Effects of Boundaries on Pulse Wave Simulations

A distributed network approach for estimating parameters of a lumped outflow boundary condition

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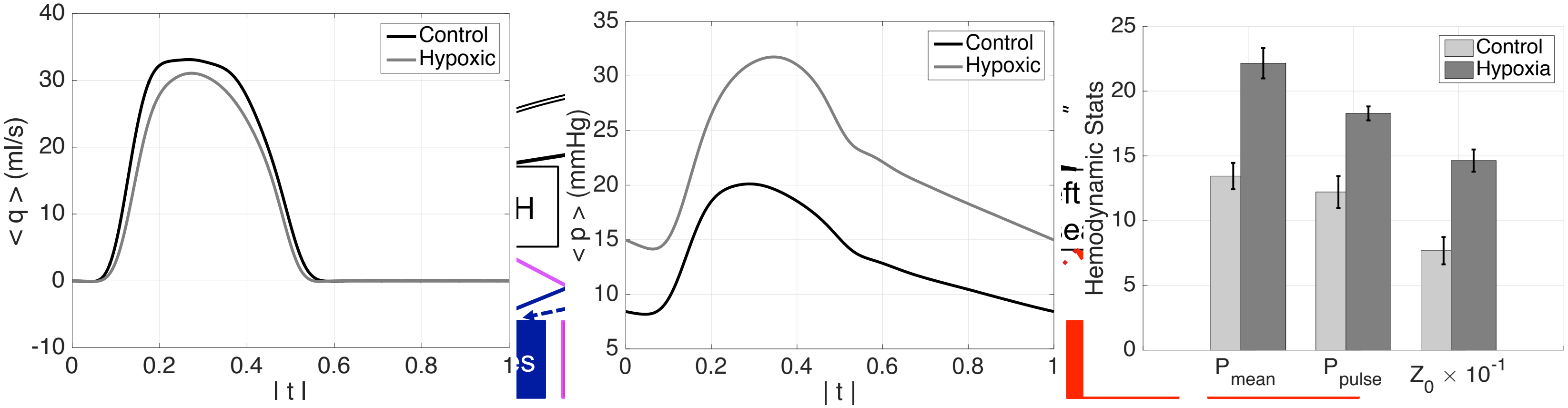
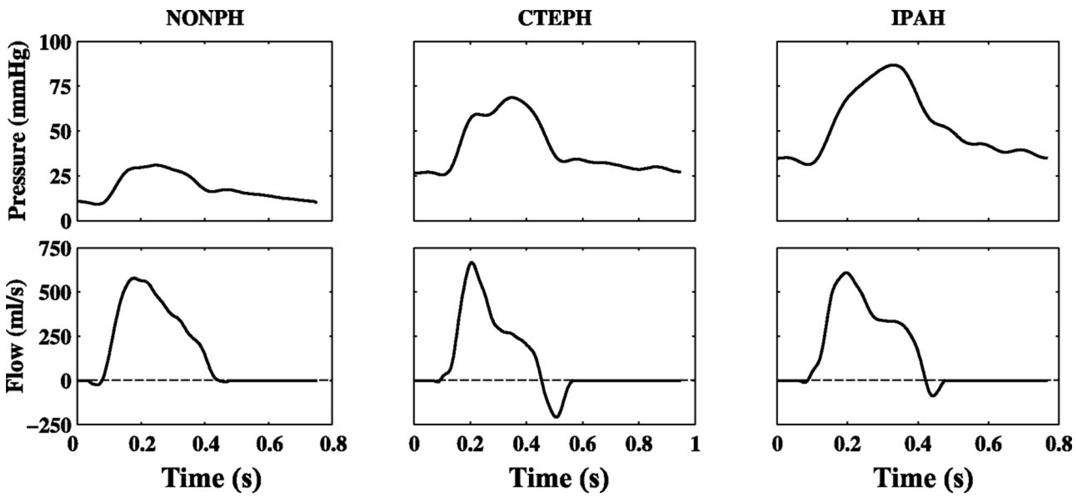
Background

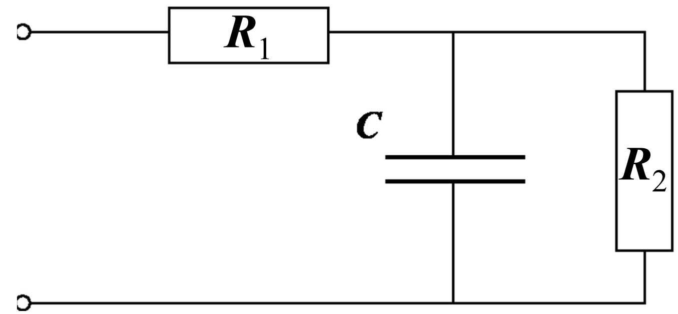


Flow and pressure waveforms in the main pulmonary artery of a mice during control and hypoxic condition. (Data courtesy Chesler Labs, University of Wisconsin.)

- Using modeling to understand hemodynamic changes due to altered conditions.
- Model adequacy for capturing the influence of affected parts of the vasculature.
- Suitable choice: 0D or 1D or multiscale?
- Most models of pulse wave propagation are multiscale 1D-0D models.

Background



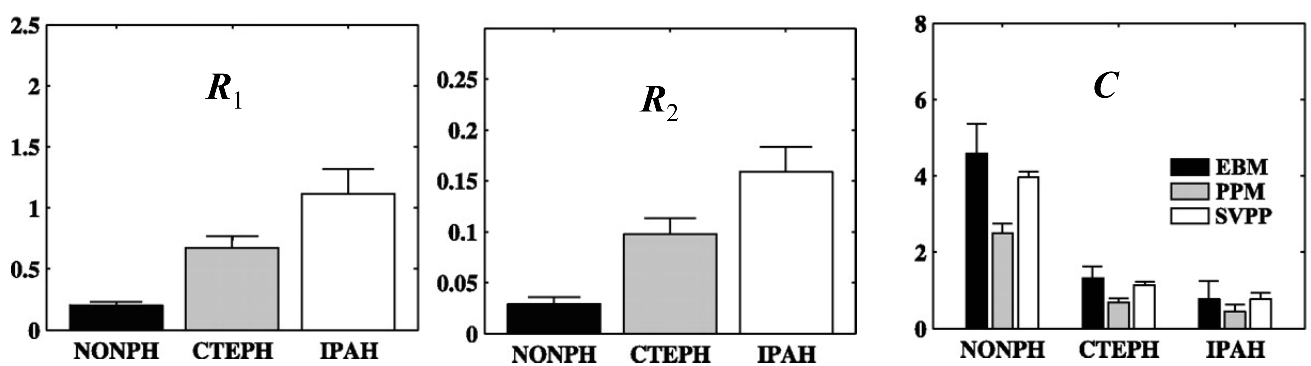


Lankhaar et al. (2006) "Quantification of right ventricular afterload in patients with and without pulmonary hypertension. Am J Physiol Heart Circ Physiol 291:H1731–H1737

