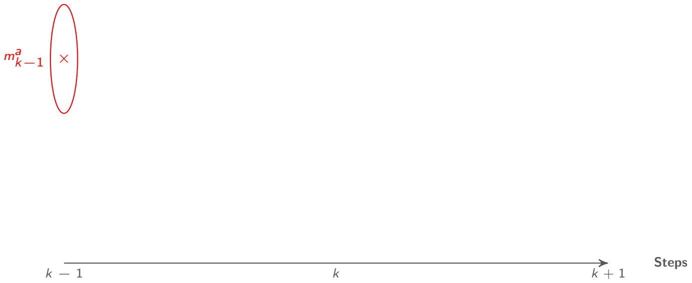
- corrects the forecast based on data

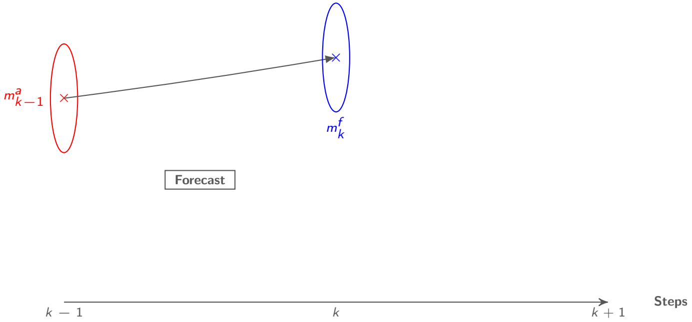
$\mathbf{F}$ is the forecast operator (forward modeling), $\eta^q \sim \mathcal{N}(0, \mathbf{Q})$ (forecast uncertainty). f denotes the forecast step, and a the analysis step.

The Kalman gain is given by

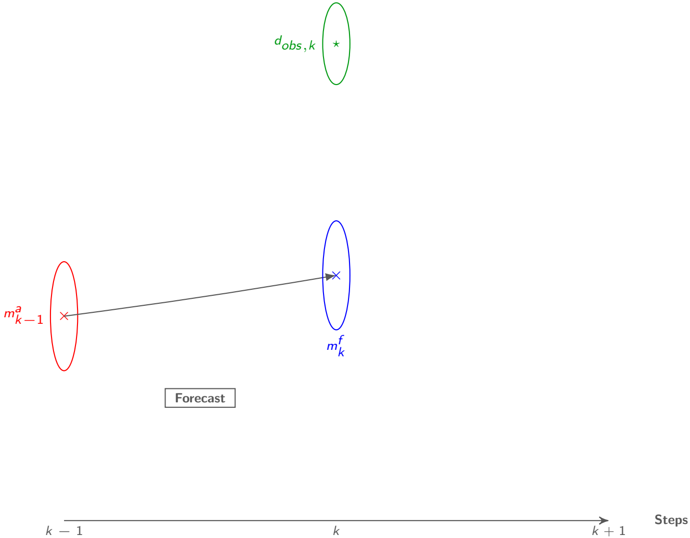
$$\mathbf{K} = \mathbf{P}^f \mathbf{H}^T (\mathbf{H}\mathbf{P}^f \mathbf{H}^T + \mathbf{R})^{-1} \tag{4}$$

Kalman Filter scheme

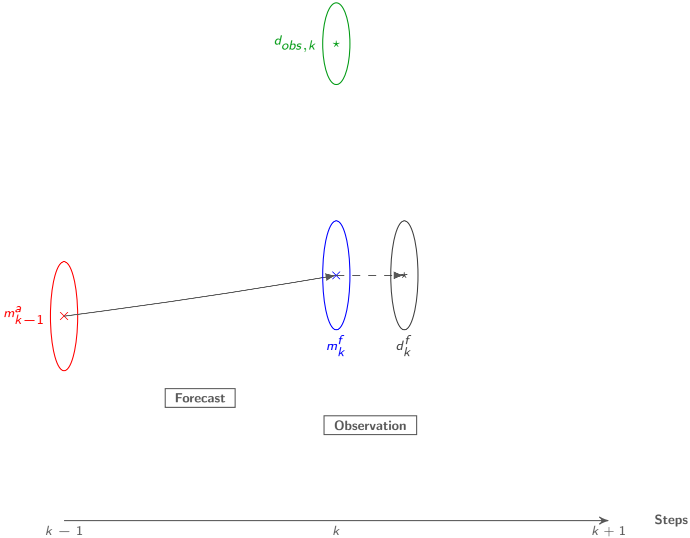


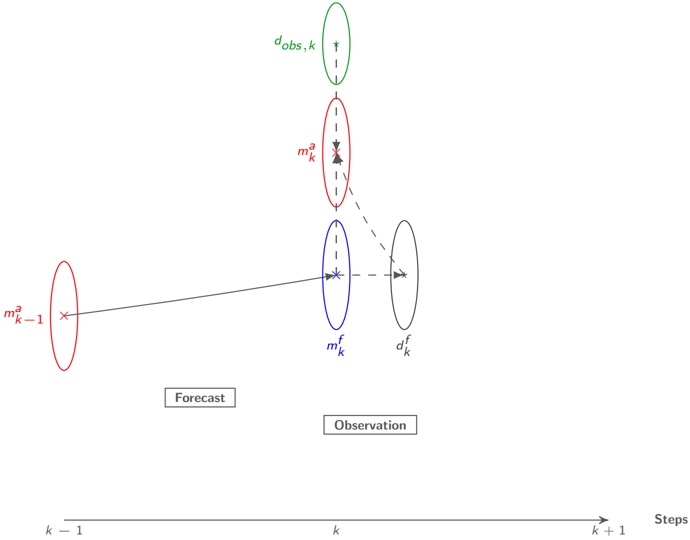

m^a_{k-1}
m^f_k
Forecast
$k-1$
k
$k+1$
Steps

Kalman Filter scheme

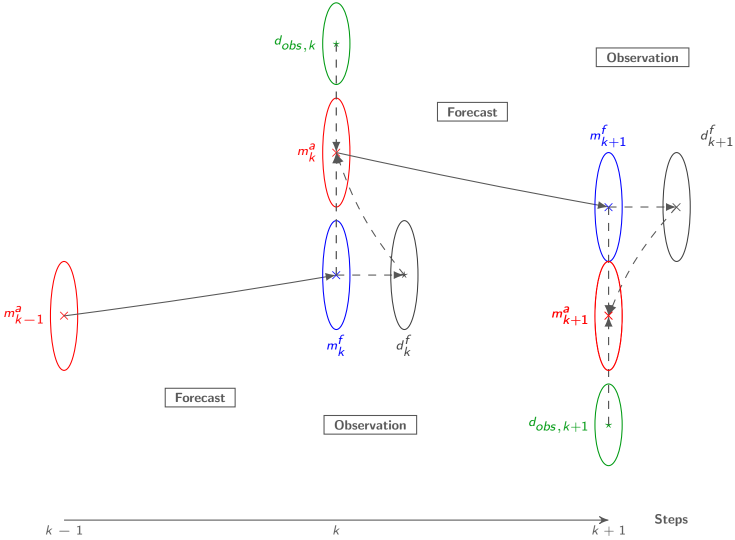


Kalman Filter scheme




$d_{obs,k}$
m_k^a
m_{k-1}^a
m_k^f
d_k^f
Forecast
Observation
$k-1$
k
$k+1$
Steps

Kalman Filter scheme



To go beyong small linear cases: Ensemble Kalman Filter (Evensen, 1994)
EnKF gives a low-rank approximation of the KF equations

Involves an ensemble of system states $\mathbf{m} = \{m^1, m^2, ..., m^{N_e}\}$ that represent both the state estimate and that estimate uncertainty

