Supported by: Federal Ministry for Economic Affairs and Energy on the basis of a decision by the German Bundestag

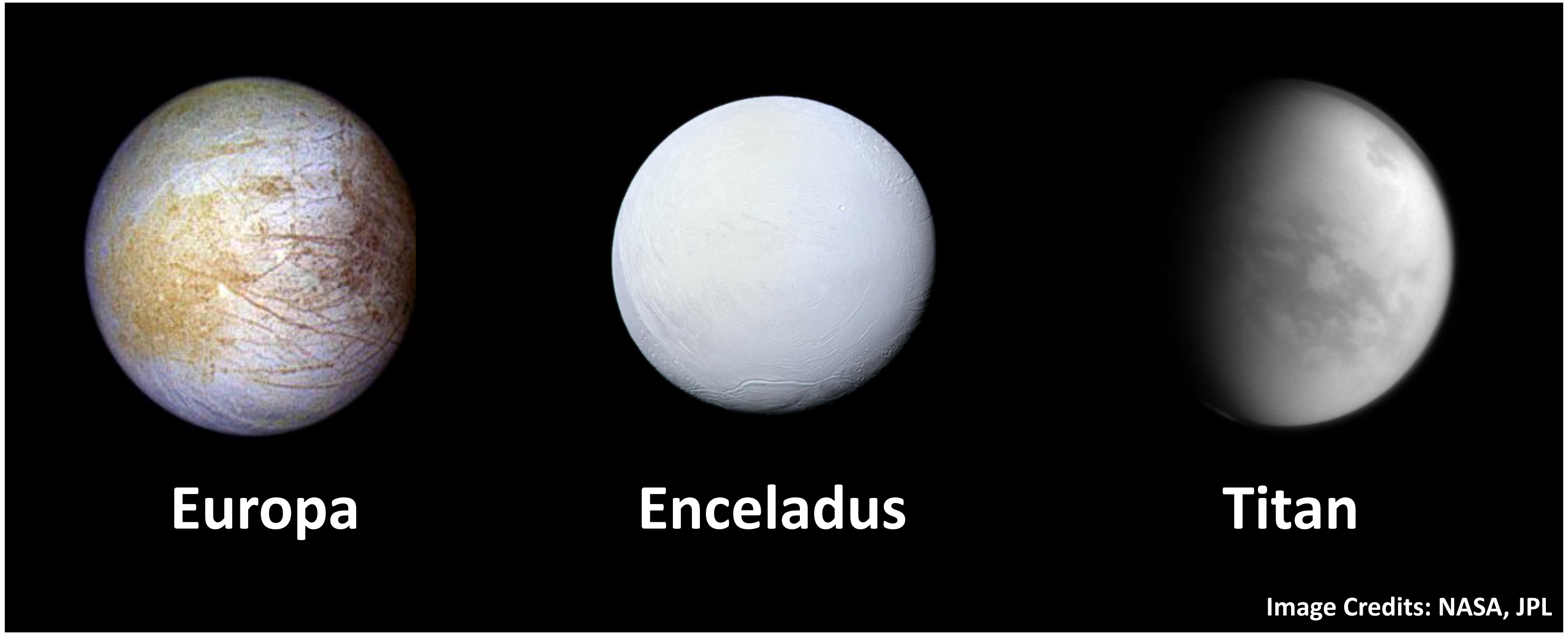
Close-contact melting in ice – Modeling and Simulation of Phase-change in the Presence of a Non-local Pressure Constraint

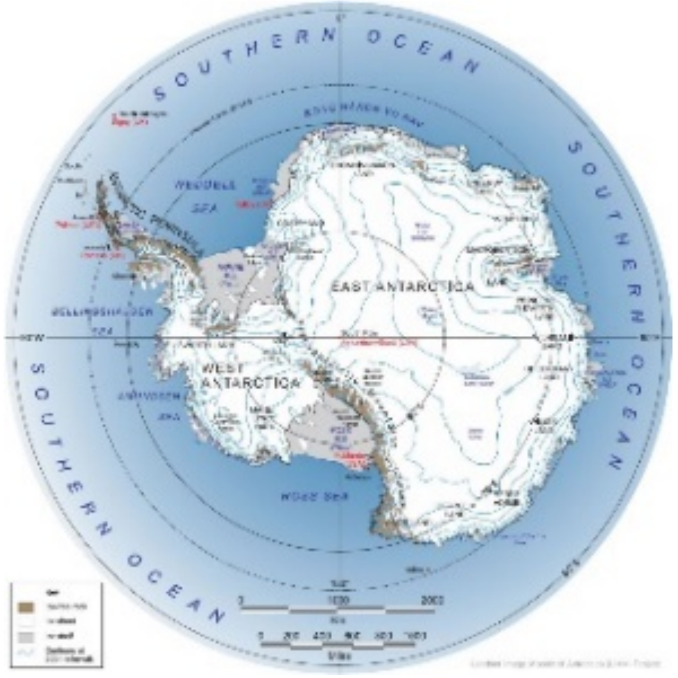
SIAM – GS
March 13th, 2019

J. Kowalski,
K. Schüller, A.G. Zimmerman, and B. Terschanski

Aachen Institute for Advanced Study in Computational Engineering Science
RWTH AACHEN UNIVERSITY

Icy Moons and the Ocean Worlds of our Solar System



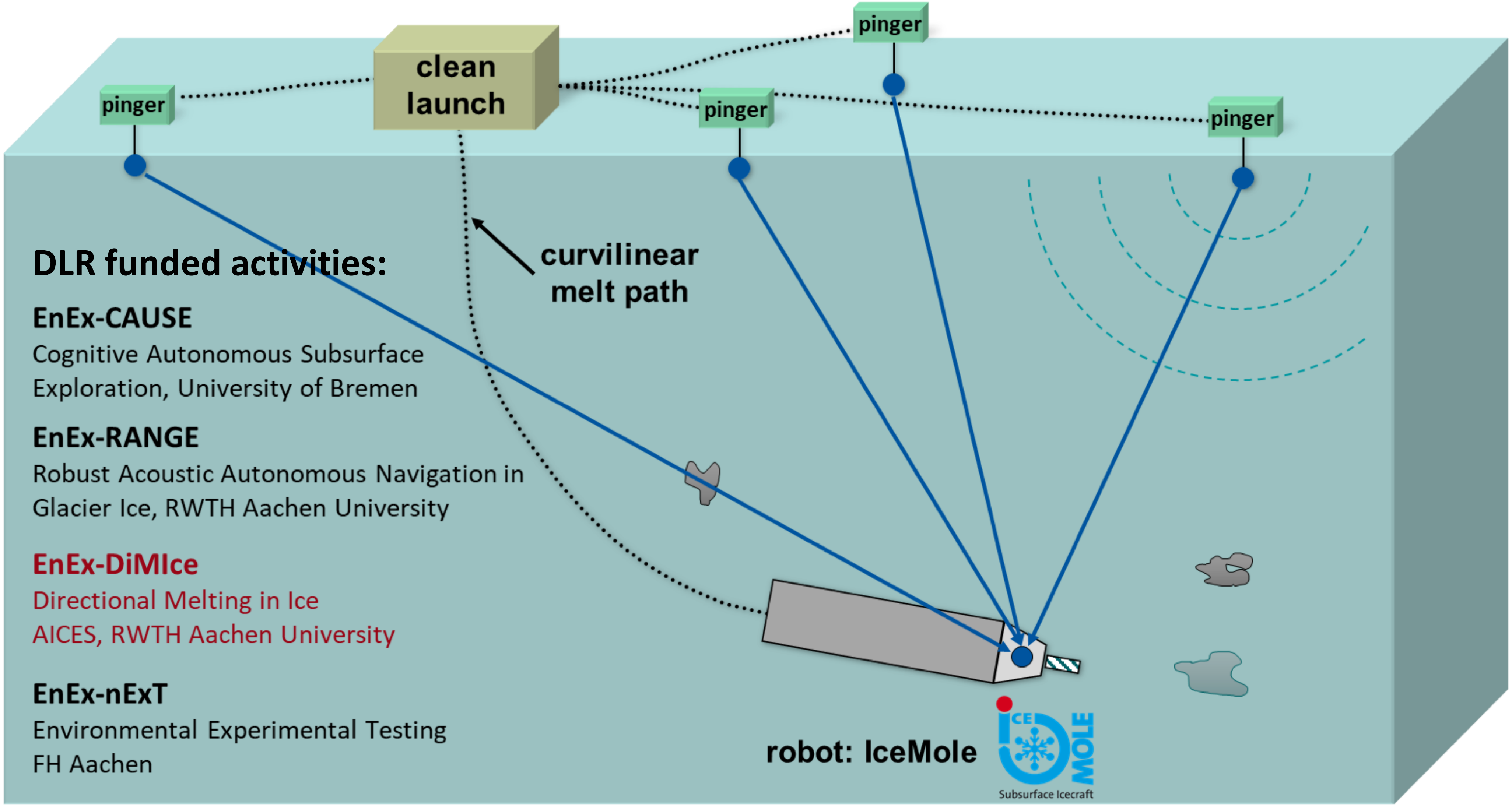


Systems that host stable, globe-girdling bodies of liquid water.