R_1: Proximal arterial resistance

R_2: Resistance in distal arterioles

C : Total arterial compliance



Snapshot

- We developed a generic 31 arterial model with lumped and distributed boundary conditions to explore these questions.
- The model mimics pulmonary arterial network with tapered main branches.
- Based on our results we propose a method to estimate parameters of a 0D outflow model from a distributed model

Typical 1D modeling approach

Governing Equations

Continuity equation

$$\frac{\partial q}{\partial x}+\frac{\partial A}{\partial t}=0, \quad \text{where} \quad q=2\pi\int_0^R u_x r\,dr$$

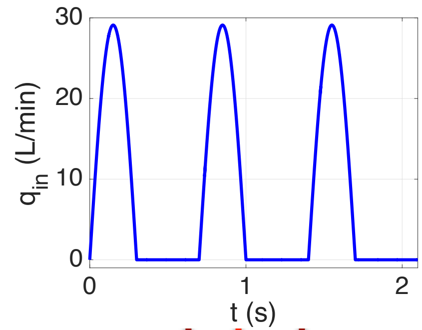
Axial momentum equation

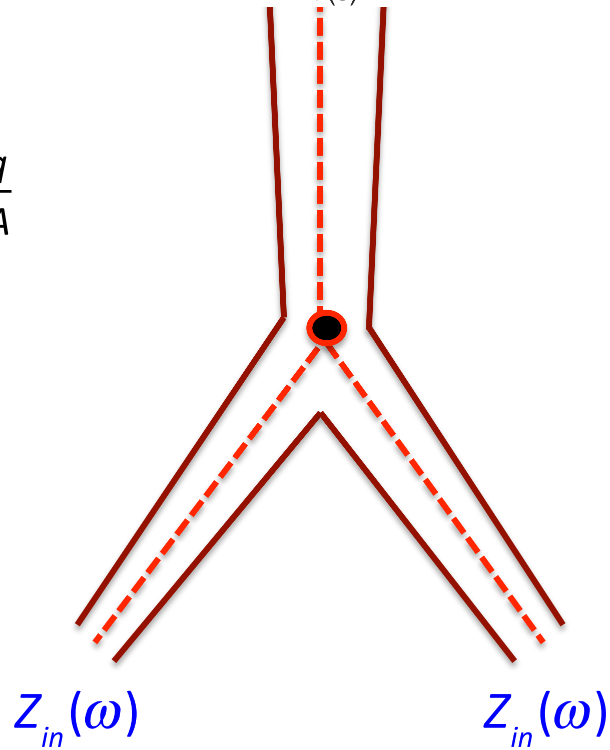
$$\frac{\partial q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{q^2}{A}\right)+\frac{A}{\rho}\frac{\partial p}{\partial x}=-\frac{2\pi\nu R}{\delta}\frac{q}{A}$$

State equation

$$p\big(A(x,t)\big)=\frac{4}{3}\frac{Eh}{r_0}\left(1-\sqrt{\frac{A_0}{A}}\right)$$

$$Z(\omega)=\frac{P(\omega)}{Q(\omega)}=|Z|e^{i\theta}$$





Boundary Conditions

Periodic Inflow Condition

$$q_{in}(t)=\begin{cases}\hat{q}\sin\left(\pi t/\tau\right) & \text{if } t<\tau,\\ 0 & \text{otherwise}\end{cases}$$

Vascular wall model

$$p(x,t)=p\big(A(x,t)\big)$$

Bifurcation Condition

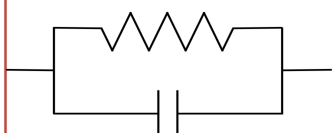
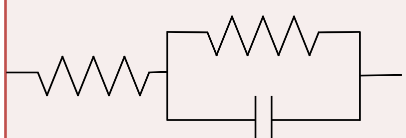
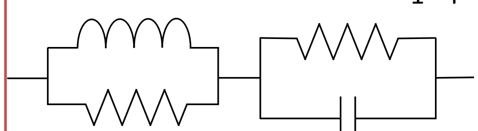
$$p_p(L,t)=p_{d_i}(0,t)$$

$$q_p(L,t)=\sum_{i=1}^{2}q_{d_i}(0,t)$$

Impedance Outflow Condition

$$p\left(L,t\right)=\frac{1}{T}\int_{-T/2}^{T/2}q\left(L,\tau\right)z_{in}\left(t-\tau\right)d\tau$$

0D, electrical analog model
(Non-physiological and ignore the wave propagation)

Model	Parameters	Pros	Cons	First Used
Pure Resistor 	R	Simplest model to use	In phase p and q, ignores vascular compliance	Olsen & Shapiro, 1967
2-Element Windkessel 	RC	Accounts for large arterial compliance and small arterioles resistance. Explains diastolic decay.	Inaccurate during systole. Inaccurate flow distribution.	Otto Frank, 1899
3-Element Windkessel 	$R_1C_TR_2$	Characteristic impedance from large arteries. Captures pressure & flow waveforms.	**Difficulty in parameter estimation**. Over/ underestimation of important parameters. Inaccurate flow distribution.	Westerhof et al. 1969
4-Element Windkessel 	$R_1C_TR_2L$	Slight improvement over e-element model	Not very useful for small vessels. Difficulty in parameter estimation	Stergiopulos et al. 1999

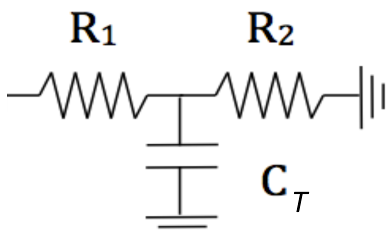
1D, structured tree models
(Based on morphometric relations and account for the wave propagation)

Model	Parameters	Pros.	Cons	First Used
Arterial structured tree	Next slide	Based physiological relations & fluid dynamical principals. Capture phase lag.	Parameter sensitivity, constant terminal impedance, periodicity, constant scaling.	Olufsen, 1998
Generalized arterial		Tiered scaling. Variable viscosity. Non-periodic flows	Parameter sensitivity, constant terminal impedance	Steele et al., 2003, Cousin & Gremaud 2013.
Two sided arterial-venous		Arterial-venous coupling. Different lengths and stiffness of venous. No terminal resistance required	Parameter sensitivity, periodicity, constant scaling. Requires venous outflow to left atrium	Qureshi et al. 2014 (**Co-authored with C. Peskin**)
arterial		Use admittance to eliminate terminal resistance condition.		Guan et al. 2016