The state estimate given by:

$$\bar{m} = \frac{1}{N_e} \sum_{i=1}^{N_e} m^i \tag{5}$$

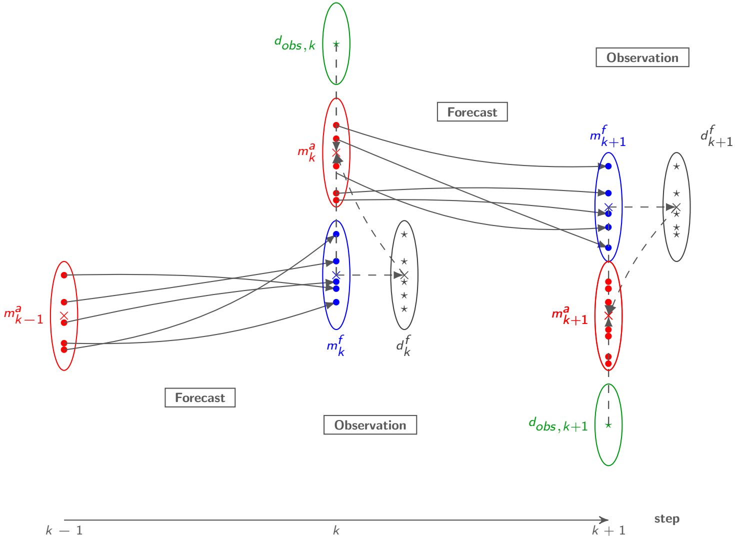
Defining the $n \times N_e$ perturbation matrix

$$M^i = m^i - \bar{m} \tag{6}$$

M columns contain ensemble members' deviation from the mean

The state estimate uncertainty given by the empirical covariance:

$$\mathbf{P}_e = \frac{1}{N_e - 1} \mathbf{M}\mathbf{M}^T \tag{7}$$



$d_{obs,k}$
Observation
Forecast
m^f_{k+1}
d^f_{k+1}
m^a_k
m^a_{k-1}
m^f_k
d^f_k
m^a_{k+1}
Forecast
Observation
$d_{obs,k+1}$
step
$k-1$
k
$k+1$

Outline

d_{obs} are wavefield data at receivers

Ensemble **m** contains subsurface models $\{m^1, m^2, ..., m^{N_e}\}$.

Defining FWI as a non-linear operator $\mathcal{F}$

$$\mathcal{F}(m, d_{obs}) = \min_{m} \quad \frac{1}{2}||d_{cal}(m) - d_{obs}||^2 \tag{8}$$

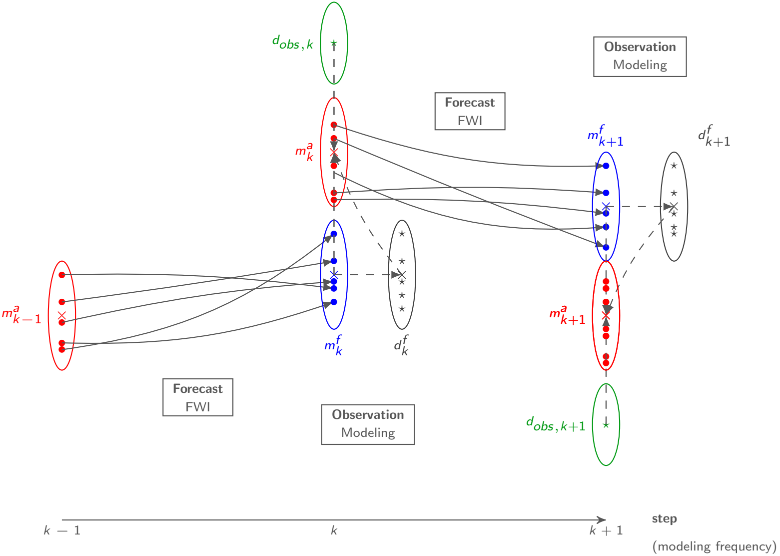
where m is the model containing the n physical parameters, $d_{cal}(m)$ is the synthetic wavefield data computed in m, d_{obs} is the observed data and $||.||$ is the Euclidean distance in the data space.

Allows to define the forecast as

$$m_k^{f,i} = \mathcal{F}(m_{k-1}^{a,i}, d_{obs,k}) \tag{9}$$

Decomposing the data in K frequency groups, allows to consider a **dynamic axis in frequency**, with $k = 1, \ldots, K$.

The K frequency bands are solved sequentially through consecutive ETKF-FWI cycles.

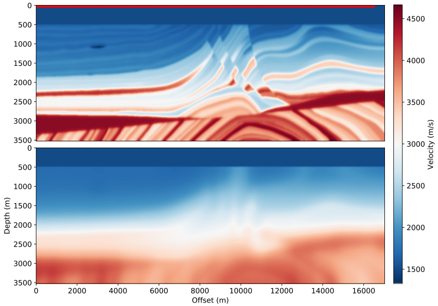

$d_{obs,k}$
Observation
Modeling
Forecast
FWI
m^f_{k+1}
d^f_{k+1}
m^a_k
m^a_{k-1}
m^a_{k+1}
m^f_k
d^f_k
Forecast
FWI
Observation
Modeling
$d_{obs,k+1}$
$k-1$
k
$k+1$
step
(modeling frequency)

Outline

Outline

Marmousi synthetic test settings

- 2D frequency-domain FWI
- Purely acoustic case
- fixed spreads surface acquisition (144 sources, 660 receivers)
- Modeling grid = Inversion grid = 141 x 681 grid points (96021 dof)
- 15 ETKF-FWI steps with 600 ensemble members
- 15 different monotonic data from 3.0 to 10.0Hz



Initial ensemble building

We want ETKF-FWI to behave well according to standard FWI results

$\rightarrow$ initial model population from a "correct" initial model $\bar{m}_0$

Generating an ensemble satisfying a covariance $\mathbf{P}$ from $\mathbf{P}^{1/2}$ is out of reach for large scale problems.

We build each ensemble member by considering N_e zero mean random vectors u_i (white noise), smoothed by a Gaussian filter

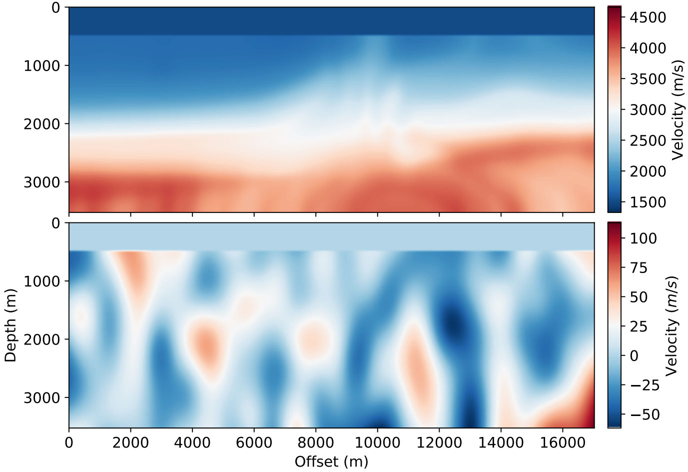
$$m_{0,i} = \bar{m}_0 + \mathcal{G} u_i, \quad (10)$$

giving a prior covariance the shape of a Gaussian squared as

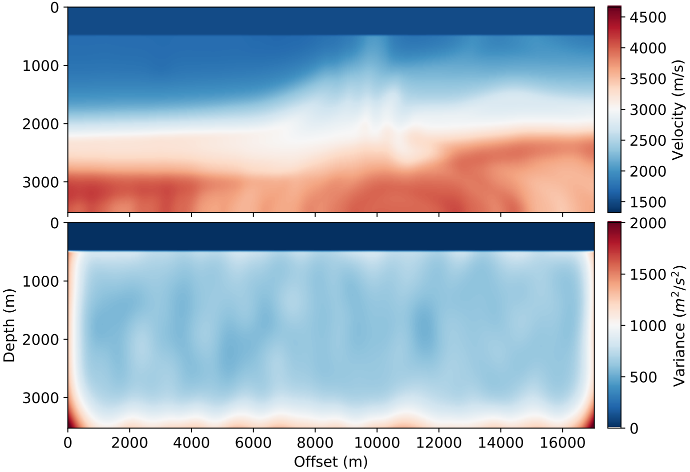
$$\mathbf{P} = \mathcal{G}\mathcal{G}^T. \quad (11)$$