Lunine, Acta Astronautica 131 (2017), 123-130.

Subglacial life exists on Earth!

Interest in Autonomous, Robotic ice exploration technologies



DLR funded activities:

EnEx-CAUSE
Cognitive Autonomous Subsurface Exploration, University of Bremen

EnEx-RANGE
Robust Acoustic Autonomous Navigation in Glacier Ice, RWTH Aachen University

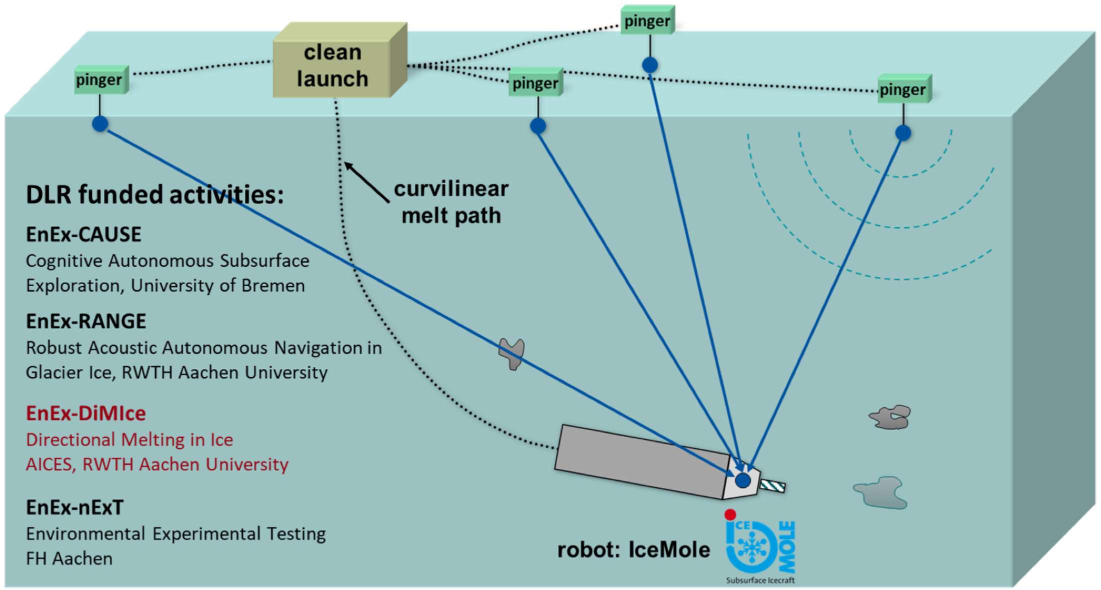
EnEx-DiMIce
Directional Melting in Ice
AICES, RWTH Aachen University

EnEx-nExT
Environmental Experimental Testing
FH Aachen

- **before 2012**
  decentral activities
- **2012 – 2015**
  EnEx-MIDGE collaboration - advancing technologies, many tests on Alpine Glaciers and subglacial sample return in Antarctica
- **since 2015**
  further development of key technologies, int. collaboration launched

Aachen Institute for Advanced Study in Computational Engineering Science
RWTH AACHEN UNIVERSITY

Modeling challenges in the context of robotic ice exploration



…

predicting the dynamics of the melting robot given its controls (heating power, ice screw induced contact force)

…

…

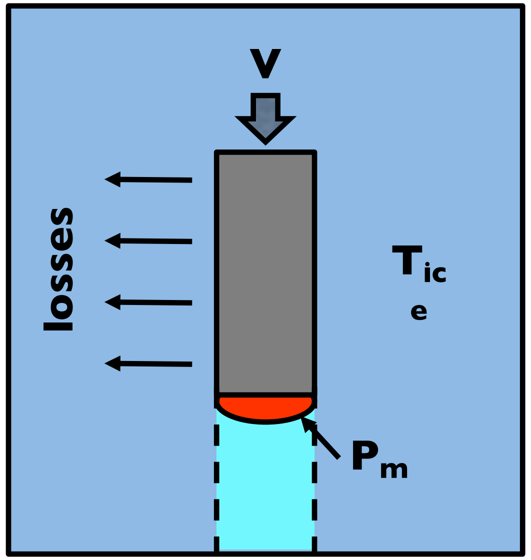
infering on the (induced or natural) biogeochemical activity to control contamination or to interpret measurements

infering on the ambient ice structure from on-line sensor data (porosity, water/salt content, dust layers, etc.)

Modeling melting robots – the 0D engineering approach (vs actual design)

0D Engineering model:

Aamot, CRREL-TR-194. 1967



energy balance:

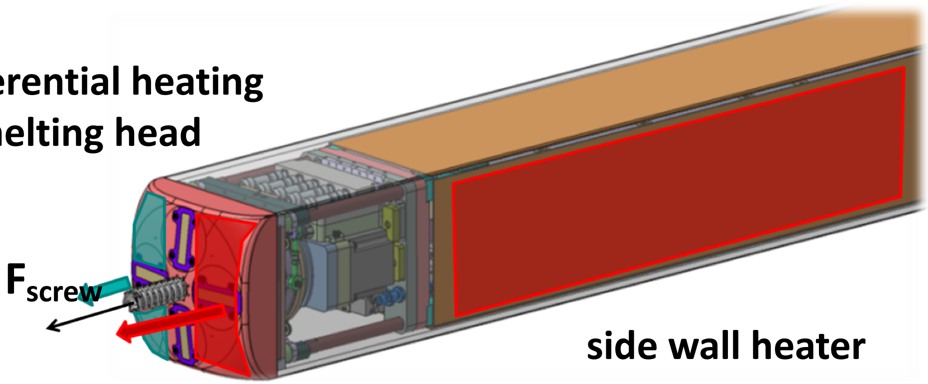
melting velocity = input power / energy needed

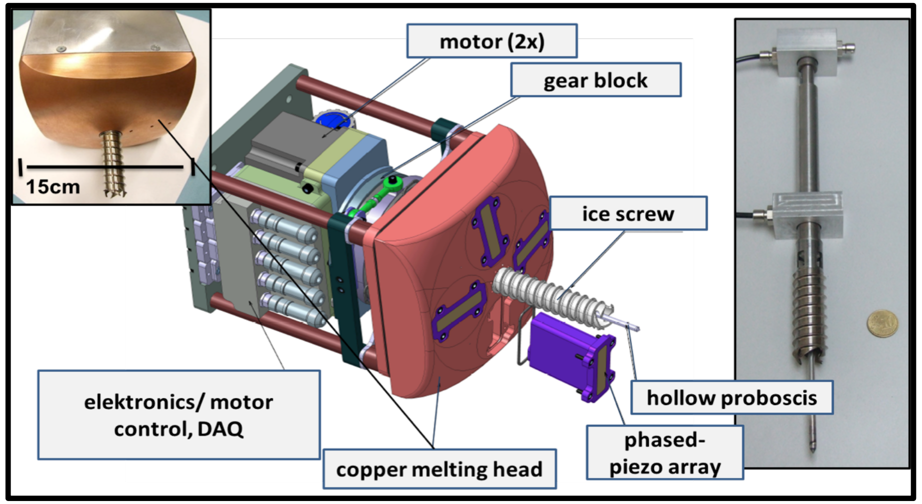
$$V = \frac{P_m}{A\rho\,(h_m + c_p(T_m - T_{\text{ice}}))}$$

V: melting velocity
P_m: input power
A: crosssection of the probe
ρ: density of the ice

c_p: specific heat capacity of the ice
T_m: melting temperature
T_{ice}: ice temperature
h_m : melting enthalpy of ice

The robots's 2015 design:





Contact force? Curve melting? Transient effects? Low gravity/temperature/pressure conditions?