Input impedance: $Z(\omega)$

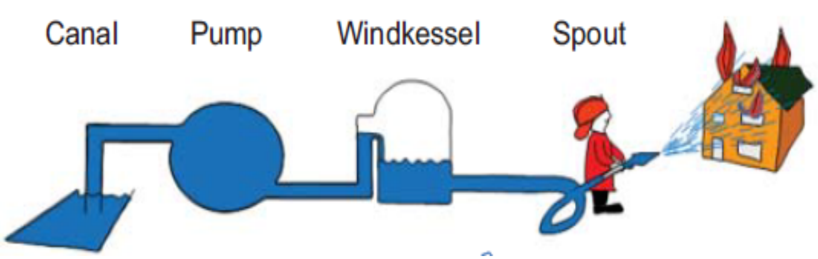
3 Element Windkessel Model

$$\phi_{WK} = \{R_1, R_2, C_T\}$$



$$\frac{dp}{dt} = R_1 \frac{dq}{dt} - \frac{p}{R_2 C_T} + \frac{R_1 + R_2}{R_2}\frac{q}{C_T}$$

$$Z_{WK}(\omega) \equiv \frac{P(\omega)}{Q(\omega)} = R_1 + \frac{R_2}{1 + i\omega R_2 C_T}$$



Binary Structured Tree Model

$$\phi_{ST} = \{r_{root}, L_{rr}, r_{\min}, \xi(r), \eta(r), \gamma(r); \mu(r), \rho\}$$

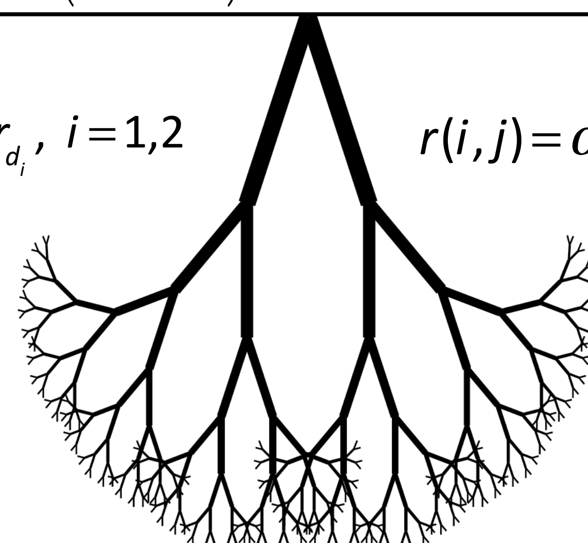
$$r_p^{\zeta} = r_{d_1}^{\zeta} + r_{d_2}^{\zeta}, \quad 2.33 \le \zeta \le 3, \qquad \gamma = \left(r_{d_1} / r_{d_2}\right)^2, \quad \gamma < 1.$$

$$\eta = \frac{r_{d_1}^2 + r_{d_2}^2}{r_p^2}, \quad \eta > 1.$$

$$\alpha = \left(1 + \gamma^{\xi/2}\right)^{-1/\xi} \quad \text{and} \quad \beta = \alpha\sqrt{\gamma}$$

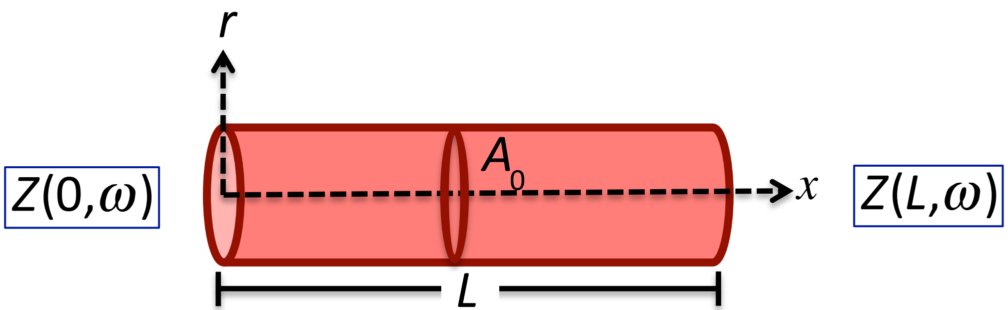
$$L_{d_i} = L_{rr} r_{d_i}, \quad i = 1,2$$

$$r(i,j) = \alpha^i \beta^j r_{root}$$



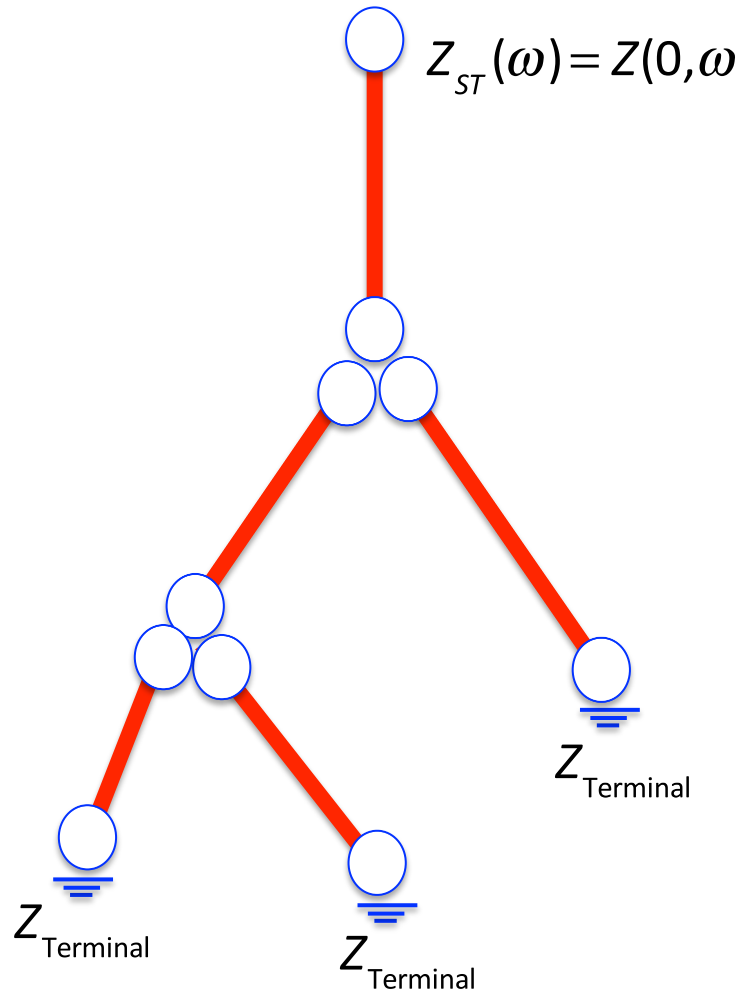
Input impedance: $Z_{ST}(\omega)$

$$Q(x,\omega)=a\cos(\omega x/c)+b\sin(\omega x/c)$$
$$P(x,\omega)=ig^{-1}\left(b\cos(\omega x/c)-a\sin(\omega x/c)\right)$$