Initial ensemble member

$$\bar{m}_0 \qquad + \qquad \mathcal{G} u_i$$



$$k = 0$$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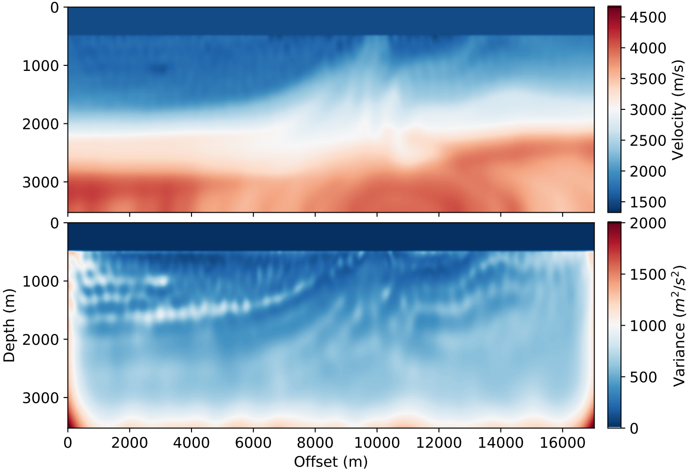
Offset (m)

$$k = 1 : 3.0 \text{ Hz}$$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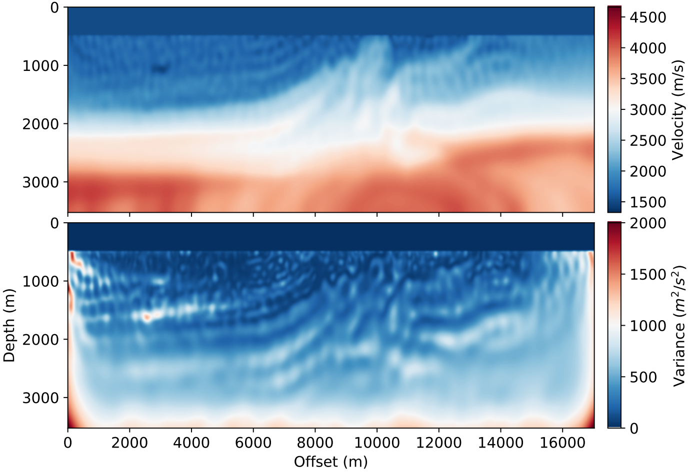
Variance (m^2/s^2)

$$k = 2 : 3.5 \text{ Hz}$$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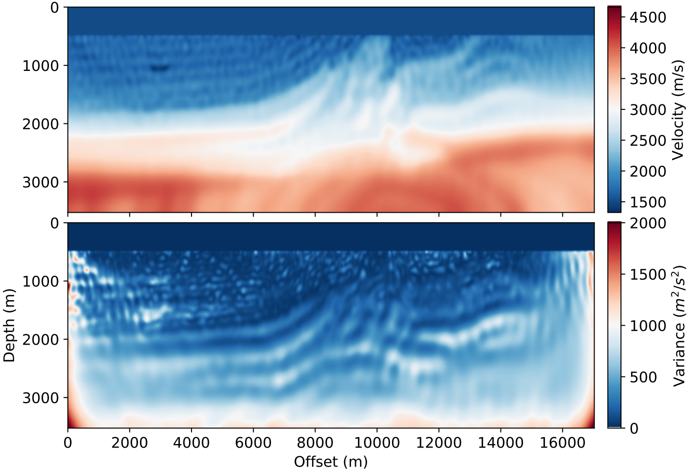
Offset (m)

$$k = 3 : 4.0 \text{ Hz}$$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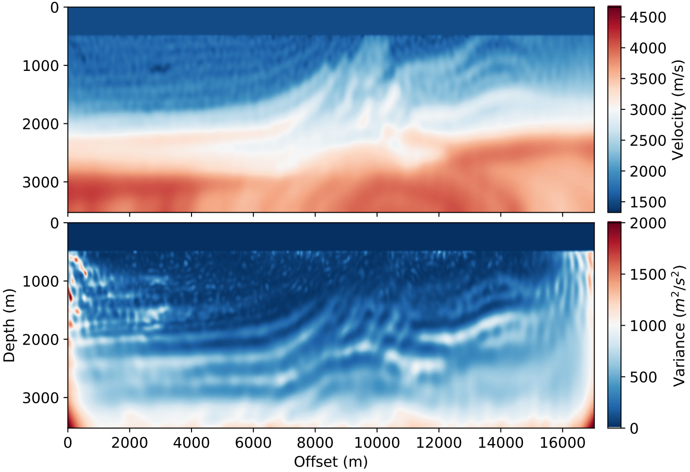
$$k = 4 : 4.5 \text{ Hz}$$



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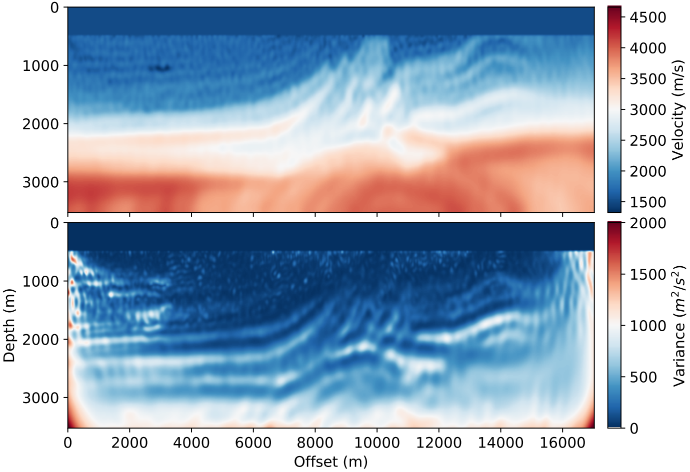
Offset (m)

$k = 5 : 5.0$ Hz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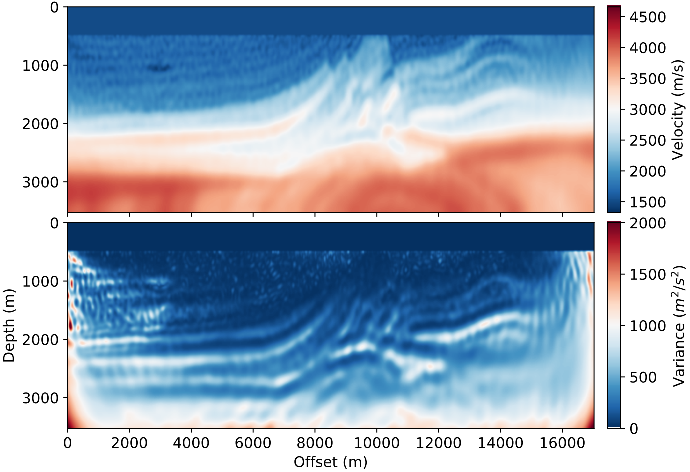
$k = 6 : 5.5$ Hz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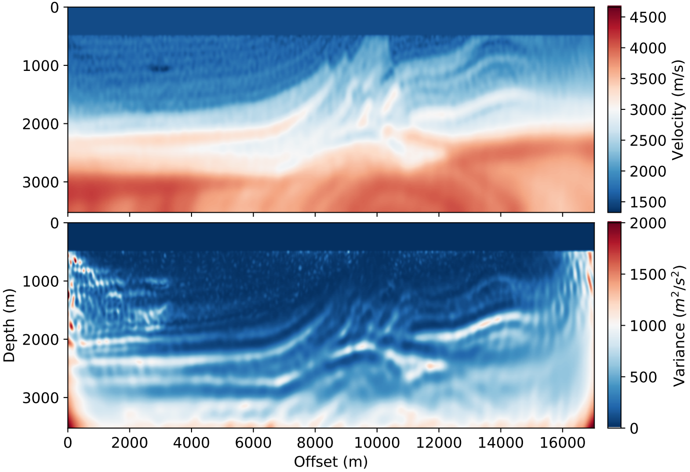
Offset (m)