Aachen Institute for Advanced Study in Computational Engineering Science
RWTH AACHEN UNIVERSITY

Modeling melting robots – the high-fidelity (but currently not feasible) 4D advanced approach

Robot motion (concentrated)

The current state of the probe is given by its center-of-mass and its attitude:

$$\xi(t) := \begin{bmatrix} X(t) \\ Q(t) \end{bmatrix}$$

First derivative yields translational and angular velocity:

$$\frac{d}{dt}\xi(t) = \begin{bmatrix} V(t) \\ \omega(t) \end{bmatrix}$$

The Euler-Newton equations allow to determine the trajectory based on applied forces:

$$\begin{aligned} m\frac{d}{dt}V(t) &= F(t, u(t)) \\ \mathbf{I}\frac{d}{dt}\omega(t) &= T(t, u(t)) - \omega \times \mathbf{I}\omega \end{aligned}$$

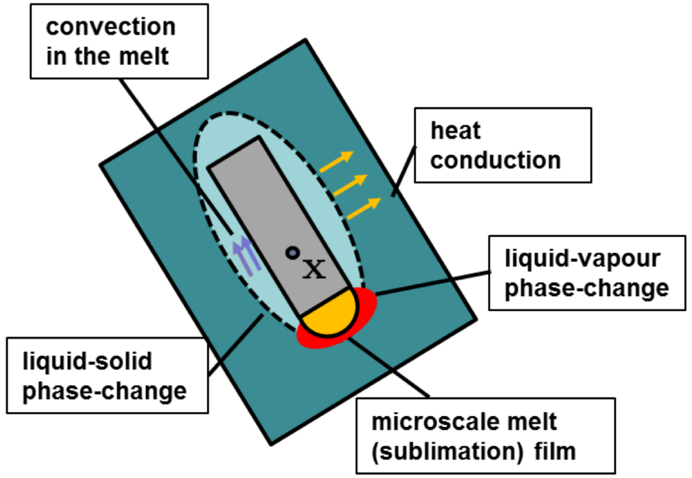
The forces depend on the position of the liquid-solid interface, hence the ambient state *u*.

two-way coupling

Cryoenvironment (distributed)

The current ambient state of the ambient is given by temperature, velocity and pressure:

$$u(t, x) := \begin{bmatrix} T(t, x) \\ v(t, x) \\ p(t, x) \end{bmatrix}$$

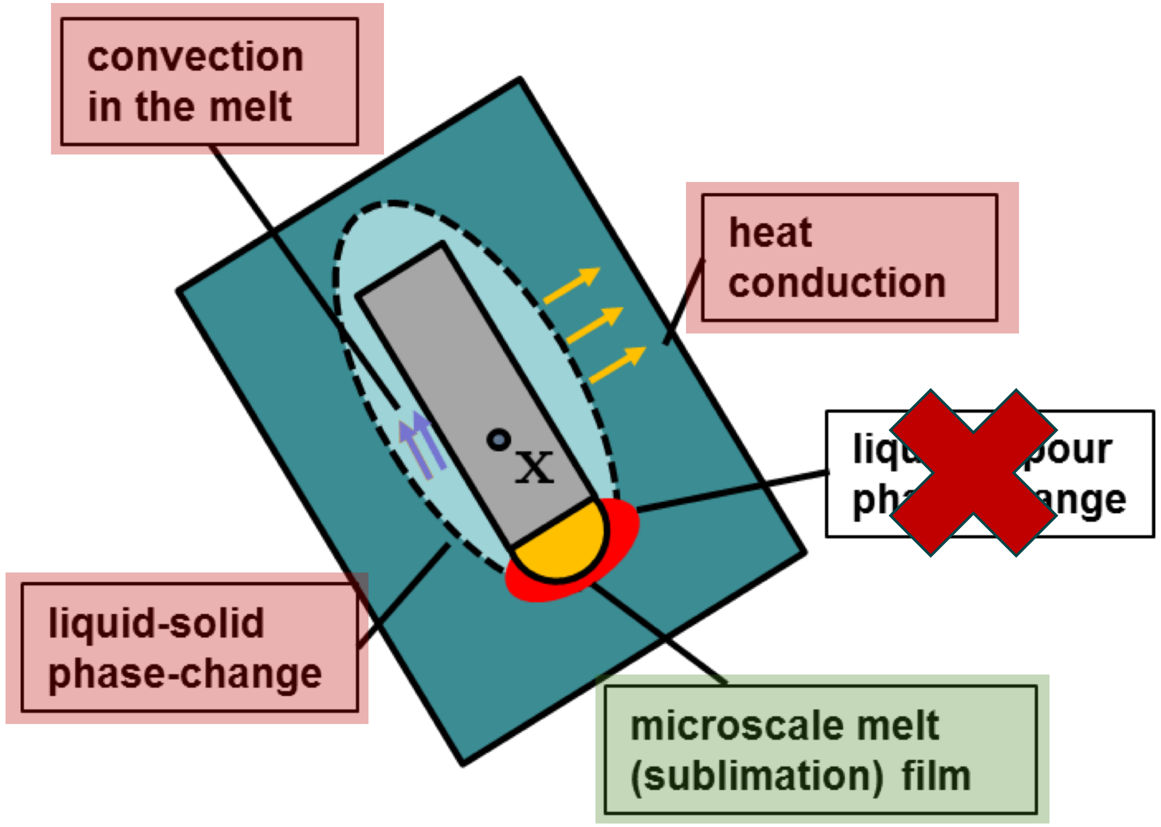


It is subject to a PDE operator

$$\frac{\partial}{\partial t}u(t, x) = \mathcal{L}\left(u(t, x), \xi(t), \frac{d}{dt}\xi(t)\right)$$

that depends on position and attitude of the probe, as well as ist translational and rotational velocity.

Divide and conquer strategy



Scales of interest
(based on a 25kg robot moving at 1m/h):

- melt film thickness ~ 10^{-6} m
- melt film time scale ~ 10^{-3} s
- heat conduction time scale ~ 10^{2} s
- heat conduction diffusion scale ~ 10^{-2} m
- mission length scale ~ 1000 m
- mission time scale ~ 40 d

Strategy:
Decouple ‚macro-scale' from ‚micro-scale' processes:

Convection-coupled solid-liquid phase-change [1]

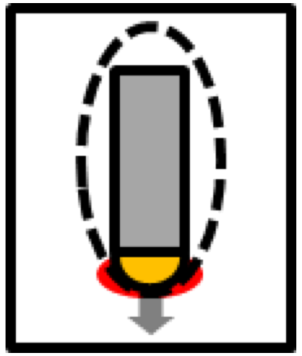
Close contact melting

Multi-scale coupled approach

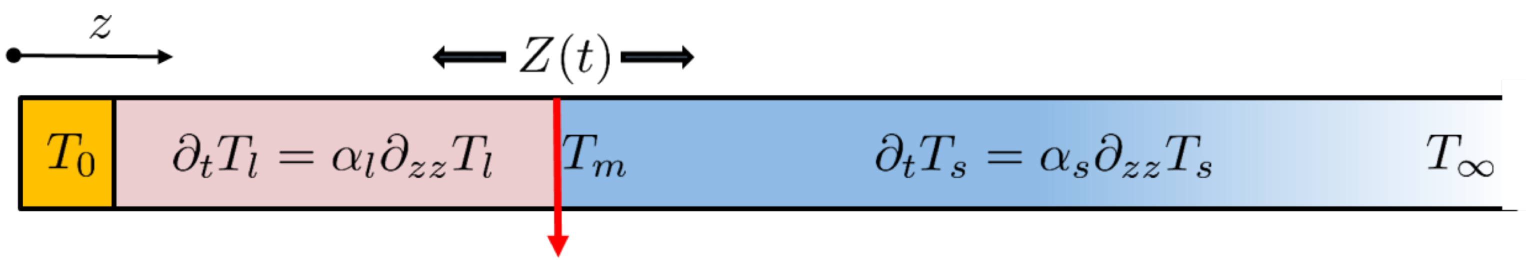
[1] Zimmerman, Kowalski SIAM CSE, 2019; Zimmerman, Kowalski 2018; Schüller, Berkels, Kowalski 2018; https://github.com/geo-fluid-dynamics/fempy

Aachen Institute for Advanced Study in Computational Engineering Science RWTH AACHEN UNIVERSITY

Modeling melting robots - processes in the microscale melt film

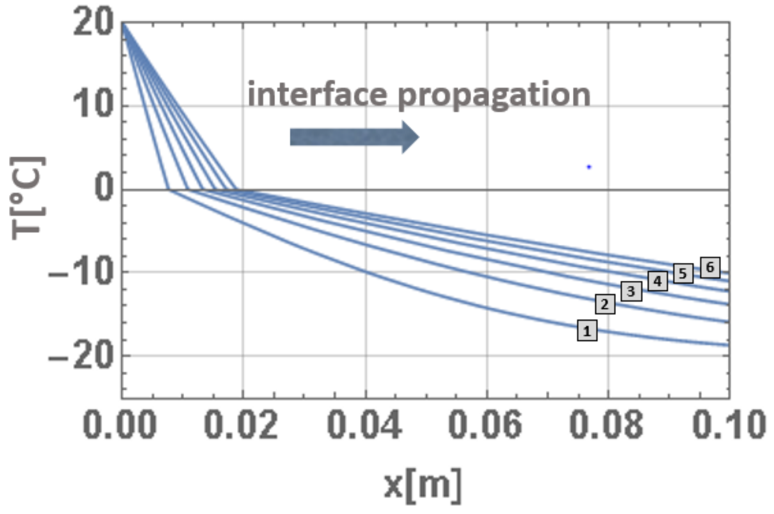


Can we leverage our knowledge on the Stefan problem?