$$Z(0,\omega)=f\left(Z(L,\omega)\right)$$

$$\frac{1}{Z_p}=\frac{1}{Z_{d_1}}+\frac{1}{Z_{d_2}}$$



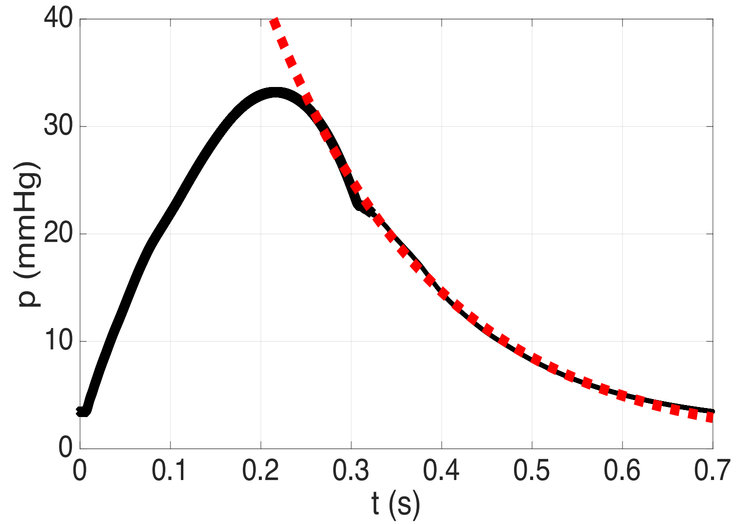
*See Olufsen et al. (2000) for detailed algorithm

Proposed approach

- Determine $\phi_{ST} : Z_{total}^{ST}(0) = \bar{p} / \bar{q}$

- Determine $R_{1j}^{mWK} = \frac{1}{N-k} \sum_{i=k}^{N} |Z_{ij}^{ST}|$

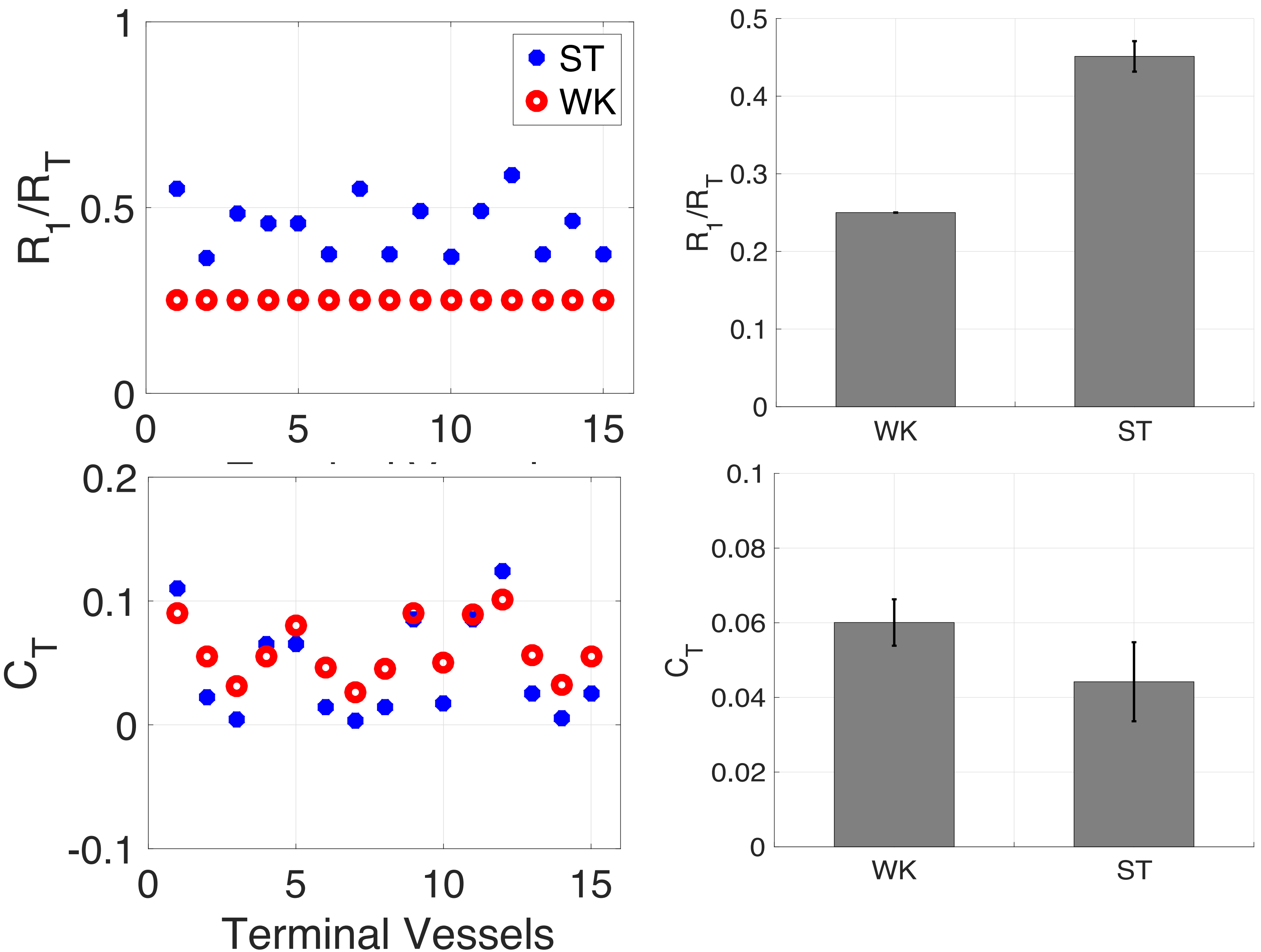
- Determine $R_{2j}^{mWK} = Z_{j}^{ST}(0) - R_{1j}^{mWK}$