$k = 7$: 6.0 Hz



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Variance (m^2/s^2)
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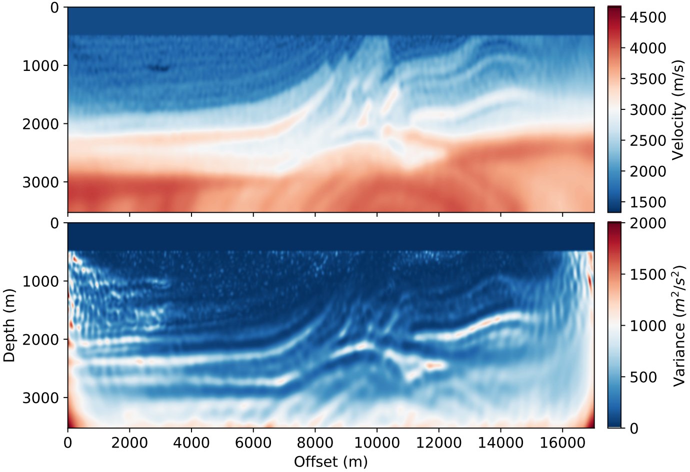
$$k = 8 : 6.5 \text{ Hz}$$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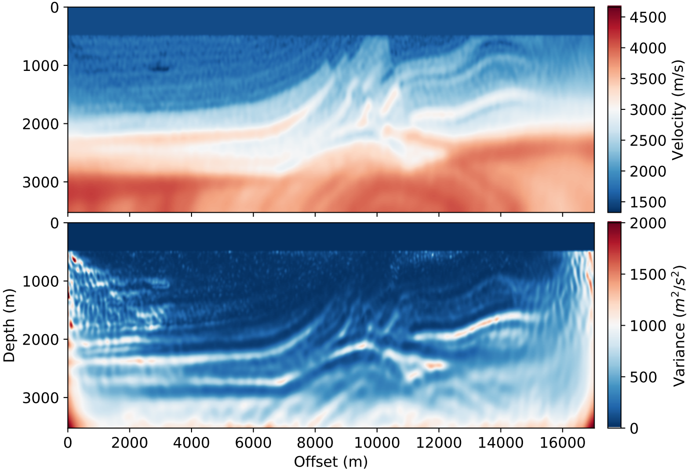
$k = 9 : 7.0$ Hz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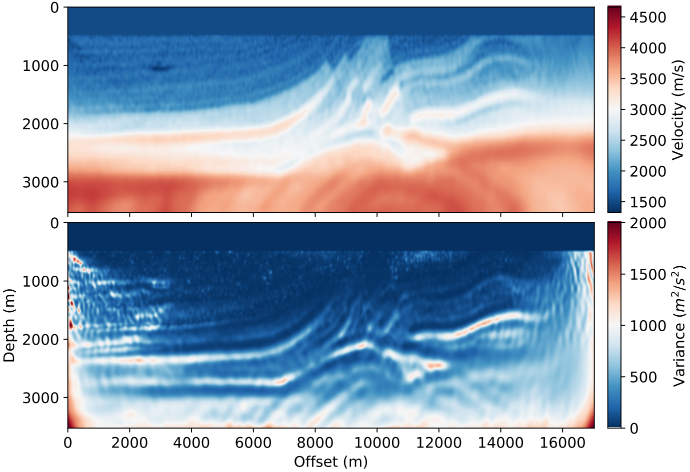
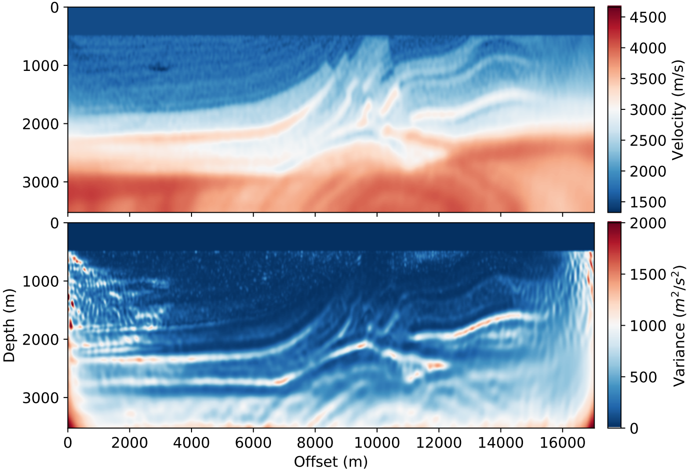
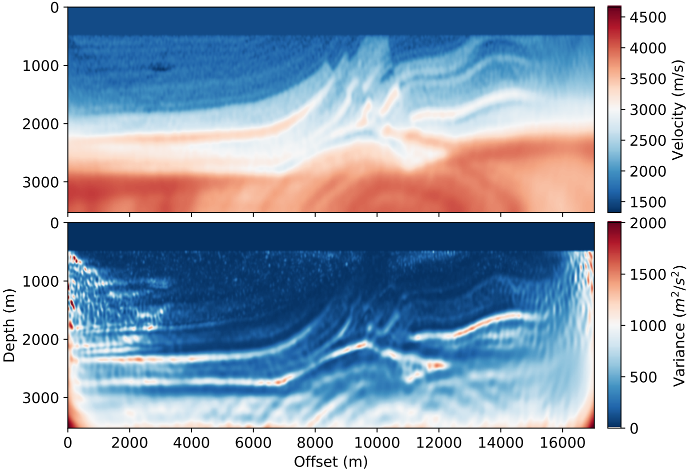
$$k = 10 : 7.5 \text{ Hz}$$



$$k = 11 : 8.0 \text{ Hz}$$



$k = 12 : 8.5$ Hz



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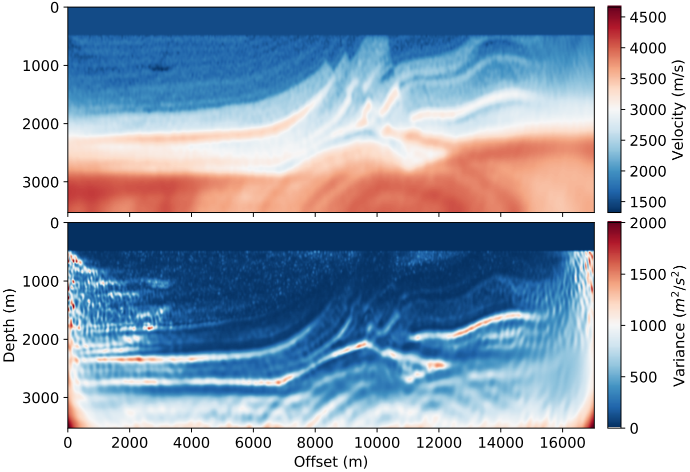
$k = 13 : 9.0$ Hz



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Variance (m^2/s^2)
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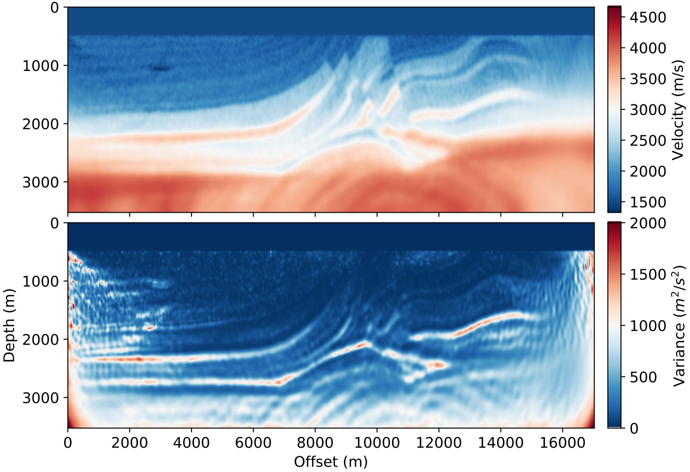
Offset (m)