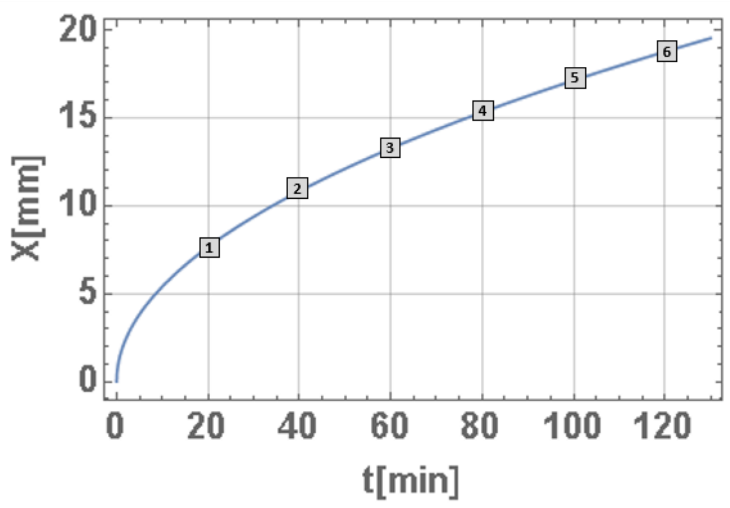


Stefan condition: $\rho L \partial_t Z = -[[q]]$

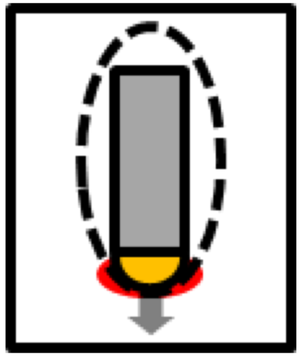
Temperature profile:



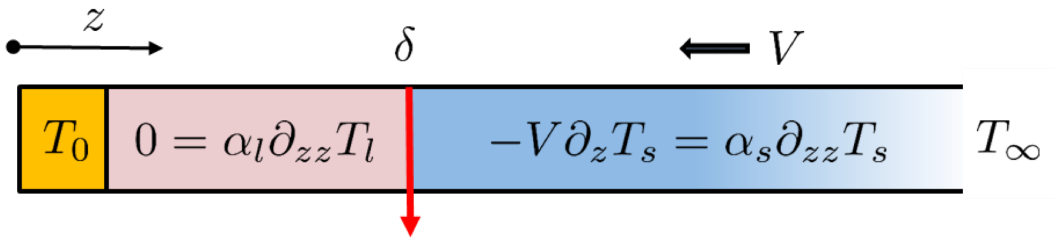
Position of the interface:



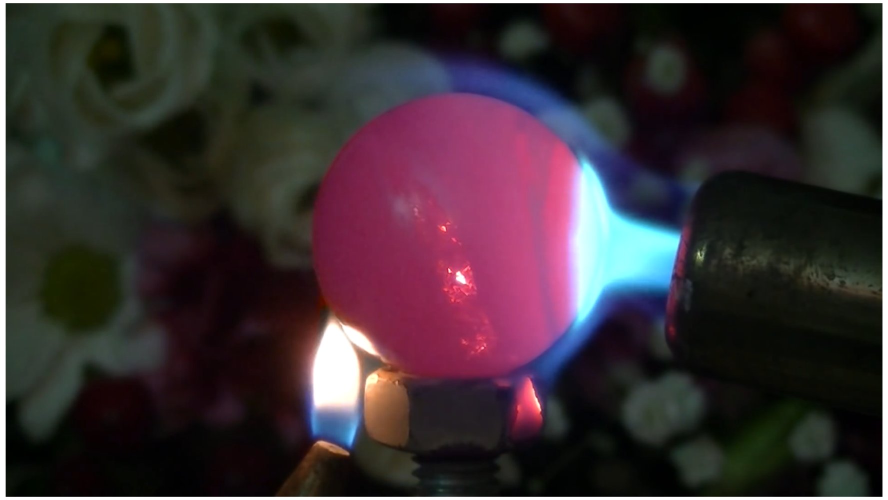
Modeling melting robots - processes in the microscale melt film



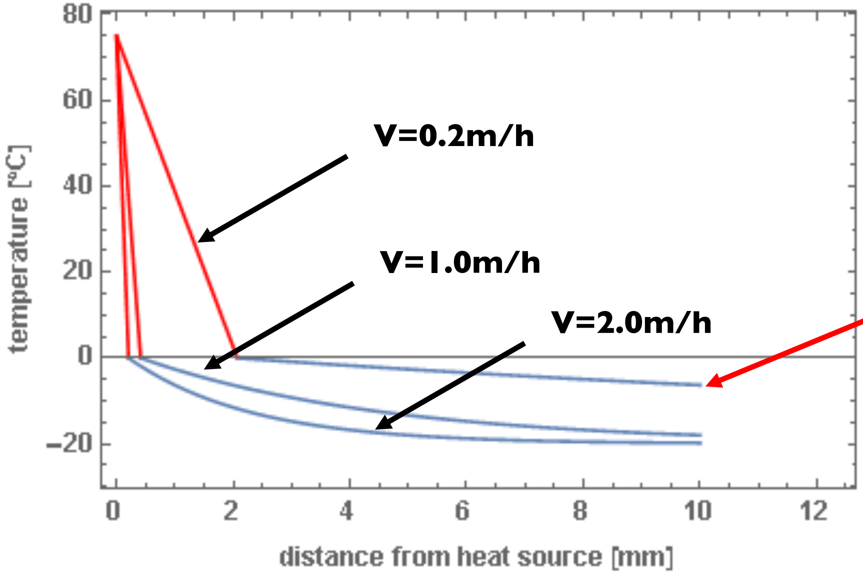
Can we pimp the Stefan problem?



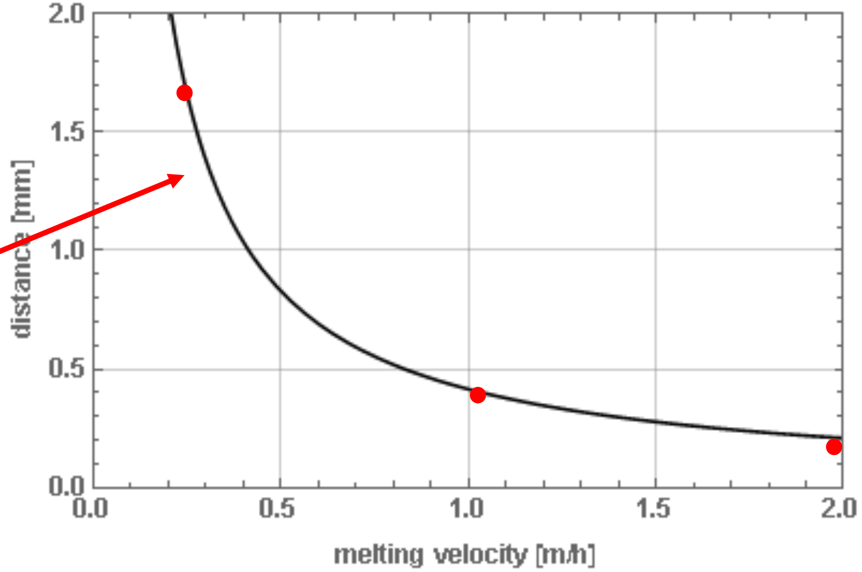
Stefan condition



Temperature profile:



Melt film thickness vs velocity:

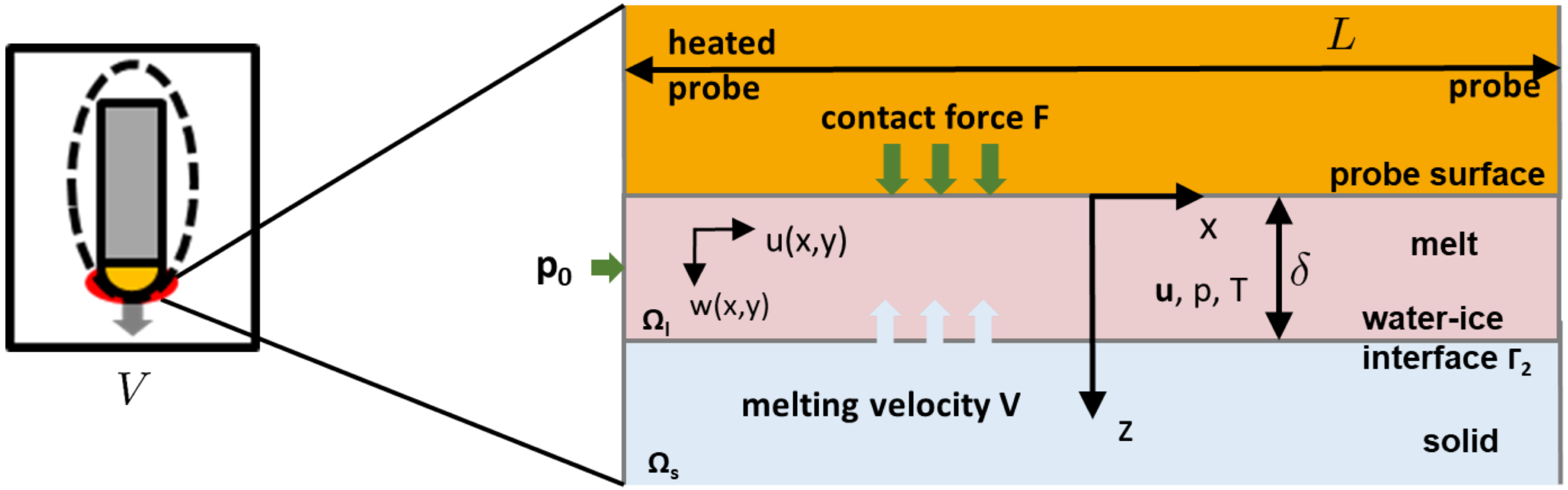


Force balance is missing:

$$\oint_{\text{surface}} p \, d\sigma = F$$

Need to resolve flow in the melt film!