- Determine $(R_T C_T)^{mWK} : P_d^{ST}(t) = P_{ds} e^{-(t - t_{ds})/R_T C_T}$

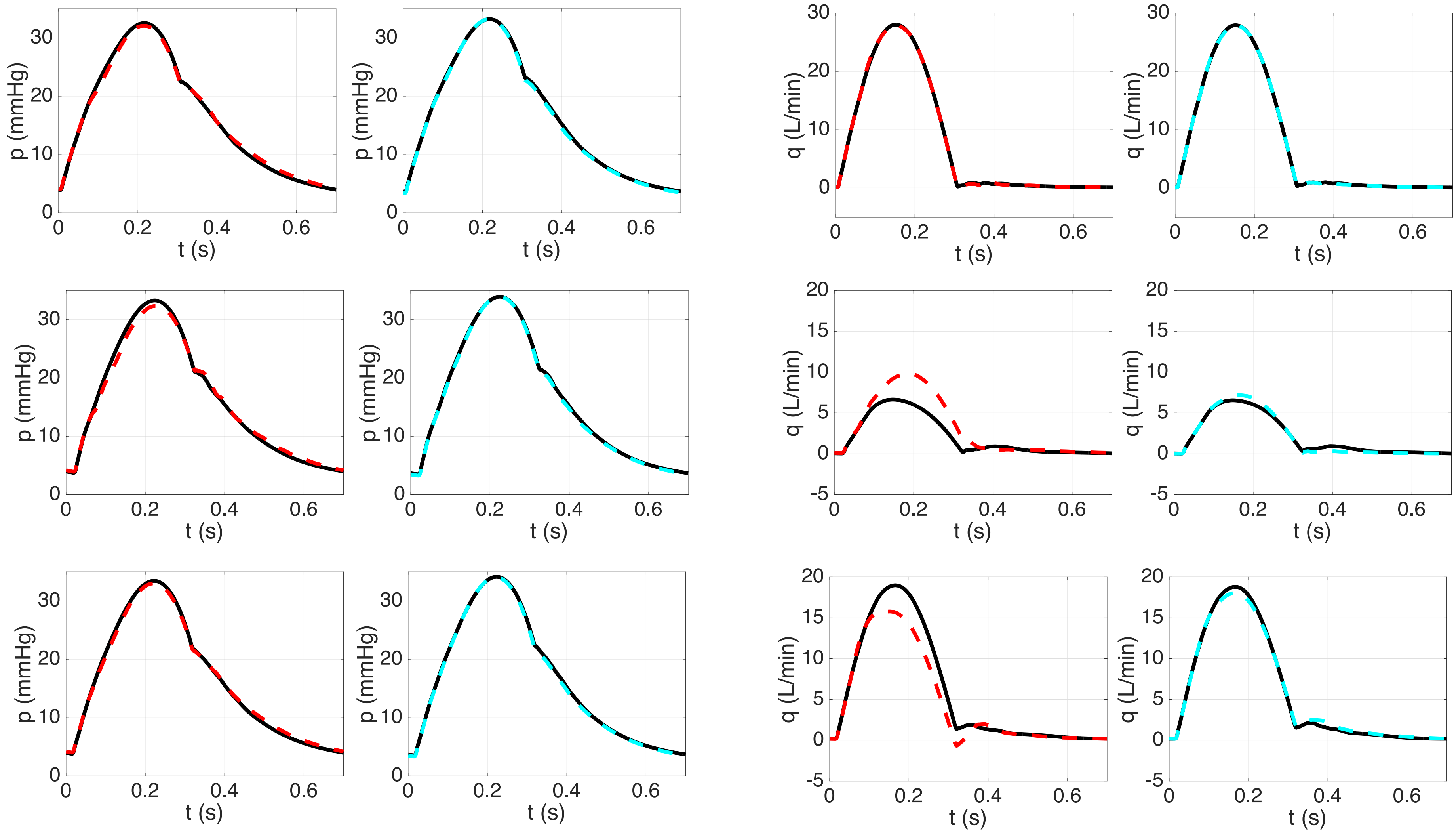
Parameter estimation for Windkessel model

Most common	$R_T = \bar{p}/\bar{q}$	$R_1/R_T = 0.2$ *McDonald & Attinger 1965	$R_T C_T = 1.34s$ *Alastruey 2008
Reymond et al. 2009	$R_T = \bar{p}/\bar{q}$	$R_1 = \frac{\rho \mathrm{PWV}}{A} = \frac{\rho}{A}\frac{13.3}{d^{0.3}}$ $0.05 \leq R_1/R_T \leq 0.4$	$C_{T_i} = C_T \frac{C_A^i}{\sum_i C_A^i}$ $R_T C_T \approx 0.28s$
C. Battista 2015	$R_T = \bar{p}/\bar{q}$	$0.01 \leq R_1/R_T \leq 0.32$	$0.97 \leq R_T C_T \leq 1.05$
Proposed Method	$R_{Tj} = Z_j^{ST}(0)$	$0.4 < R_1 / R_T < 0.6$	$R_T C_T = 1.34s$



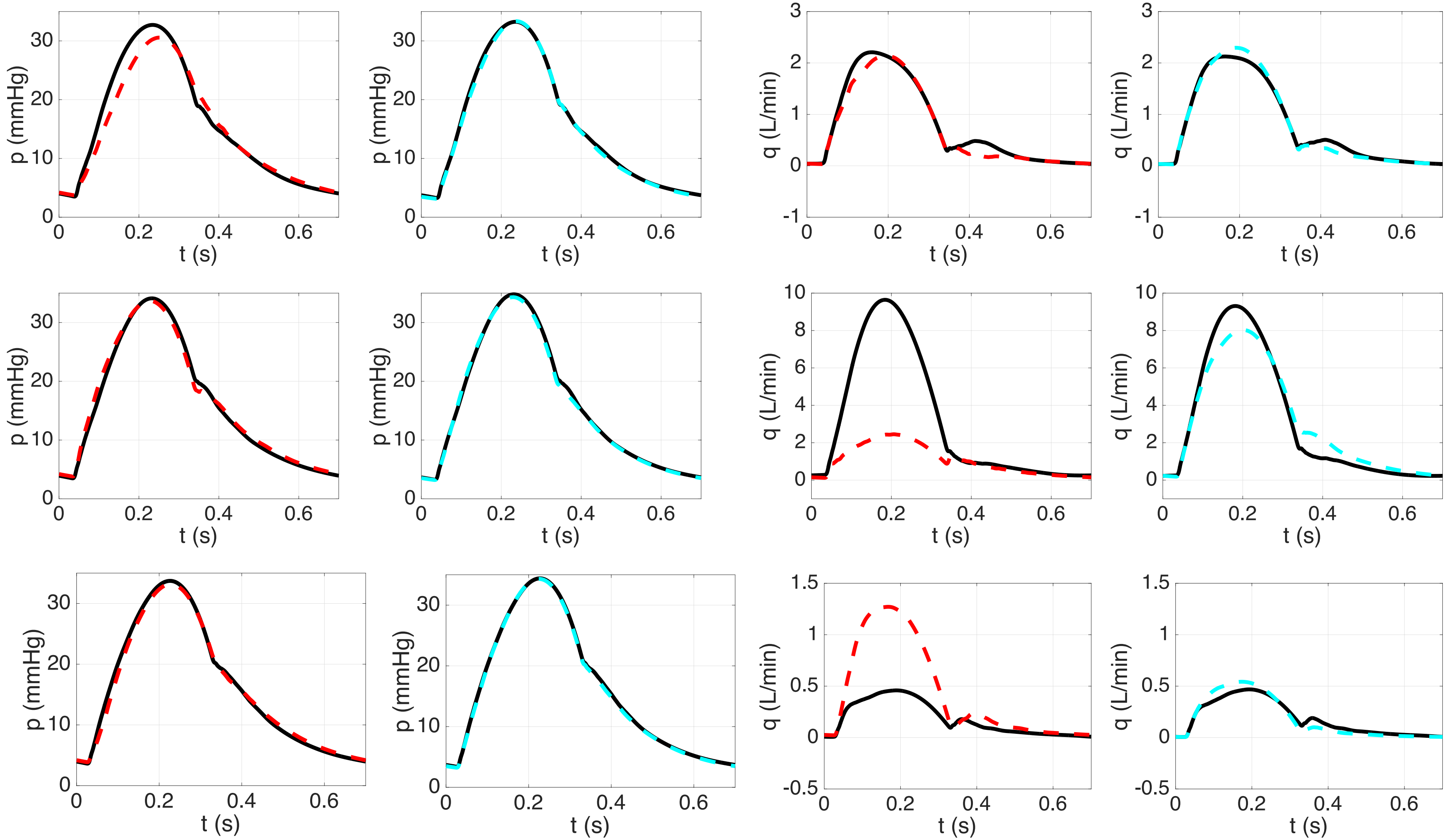
ST
WK
R_1/R_T
C_T
Terminal Vessels
WK
ST