$k = 14 : 9.5$ Hz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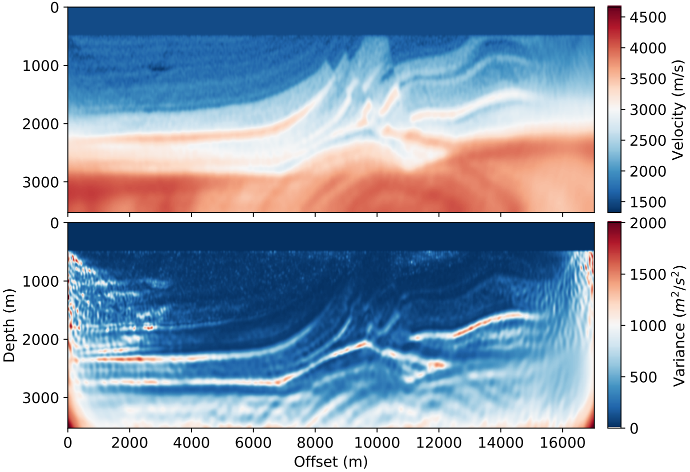
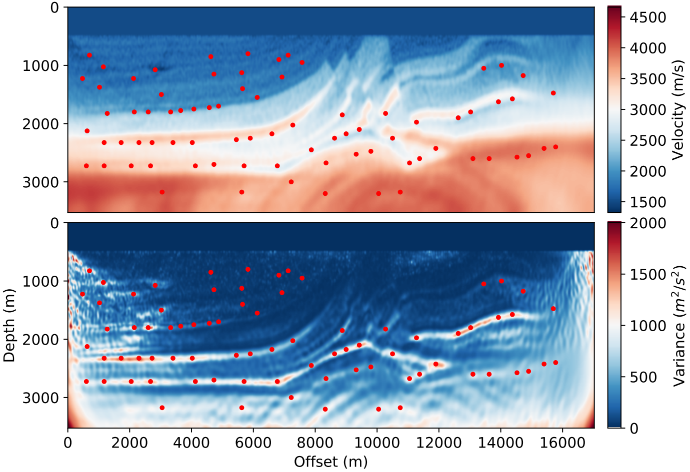
Depth (m)
2000
1500
1000
500
0
Variance (m^2/s^2)
0
2000
4000
6000
8000
10000
12000
14000
16000
Offset (m)

$k = 15 : 10.0$ Hz


0
1000
2000
3000
4500
4000
3500
3000
2500
2000
1500
Velocity (m/s)
0
1000
2000
3000
Depth (m)
2000
1500
1000
500
0
Variance (m^2/s^2)
0
2000
4000
6000
8000
10000
12000
14000
16000
Offset (m)

$k = 15 : 10.0$ Hz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4500
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3500
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2500
2000
1500
Velocity (m/s)
0
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2000
3000
Depth (m)
2000
1500
1000
500
0
Variance (m^2/s^2)
0
2000
4000
6000
8000
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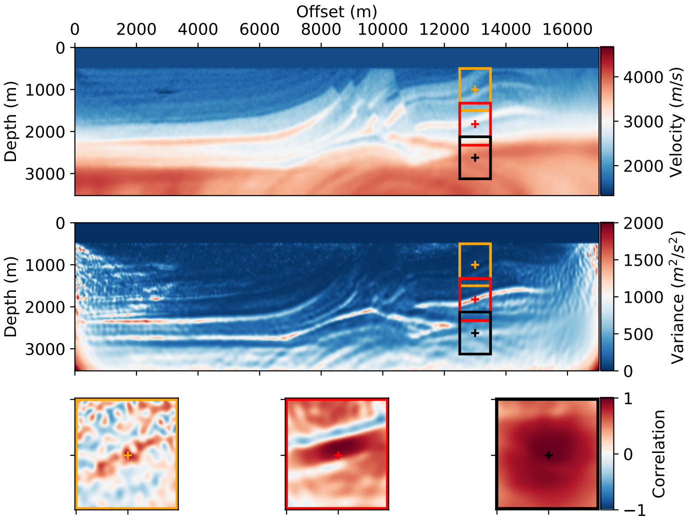
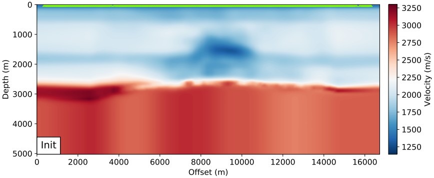
12000
14000
16000
Offset (m)


Offset (m)
0
2000
4000
6000
8000
10000
12000
14000
16000
Depth (m)
0
1000
2000
3000
Velocity (m/s)
4000
3000
2000
Variance (m^2/s^2)
2000
1500
1000
500
0
Correlation
1
0
−1

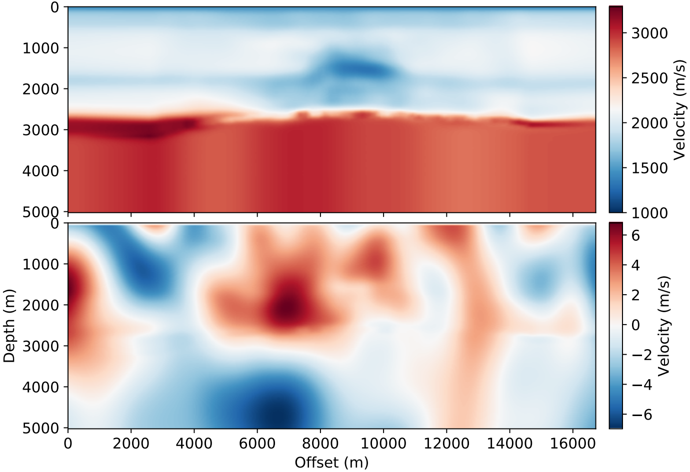
Outline

Application on the Valhall OBC field-data

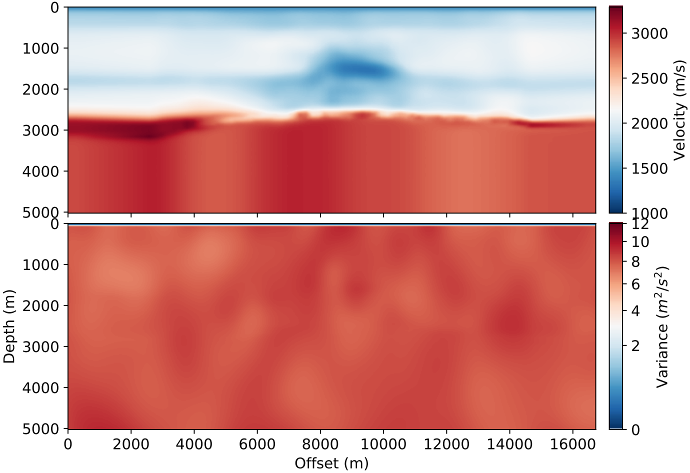
- 2D line extracted from OBC data from the Valhall field, provided by AkerBP
- 2D frequency-domain FWI
- Modeling done with FD visco-acoustic in VTI medium
- fixed spreads surface acquisition (192 sources, 315 receivers)
- Modeling grid = Inversion grid = 201 x 669 grid points (134469 dof)
- 6 ETKF-FWI cycles with 600 ensemble members
- 6 different frequency groups (3 to 4 frequencies) from 3.6 to 7.0Hz



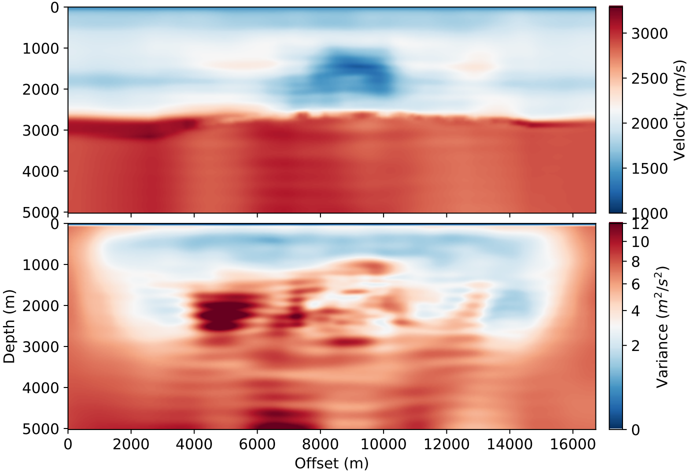
$$\bar{m}_0 \quad + \quad \mathcal{G} u_i$$



$$k = 0$$


0
1000
2000
3000
4000
5000
3000
2500
2000
1500
1000
Velocity (m/s)
0
1000
2000
3000
4000
5000
Depth (m)
12
10
8
6
4
2
0
Variance (m^2/s^2)
0
2000
4000
6000
8000
10000
12000
14000
16000
Offset (m)