Modeling melting robots - processes in the microscale melt film



Water-ice interface conditions:
- no-slip
- inflow according to melting rate
- melting temperature
- Stefan condition

Heat source surface:
- no-slip
- no inflow
- temperature or heat flux

Ω_l: Mass, momentum and energy balance:

$$\begin{aligned} \nabla \cdot \mathbf{u} &= 0 \\ \partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} &= -\frac{1}{\rho}\nabla p + \nu \triangle \mathbf{u} \\ \partial_t T_l + (\mathbf{u} \cdot \nabla) T_l &= \alpha_l \triangle T_l \end{aligned}$$

Unknowns: u, T, p, V, δ

Ω_s: Heat equation in the solid ice

$$-\mathbf{V}\partial_z T_s = \alpha_s \partial_{zz} T_s$$

Model closure:
- Stefan condition at the interface
- Newton‘s third law

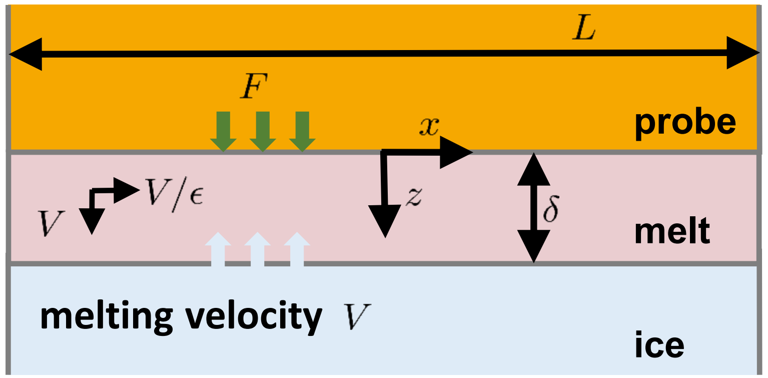
Aachen Institute for Advanced Study in Computational Engineering Science
RWTH AACHEN UNIVERSITY

Semi-analytical model solution based on dimensional reduction

Scaling:

$$x = L\tilde{x} \qquad u = V/\epsilon\tilde{u} \qquad t = L/V\tilde{t}$$

$$z = \delta\tilde{z} \qquad w = V\tilde{w} \qquad p = \frac{\mu V/\epsilon}{\delta}\tilde{p}$$



Dimensionless groups:

thin film parameter: $\epsilon = \delta/L$ $<<$ Reynolds number: $Re = VL/\nu$ $<$ Peclet number: $Pe = VL/\alpha$

Dimensionless system:

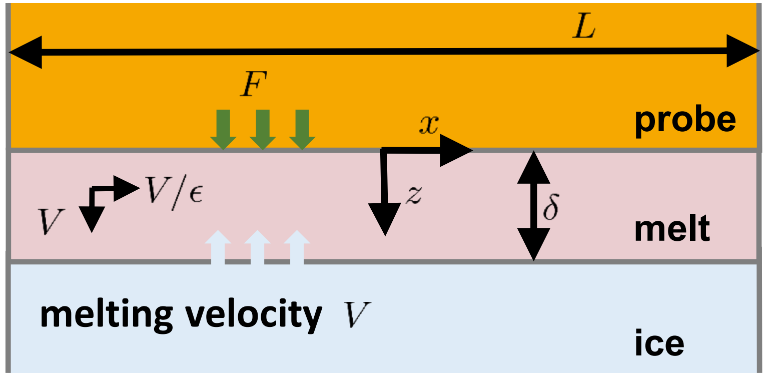
$$\begin{aligned}
\partial_x u + \partial_z w &= 0 \\
\epsilon Re(\epsilon\partial_t u + u\partial_x u + w\partial_z u) &= -\epsilon d_x p + \epsilon^2\partial_x^2 u + \partial_z^2 u \\
\epsilon^2 Re(\epsilon\partial_t w + w\partial_x u + w\partial_z w) &= -d_z p + \epsilon^2\partial_x^2 w + \partial_z^2 w \\
\epsilon Pe(\epsilon\partial_t T + u\partial_x T + w\partial_z T) &= \epsilon^2\partial_x^2 T + \partial_z^2 T
\end{aligned}$$

Aachen Institute for Advanced Study in Computational Engineering Science | RWTH AACHEN UNIVERSITY

Semi-analytical model solution based on dimensional reduction

Scaling:

$$x = L\tilde{x} \qquad u = V/\epsilon\tilde{u} \qquad t = L/V\tilde{t}$$
$$z = \delta\tilde{z} \qquad w = V\tilde{w} \qquad p = \frac{\mu V/\epsilon}{\delta}\tilde{p}$$



Dimensionless groups:

thin film parameter: $\epsilon = \delta/L$ $<<$ Reynolds number: $Re = VL/\nu$ $<$ Peclet number: $Pe = VL/\alpha$

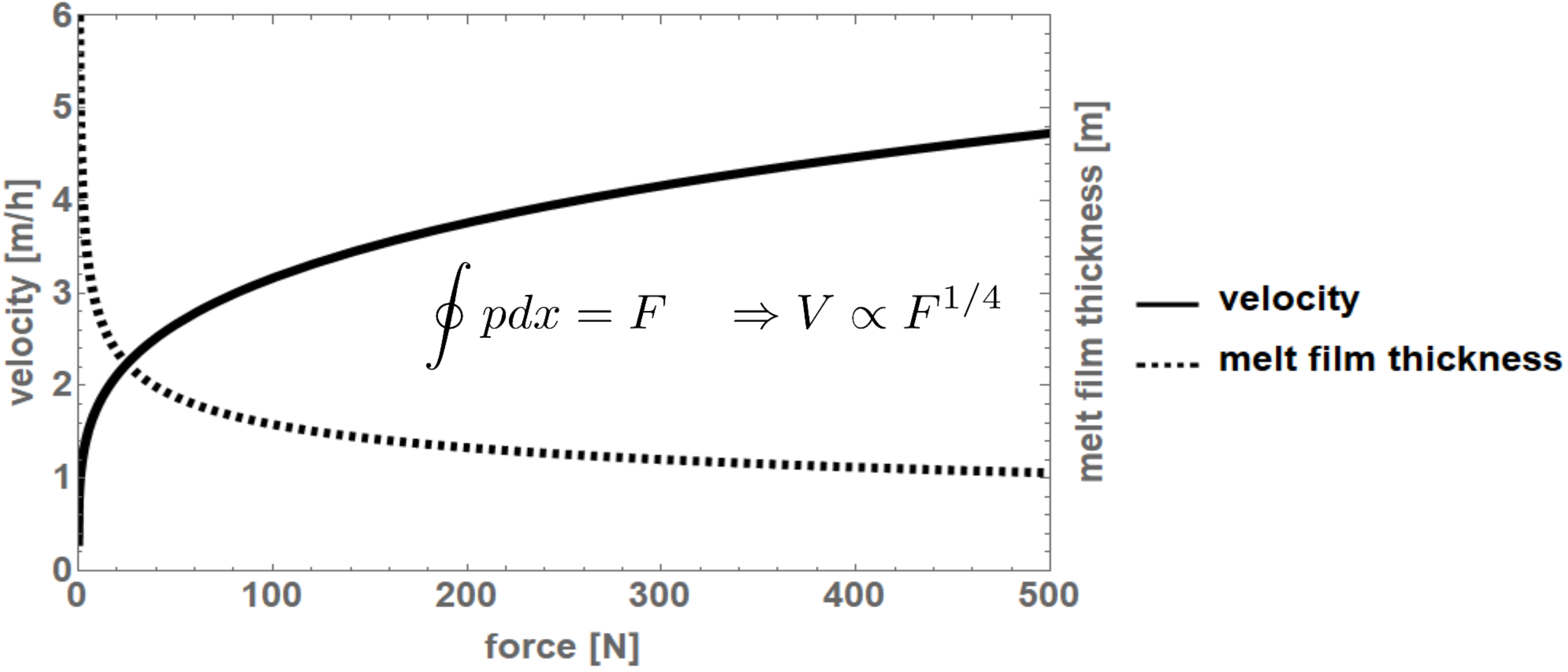
Dimensionless system:

$$\begin{aligned} \partial_x u + \partial_z w &= 0 \\ 0 &= -\epsilon d_x p + \partial_z^2 u \\ 0 &= -d_z p \\ \epsilon Pe(u\partial_x T + w\partial_z T) &= \partial_z^2 T \end{aligned}$$

Dimensional reduction gives rise to lubrication theory coupled to a Stefan Problem!

> Close Contact Melting Theory (More general than melting robots!!!)