Results (Proximal vessels)



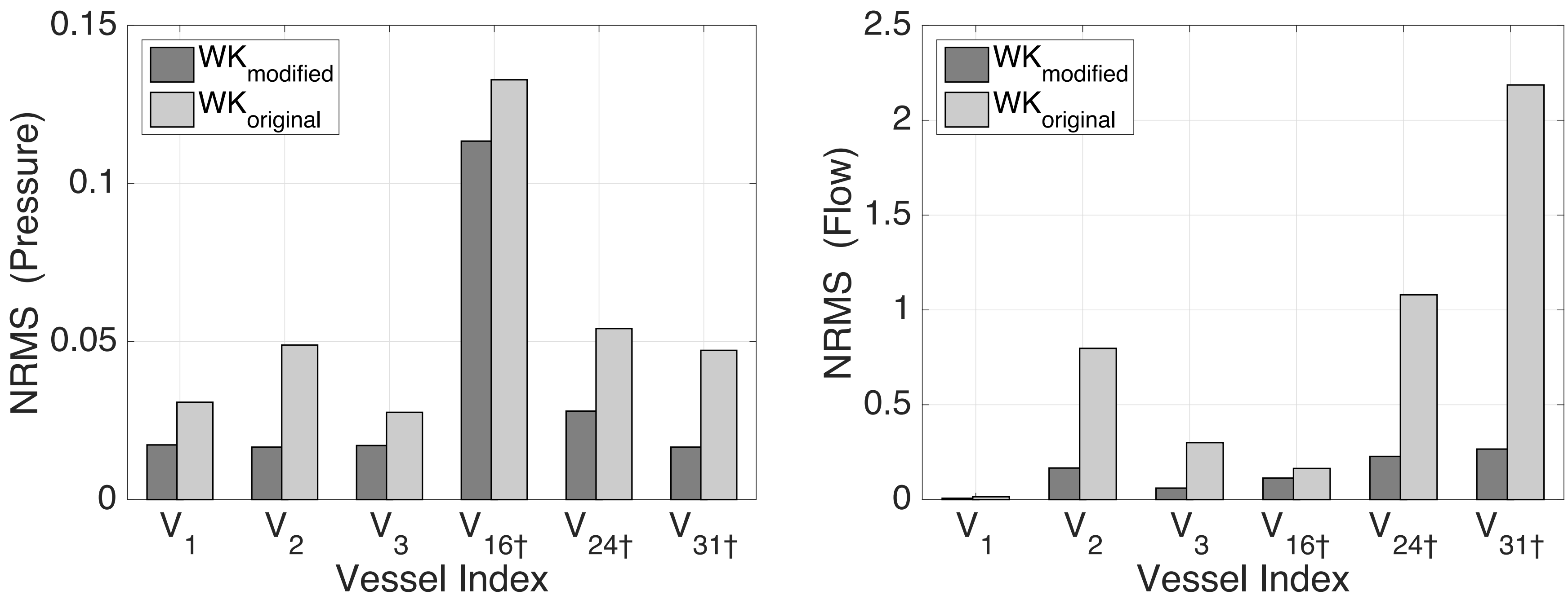
Results from a 31 arterial model (including 7 tapered anatomical vessels) with structured tree (**Solid Black -**), conventional WK (**Dashed Red --**) and modified WK approach (Dashed Cyan **--**)

Result (Terminal vessels)