$k = 1$: 3.6 - 3.9 - 4.3 Hz



0
1000
2000
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4000
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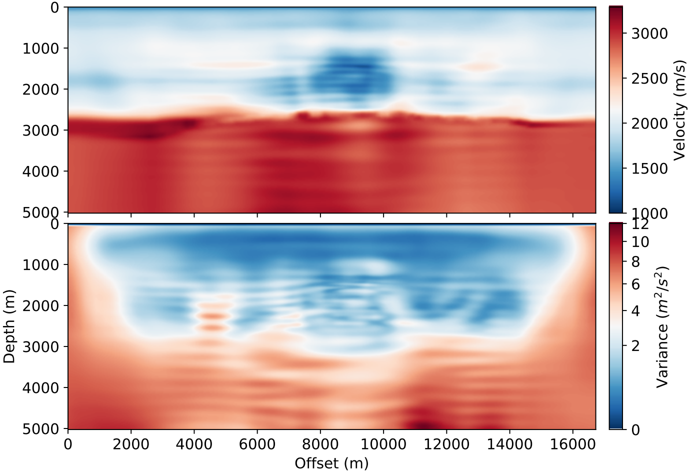
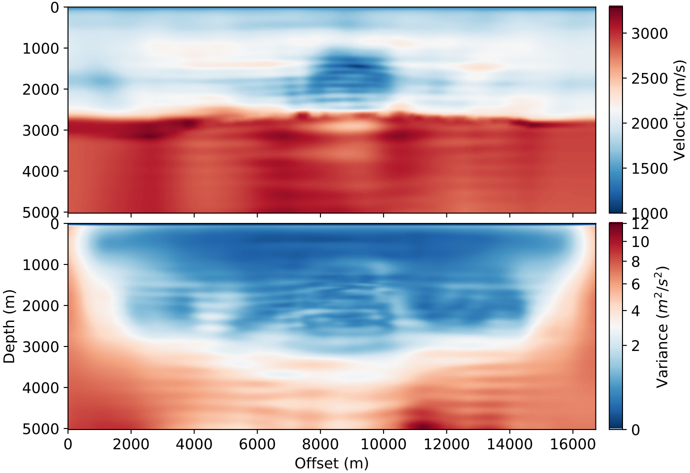
3000
2500
2000
1500
1000
Velocity (m/s)
Depth (m)
12
10
8
6
4
2
0
Variance (m^2/s^2)
0
2000
4000
6000
8000
10000
12000
14000
16000
Offset (m)

$$k = 2 : 4.1 - 4.4 - 4.8 \text{ Hz}$$



$k = 3$: 4.6 - 4.9 - 5.3 Hz


0
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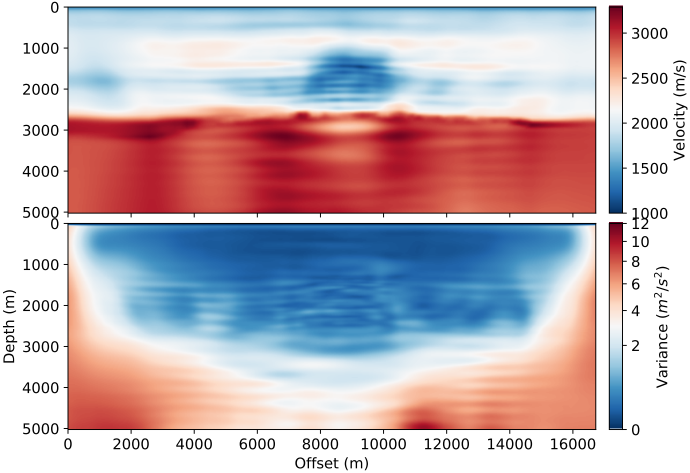
Depth (m)
0
2000
4000
6000
8000
10000
12000
14000
16000
Offset (m)
3000
2500
2000
1500
1000
Velocity (m/s)
12
10
8
6
4
2
0
Variance (m^2/s^2)

$k = 4$: 5.0 - 5.4 - 5.8 Hz



0
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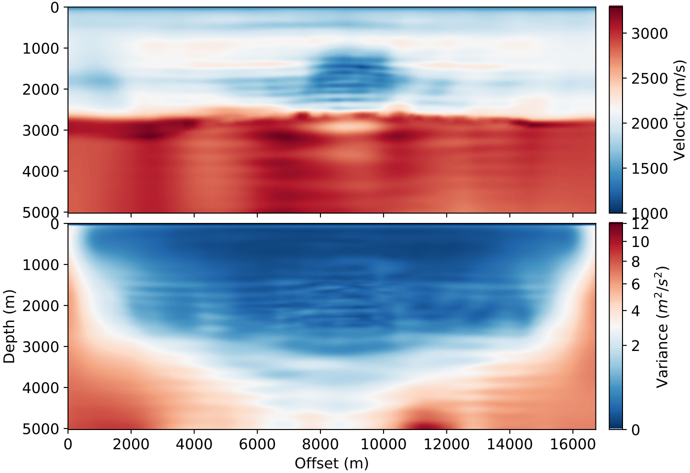
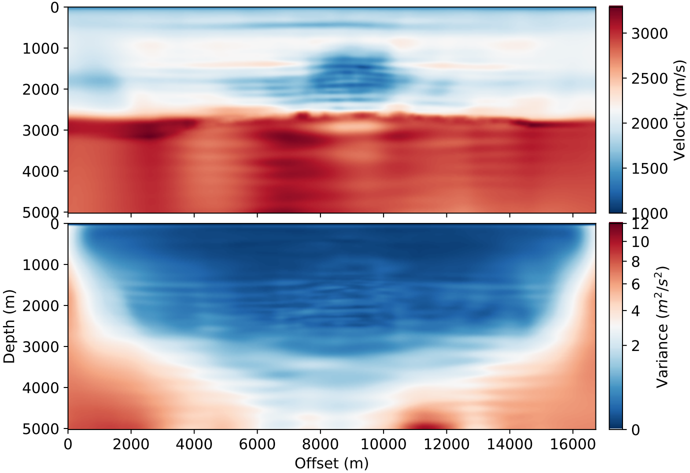
Depth (m)
0
2000
4000
6000
8000
10000
12000
14000
16000
Offset (m)
3000
2500
2000
1500
1000
Velocity (m/s)
12
10
8
6
4
2
0
Variance (m^2/s^2)

$$k = 5 : 5.5 \text{ - } 5.9 \text{ - } 6.3 \text{ Hz}$$



$k = 6$: 5.5 - 5.9 - 6.4 - 7.0 Hz



Outline

Conclusions

Uncertainty estimation with ETKF-FWI is possible on industrial field-data (2D).

This application shows:

- variance and correlation information consistent with theoretical intuitions
- local variance peaks located on interfaces
- parameter estimate converge toward typical FWI result
- uncertainty all along the inversion cycles

Reading of variance and correlation maps is straightforward

Perspectives

Current focus:

- multiparameter ETKF-FWI

Long term:

- alternative to the current dynamic axis
- ETKF-FWI on hyperparameters

Future work:

- multi-physics joint inversion
- time-lapse application
- test alternatives to the ETKF

Acknowledgments

Thanks for the support from

- 2018 SEISCOPE sponsors (`http://seiscope2.osug.fr`):
  AKERBP, CGG, CHEVRON, EXXON-MOBIL, JGI, PETROBRAS, SCHLUMBERGER, SHELL, SINOPEC, EQUINOR and TOTAL.
- CIMENT (Froggy) computing center *https://ciment.ujf-grenoble.fr*
- CINES/IDRIS/TGCC computing center (allocation 046091 made by GENCI)

Thank you for your attention! Any questions?

Bishop, C. H., Etherton, B. J., and Majumdar, S. J. (2001). Adaptive sampling with the ensemble transform kalman filter. part i: Theoretical aspects. *Monthly weather review*, 129(3):420–436.