Semi-analytical model solution based on dimensional reduction

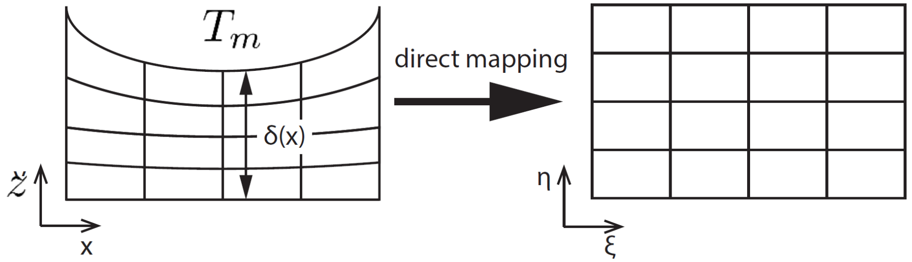


Mixed-dimensional computational model for close-contact processes

Energy balance:

$$\epsilon Pe(u\partial_x T + w\partial_z T) = \partial_z^2 T$$

Transformation:



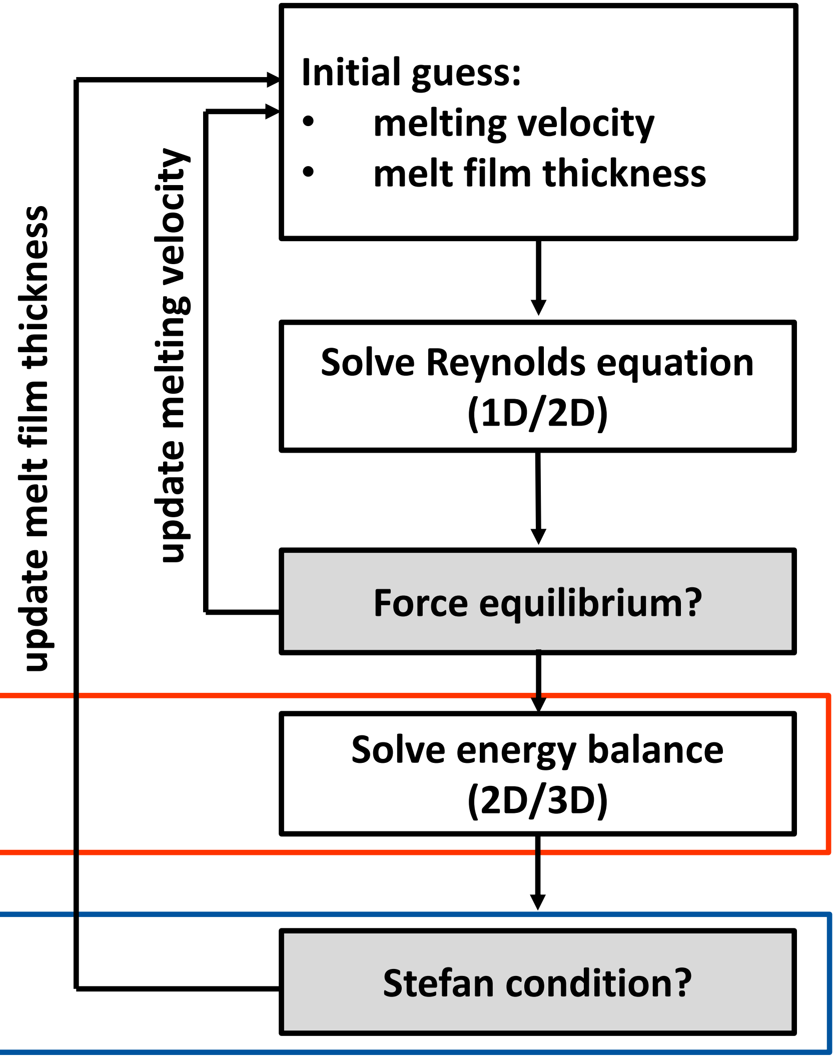
$$\xi = x \qquad \eta = z/\delta(x)$$

$$\begin{aligned} u\partial_x T + w\partial_z T &= u\partial_\xi T + u\partial_\eta T \tfrac{1}{\delta}\left(w - u\eta\partial_\xi \delta\right) \\ \partial_{zz} T &= \partial_{\xi\xi} T \left(\frac{d\xi}{dz}\right)^2 + 2\partial_{\xi\eta} T \left(\frac{d\xi}{dz}\frac{d\eta}{dz}\right) + \\ & \quad \partial_{\eta\eta} T \left(\frac{d\eta}{dz}\right)^2 = \tfrac{1}{\delta^2}\partial_{\eta\eta} T \end{aligned}$$

Solved using FDM

Stefan condition (for each x):

$$\partial_z T|_{z=\delta(x)} = \frac{\rho v(x, \delta(x))}{\kappa_l}\left(h_m + c_p(T_m - T_s)\right)$$