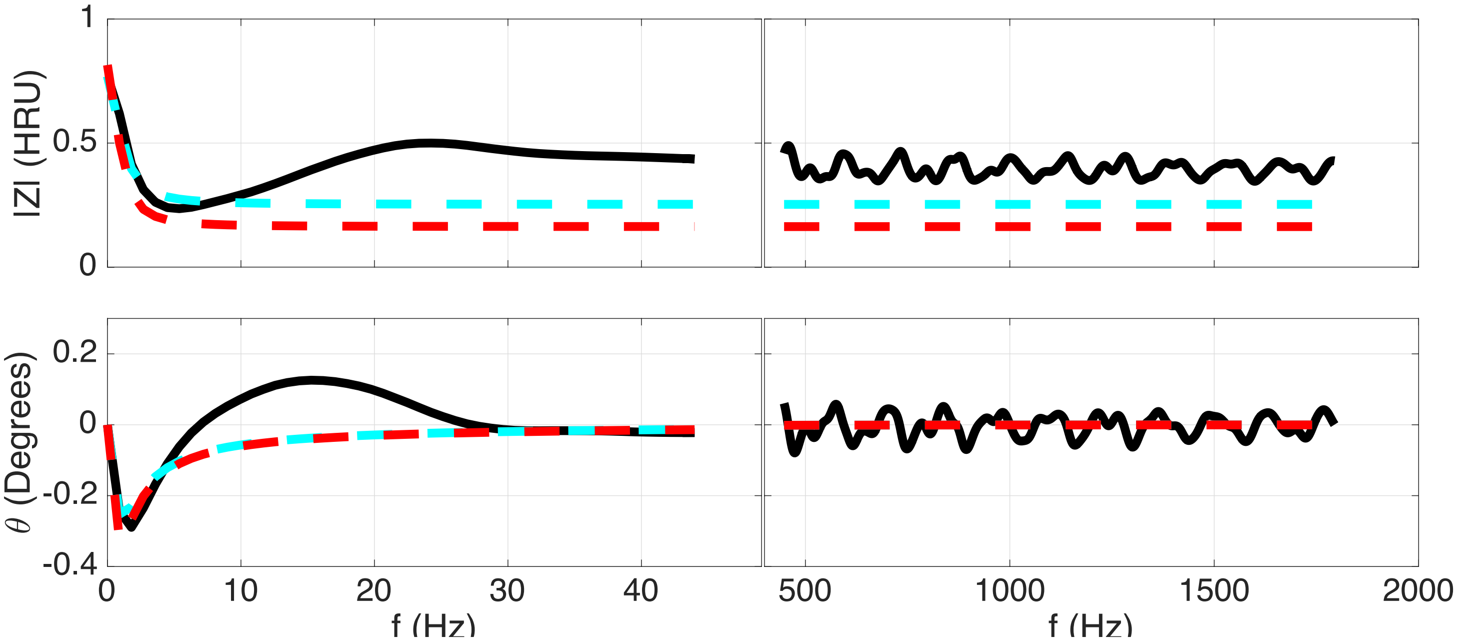


Results from a 31 arterial model (including 7 tapered anatomical vessels) with structured tree (**Solid Black -**), conventional WK (**Dashed Red --**) and modified WK approach (Dashed Cyan --)

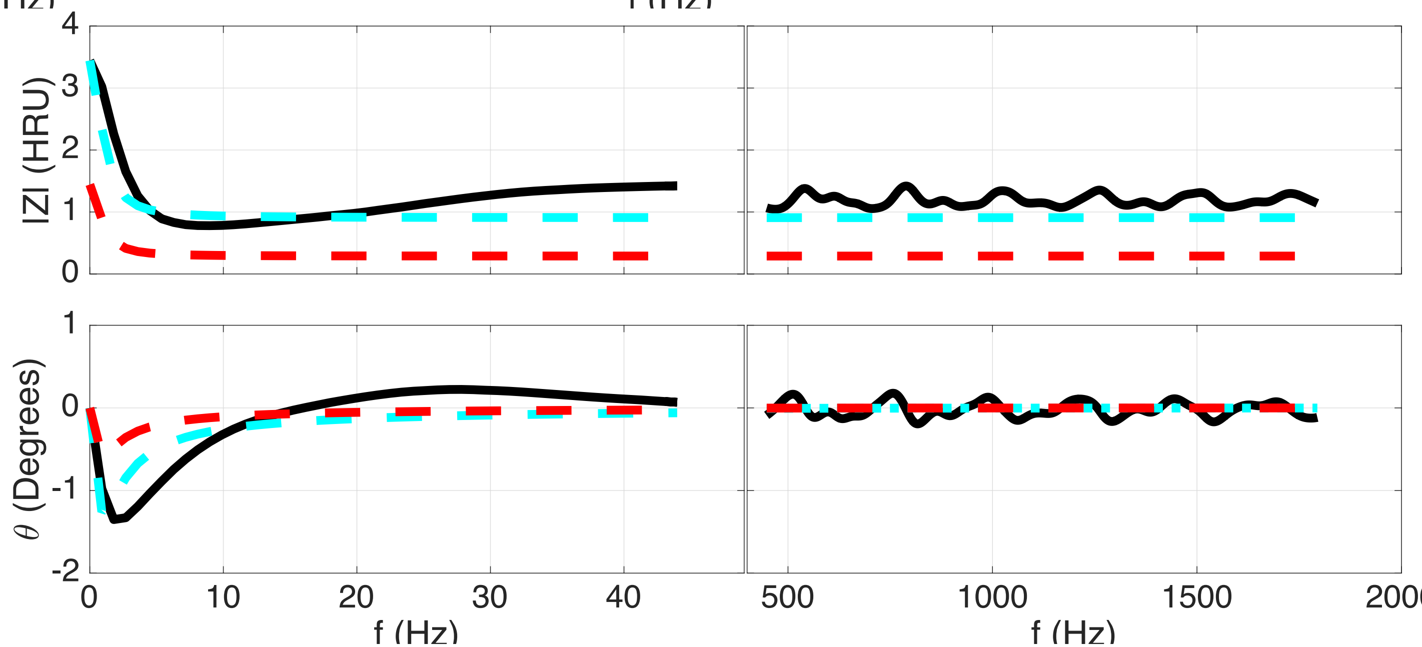
Error quantification



Root mean square error between pressure and flow, using conventional and structured tree based modified approach of parameter estimation for three element Windkessel. Error quantification is shown the proximal and distal vessels. (†: Terminal vessels)