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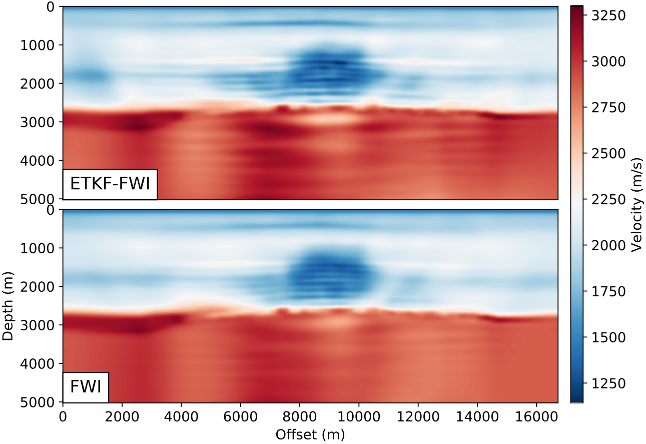
ETKF-FWI vs FWI



EnKF Forecast

The forecast ensemble is given by

$$m_{k+1}^{f}{}^{(i)} = \mathcal{F}_k(m_k{}^{(i)}) + \eta_k \qquad i = 1, 2, \ldots, N_e, \tag{12}$$

where the forecast operator $\mathcal{F}$ can be non-linear.

$\mathbf{P}^f$ is approximated by empirical covariance

$$\mathbf{P}_e^f = \frac{1}{N_e - 1}\mathbf{M}^f \mathbf{M}^{f\,T} \tag{13}$$

avoiding explicit forecast of covariance matrix.

Instead carried through the ensemble trajectory and repartition!

EnKF Analysis

Because analysis outputs are

$$\bar{m}^a = \frac{1}{N_e}\sum_{i=1}^{N_e} m_i^a, \qquad \mathbf{P}^a = \frac{1}{N_e - 1}\mathbf{M}^a{\mathbf{M}^a}^T \tag{14}$$

analysis can be performed by updating the whole ensemble

$$\{\mathbf{m}^f = \bar{m}^f + \mathbf{M}^f\} \rightarrow \{\mathbf{m}^a = \bar{m}^a + \mathbf{M}^f\} \tag{15}$$

Formalism proposed by Bishop et al. (2001): Ensemble Transform Kalman Filter(ETKF)

$\mathbf{P}^f$ is rank limited but overdetermined on $\mathcal{S}$ (arbitrary subspace spanned by $\mathbf{M}^f$)
Allowing to define the analysis error covariance on $\mathcal{S}$ (Hunt et al., 2007)

$$\tilde{\mathbf{P}}^a = \{(N_e - 1)\mathbf{I} + {\mathbf{M}^f}^T\mathbf{H}^T\mathbf{R}^{-1}\mathbf{M}^f\mathbf{H}\}^{-1} \tag{16}$$

satisfying

$$\mathbf{P}^a = \frac{1}{N_e - 1}\mathbf{M}^f\tilde{\mathbf{P}}^a{\mathbf{M}^f}^T \tag{17}$$

EnKF Analysis

Analysis is completed by

$$\bar{m}^a = \bar{m}^f + \mathbf{M}^f \tilde{\mathbf{P}^a} {\mathbf{M}^f}^T \mathbf{H}^T \mathbf{R}^{-1}(d_{obs} - \mathbf{H}\bar{m}) \tag{18}$$

and

$$\mathbf{M}^a = \mathbf{M}^f \mathbf{W} \tag{19}$$

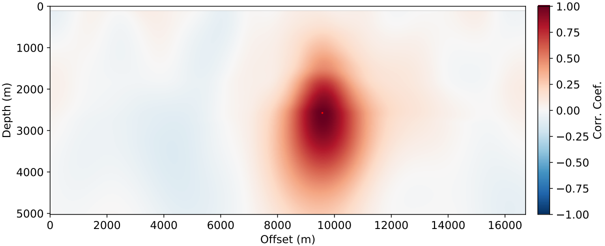
where $\mathbf{W}$ is defined as symmetric square root of $(N_e - 1)\tilde{\mathbf{P}^a}$; $\mathbf{W}\mathbf{W}^T = (N_e - 1)\tilde{\mathbf{P}^a}$

This allows to approximate

$$\mathbf{K} = \mathbf{H}\mathbf{P}^f(\mathbf{H}\mathbf{P}^f\mathbf{H}^T + \mathbf{R})^{-1} \approx \mathbf{M}^f \tilde{\mathbf{P}^a} {\mathbf{M}^f}^T \mathbf{H}^T \mathbf{R}^{-1} \tag{20}$$

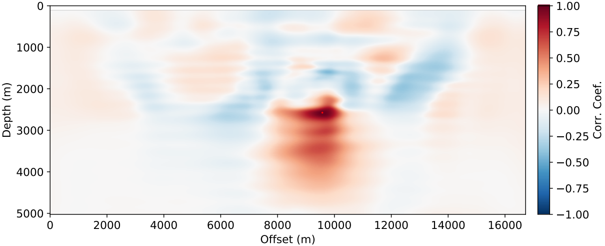
Resolution analysis

$$k = 0$$



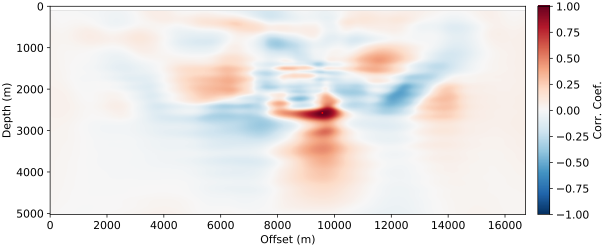
Resolution analysis

$k = 1$: 3.6 - 3.9 - 4.3 Hz



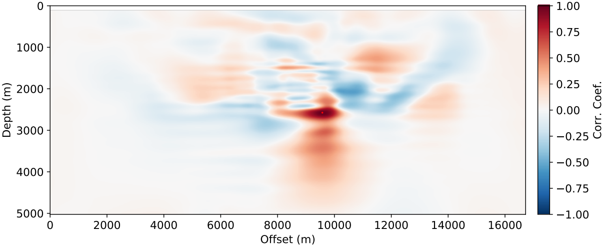
Resolution analysis

$k = 2$: 4.1 - 4.4 - 4.8 Hz

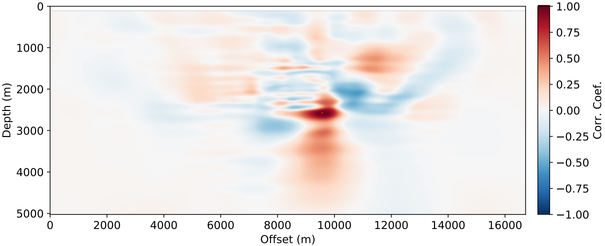


Resolution analysis

$k = 3$: 4.6 - 4.9 - 5.3 Hz



$k = 4$: 5.0 - 5.4 - 5.8 Hz



0
1000
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Depth (m)
0
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6000
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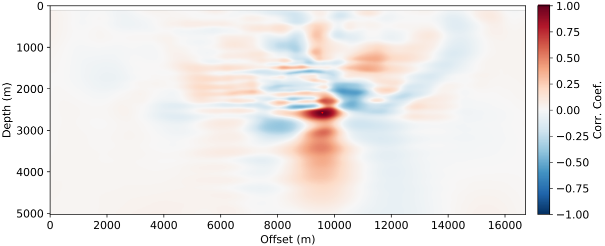
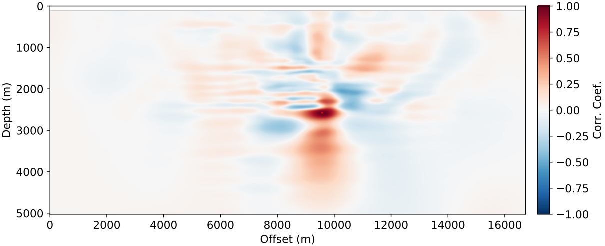
12000
14000
16000
Offset (m)
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
Corr. Coef.