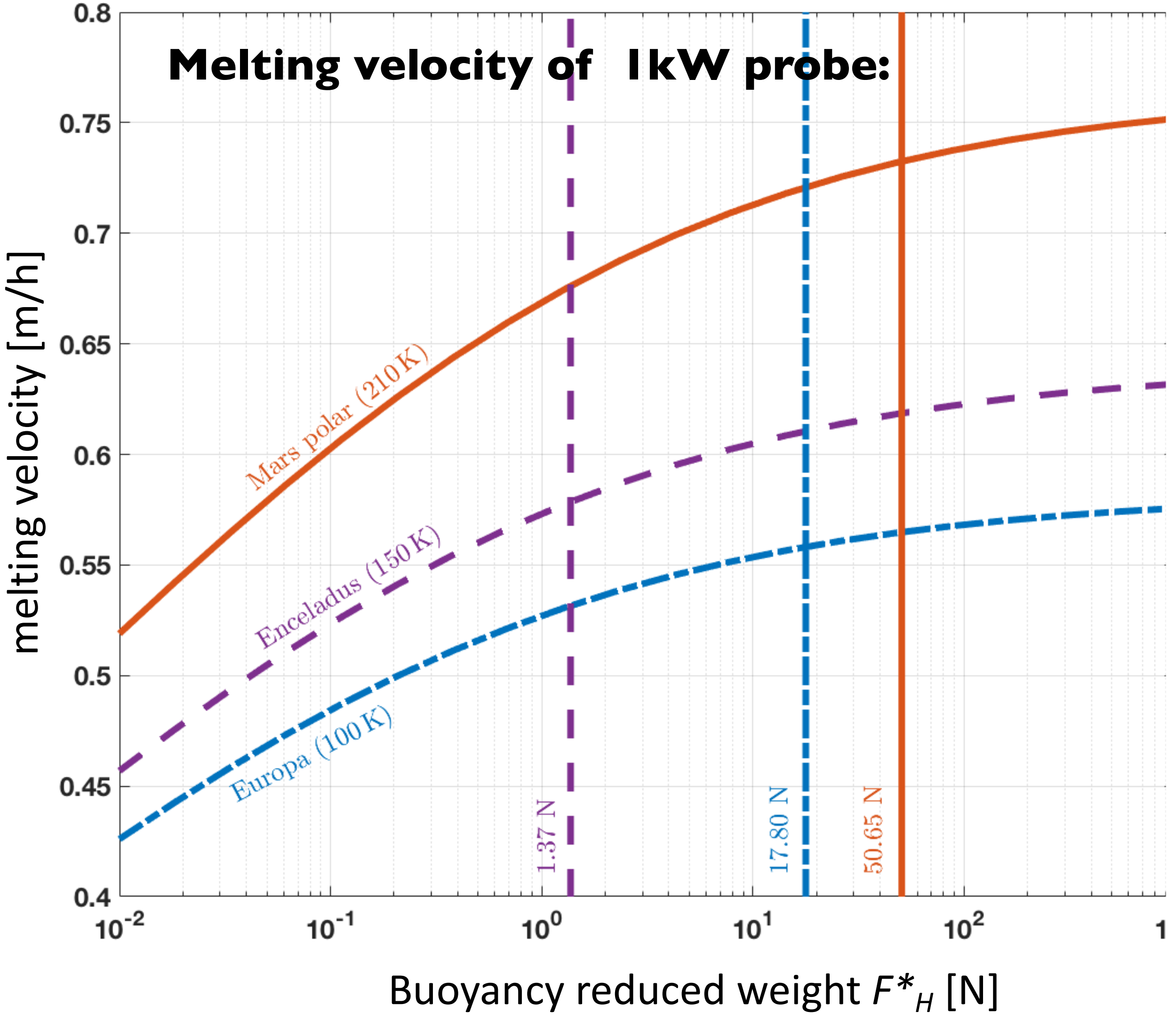


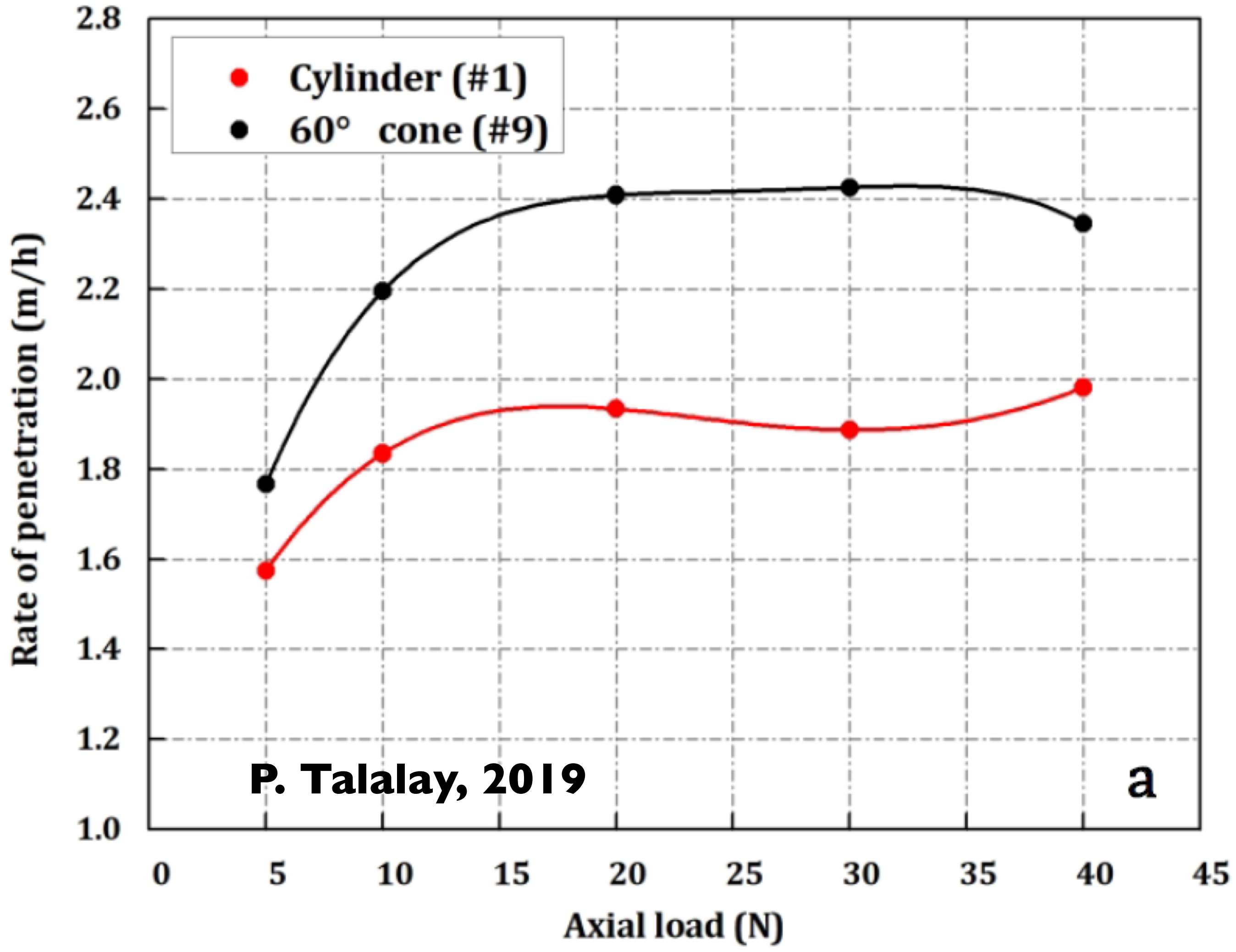
Schüller, Kowalski, 2017, Int Heat Mass Trans,115, 1276-1287.

- **temperature field (2D/3D):** finite differences and upwind discretization
- **pressure equation (1D/2D):** finite differences
- **velocity update:** based on bisection
- **delta update:** based on deviation between approximated normal heat flux at the interface and physically consistent heat flux (Stefan condition) + relaxation

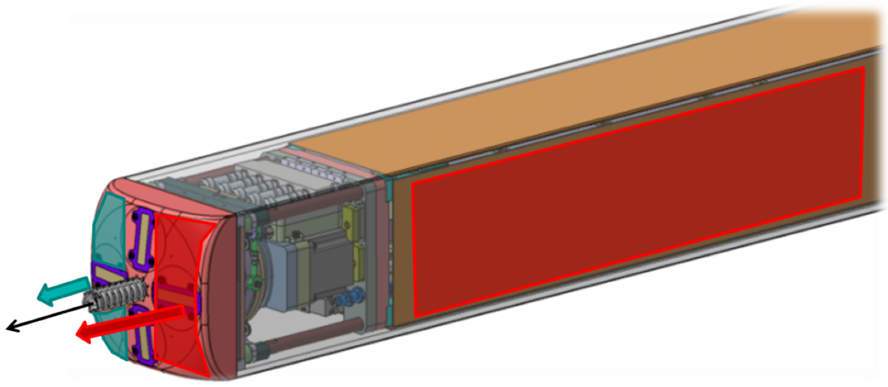
Melting velocity as a function of contact force

Schüller, Kowalski, 2019, Icarus 317, 92, 1-9.





Close Contact melting – rotational melting modes

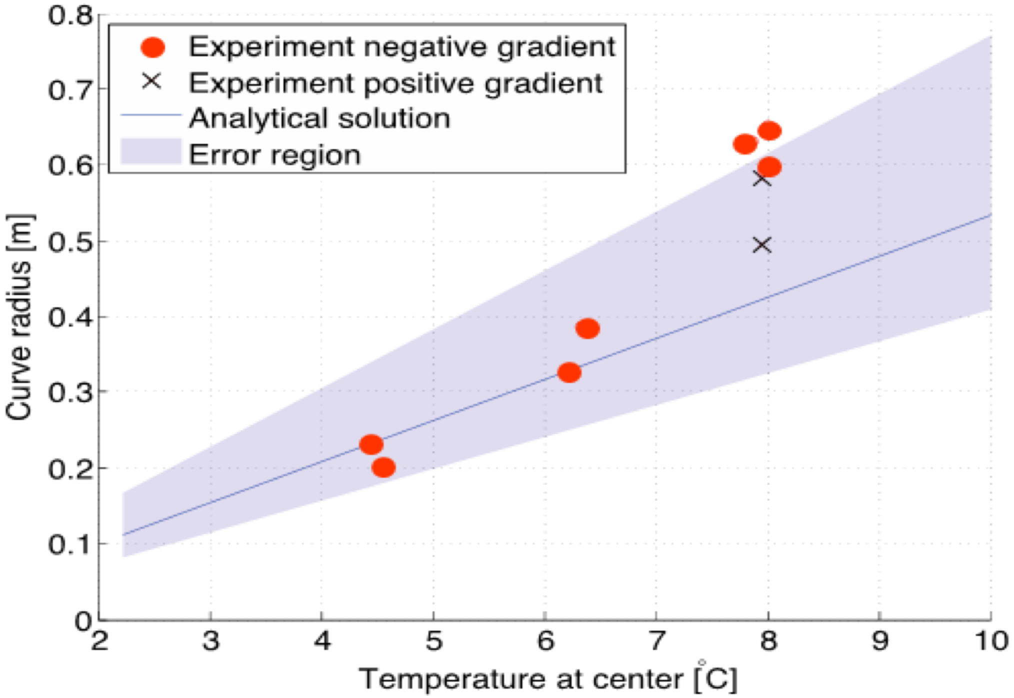


Idea:
Parametrization rotating motion

$$V(x) = V_0 \left(1 - \frac{x}{r_c}\right)$$

Additional closure:
Vanishing torque $\oint xpdx = 0$

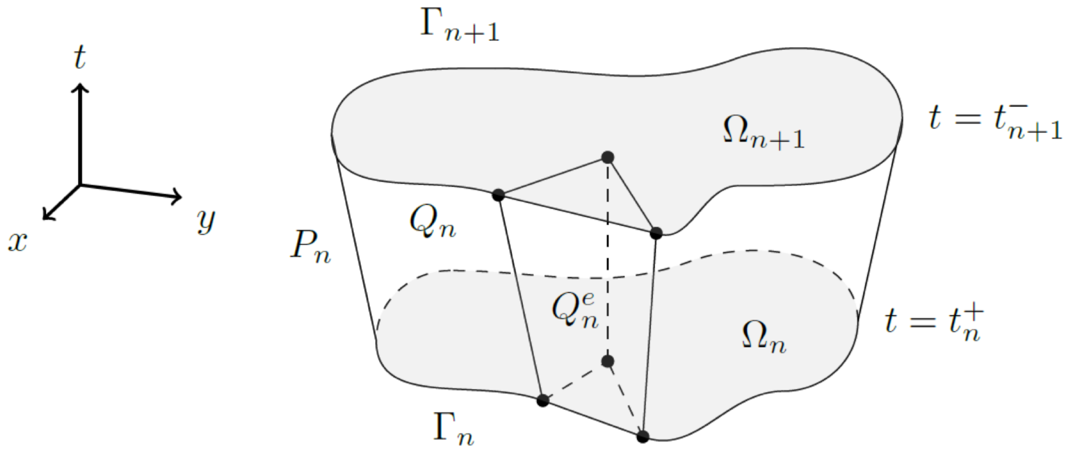
Experiments with constant temperature gradient:

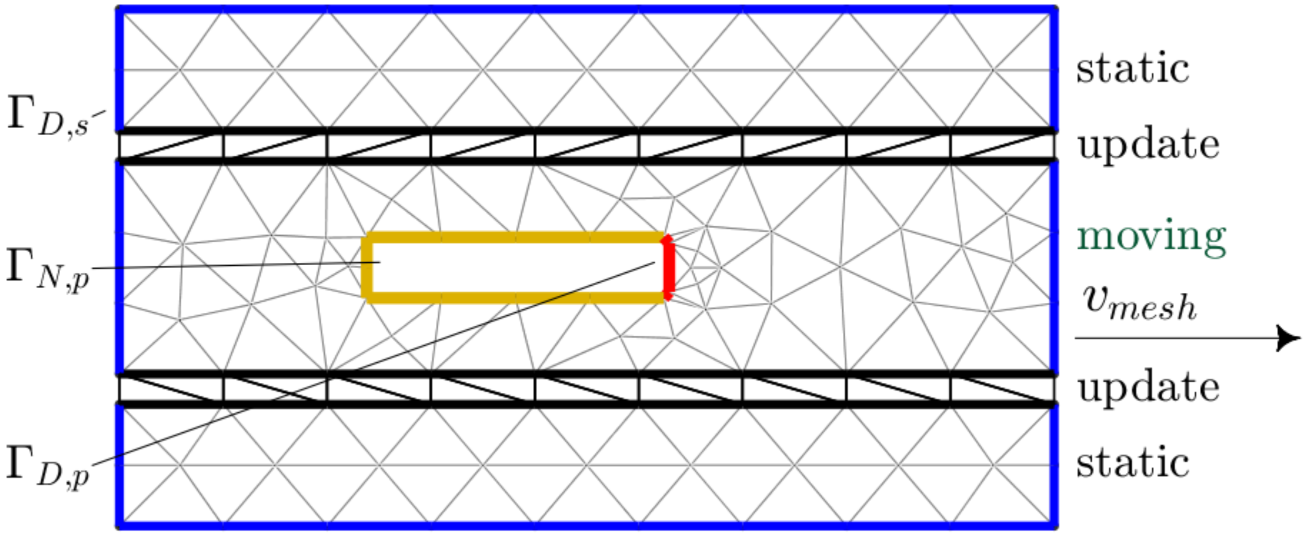


Schüller, Kowalski, 2016, Int Heat Mass Trans, 92, 884-892.

Multi-scale coupling – idea and approach

Utilize space-time FE implementation:



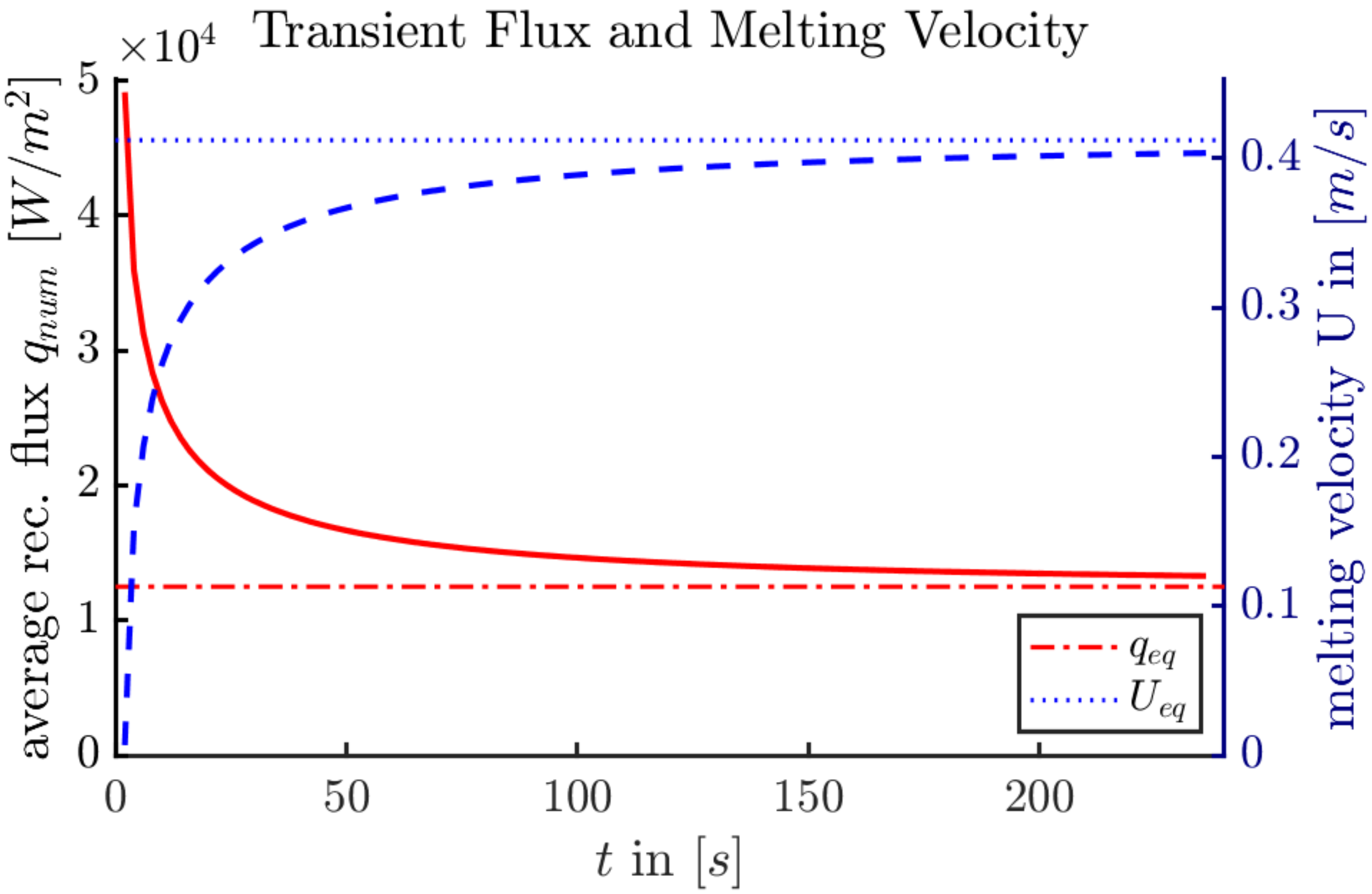


- in-house code at CATS, RWTH (Behr/Elgeti)
- parallelized on RWTH compute cluster and FZ Jülich JUREKA
- includes hybridized dynamic and static mesh capabilities: F. Key et al. (2018), Computers and Fluids 172, 352-361.

Coupling strategy:

- Macro2Micro: use ‚real‘ heat flux in the CCM model closure
- Micro2Macro: update melting velocity (dynamic mesh velocity)

Multi-scale coupling – preliminary results – response to a power ramp-up

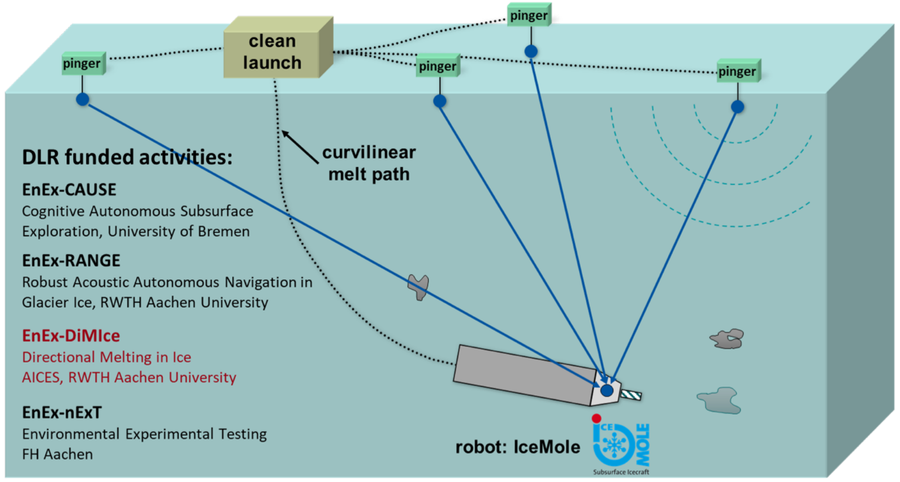


Conclusions and Outlook

Conclusions

- **Developed an easy to implement dimensionally reduced model for general CCM situations**
- **Proposed a coupling strategy that enables us to compute transient CCM processes**
- **convergence and plausibility checks are promising**
- **Model can be used for design studies extrapolation to extreme environments**

Next steps



predicting the dynamics of the melting robot given its controls (heating power, ice screw induced contact force)

infering on the ambient ice structure from on-line sensor data (porosity, water/salt content, dust layers, etc.)

Aachen Institute for Advanced Study in Computational Engineering Science
RWTH AACHEN UNIVERSITY

Thank you

Aachen Institute for Advanced Study in Computational Engineering Science
RWTH AACHEN UNIVERSITY