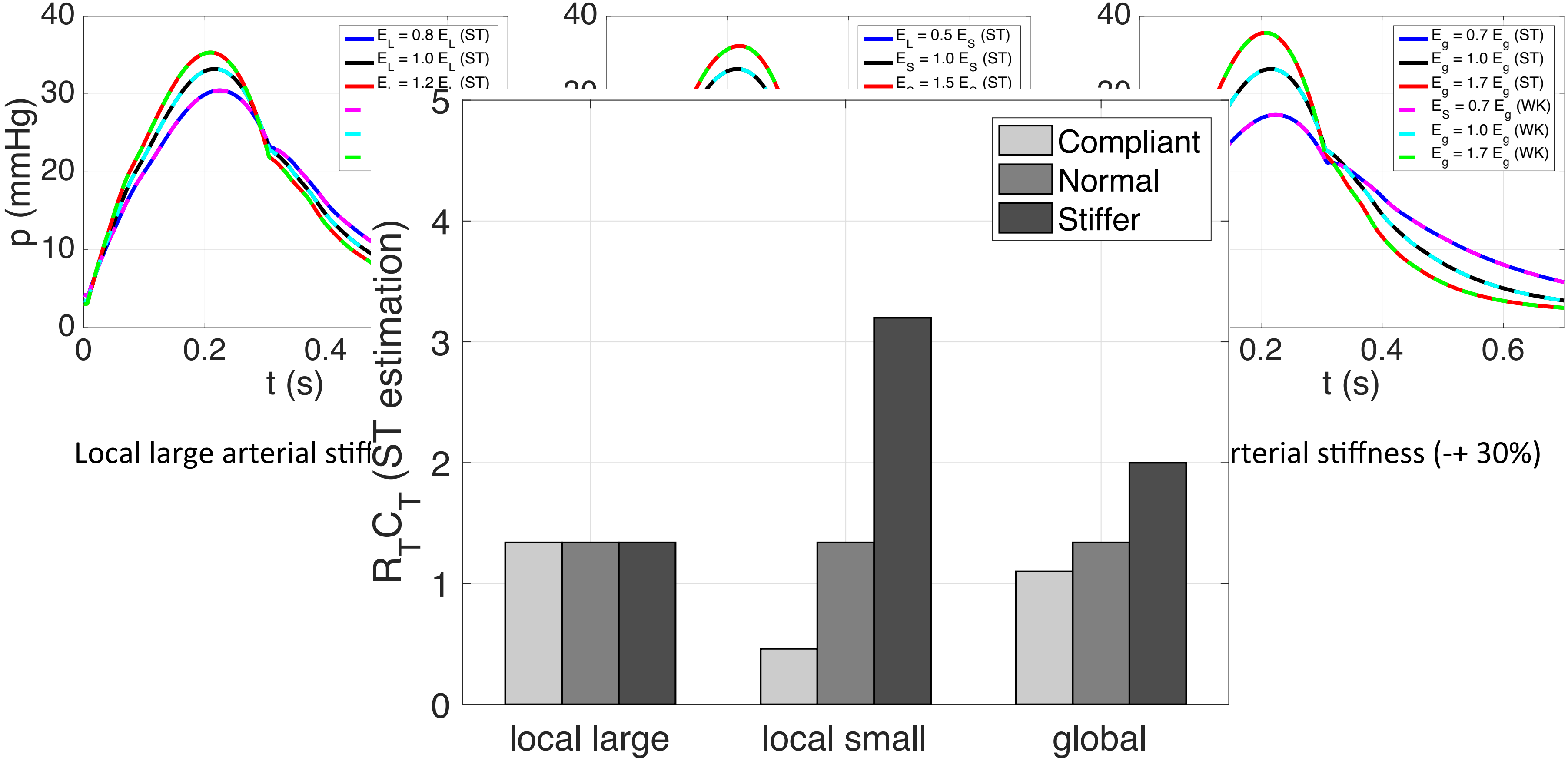


Impedance phase and modulus from terminal vessel **V16** at different frequency range. Structured tree (**Solid Black -**), conventional WK (**Dashed Red --**) and modified WK approach (Dashed Cyan --)



Impedance phase and modulus from terminal vessel **V31** at different frequency range. Structured tree (**Solid Black -**), conventional WK (**Dashed Red --**) and modified WK approach (Dashed Cyan --)

Effects of local and global stiffening



Concluding remarks

- Modified approach allows:
  - Improved flow distribution in the vasculature.
  - Automatic determination of total resistance underneath each terminal vessel.
  - Automatic determination of relation between R1, R2 and CT.
- The proposed approach needs to be validated using real data and a subject specific model.

Key References

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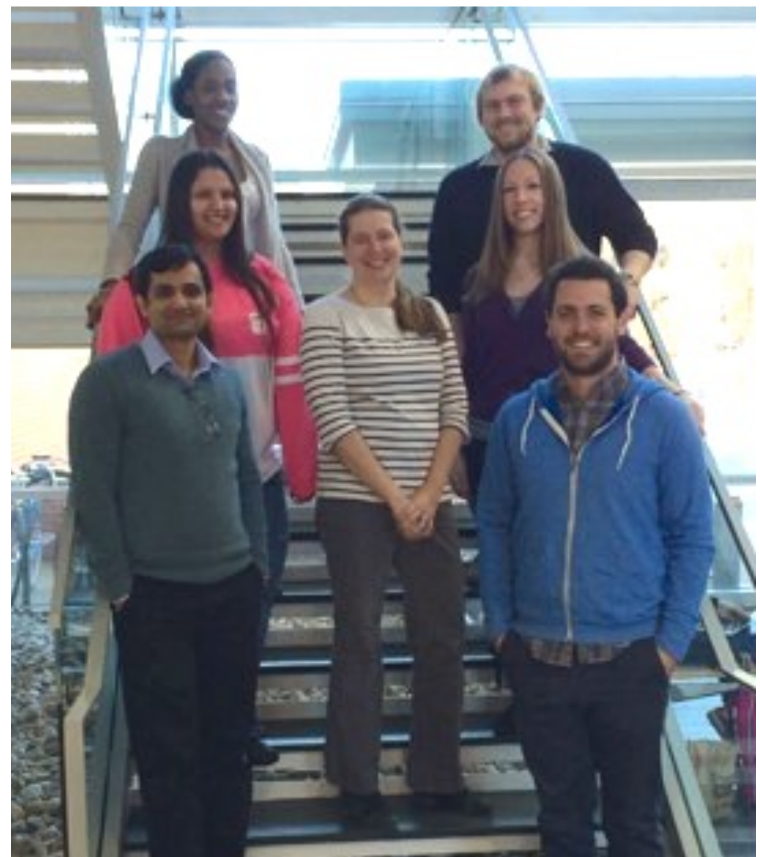

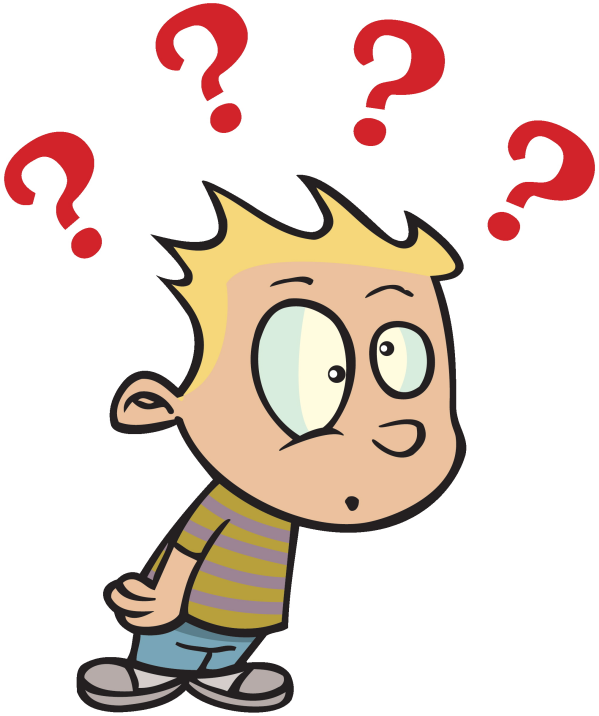

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