Resolution analysis

$k = 5$: 5.5 - 5.9 - 6.3 Hz

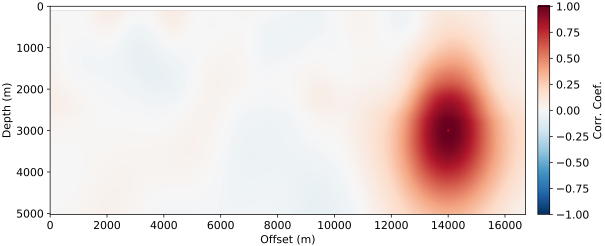


$k = 6$: 5.5 - 5.9 - 6.4 - 7.0 Hz



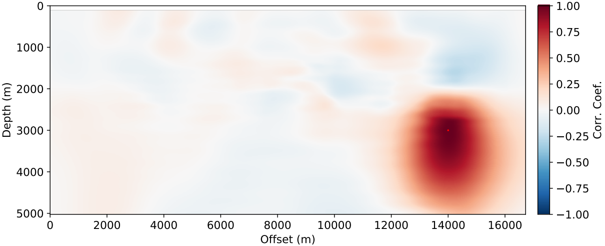
Resolution analysis

$$k = 0$$

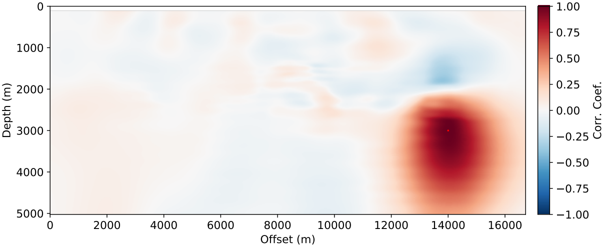


Resolution analysis

$k = 1$: 3.6 - 3.9 - 4.3 Hz



$k = 2$: 4.1 - 4.4 - 4.8 Hz



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Depth (m)
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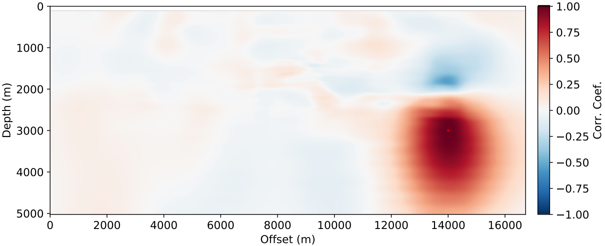
16000
Offset (m)
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
Corr. Coef.

$k = 3$: 4.6 - 4.9 - 5.3 Hz


0
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Depth (m)
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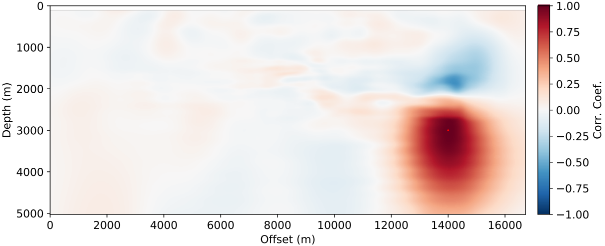
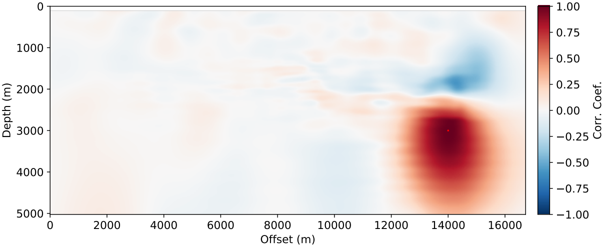
12000
14000
16000
Offset (m)
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
Corr. Coef.

Resolution analysis

$k = 4$: 5.0 - 5.4 - 5.8 Hz



$k = 5$: 5.5 - 5.9 - 6.3 Hz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Depth (m)
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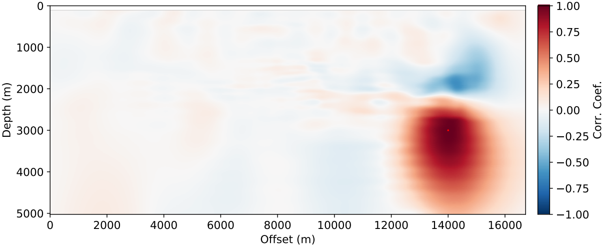
Offset (m)
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
Corr. Coef.

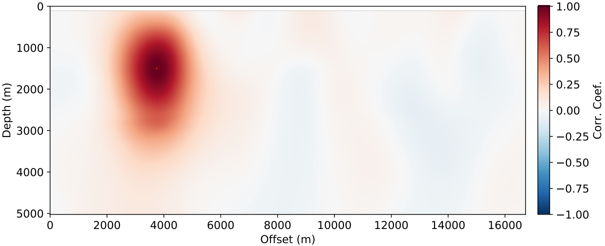
Resolution analysis

$$k = 6 : 5.5 \text{ - } 5.9 \text{ - } 6.4 \text{ - } 7.0 \text{ Hz}$$

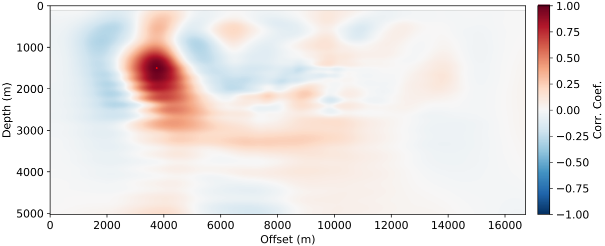


Resolution analysis

$$k = 0$$



$k = 1$: 3.6 - 3.9 - 4.3 Hz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Depth (m)
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12000
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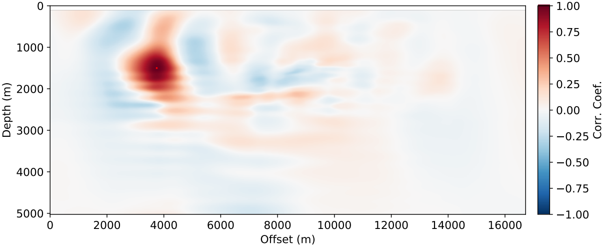
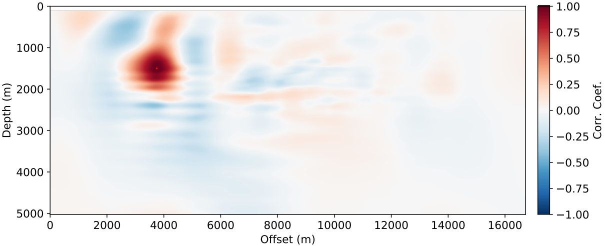
16000
Offset (m)
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
Corr. Coef.

Resolution analysis

$k = 2$: 4.1 - 4.4 - 4.8 Hz



$k = 3$: 4.6 - 4.9 - 5.3 Hz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Depth (m)
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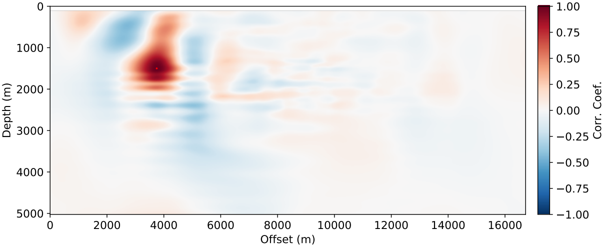
16000
Offset (m)
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
Corr. Coef.

$k = 4$: 5.0 - 5.4 - 5.8 Hz


Depth (m)
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Offset (m)
0
2000
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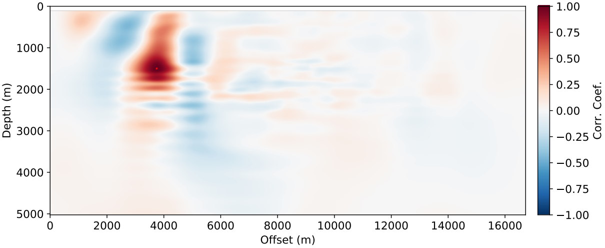
6000
8000
10000
12000
14000
16000
Corr. Coef.
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00

$k = 5$: 5.5 - 5.9 - 6.3 Hz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Depth (m)
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16000
Offset (m)
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
Corr. Coef.

Resolution analysis

$$k = 6 : 5.5 \text{ - } 5.9 \text{ - } 6.4 \text{ - } 7.0 \text{ Hz}$$